Robotics

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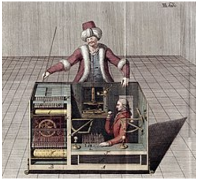
ENSIT

Introduction

- Historical perspective
  - A great interest in "automatic/robotic" systems even in mythology:
    - Talos, created by Hephaistos and offered to the king Minos
    - In Iliad, Hephaistosis referred as creator of technical artificial creatures
    - 384-322 BJC, Aristote is speaking about machines doing human work

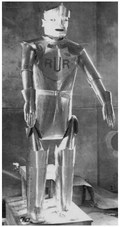


- First realizations
  - In Egypt: moving jaw of Anubis mask, moving arm of Amon's statue to designate the new pharaon
  - In Alexandria: fountains with moving birds (hydraulic systems)
  - Early mechanical automatons in the 9th/10th century (mainly in the Arabic world)
  - Clocks and automatons in the 13th/14th century (Europe)
  - Industrial automatons: 18th/19th century (e.g. Vaucanson, Jacquard)

Introduction

- Historical perspective
  - First use of the word Robot (means forced labor or serf in Czech) in the play R.U.R. (Rossum's Universal Robots) by Karel Capek (1890-1938) in January 1921.

  In R.U.R., Capek poses a paradise, where the machines initially bring so many benefits but in the end bring an equal amount of blight in the form of unemployment and social unrest



RUR

- Science fiction
  - Often a bad image: men against robots, dystopic society, etc. More and more a good image.

Formal definition (Robot Institute of America)

A reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through various programmed motions for the performance of a variety of tasks

Robots and their image

- Robots have a bad image (1930-1960)
  - Robots take human works
  - Robots are dangerous since potentially independent and more intelligent than we are
- Robots have a better image (1960-today)
  - Robots can make things that human can not do (space, etc.)
  - Human can do things that robots can not do (we still are clever)
  - Robots can be games
  - Robots can be good or bad

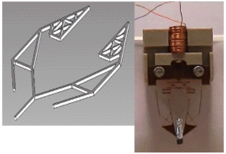
Robotics industry (1/many)

- Where are the robots ?
  - France:
    - 61% in automotive industry
    - 14% in chemical industry
    - ...
    - 4% in electricity industry
    - 3% in food industry

- What kind of robots ?
  - Industry: ground fixed robots: manipulators, arm robots, ...
  - Private individuals: mobile robots: service, games, ...
- Future of robots:
  - Industrial mobile robotics
  - Medical robotics
  - Service robots (growing field)

Robotics industry... today (2/many)

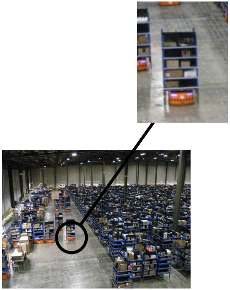
Vacuum cleaner



Micromanipulator

Surgical robot

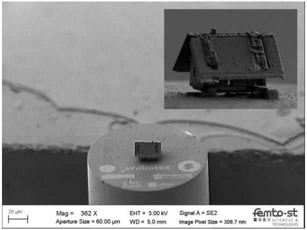
Forest robot



Kuka robot for automotive industry

Hollywood robots

… AND IN THE FUTURE ? (2 BIS/MANY)

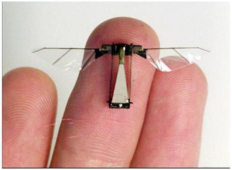


Nanorobotics (FEMTO-ST, Fr)

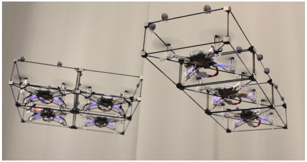
Military robots (Black Hornet, FLIR Systems, No)
2015 : Autonomous weapons: an open letter from AI & Robotics researchers [link]

Bio-inspired robots
Spot from Boston Dynamics

Exoskeleton



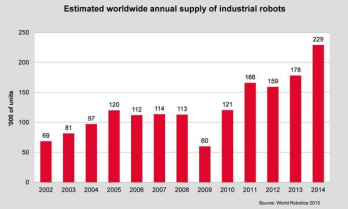
Microrobotics (Harvard, USA)



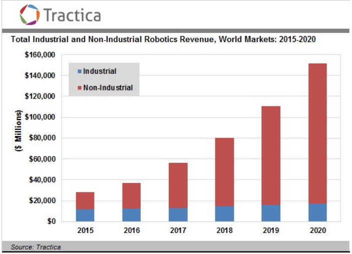
Modular robots

Robotics industry (3/many)

- Past



- Future



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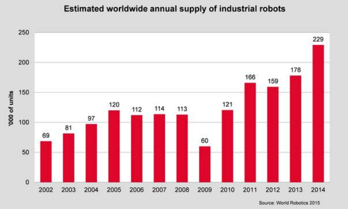

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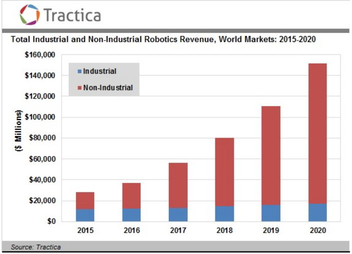
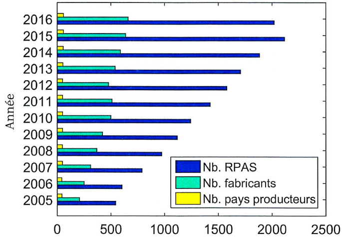


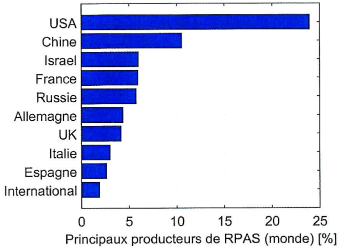
- Enova Robotics, Sousse, Tunisia

Robotics industry: UAVs (4/many)

- UAV's Manufacturer

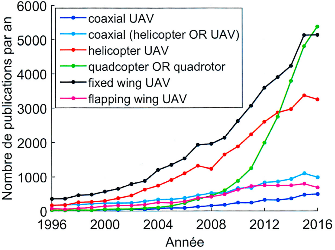


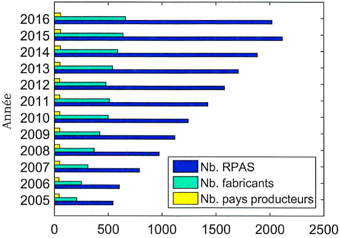
- UAVs by countries



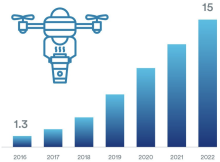
Robotics industry: UAVs (5/many)

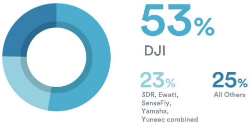
- Development of the drone's industry:





