Robotics

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Introduction

- Historical perspective
  - First use of the word Robot (means forced labor or serf in Czech) in the play R.U.R. (Rossum's Universal Robots) by Karel Capek (1890-1938) in January 1921.

  In R.U.R., Capek poses a paradise, where the machines initially bring so many benefits but in the end bring an equal amount of blight in the form of unemployment and social unrest

Metropolis, Fritz Lang, 1927

- Science fiction
  - Often a bad image: men against robots, dystopic society, etc. More and more a good image.

Formal definition (Robot Institute of America)

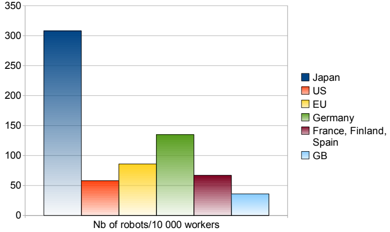
A reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through various programmed motions for the performance of a variety of tasks

Robots and their image

- Robots have a bad image (1930-1960)
  - Robots take human works
  - Robots are dangerous since potentially independent and more intelligent than we are
- Robots have a better image (1960-today)
  - Robots can make things that human can not do (space, etc.)
  - Human can do things that robots can not do (we still are clever)
  - Robots can be games
  - Robots can be good or bad

Robotics industry (1/many)

- Number of robots for every 10 000 workers:



- 70% of robots in companies with more than 1000 employees
- 17% of robots in companies with less than 300 employees
- In 2002, 95% of robots > 30k€ and 32% of robots > 60k€
- 79% of decrease of the mean price between 1990 and 2002
- Big robots manufacturers: ABB (S), KUKA (G), Fanuc (JP), etc.

Robotics industry (2/many)

- Where are the robots ?
  - France:
    - 61% in automotive industry
    - 14% in chemical industry
    - ...
    - 4% in electricity industry
    - 3% in food industry

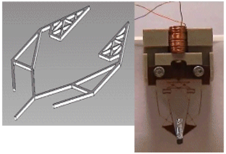
- What kind of robots ?
  - Industry: ground fixed robots: manipulators, arm robots, ...
  - Private individuals: mobile robots: service, games, ...
- Future of robots:
  - Industrial mobile robotics
  - Medical robotics
  - Service robots (growing field)

Robotics industry (3/many)

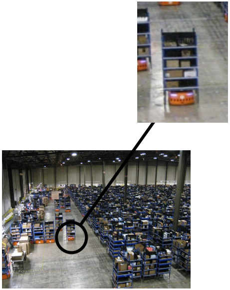
Vacuum cleaner (Kärcher)

Forest robot

Kuka robot for automotive industry



Micromanipulator

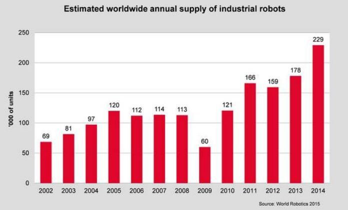


Surgical robot

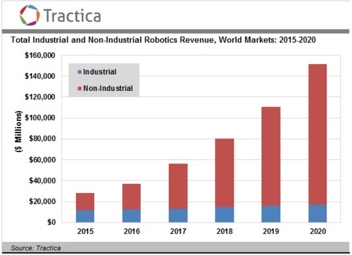
Hollywood robots

Robotics industry (4/many)

- Past

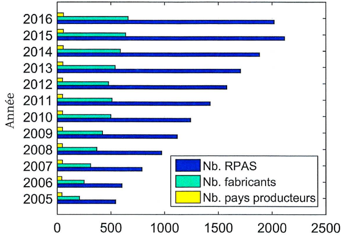


- Future

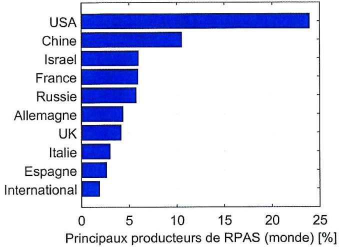


Robotics industry: UAVs (5/many)

- UAV's Manufacturer

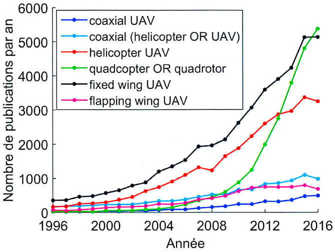


- UAVs by countries

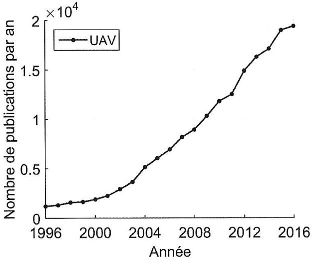


Robotics industry: UAVs (6/many)

- Publications indicates future ?



- By keywords



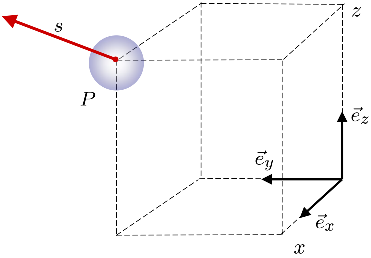
Outline

- Basic mechanics for robotics
  - Space representation
    frames, coordinate transformation, etc.
  - Force and torques
- Modelisation
- Control for robots
  - All potential problems:
    Oscillations, dry friction, saturations, etc.
  - Linear approaches
  - Nonlinear approaches

Outline

Position and speed in a fixed frame

- The **position** of some point P in the **fixed** frame $\mathcal{F}(o, \vec{e}_x, \vec{e}_y, \vec{e}_z)$ is the vector $p = (x, y, z)^T$



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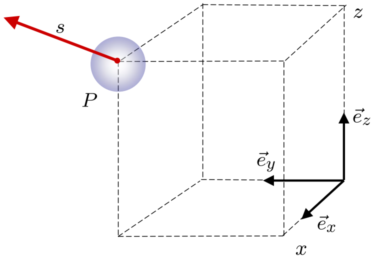

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- The rotation resulting from 2 successive rotations R_1 and then R_2 is $R_2 R_1$

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- Skew-symmetric matrices and rotations

$$u^\times \sin\theta + (I - uu^T)\cos\theta + uu^T \text{ and } \exp((u\theta)^\times)$$

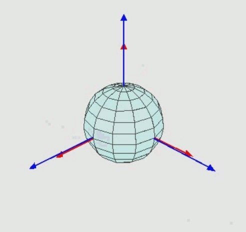
is the rotation of angle θ leaving axis u fixed

Attitude representation

Angles

W.R. Hamilton (1805-1865)

- Attitude:



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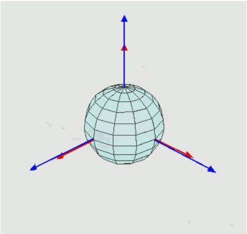

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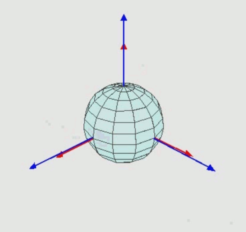

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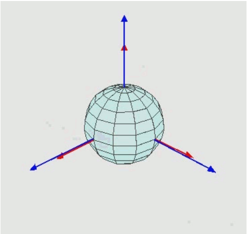

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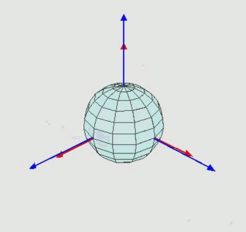

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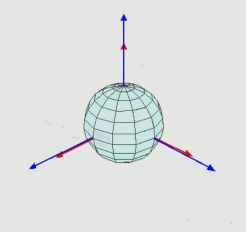

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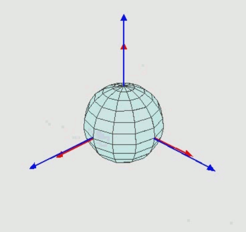

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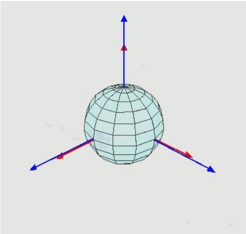

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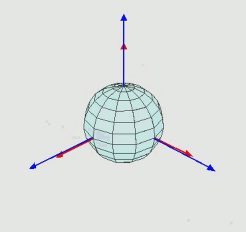

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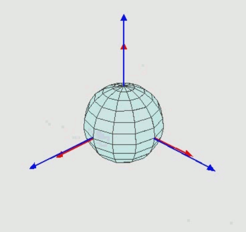
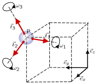
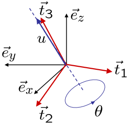
- Engineering and robotics communities typically use 3-1-3 Euler angles







Attitude representation

Angles

W.R. Hamilton (1805-1865)

- Attitude:
  - equivalent of position for angles: what is the orientation of an object w.r.t. the ground ?
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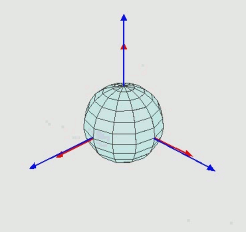

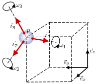

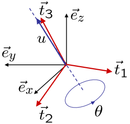

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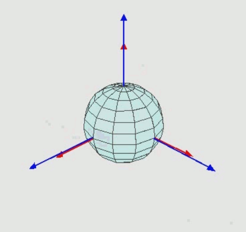
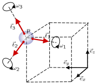
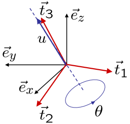


- **Quaternions**





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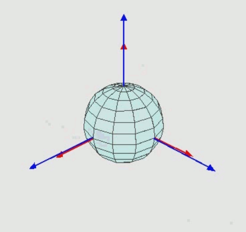

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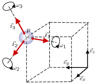

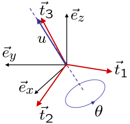

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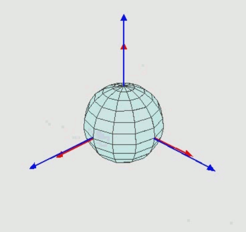

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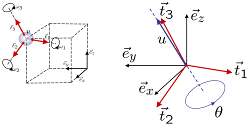

- u fixed by rotation of angle θ

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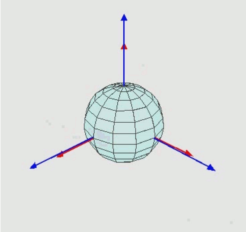

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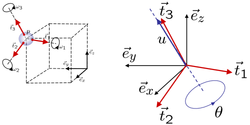


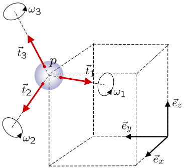
- u fixed by rotation of angle θ
- the quaternion is:

$$q = \begin{pmatrix} u_x \sin\theta/2 \\ u_y \sin\theta/2 \\ u_z \sin\theta/2 \\ \cos\theta/2 \end{pmatrix} = \begin{pmatrix} \vec{q} \\ q_0 \end{pmatrix}$$