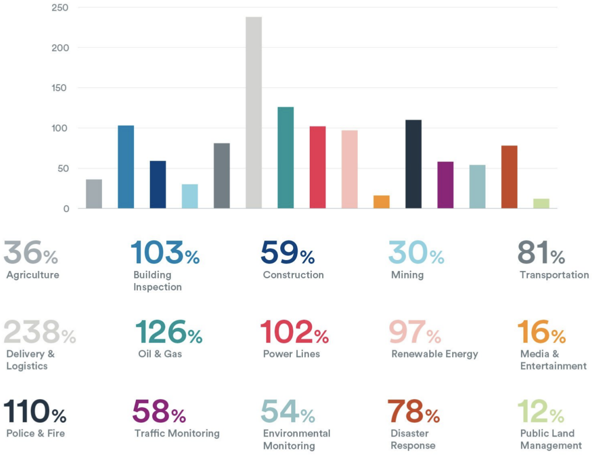
- Foundation of DJI: 2006





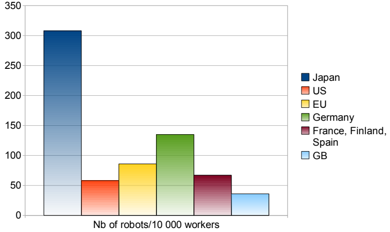
- Very competitive market with a high technological level of intergration
- Commercial margin of 10% to 15% (more than 50% on iPhone)

Robotics industry: UAVs (6/many)



Robolution (1/many)

- Number of robots for every 10 000 workers:



- 70% of robots in companies with more than 1000 employees
- 17% of robots in companies with less than 300 employees
- In 2002, 95% of robots > 30k€ and 32% of robots > 60k€
- 79% of decrease of the mean price between 1990 and 2002
- Big robots manufacturers: ABB (S), KUKA (G), Fanuc (JP), etc.

Robolution (2/many)

- Robotics enables 90% of cost reduction (60% for delocation)
- Each new robot destroys 6.2 jobs [MIT/Boston 1990-2007, 2017]
- 47% of jobs in the US, 50% of jobs in Europe have a high risk of being replaced by robots in the next 20 years [Oxford, 2013] ... but only 9% according [OCDE, 2016]
- Poor countries are more vulnerable, especially world factories (85% of the jobs in Ethiopia, 77% in Chine [World Bank])
- Sectors with high impact: Administration et Production
- Winner sectors: Finance, Maths/Sciences, Education
- No link between unemployment and robots
- Helps to relocate jobs in countries where the consumers are
- Very few studies on created jobs (compared to destroyed jobs)
- 800 000 direct jobs in robotics in 2020 and more than 2 millions in connected domains (electronic, energy, agriculture, etc.)

Robolution (3/many)

- What about the previous industrial revolution ?
  - Machines have created more jobs than they have replaced in the last 140 years
  - Working is getting less and less exhausting
  - Increase of new jobs (+580% éducation)
  - But we had fears, as in any big change periods:
    - 1675 : Destruction of machines by weavers (England), 1788 : 2000 workers break weaving machines (France), 1811-1812 : Luddism (Angleterre)
    - 1858 : Karl Marx is prophesies the replacement of the humans by machines
    - 1930 : John Maynard Keynes invents the term "technological unemployment"

Outline

- Basic mechanics for robotics
  - Space representation
    frames, coordinate transformation, etc.
  - Force and torques
- Modelisation
- Control for robots
  - All potential problems:
    Oscillations, dry friction, saturations, etc.
  - Linear approaches
  - Nonlinear approaches

Outline

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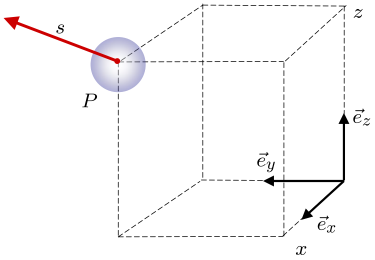
Position and speed

- The **position** of some point P in the **fixed** frame $\mathcal{F}(o, \vec{e}_x, \vec{e}_y, \vec{e}_z)$ is the vector $\vec{p} = (x, y, z)^T$



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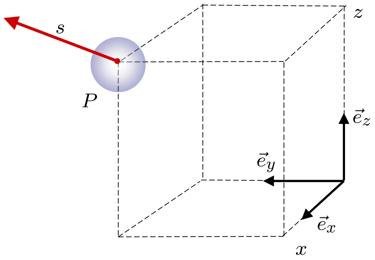
- The **speed** of P in $\mathcal{F}$ is the vector $\vec{s} = \dot{\vec{p}} = (\dot{x}, \dot{y}, \dot{z})^T$



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  - a unit vector $\vec{u} = (u_x, u_y, u_z)^T$:

$$\begin{pmatrix} u_x^2 + (1-u_x^2)c_\theta & u_x u_y(1-c_\theta) - u_z s_\theta & u_x u_z(1-c_\theta) + u_y s_\theta \\ u_x u_y(1-c_\theta) + u_z s_\theta & u_y^2 + (1-u_y^2)c_\theta & u_y u_z(1-c_\theta) - u_x s_\theta \\ u_x u_z(1-c_\theta) - u_y s_\theta & u_y u_z(1-c_\theta) + u_x s_\theta & u_z^2 + (1-u_z^2)c_\theta \end{pmatrix}$$

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- The rotation resulting from 2 successive rotations R_1 and then R_2 is $R_2 R_1$

Products and associated tools

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- Skew-symmetric matrices and rotations

$$u^\times \sin\theta + (I - uu^T)\cos\theta + uu^T = \exp((u\theta)^\times)$$

is the rotation of angle θ leaving axis u fixed

Attitude representation: angles

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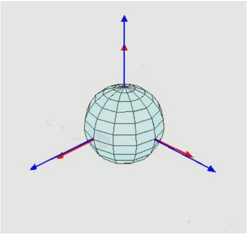

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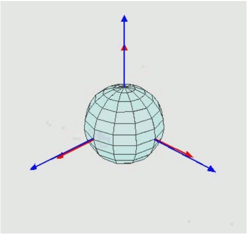

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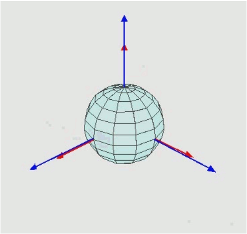

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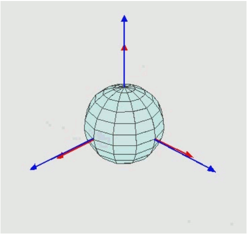

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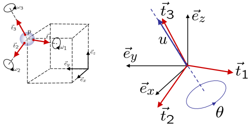

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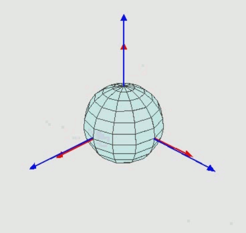

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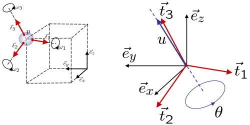

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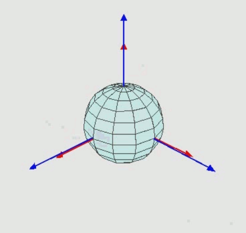