Attitude representation

Angular velocities

- The angular velocity $\omega = (\omega_1, \omega_2, \omega_3)^T$ represents the rotation speed w.r.t. each axis of the body frame



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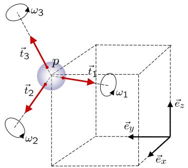

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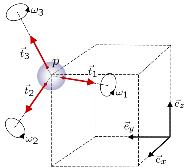

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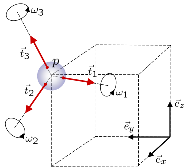

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$$\dot{R} = R\omega^{\times}$$

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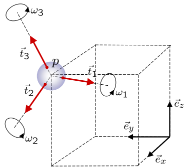

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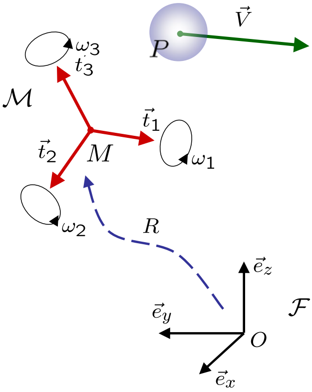
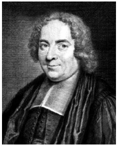
$$\begin{aligned} \dot{\vec{q}} &= \frac{1}{2}\Omega(\vec{\omega})q \\ &= \frac{1}{2}\Xi(q)\vec{\omega} \end{aligned} \quad \text{with} \quad \begin{cases} \Omega(\vec{\omega}) = \begin{pmatrix} 0 & -\vec{\omega}^T \\ \vec{\omega} & -\vec{\omega}^{\times} \end{pmatrix} \\ \Xi(q) = \begin{pmatrix} -\vec{q}^T \\ I_{3\times 3}q_0 + \vec{q}^{\times} \end{pmatrix} \end{cases}$$

Moving frames





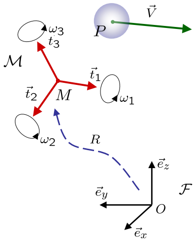
P. Varignon (1654-1722)

Varignon's formula

$$\frac{d\vec{U}}{dt}^{\mathcal{M}} = \frac{d\vec{U}}{dt}^{\mathcal{F}} + \Omega^{\mathcal{F}/\mathcal{M}} \times \vec{U}^{\mathcal{F}}$$

Moving frames

- $\mathcal{F} := (O, \vec{e}_x, \vec{e}_y, \vec{e}_z)$ fixed frame



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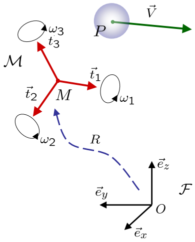

Moving frames

Robotics

N. Marchand

Introduction

Outline

Mechanics

Kinematics

Arm robots

Inner-loop

Geometrical model

Kinematic model

Dynamical model

Conclusion

Path planning

Workspace and obstacles

path planning

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Visual servoing

RX90 robot

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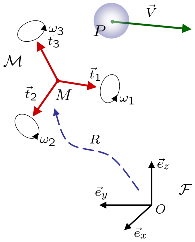

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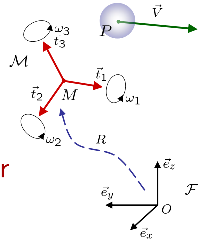

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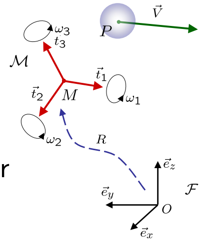

$$\frac{d\vec{OP}}{dt}^{\mathcal{F}} = \frac{d\vec{OM}}{dt}^{\mathcal{F}} + \frac{d\vec{MP}}{dt}^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \vec{MP}$$

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- **Velocities**:
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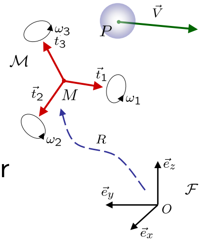

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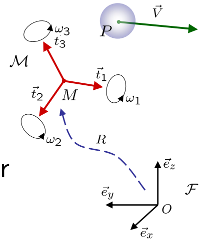

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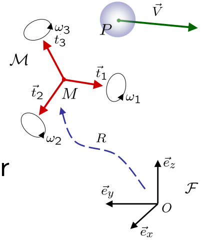

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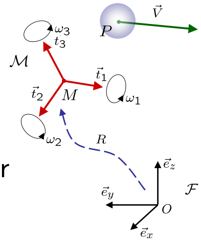

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  - Relative velocity
  - Due to the rotation of $\mathcal{M}$ w.r.t. $\mathcal{F}$

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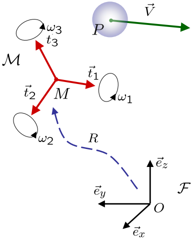

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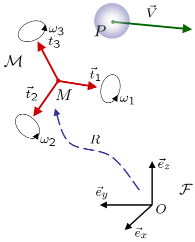

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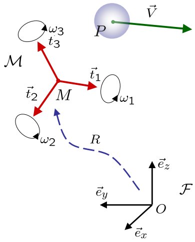

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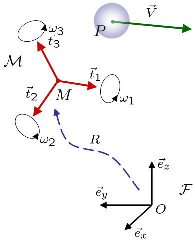

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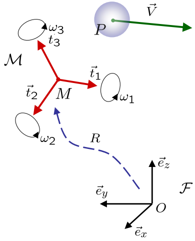
- **Acceleration**:

$$\ddot{P}^{\mathcal{F}} := \frac{d\dot{P}^{\mathcal{F}}{}^{\mathcal{F}}}{dt} = \frac{d\dot{P}^{\mathcal{M}}{}^{\mathcal{F}}}{dt} + \frac{d\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}}}{dt}$$



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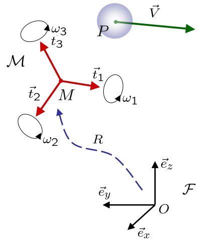

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- $\frac{d\dot{P}^{\mathcal{M}}}{dt}^{\mathcal{F}} = \ddot{P}^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{P}^{\mathcal{M}}$ (Varignon's formula)

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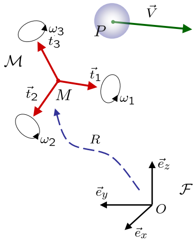

$$\ddot{P}^{\mathcal{F}} := \frac{d\dot{P}^{\mathcal{F}}}{dt}^{\mathcal{F}} = \frac{d\dot{P}^{\mathcal{M}}}{dt}^{\mathcal{F}} + \frac{d\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}}}{dt}$$

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all together:

$$\ddot{P}^{\mathcal{M}} = \ddot{P}^{\mathcal{F}} - 2\Omega \times \dot{P}^{\mathcal{M}} - \dot{\Omega} \times P^{\mathcal{F}} - \Omega \times (\Omega \times P^{\mathcal{F}})$$

Moving frames

- $\mathcal{F} := (O, \vec{e}_x, \vec{e}_y, \vec{e}_z)$ fixed frame
- $\mathcal{M} := (M, \vec{t}_1, \vec{t}_2, \vec{t}_3)$: mobile frame
- R: rotation matrix s.t. $\mathcal{M} = R\mathcal{F}$
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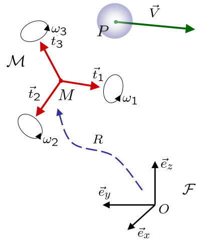

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- Coriolis effect

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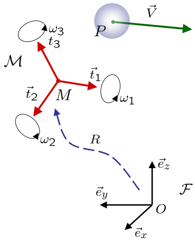

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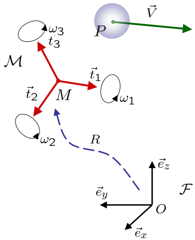
- Coriolis effect
- Euler effect (tangent acceleration)

Moving frames



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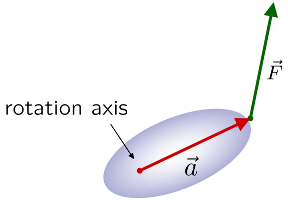
- Coriolis effect
- Euler effect (tangent acceleration)
- Centrifugal effect

Forces and torques

- A force ... everybody knows what it is:

  a vector, denoted $\vec{F}$



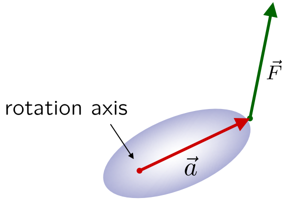
Forces and torques

- A force ... everybody knows what it is:

  a vector, denoted $\vec{F}$

- A torque (moment or moment of force):

$$\vec{\tau} = \vec{p} \times \vec{F}$$



Conservation of linear momentum

Linear Momentum

$$P := \sum_i m_i \vec{v}_i \in \mathbb{R}^3$$

where i denotes the index of the element composing the system, m_i it's mass and v_i it's speed (in a fixed frame)

- Single body system:

$$P = M\vec{v}$$

Newton's second law (conservation of the linear momentum)

$$\sum \vec{F} = \frac{dP}{dt}^{\mathcal{F}}$$

I. Newton (1643-1727)

Conservation of angular momentum

Angular Momentum

$$L := \vec{p} \times P \in \mathbb{R}^3$$

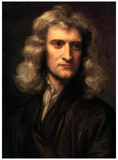
where $\vec{p}$ denotes the position vector and P the linear momentum

Conservation of the angular momentum

$$\sum \vec{\tau} = \frac{dL}{dt}^{\mathcal{F}}$$

- In a moving frame (Varignon's formula):

$$\frac{dL}{dt}^{\mathcal{F}} = \frac{dL}{dt}^{\mathcal{M}} + \Omega \times L$$



I. Newton (1643-1727)

Conservation of angular momentum

Inertia momentum

$$dJ = r^2 dm$$

where r is the distance of the elementary mass dm to the rotation axis

One has:

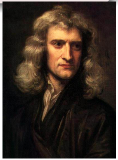
Conservation of the angular momentum

$$\sum \vec{\tau} = J \frac{d\Omega}{dt}^{\mathcal{F}}$$

with $J = \int_{\text{rigid body}} dJ$

- In a moving frame (Varignon's formula):

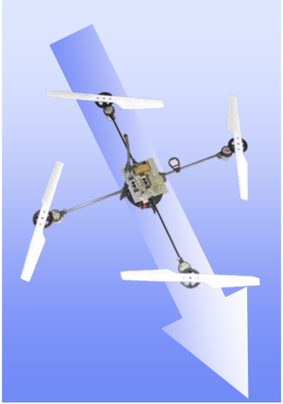
$$J\frac{d\Omega}{dt}^{\mathcal{F}} = J\frac{d\Omega}{dt}^{\mathcal{M}} + \Omega \times L$$



I. Newton (1643-1727)

Example: the X4 helicopter

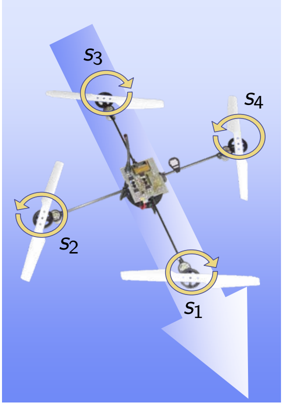
How it works ?



Example: the X4 helicopter

How it works ?

- 4 fixed rotors with controlled rotation speed s_i



Example: the X4 helicopter

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- 4 fixed rotors with controlled rotation speed s_i
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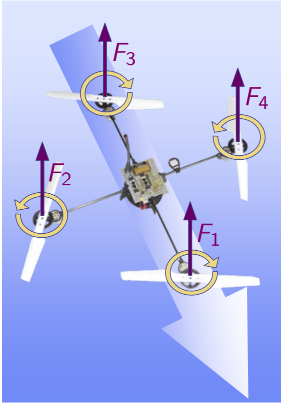

Example: the X4 helicopter

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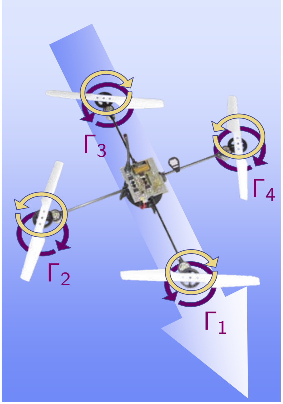

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How it works ?

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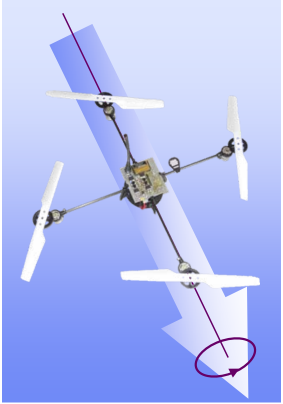
- **Roll movement**



Example: the X4 helicopter

How it works ?

- 4 fixed rotors with controlled rotation speed s_i
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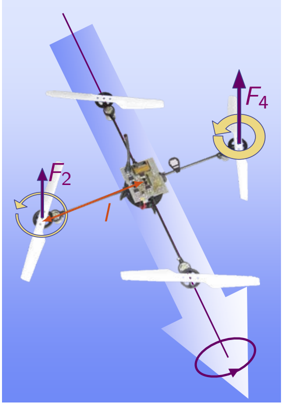
- **Roll movement** generated with a dissymmetry between left and right forces:

$$\Gamma_r = l(F_4 - F_2)$$



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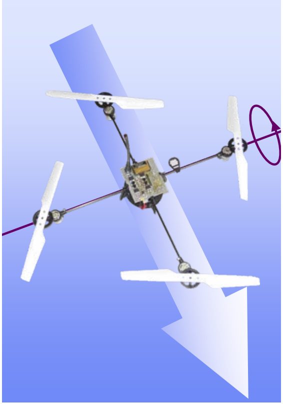

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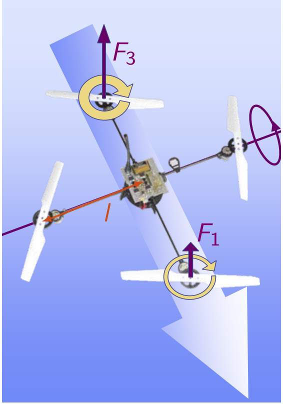

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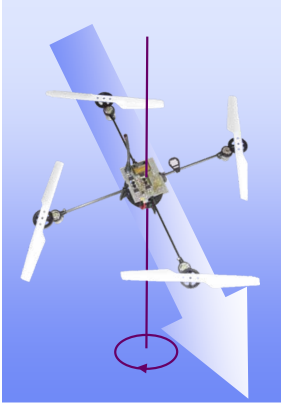
- **Yaw movement**



Example: the X4 helicopter

How it works ?

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- **Pitch movement** generated with a dissymmetry between front and rear forces:
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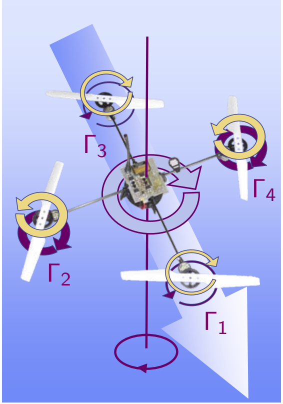
- **Yaw movement** generated with a dissymmetry between front/rear and left/right torques:
$$\Gamma_y = \Gamma_1 + \Gamma_3 - \Gamma_2 - \Gamma_4$$



Example: the X4 helicopter

Building a model (1/3)

- **Electrical motor**: A 2nd order system with friction and saturation

Example: the X4 helicopter

Building a model (1/3)

- **Electrical motor**: A 2nd order system with friction and saturation usually *approximated* by a 1rst order system:

$$\dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r}\tau_{\text{load}} + \frac{k_m}{J_r R}\,\text{sat}_{\bar{U}_i}(U_i) \quad i \in \{1,2,3,4\} \; (1)$$

s_i: rotation speed

U_i: voltage applied to the motor; real control variable

τ_{load}: motor load: $\tau_{\text{load}} = k_{gearbox}\kappa\,|s_i|\,s_i$ with κ drag coefficient

Example: the X4 helicopter

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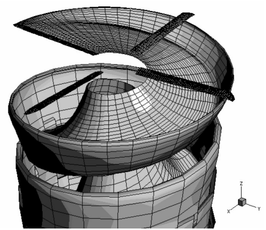
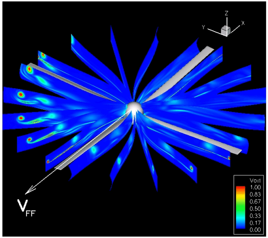
- **Aerodynamical forces and torques**: Very complex models exist





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- **Aerodynamical forces and torques**: Very complex models exist but overcomplicated for control, better use the *simplified* model:

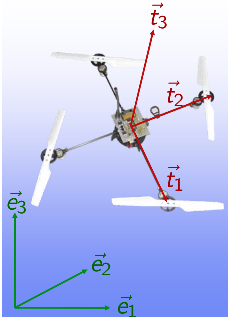
$$\begin{aligned} F_i &= b s_i^2 \\ \Gamma_r &= lb(s_4^2 - s_2^2) \\ \Gamma_p &= lb(s_1^2 - s_3^2) \\ \Gamma_y &= \kappa(s_1^2 + s_3^2 - s_2^2 - s_4^2) \end{aligned} \qquad i \in \{1,2,3,4\} \qquad (2)$$

b: thrust coefficient, κ: drag coefficient

Example: the X4 helicopter

Building a model (2/3)

- Two frames
  - a fixed frame $E(\vec{e}_1, \vec{e}_2, \vec{e}_3)$
  - a frame attached to the X4 $T(\vec{t}_1, \vec{t}_2, \vec{t}_3)$



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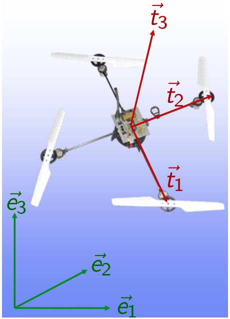
  - a rotation matrix R from T to E



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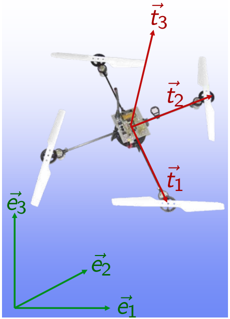

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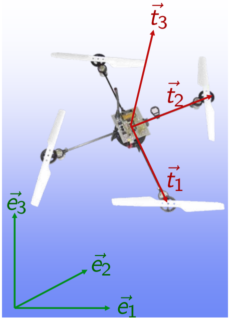
    - position $\vec{p}$
    - velocity $\vec{v}$



Example: the X4 helicopter

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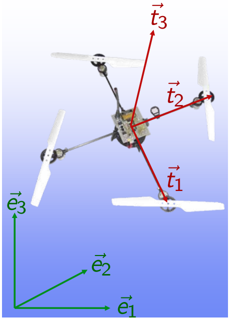

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- Frame change
  - a rotation matrix R from T to E
- State variables:
  - Cartesian coordinates (in E)
    - position $\vec{p}$
    - velocity $\vec{v}$
  - Attitude coordinates:
    - angular velocity $\vec{\omega}$ in the moving frame T
    - *either:* Euler angles three successive rotations about $\vec{t}_3$, $\vec{t}_1$ and $\vec{t}_3$ of angles angles ϕ, θ and ψ giving R
    - *or:* Quaternion representation $(q_0, \vec{q}) = (\cos\beta/2, \vec{u}\sin\beta/2)$ represent a rotation of angle β about $\vec{u}$

Example: the X4 helicopter

Building a model (3/3)

- **Cartesian coordinates**:

$$\left\{ \begin{array}{rcl} \dot{\vec{p}} & = & \vec{v} \\ m\dot{\vec{v}} & = & -mg\vec{e}_3 + R(\sum_i F_i(s_i)\vec{t}_3) \end{array} \right. \tag{3}$$

Example: the X4 helicopter

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- **Attitude**:

Example: the X4 helicopter

Building a model (3/3)

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- **Attitude**:
  - Euler angles formalism:

$$\begin{cases} \dot{R} &= R\vec{\omega}^\times \\ J\dot{\vec{\omega}} &= -\vec{\omega}^\times J\vec{\omega} + \Gamma_{tot} \end{cases} \quad \text{with } \vec{\omega}^\times = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \tag{4}$$

$\vec{\omega}^\times$ is the skew symmetric tensor associated to $\vec{\omega}$

Example: the X4 helicopter

Building a model (3/3)

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$$\begin{cases} \dot{\vec{p}} &= \vec{v} \\ m\dot{\vec{v}} &= -mg\vec{e}_3 + R(\sum_i F_i(s_i)\vec{t}_3) \end{cases} \quad (3)$$

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  $\vec{\omega}^\times$ is the skew symmetric tensor associated to $\vec{\omega}$
  - Quaternion formalism:

$$\begin{cases} \dot{\vec{q}} &= \frac{1}{2}\Omega(\vec{\omega})q \\ &= \frac{1}{2}\Xi(q)\vec{\omega} \\ J\dot{\vec{\omega}} &= -\vec{\omega}^\times J\vec{\omega} + \Gamma_{tot} \end{cases} \quad \text{with} \begin{cases} \Omega(\vec{\omega}) = \begin{pmatrix} 0 & -\vec{\omega}^T \\ \vec{\omega} & -\vec{\omega}^\times \end{pmatrix} \\ \Xi(q) = \begin{pmatrix} -\vec{q}^T \\ I_{3\times 3}q_0 + \vec{q}^\times \end{pmatrix} \end{cases} \quad (5)$$