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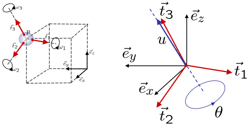

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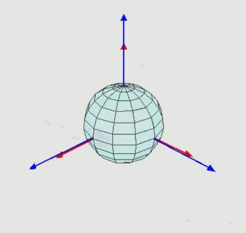

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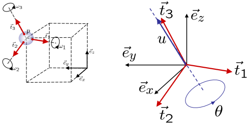

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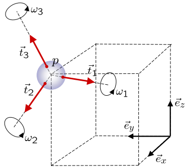

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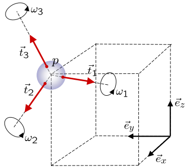

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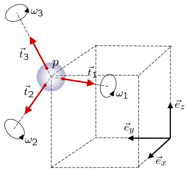

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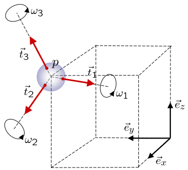

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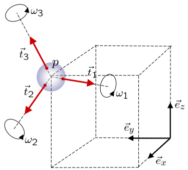

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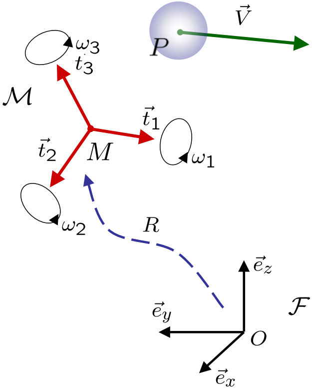
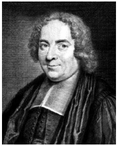
$$\begin{aligned} \dot{\vec{q}} &= \frac{1}{2}\Omega(\vec{\omega})q \\ &= \frac{1}{2}\Xi(q)\vec{\omega} \end{aligned} \quad \text{with} \quad \begin{cases} \Omega(\vec{\omega}) = \begin{pmatrix} 0 & -\vec{\omega}^T \\ \vec{\omega} & -\vec{\omega}^{\times} \end{pmatrix} \\ \Xi(q) = \begin{pmatrix} -\vec{q}^T \\ I_{3\times 3}q_0 + \vec{q}^{\times} \end{pmatrix} \end{cases}$$

Moving frames





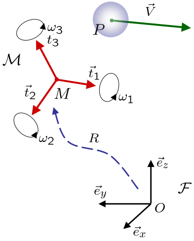
P. Varignon (1654-1722)

Varignon's formula

$$\frac{d\vec{U}^{\mathcal{M}}}{dt} = \frac{d\vec{U}^{\mathcal{F}}}{dt} + \Omega^{\mathcal{F}/\mathcal{M}} \times \vec{U}^{\mathcal{F}}$$

Moving frames

- $\mathcal{F} := (O, \vec{e}_x, \vec{e}_y, \vec{e}_z)$ fixed inertial frame



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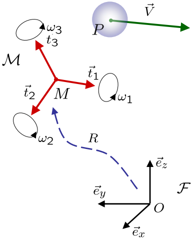

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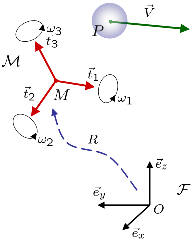

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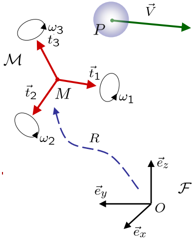

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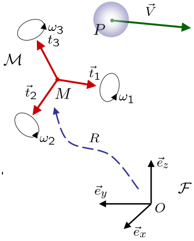

- **Velocities**:
  - Absolute velocity

$$\left.\frac{d\vec{OP}}{dt}\right|^{\mathcal{F}} = \left.\frac{d\vec{OM}}{dt}\right|^{\mathcal{F}} + \left.\frac{d\vec{MP}}{dt}\right|^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \vec{MP}$$

  - Speed of $\mathcal{M}$ w.r.t $\mathcal{F}$
  - Relative velocity
  - Due to the rotation of $\mathcal{M}$ w.r.t. $\mathcal{F}$

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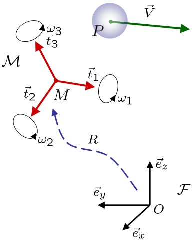

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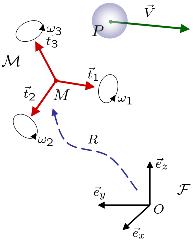

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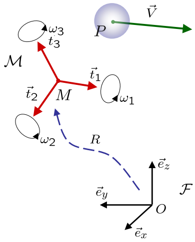

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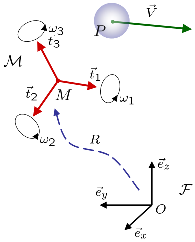

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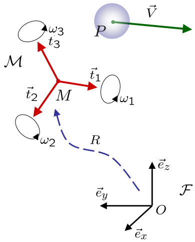
- **Acceleration**:



$$\ddot{P}^{\mathcal{F}} := \left(\frac{d\dot{P}^{\mathcal{F}}}{dt}\right)^{\mathcal{F}} = \frac{d\dot{P}^{\mathcal{M}}}{dt}^{\mathcal{F}} + \frac{d\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}}}{dt}$$

- $\dfrac{d\dot{P}^{\mathcal{M}}}{dt}^{\mathcal{F}} = \ddot{P}^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{P}^{\mathcal{M}}$ (Varignon's formula)
- $\dfrac{d\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}}}{dt} = \dot{\Omega}^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{P}^{\mathcal{F}}$

$$= \dot{\Omega}^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}} + \Omega^{\mathcal{M}/\mathcal{F}} \times \dot{P}^{\mathcal{M}} + \Omega^{\mathcal{M}/\mathcal{F}} \times (\Omega^{\mathcal{M}/\mathcal{F}} \times P^{\mathcal{F}})$$

all together:

$$\ddot{P}^{\mathcal{M}} = \ddot{P}^{\mathcal{F}} - 2\Omega \times \dot{P}^{\mathcal{M}} - \dot{\Omega} \times P^{\mathcal{F}} - \Omega \times (\Omega \times P^{\mathcal{F}})$$

- Coriolis effect
- Euler effect (tangent acceleration)
- Centrifugal effect

Newton's laws

Consider:

- an inertial frame $\mathcal{F}$
- a body of mass $m := \sum_i m_i$ composed of elements located in $\vec{p}_i$ with speed $\vec{v}_i$ in $\mathcal{F}$
- *or* a body of mass $m := \int_{\text{body}} dm$ composed of elementary part located in $\vec{p}_{dm}$ with speed $\vec{v}_{dm}$ in $\mathcal{F}$
- $\vec{p} := \dfrac{\sum_i m_i \vec{p}_i}{m}$ defines the position of its center of mass G in $\mathcal{F}$
- *or* $\vec{p} := \dfrac{\int_{\text{body}} dm \vec{p}_{dm}}{m}$ defines the position of its center of mass G in $\mathcal{F}$
- $\vec{v} := \dot{\vec{p}}$ defines speed of the center of mass
- $\vec{r}_i := (\vec{p}_i - \vec{p})$ (resp. $\vec{r}_{dm} := (\vec{p}_{dm} - \vec{p})$)