Example: the X4 helicopter

Building a model (3/3)

- **Cartesian coordinates**:

$$\begin{cases} \dot{\vec{p}} &= \vec{v} \\ m\dot{\vec{v}} &= -mg\vec{e}_3 + R(\sum_i F_i(s_i)\vec{t}_3) \end{cases} \tag{3}$$

- **Attitude**:
  - Euler angles formalism:

$$\begin{cases} \dot{R} &= R\vec{\omega}^\times \\ J\dot{\vec{\omega}} &= -\vec{\omega}^\times J\vec{\omega} + \Gamma_{\text{tot}} \end{cases} \quad \text{with } \vec{\omega}^\times = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \tag{4}$$

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where $\Gamma_{\text{tot}} = \underbrace{-\sum_i I_r \vec{\omega}^\times \vec{t}_3 s_i}_{\text{gyroscopic torque}} + \Gamma_{pert} + \begin{pmatrix} \Gamma_r(s_2, s_4) \\ \Gamma_p(s_1, s_3) \\ \Gamma_y(s_1, s_2, s_3, s_4) \end{pmatrix}$

Example: the X4 helicopter

Review of the nonlinearities

$$\left\{\begin{array}{rcl}
\dot{s}_i &=& -\dfrac{k_m^2}{J_r R}s_i - \dfrac{k_{gearbox}\kappa}{J_r}\left|s_i\right| s_i + \dfrac{k_m}{J_r R}\,\mathrm{sat}_{\bar{U}_i}(U_i) \\
\dot{\vec{p}} &=& \vec{v} \\
m\dot{\vec{v}} &=& -mg\,\vec{e}_3 + R\begin{pmatrix} 0 \\ 0 \\ \sum_i F_i(s_i) \end{pmatrix} \\
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\end{cases}
$$

In red: the nonlinearities

In blue: where the control variables act

Example: the X4 helicopter

Identification of the parameters

- **Electrical motor**:
  - For small input steps, the system behaves very close to a **linear** first order system
  - Hence, use linear identification tools
  - $\bar{U}_i$ is found on the data-sheet of the motor (damage avoidance)

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Values of the parameters

- **Motor parameters**:

parameter	description	value	unit
k_m	motor constant	4.3×10^{-3}	N.m/A
J_r	rotor inertia	3.4×10^{-5}	J.g.m^2
R	motor resistance	0.67	Ω
$k_{gearbox}$	gearbox ratio	2.7×10^{-3}	-
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- **Body parameters**:

parameter	description	value	unit
J	inertia matrix	$\begin{pmatrix} 14.6 \times 10^{-3} & 0 & 0 \\ 0 & 7.8 \times 10^{-3} & 0 \\ 0 & 0 & 7.8 \times 10^{-3} \end{pmatrix}$	kg.m^2
m	mass of the UAV	0.458	kg
l	radius of the UAV	22.5	cm
g	gravity	9.81	m/s^2

Outline

Jointed-arm robots

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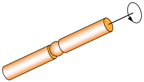

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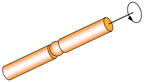

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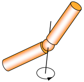

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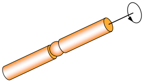

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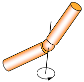

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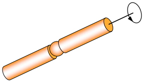

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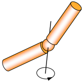

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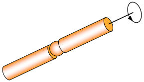

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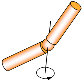

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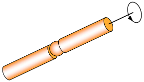

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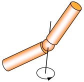

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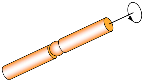

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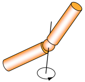

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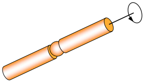

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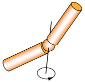

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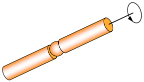

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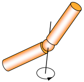

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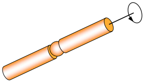

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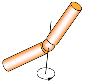

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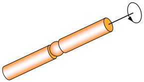

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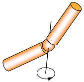

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- Many tasks can be performed with less than 6 dof: "pick and place" needs only 4 dof

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Outline

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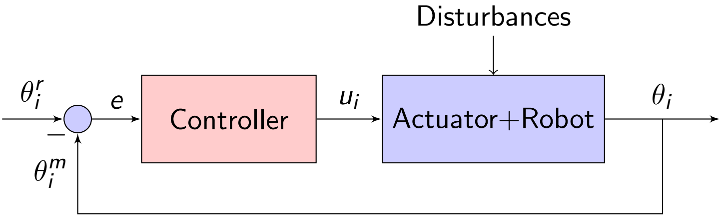

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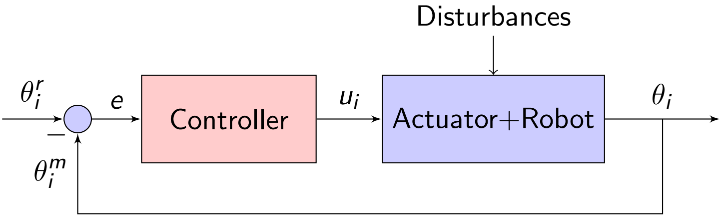

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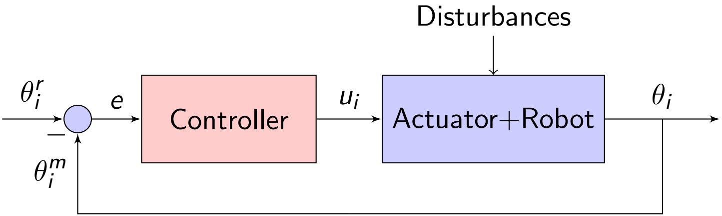

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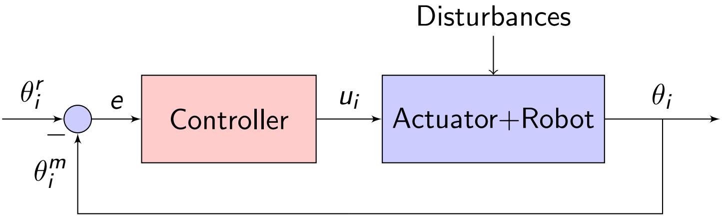

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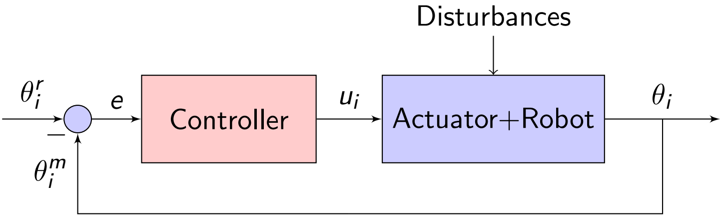

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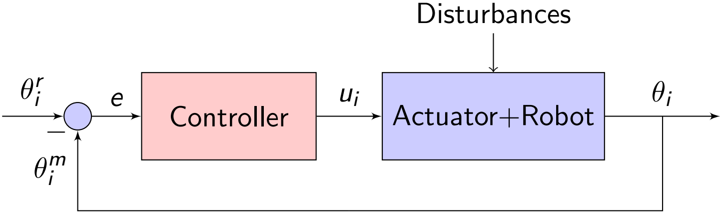

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  - anti-windup to tackle saturations

Inner control loop

Anti-windup PID

- We go back to the **X4** example and focus on the **rotors**:

$$\left\{ \dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \tau_{\text{load}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i) \right.$$

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  - **neglecting the voltage limitations $\bar{U}_i$**

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Anti-windup PID

- The so obtained **system is linear**

$$\frac{s_i(s)}{U_i(s)} = \frac{\frac{1}{k_m}}{1+\frac{J_r R}{k_m^2}s} = \frac{G}{1+\tau s}$$

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- Taking $K_i = \dfrac{1}{\tau_{CL} G}$ and $K_p = \tau K_i$, the closed loop system is:

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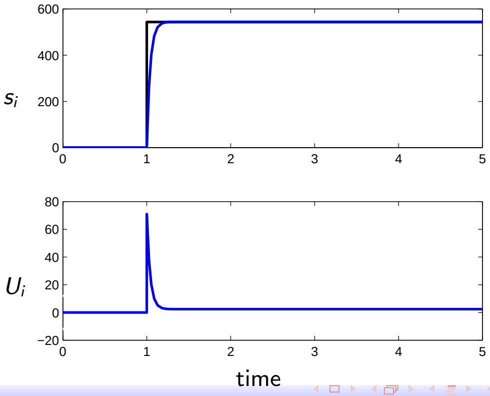

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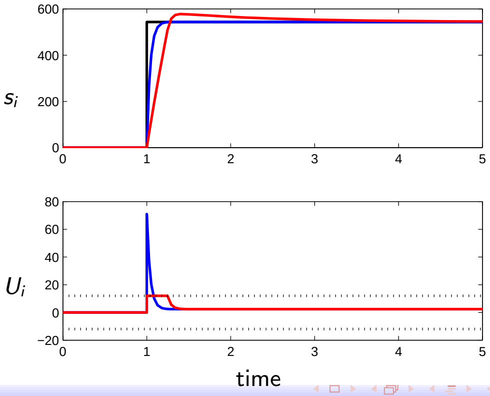

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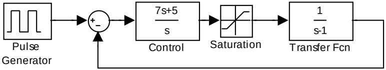

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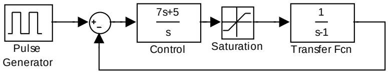

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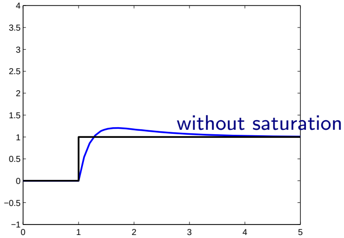

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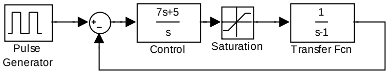

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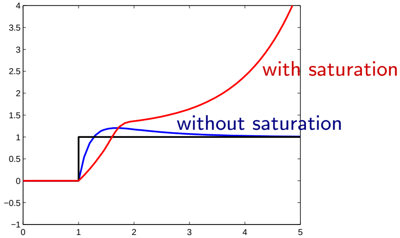

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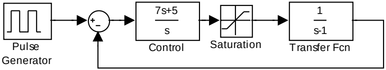

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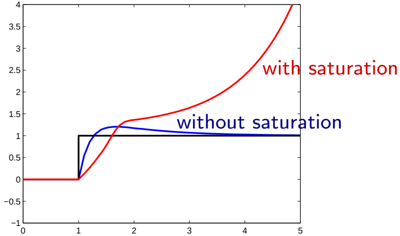
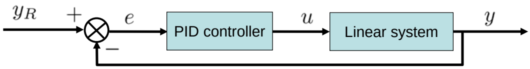