Linear Momentum

$$\vec{P} := \sum_i m_i \vec{v}_i = m\vec{v} \in \mathbb{R}^3$$

$$\vec{P} := \int_{\text{body}} \vec{v}_{dm} dm \in \mathbb{R}^3$$

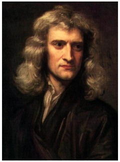
Angular Momentum

$$\vec{L} := \sum_i m_i (\vec{p}_i - \vec{p}) \times \vec{v}_i$$

$$\vec{L} := \int_{\text{body}} (\vec{p}_{dm} - \vec{p}) \times \vec{v}_{dm} dm$$

$$= \underbrace{\int_{\text{body}} ||\vec{r}_{dm}||^2 \, dm}_{J:\ \text{moment of inertia}} \quad \vec{\omega}$$

Newton's laws


I. Newton (1643-1727)


J. L. Lagrange (1736-1813)

Consider:

- a rigid body
- an inertial frame $\mathcal{F}$
- a moving frame $\mathcal{M}$ centered in the center of mass and aligned with the main axis of the rigid body
- Let $\vec{F}_i$'s be forces applying on the body with moment arm $\vec{a}_i$

Newton's second law

$$\sum \vec{F} = \frac{d\vec{P}^{\mathcal{F}}}{dt}$$

Conservation of the angular momentum

$$\sum \vec{\tau} = \frac{d\vec{L}^{\mathcal{F}}}{dt}$$

- In a moving frame (Varignon's formula):

$$\frac{d\vec{L}^{\mathcal{F}}}{dt} = \frac{d\vec{L}^{\mathcal{M}}}{dt} + \Omega \times \vec{L}$$

Menu

How it works

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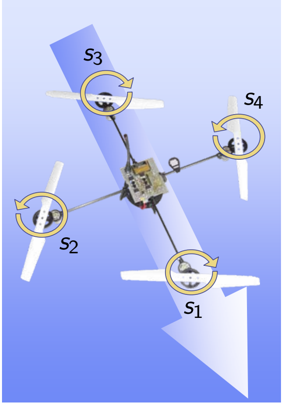
- 4 fixed rotors with controlled rotation speed s_i



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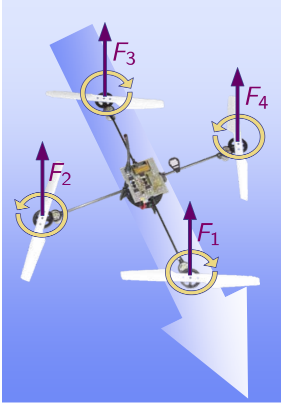

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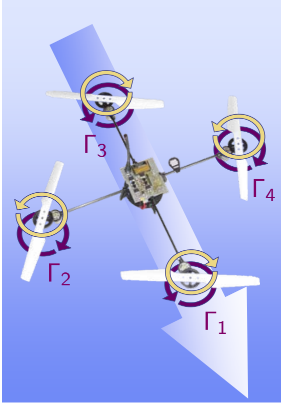

How it works

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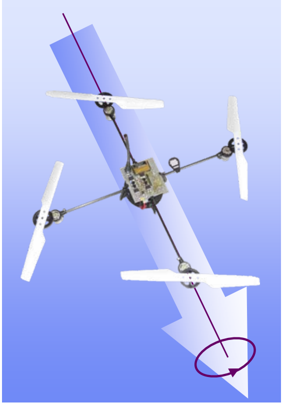

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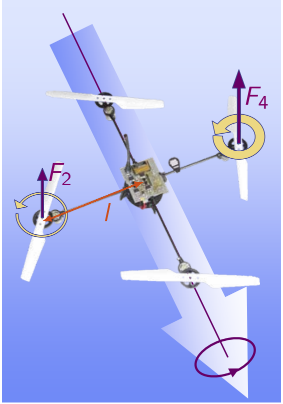

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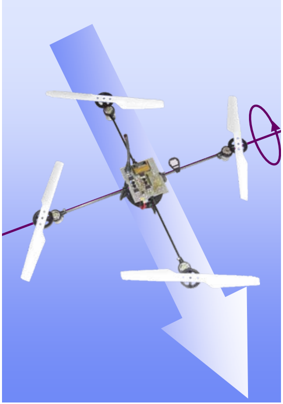

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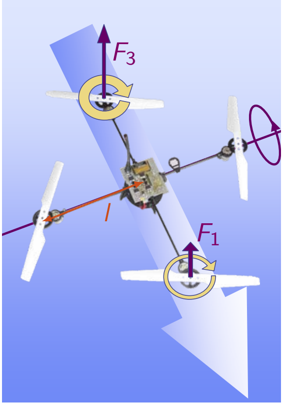

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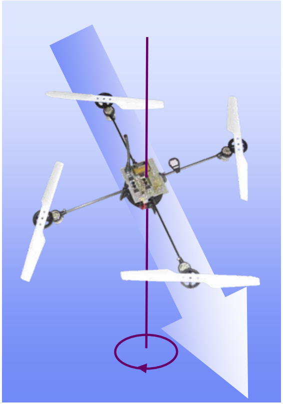

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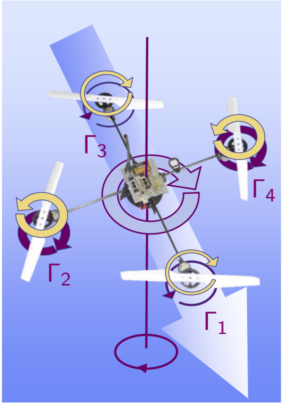
$$\Gamma_y = \Gamma_1 + \Gamma_3 - \Gamma_2 - \Gamma_4$$



Modeling the quadrotor

Actuation and aerodynamics

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s_i: rotation speed

U_i: voltage applied to the motor; real control variable

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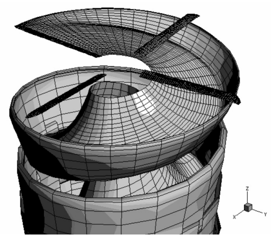

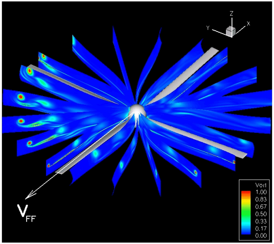

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- **Aerodynamical forces and torques**: Very complex models exist but overcomplicated for control, better use the *simplified* model:

$$\begin{aligned} F_i &= c_T s_i^2 \\ \Gamma_r &= l c_T (s_4^2 - s_2^2) \\ \Gamma_p &= l c_T (s_1^2 - s_3^2) \\ \Gamma_y &= l c_D (s_1^2 + s_3^2 - s_2^2 - s_4^2) \end{aligned} \qquad i \in \{1,2,3,4\}$$

c_T: thrust coefficient, c_D: drag coefficient

Frames and variables

- Two frames
  - a fixed frame $\mathcal{E}(\vec{e}_1, \vec{e}_2, \vec{e}_3)$
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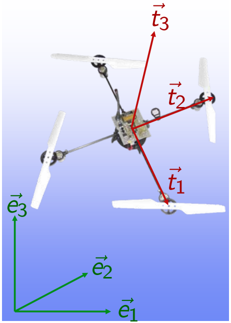