- **Saturations may lead to instability** especially in the presence of integrators in the loop

Inner control loop

Anti-windup PID

- Consider a **linear system with a PID** controller:



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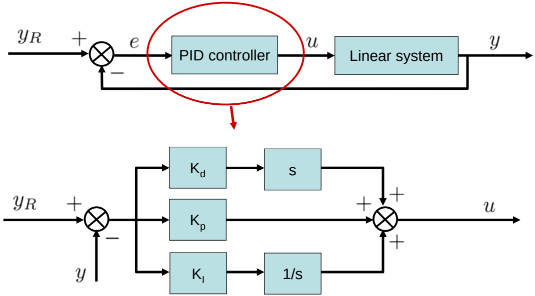

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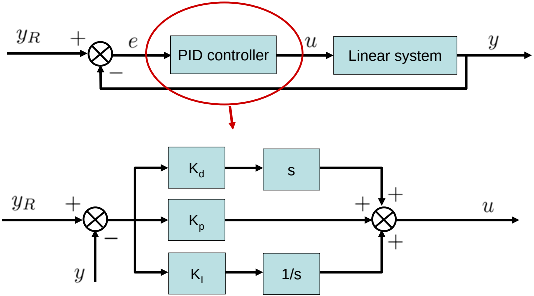

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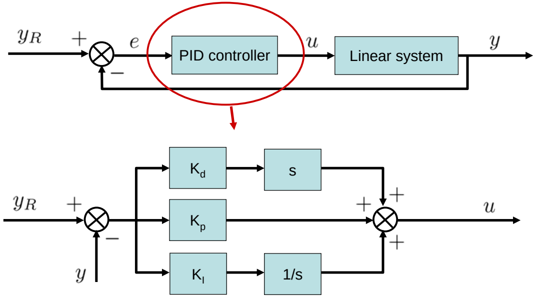
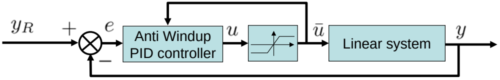


- The instability comes from the **integration** of the error
- **Key idea:** soften the integral effect when the control is saturated

Inner control loop

Anti-windup PID

- Structure of the **PID controller with anti-windup**:



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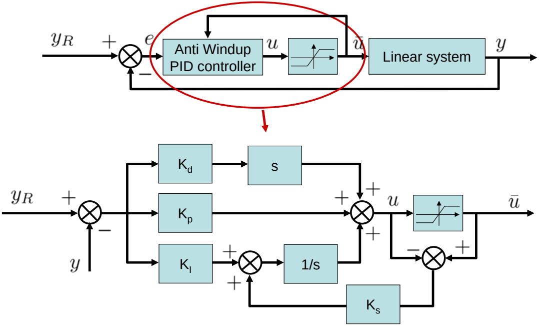

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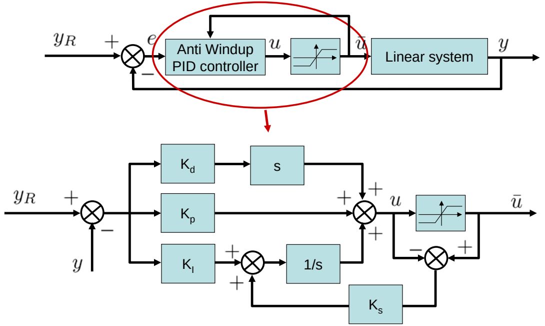

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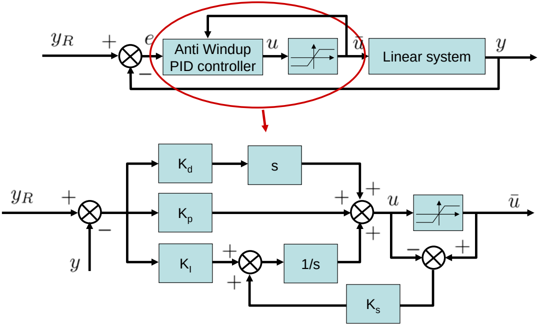

- If $u = \bar{u}$, that is if u is not saturated, **then the PID controller with anti-windup is identical to the classical PID controller**
- **If u is saturated** ($u \neq \bar{u}$), K_s **tunes the reduction of the integral effect** of the PID

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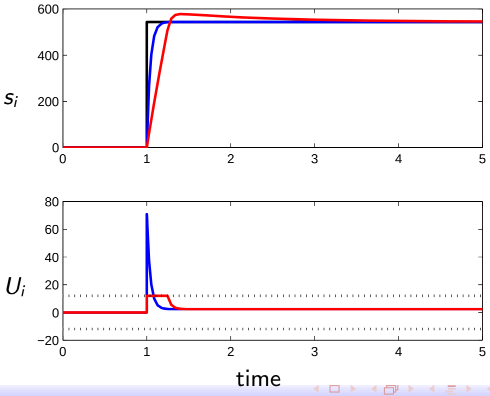

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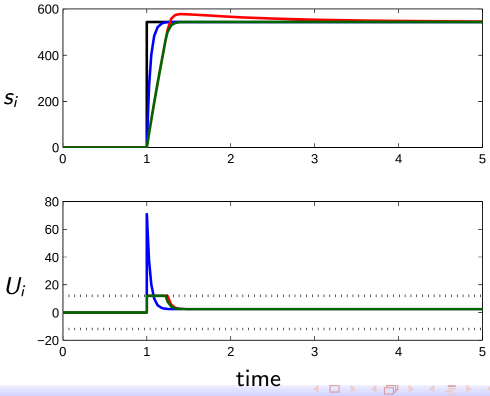

Inner control loop

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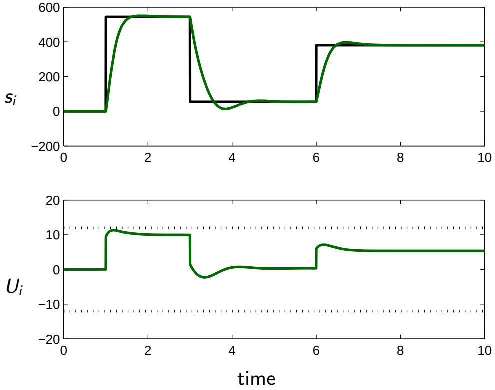

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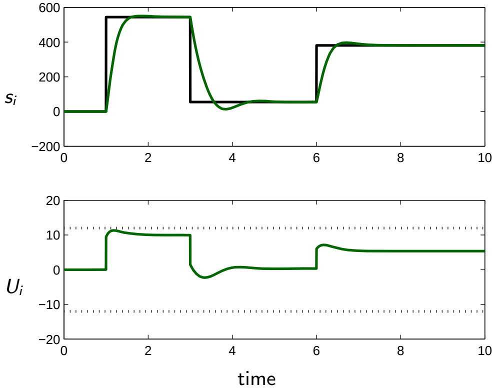

- The controller is well tuned near s_i^d but **not very good a large range of use**

Inner control loop

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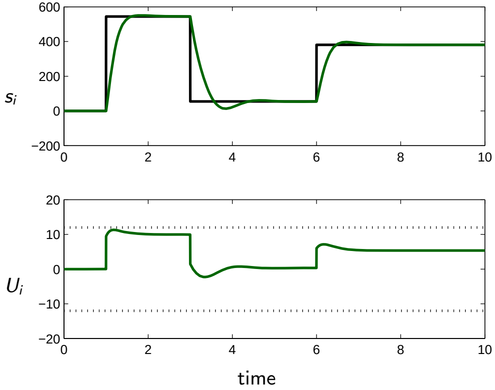

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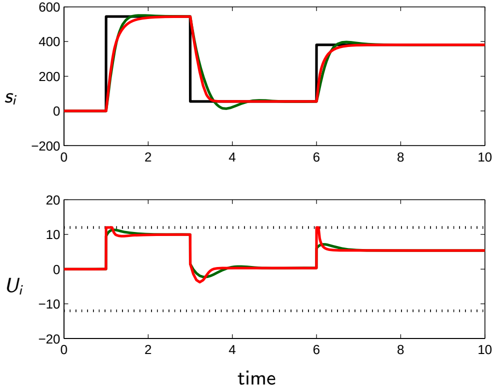

- **The rotors are now well controlled**

Outline

Jointed-arm robots

- Characteristic variables:

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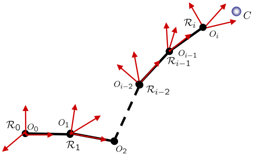
- Despite the limitations, this approach is widely used (oversized robots)

Computation of the geometrical model

Combination of rotations and translations



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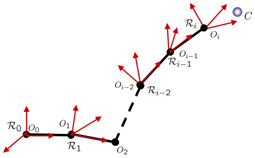

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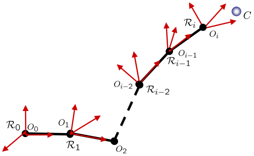

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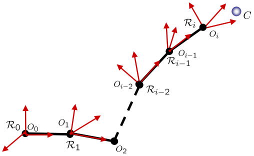

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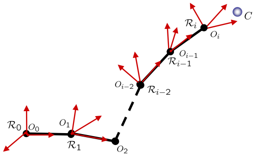
Computation of the geometrical model

Combination of rotations and translations



- Let X be the orientation and position of the last segment in $\mathcal{R}_0$ (usually variable to control)
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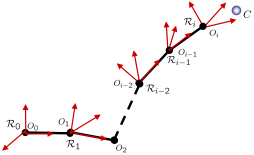

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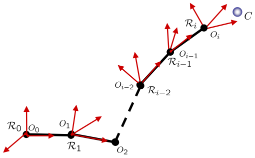

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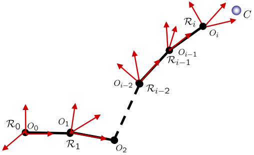

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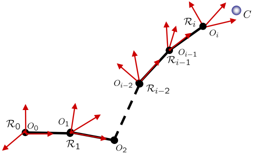

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  - $R^{i+1}_i = {R^i_{i+1}}^T$, $\det R^{i+1}_i = 1$

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$$\text{Rot} = \begin{pmatrix} R & 0_{3\times 1} \\ 0_{1\times 3} & 1 \end{pmatrix}$$

Note that still $R^{-1} = R^T$ and $det(R) = 1$

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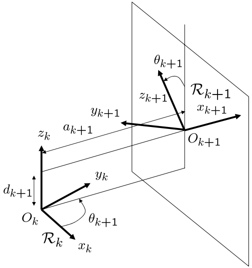
Denavit-Hartenberg's convention



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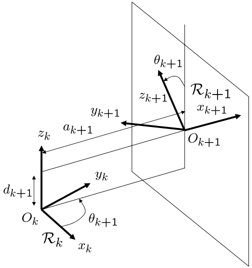

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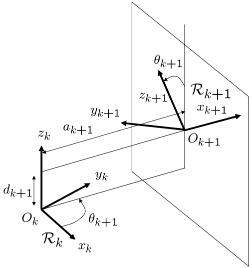

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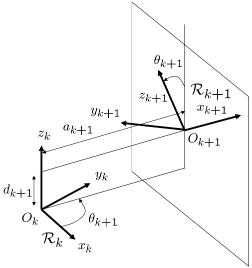

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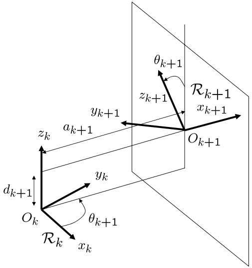

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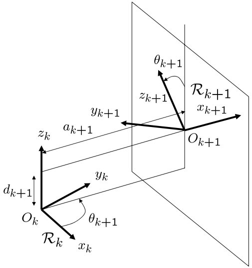

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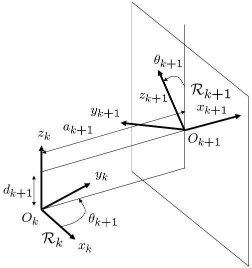

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- The DH parametrization **always exists and is unique**

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    - inertia may cause overshoot or oscillations

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  - The matrix transformation of the Denavit-Hartenberg's convention is: $R_2 \cdot T_2 \cdot T_1 \cdot R_1$

Outline

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- The kinematic model can also be obtained using the composition of speed and decomposing the Denavit-Hartenberg's parametrization:

$$R(z,\theta)T(z,d)T(x^+,a)R(x^+,\alpha)$$

Kinematic model of robots

- Express the infinitesimal mouvement dX as a function of speed of the actuators $\frac{d\theta}{dt}$
- Sometimes called "velocity kinematics"
- Assumes that, thanks to inner-loops, actuators speeds can be assumed to be control variables
- The kinematic model is "simply" the derivation of the geometric model $X = f(\theta_0, \theta_1, \cdots, \theta_n)$:

$$\dot{X} = \frac{\partial f}{\partial \theta}\dot{\theta}$$

- $J := \frac{\partial f}{\partial \theta}$ is called the *Jacobian* of the robot
- J represents the instantaneous transformation between a vector of joint velocities and the linear and angular velocities of the end-effector
- J can be decomposed into J_v and J_ω so that:

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- Fastidious in many cases but systematic ! See books for that

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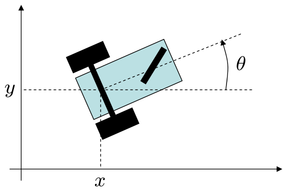
- **The kinematic model is a state space representation of a controlled system**

Example of kinematic model

- Example: the car in the plane



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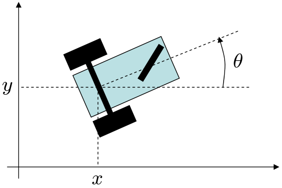

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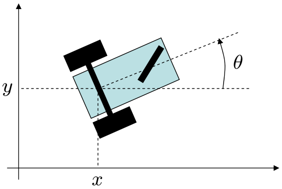

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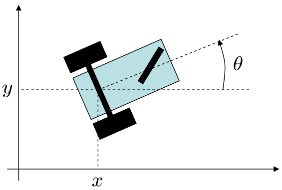

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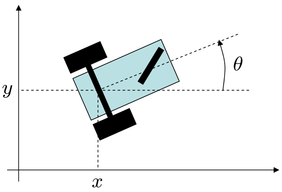

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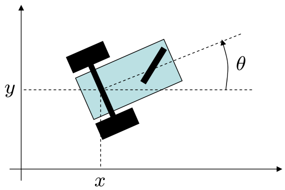

$$\dot{x} = \frac{V_l + V_r}{2}\cos\theta$$

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$$\dot{\theta} = \frac{V_r - V_l}{d}$$

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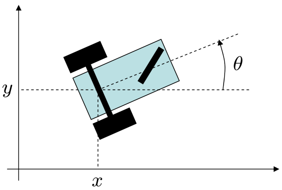
  - Is this system underactuated or overactuated ? Explain why



$$\begin{aligned}\dot{x} &= \frac{V_l + V_r}{2}\cos\theta\\ \dot{y} &= \frac{V_l + V_r}{2}\sin\theta\\ \dot{\theta} &= \frac{V_r - V_l}{d}\end{aligned}$$

$$J = \frac{1}{2}\begin{pmatrix}\cos\theta & \cos\theta\\ \sin\theta & \sin\theta\\ -\frac{2}{d} & \frac{2}{d}\end{pmatrix}$$

Relation between workspace forces and joint torques

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- the Jacobian must be derived at each origin O_i of each link frame

Kinematic redundancy

When a robot is given by its kinematic model $\dot{X} = J\dot{\theta}$

- J is usually $n \times p$ with $X \in \mathbb{R}^n$ and $\theta \in \mathbb{R}^p$
- $r = p - n$ is called the **kinematic redundancy number**
- When $r < 0$, the robot is underactuated, usually the case with mobile robots $\Rightarrow$ advanced control
- When $r > 0$, the robot is overactuated. It has redundancy.

For a robot with redundancy, one can write:

- $J = \begin{pmatrix} J_n & J_{p-n} \end{pmatrix}$ with J_n invertible

Control through the kinematic equation

Control with J^t Take a robot given by its kinematic model $\dot{X} = J\dot{\theta}$

- Control with J^t
  - Apply a fictive force $F = K(X - X_d)$ with K positive and symmetric
  - Take $\dot{\theta} = J^t F = J^t K(X - X_d) = J^t Ke$
  - Then the elastic potential $\Phi(e) = \frac{1}{2}e^t Ke$ is such that

$$\dot{\Phi}(e) = -e^t KJJ^t Ke < 0$$

  - $e \rightharpoonup 0$, $X \rightharpoonup X_d$
  - Automatically handles redundancy

Control through the kinematic equation

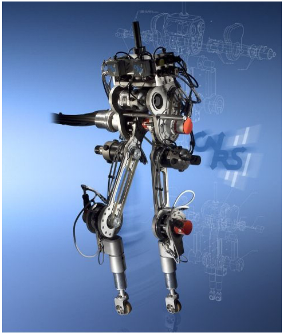
Control with J^+ Take a robot given by its kinematic model $\dot{X} = J\dot{\theta}$

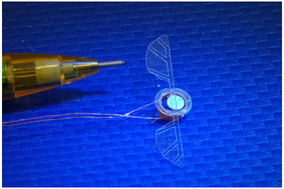
- Control with $J^+ := J^t(JJ^t)^{-1}$
  - J^+ is the Moore-Penrose pseudo-inverse (`pinv` in Matlab)
  - Can be obtained through SVD decomposition. $J = U\Delta V^t$, Δ diagonal $\Longrightarrow J^+ = V\Delta^+U^t$, Δ^+ is the inverse of the nonzero coefficient of Δ
  - Taking $\dot{\theta} = J^+\dot{X}$ minimizes the energy $\dot{\theta}^t\dot{\theta}$
  - Taking $\dot{\theta} = J_M^+\dot{X}$ with $J_M^+ := M^{-1}J^t(JM^{-1}J^t)^{-1}$ minimizes the kinetic energy $T = \dfrac{1}{2}\dot{\theta}^t M(\theta)\dot{\theta}$

Outline

Dynamical model of robots

- Express the accelerations of movement as a function of the actuation variables





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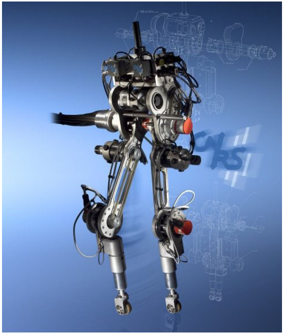

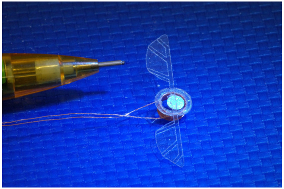

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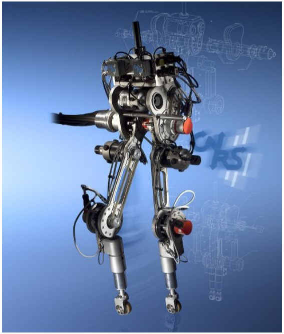

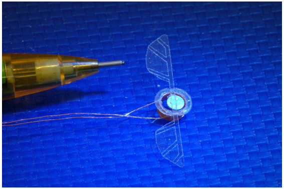

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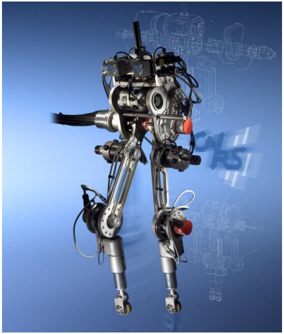

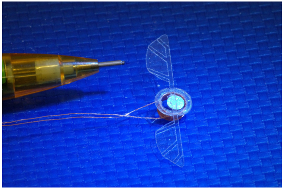

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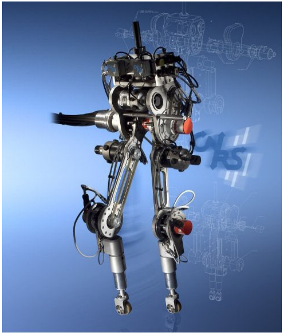

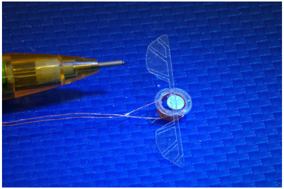

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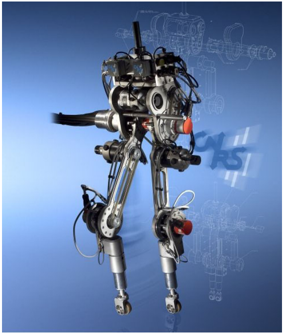

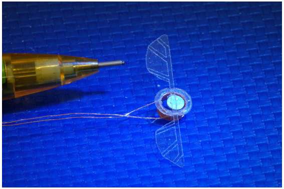

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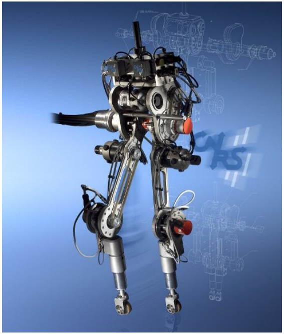