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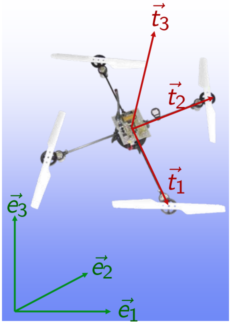

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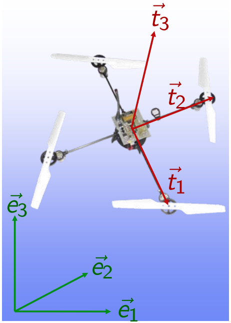

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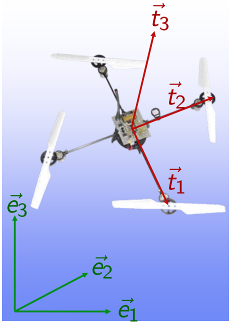

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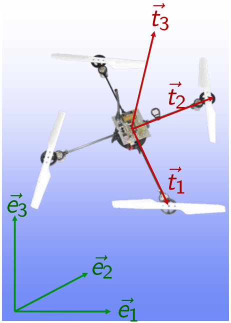

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  - Attitude coordinates:
    - angular velocity $\vec{\omega}$ in the moving frame $\mathcal{T}$
    - *either:* Euler angles three successive rotations about $\vec{t}_3$, $\vec{t}_1$ and $\vec{t}_3$ of angles angles ϕ, θ and ψ giving R
    - *or:* Quaternion representation $(q_0, \vec{q}) = (\cos\beta/2, \vec{u}\sin\beta/2)$ represent a rotation of angle β about $\vec{u}$

Newton's laws

- **Cartesian coordinates**:

$$\begin{cases} \dot{\vec{p}} = \vec{v} \\ m\dot{\vec{v}} = -mg\vec{e}_3 + R \cdot \underbrace{\sum_i F_i(s_i)\vec{t}_3}_{\vec{T}\ :\ \text{control thrust}} + \vec{F}_{ext} \end{cases}$$

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where $\vec{\Gamma}_c = \begin{pmatrix} \Gamma_r(s_2, s_4) \\ \Gamma_p(s_1, s_3) \\ \Gamma_y(s_1, s_2, s_3, s_4) \end{pmatrix}$ are the control torques

The wronskian matrix

- Consider the 1-2-3 Euler angles (ϕ, θ, ψ)
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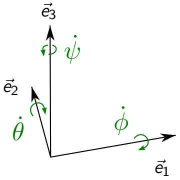
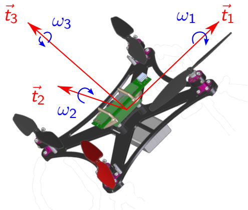
- The relation between the time derivative of the Euler angles and the angular velocity is:

$$\vec{\omega} = \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} + R_z \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_z R_y \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} = W^{-1} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$





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- W is called the **wronskian matrix** given by (for 1-2-3 Euler angles):

$$W = \begin{pmatrix} 0 & \frac{s_\phi}{c_\theta} & \frac{c_\phi}{c_\theta} \\ 0 & c_\phi & -s_\phi \\ 1 & s_\phi t_\theta & c_\phi t_\theta \end{pmatrix}$$

- This matrix is singular for $\theta = \pi/2 + k\pi$

A first model: review of nonlinearities

$$
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In red: nonlinearities

In blue: where the control variables act

Parameter identification

- **Electrical motor**:
  - For small input steps, the system behaves very close to a **linear** first order system
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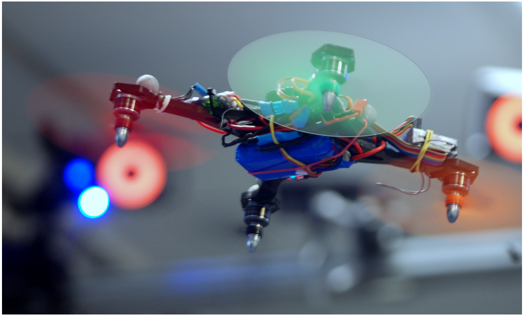
The flapping effect

- The thrust was assumed to be $\sum_i F_i(s_i)\vec{t}_3$, that is colinear to $\vec{t}_3$
- It has been proved to be false because it neglects the effect of the apparent wind speed, this is the **flapping effect**



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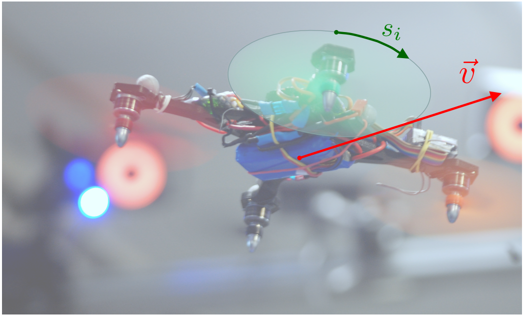

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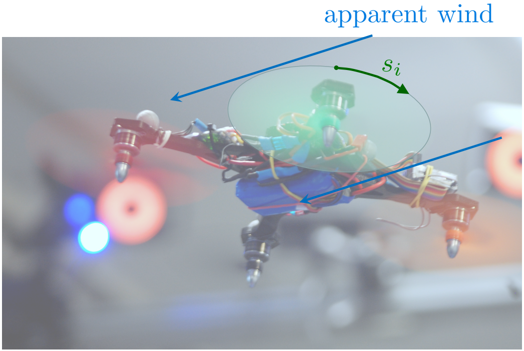

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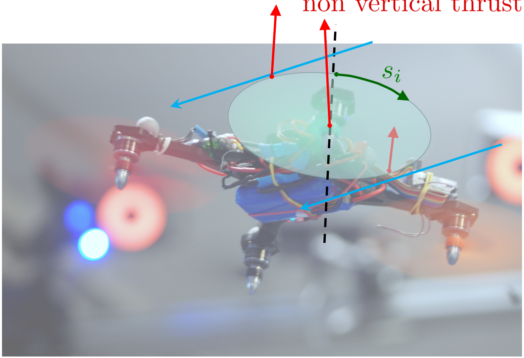

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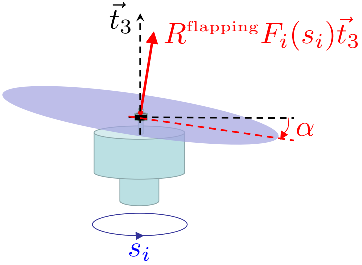