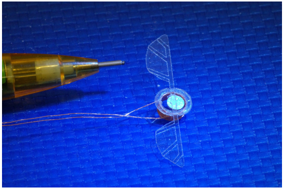

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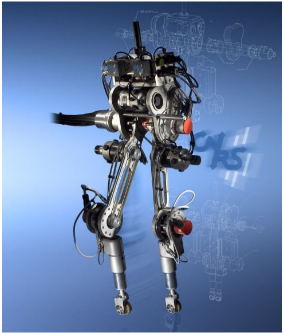

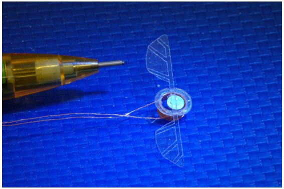

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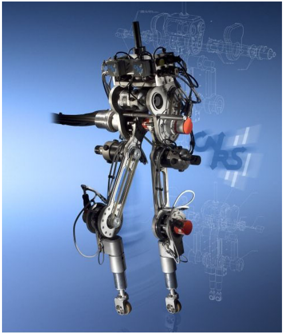
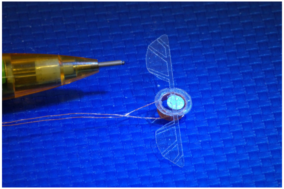
- Widely used for flying or diving robots (UAVs, AUVs, etc.) or walking robots





Dynamical models of robots

n-link manipulator

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- An important literature on the control of this type of systems can be found

Dynamical models of robots

flying and diving robots

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$$\left\{\begin{array}{rcl}\dot{\vec{p}} & = & \vec{v} \\ m\dot{\vec{v}} & = & -mg\vec{e}_3 + R\begin{pmatrix}F_x \\ F_y \\ F_z\end{pmatrix} \\ \dot{R} & = & R\vec{\omega}^\times \\ J\dot{\vec{\omega}} & = & -\vec{\omega}^\times J\vec{\omega} + \begin{pmatrix}\Gamma_r \\ \Gamma_p \\ \Gamma_y\end{pmatrix}\end{array}\right.$$

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  - The number of available controls depends upon the system

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- **Dynamical model (state space representation)**:

  Robot acceleration $= f$(position and speed,forces/torques)

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  - Gradient descent or Probabilistic roadmaps to generate the path

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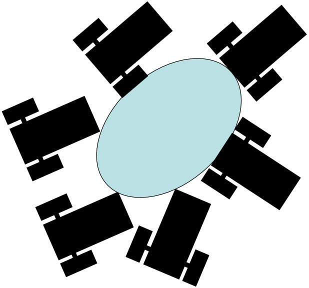
Example: the car



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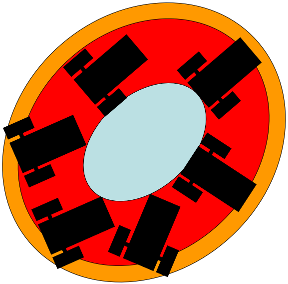

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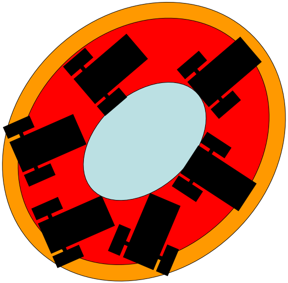

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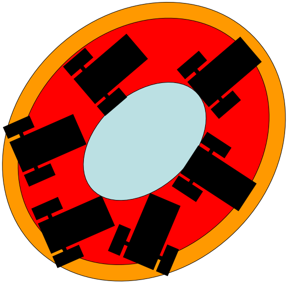

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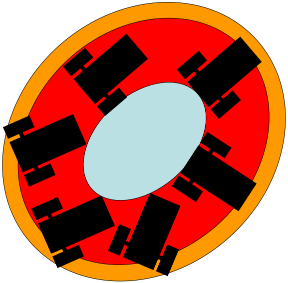

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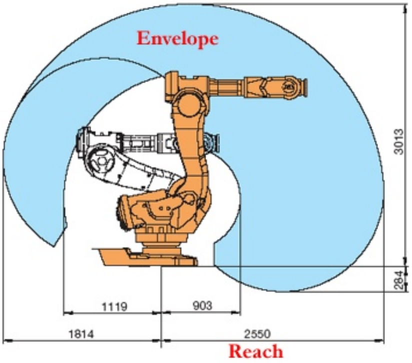
- Numerical simulation can easily solve this problem (systematic simulation)

Example: arm robot



Outline

A recall on gradient descent

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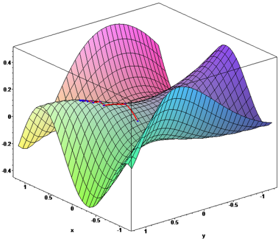

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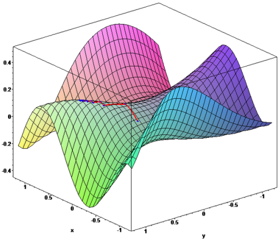

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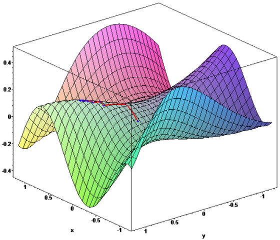
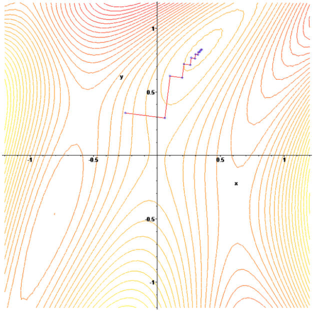

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- No more slope (almost the same as previous condition)

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- Quasi-Newton method:

$$z_{k+1} = z_k - \rho_k B_k \nabla F(z_k)$$

B_k: approximation of the Hessian

http://en.wikipedia.org/wiki/Quasi-Newton_method)

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- The repulsive field will create a fictitious force that will prevent collisions by repelling the robot from the obstacles

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From attractive/repulsive forces to actuator torques

- The total joint torques acting on a robot is the sum of the torques from all attractive and repulsive potentials:

$$\tau(\theta) = \sum_i J_{O_i}^T(\theta)\left(F_{att_i}(\theta) + F_{rep_i}(\theta)\right)$$

Gradient descent

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  4. Sum the joint torques in the configuration space
  5. Use gradient descent to reach your target configuration

Gradient descent

1. $i = 0,\ \theta[0] = \theta_0$
2. if $||\theta[i] - \theta_f|| > \varepsilon$, then:
    - $\theta[i+1] = \theta[i] + \alpha[i] \dfrac{\tau(\theta[i])}{||\tau(\theta[i])||}$
    - $i = i + 1$
    - goto 2

   else:
    - return $\theta[0], \ldots, \theta[i]$

- Many other algorithm are possible
    - steepest descent (gradient) (Euler)
    - Newton
    - ... see optimization books
- the $\theta[0], \ldots, \theta[i]$ are the successive configuration to track = path
- It is possible to add random to escape local minima

Probabilistic roadmap

- Randomly sample the configuration space

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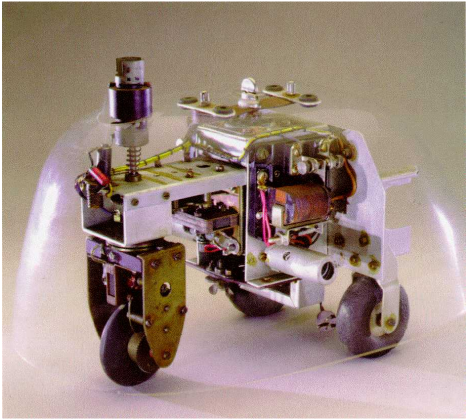
- Predictive control can also be used to handle constraints "on line"
- Adding fictive force is a very power tool also widely used in formation control or robotics with communication constraints (mainly range)

Mobile robotics

- Born in the 50s, aiming to *autonomously moving* robots



Grey Walter's "Turtle" (machina speculatrix): attracted by light

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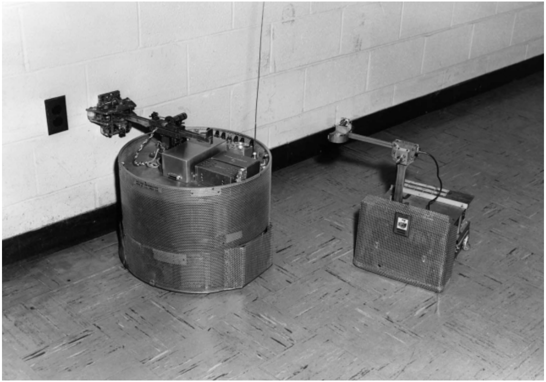

John Hopkins Univ. "Beast" robot: first use of transistor based sensing (ultrasound and photodiodes)

Mobile robotics

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Shakey robot from Stanford Univ.

Mobile robotics

- Bio inspired locomotion: first biped robot

Honda E0 first biped robot (1986)

Mobile robotics

- Bio inspired locomotion: first biped walk

Rabbit robot CNRS-Grenoble (2004)

Mobile robotics

- Bio inspired locomotion: more about mobility

Boston Dynamics (SoftBank)

Mobile robotics

- SLAM: Simultaneous localization and mapping

https://github.com/erik-nelson/blam

Mobile robotics

- SLAM: Simultaneous localization and mapping

Mobile robotics

- Aerial robotics

Mobile robotics

- Bionics

Introduction to visual servoing

- An arm robot equipped with a camera

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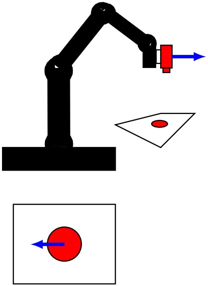

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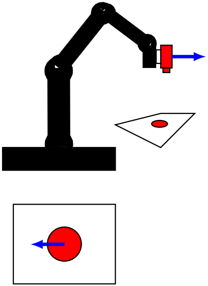

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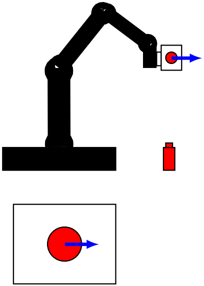


Introduction to visual servoing

The key points

- Being able to extract feature from the image: "recognize" points of the object
- Being able to characterize the relation between the robot movement and the image changes

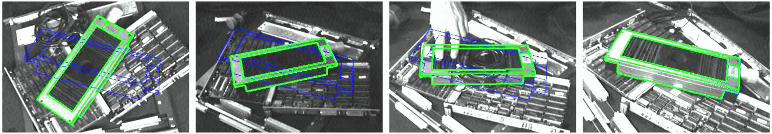
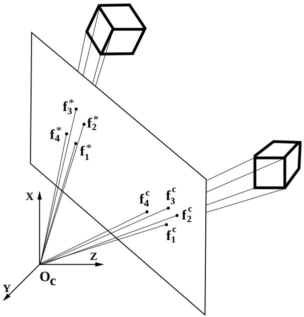