Modeling more into details: the flapping effect

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- a_ω and b_ω represent the contribution of the rotational speed of the body to the flapping effect

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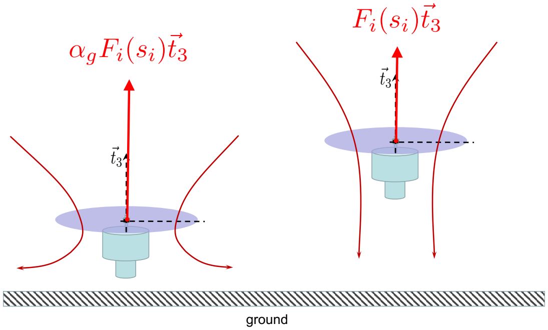
- Unfortunately, c_T is not constant but depends upon
  - the density of the air, therefore of the temperature
  - the ground distance : it is the ground effect, $\alpha_g(z) \geq 1$



Rotors effects

- Each rotor may be thought of as a rigid disc rotating around the vertical axis the body frame, with angular velocity s_i. The rotor's axis of rotation is itself moving with the angular velocity of the frame. This leads to the following **gyroscopic torque** :

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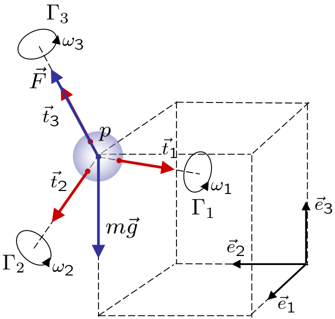
- Each rotor produces a counter rotating torque that can be expressed as:

$$\begin{aligned} s_{res} &:= \sum_i (-1)^i \, |s_i| \\ \vec{\Gamma}_I &= I_r \dot{s}_{res} \vec{t}_3 \end{aligned}$$

Other effects



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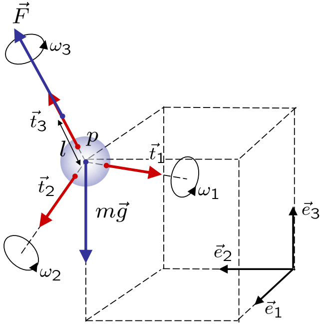

- Superposition of thrust center and mass center
- External forces
- Air friction: $-K_v \, ||\vec{v}|| \, \vec{v}$

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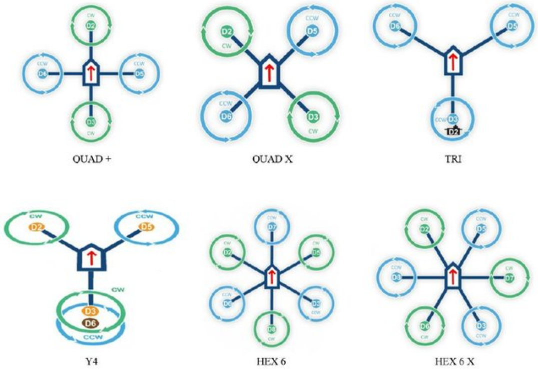

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- With flapping appears coupling phenomenon: the thrust affects the yaw movement and the drag affects thrust/roll/pitch movements

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Complete model

Robotics

N. Marchand

Introduction

Modeling

Cartesian coordinates

Orientation

Frames

Newton

X4

Kinematics

Arm robots

Inner-loop

Geometrical model

Kinematic model

Dynamical model

Conclusion

Path planning

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Outline

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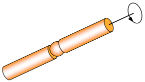

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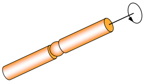

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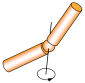

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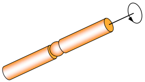

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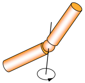

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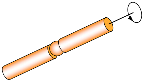

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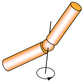

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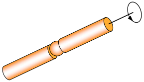

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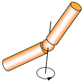

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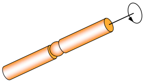

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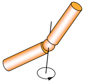

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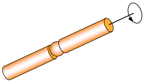

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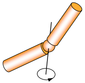

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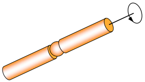

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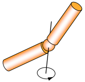

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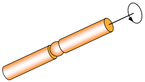

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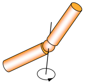

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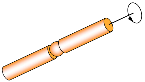

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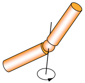

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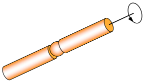

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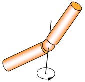

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- Many tasks can be performed with less than 6 dof: "pick and place" needs only 4 dof

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- Actuator's dynamics
- Robot's dynamics

Outline

Inner control loop

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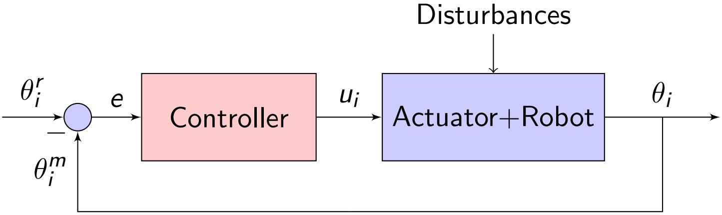

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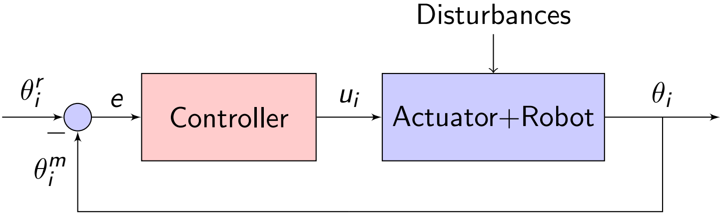

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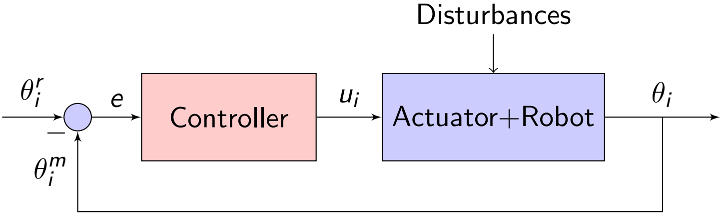

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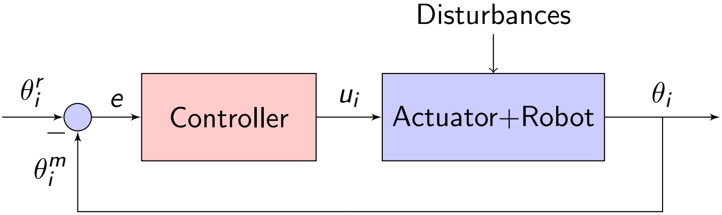

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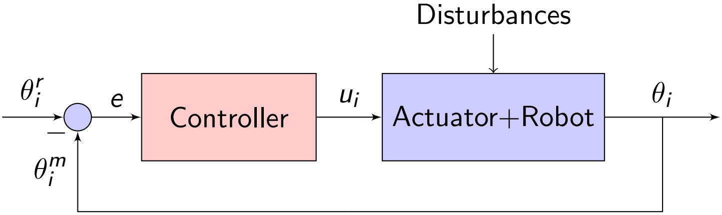