Image based visual servoing

The interaction matrix



The interaction matrix links the mouvement of O_c (lateral and rotational) to the movement of the feature points (f_i^c)

Introduction to visual servoing

A short mathematical background

- Positioning error:

$$e(t) = s(q(t), a) - s^\star$$

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  - $L_s \in \mathbb{R}^{k\times 6}$: interaction matrix (Jacobian)

Control in visual servoing

A simple control approach

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- Practically, L_s is never known perfectly and we use an approximation

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- Derivating, we get

$$\begin{aligned} \dot{x} &= \dot{X}/Z - X\dot{Z}/Z^2 = (\dot{X} - x\dot{Z})/Z \\ \dot{y} &= \dot{Y}/Z - Y\dot{Z}/Z^2 = (\dot{Y} - y\dot{Z})/Z \end{aligned}$$

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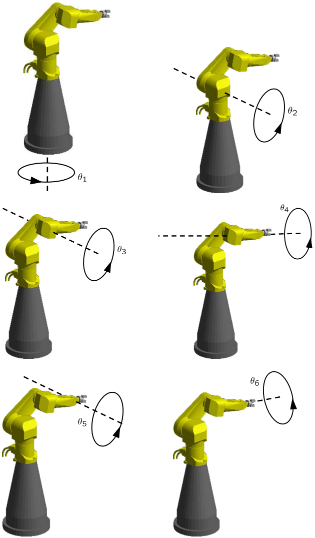
- To control six degrees of freedom, at least three points are required (p_1, p_2, p_3)
- Camera parameters can be obtained by calibration

Image-based visual stereo servoing

- We assume now that we have two cameras

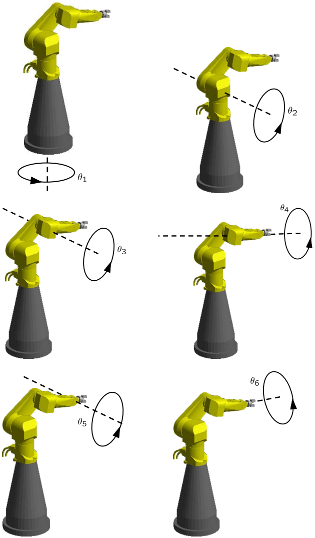
Stäubli RX90 robot

- 6 angles with constraints:



Stäubli RX90 robot

- 6 angles with constraints:
  - $-160 \leq \theta_1 \leq 160$: waist angle



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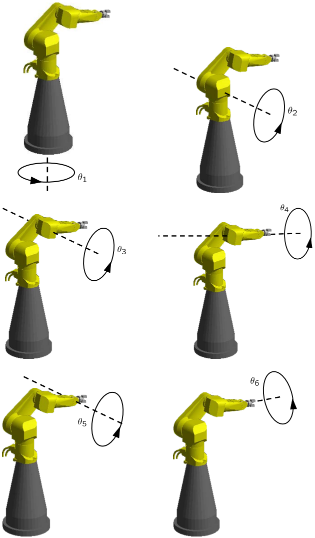

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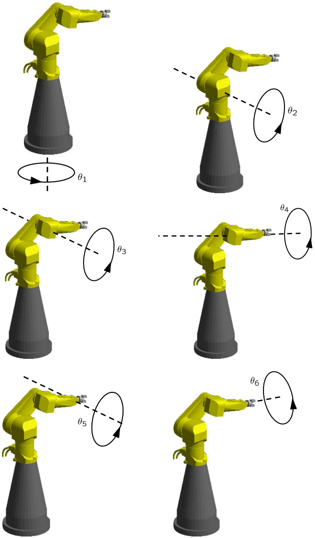

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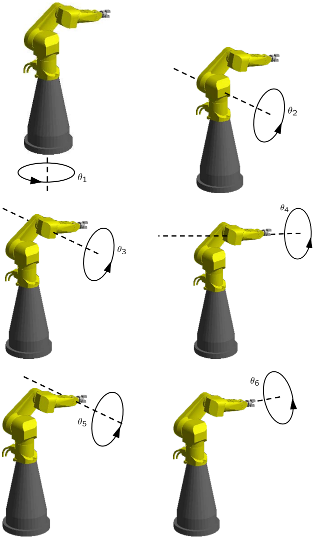

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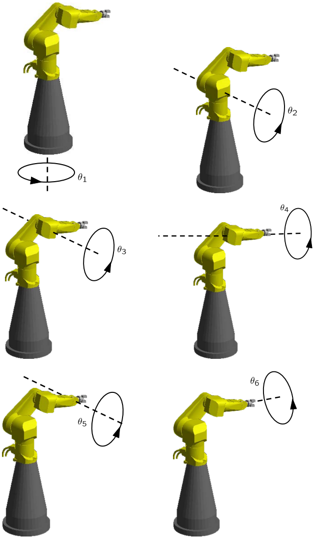

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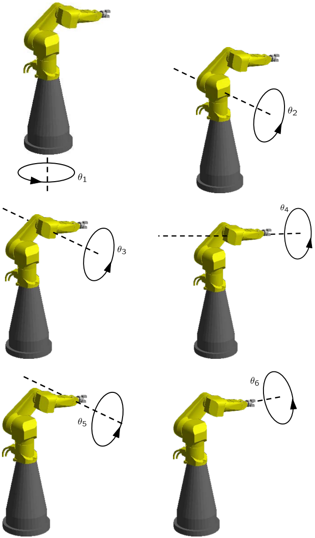

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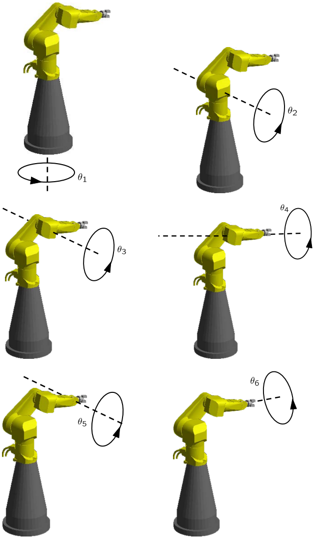

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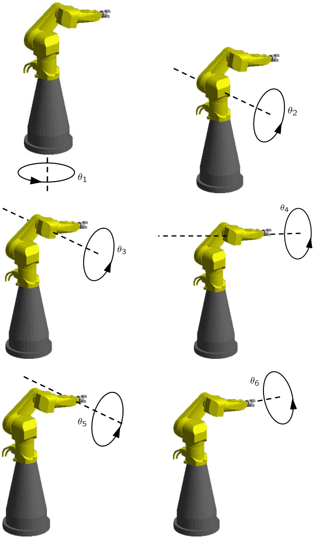
  - $\theta_{10} = 90$



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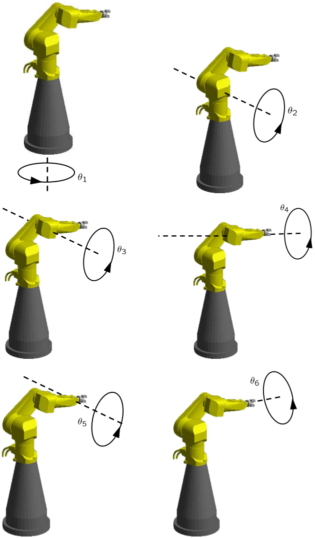

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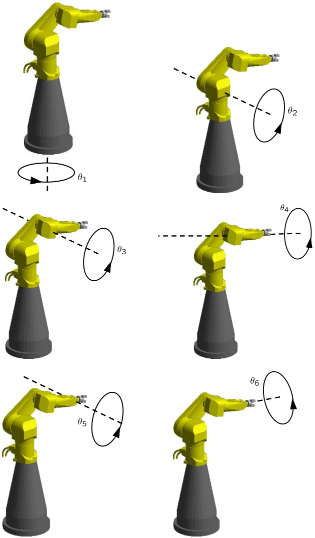
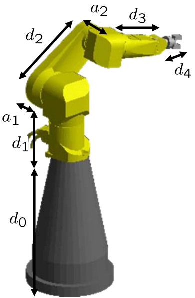
  - $\theta_{40} = \theta_{50} = \theta_{60} = 0$



Stäubli RX90 robot

- Stäubli RX90 robot

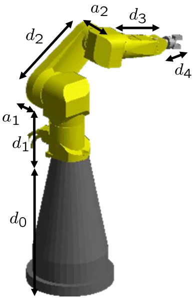
d_0	d_1	a_1	d_2	a_2	d_3	d_4
	177					



Stäubli RX90 robot

- Stäubli RX90 robot
  - What are the number of DOF ?

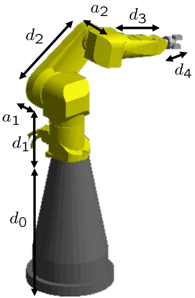
d_0	d_1	a_1	d_2	a_2	d_3	d_4
	177					



Stäubli RX90 robot

- Stäubli RX90 robot
  - What are the number of DOF ?
  - Compute the forward kinematic (geometrical model) of the wrist

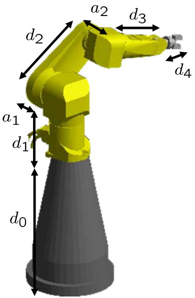
d_0	d_1	a_1	d_2	a_2	d_3	d_4
	177					



Stäubli RX90 robot

- Stäubli RX90 robot
  - What are the number of DOF ?
  - Compute the forward kinematic (geometrical model) of the wrist
  - Compute the inverse kinematic

d_0	d_1	a_1	d_2	a_2	d_3	d_4
	177					



Stäubli RX90 robot

1. Compute U_{att} and U_{rep} as function of the current position $p = (x, y, z)$ and the desired final position $p_f = (x_f, y_f, z_f)$
2. Compute $U(p, p_f) = U_{att} + U_{rep} \in \mathbb{R}^+$
3. Compute $\nabla U \in R^3$, the derivative of U w.r.t. p
   - ∇U can be computed analytically
   - ∇U can be computed numerically for ε small:

$$\nabla U \approx \begin{pmatrix} \frac{U(x-\varepsilon, y, z, p_f) - U(x+\varepsilon, y, z, p_f)}{2\varepsilon} \\ \frac{U(x, y-\varepsilon, z, p_f) - U(x, y+\varepsilon, z, p_f)}{2\varepsilon} \\ \frac{U(x, y, z-\varepsilon, p_f) - U(x, y, z+\varepsilon, p_f)}{2\varepsilon} \end{pmatrix}$$

4. Program an iterative routine with the following iteration:

$$p_{k+1} = p_k - \gamma \nabla U(p_k, p_f)$$

5. Start the program at your initial position p_0 and stop the program when p_k is close to p_f
6. The successive p_0, p_1, ... give you the path