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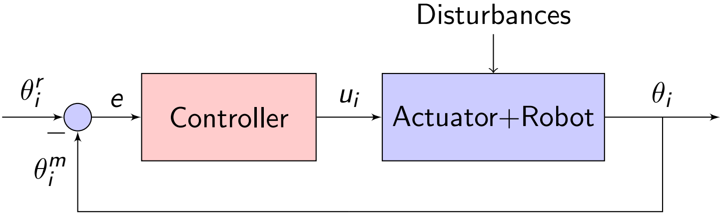

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  - anti-windup to tackle saturations

Inner control loop

Anti-windup PID

- We go back to the **X4** example and focus on the **rotors**:

$$\begin{cases} \dot{s}_i = -\frac{k_m^2}{J_r R} s_i - \frac{1}{J_r} \tau_{\text{load}} + \frac{k_m}{J_r R} \operatorname{sat}_{\bar{U}_i}(U_i) \end{cases}$$

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  - **neglecting the voltage limitations** $\bar{U}_i$

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- The so obtained **system is linear**

$$\frac{s_i(s)}{U_i(s)} = \frac{\frac{1}{k_m}}{1+\frac{J_r R}{k_m^2}s} = \frac{G}{1+\tau s}$$

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- Taking $K_i = \dfrac{1}{\tau_{CL} G}$ and $K_p = \tau K_i$, the closed loop system is:

$$\frac{s_i(s)}{U_i(s)} = \frac{1}{1 + \tau_{CL} s}$$

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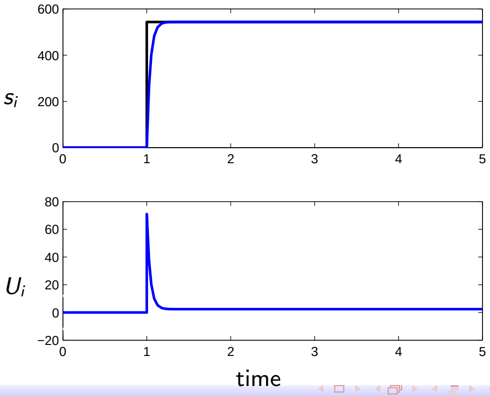

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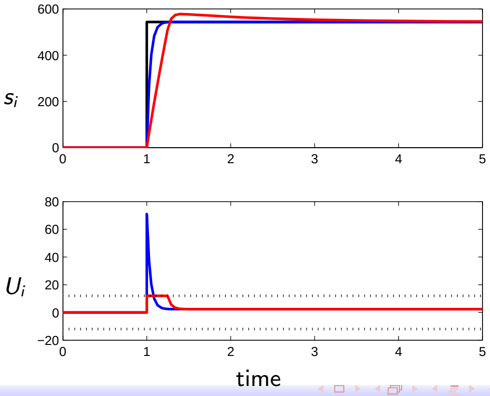

Inner control loop

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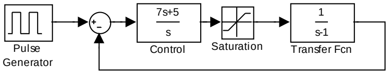

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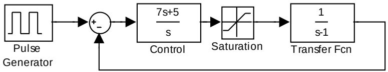

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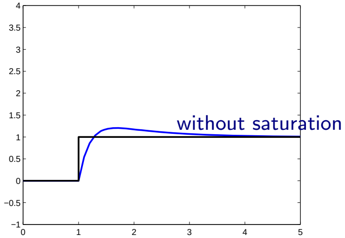

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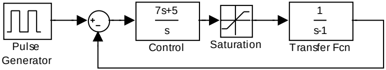

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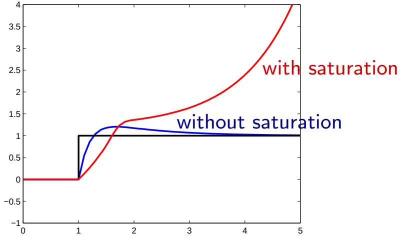

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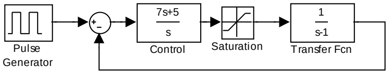

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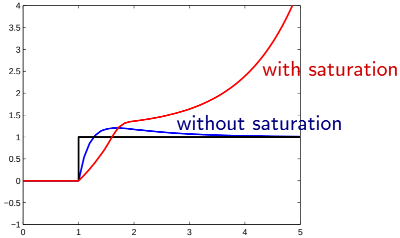

- **Saturations may lead to instability** especially in the presence of integrators in the loop

Inner control loop

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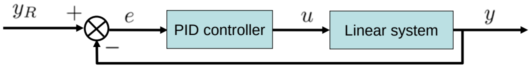
- Consider a **linear system with a PID** controller:



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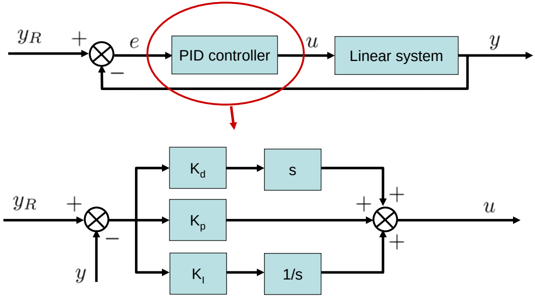

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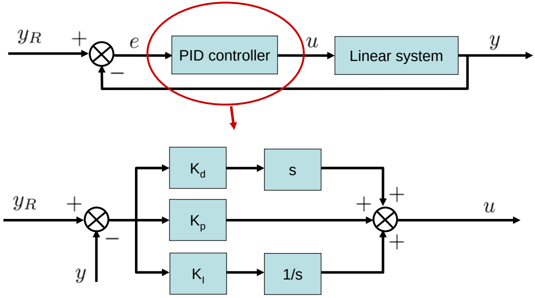

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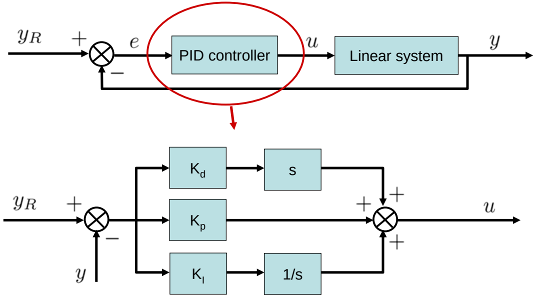
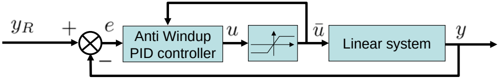


- The instability comes from the **integration** of the error
- **Key idea:** soften the integral effect when the control is saturated

Inner control loop

Anti-windup PID

- Structure of the **PID controller with anti-windup**:



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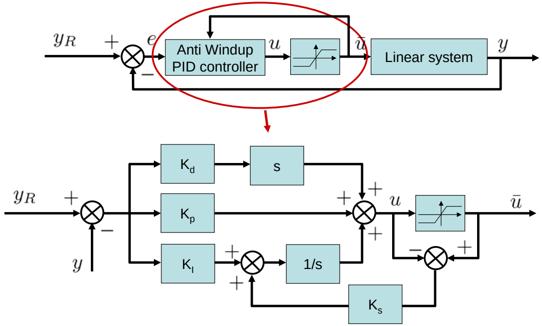

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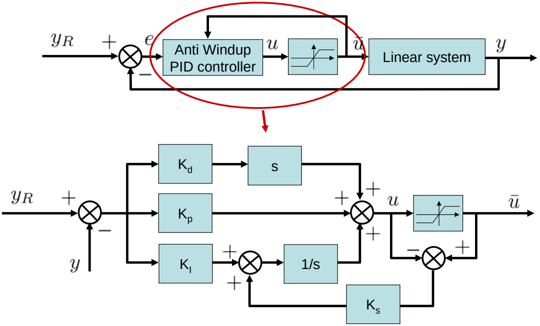

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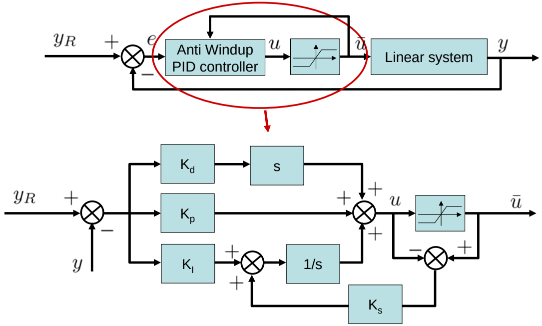

- If $u = \bar{u}$, that is if u is not saturated, **then the PID controller with anti-windup is identical to the classical PID controller**
- **If u is saturated** $(u \neq \bar{u})$, K_s **tunes the reduction of the integral effect** of the PID

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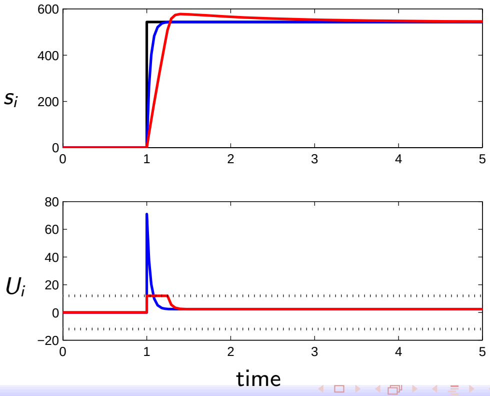

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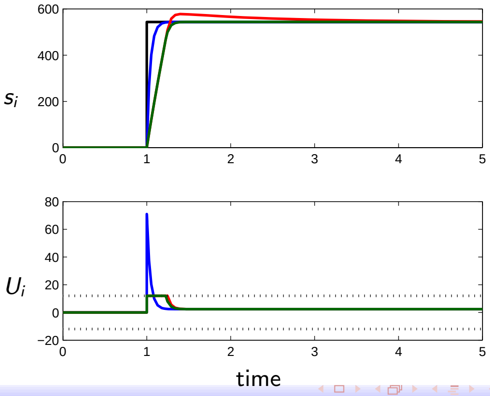

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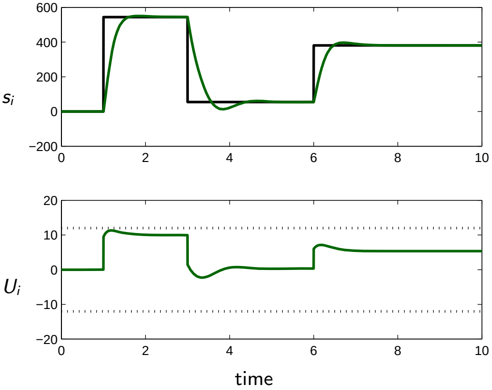

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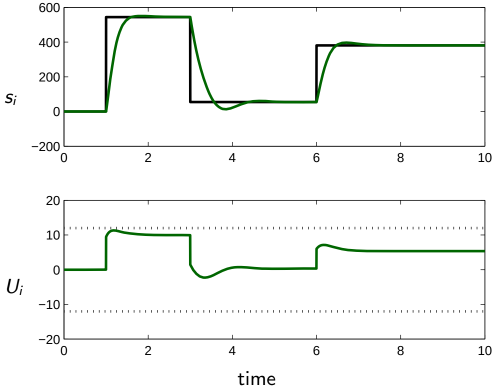

- The controller is well tuned near s_i^d but **not very good a large range of use**

Inner control loop

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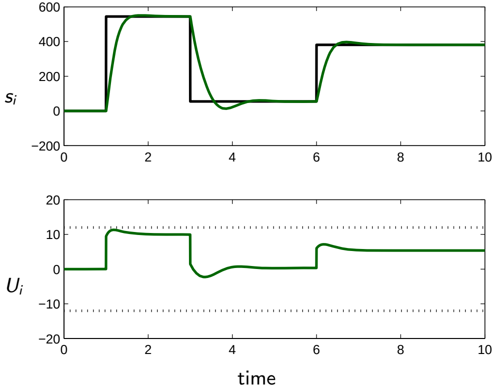

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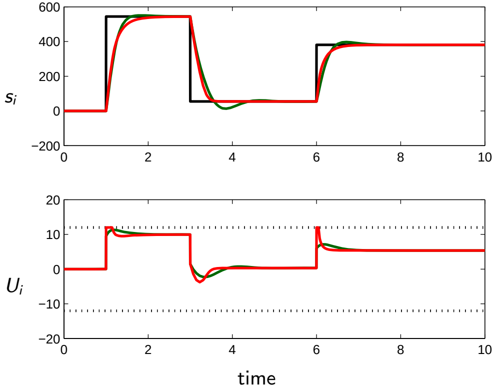

- **The rotors are now well controlled...almost**

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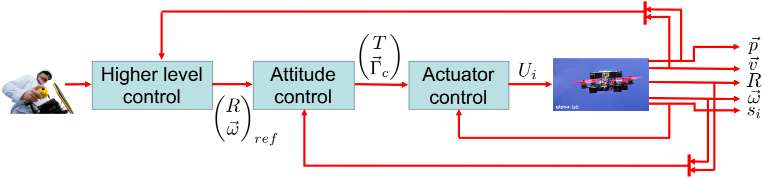

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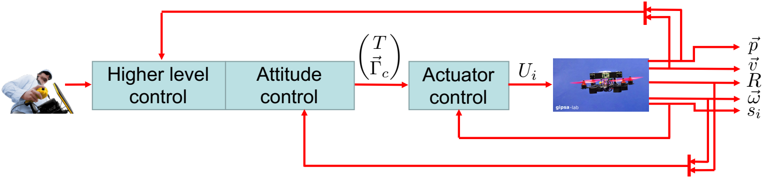

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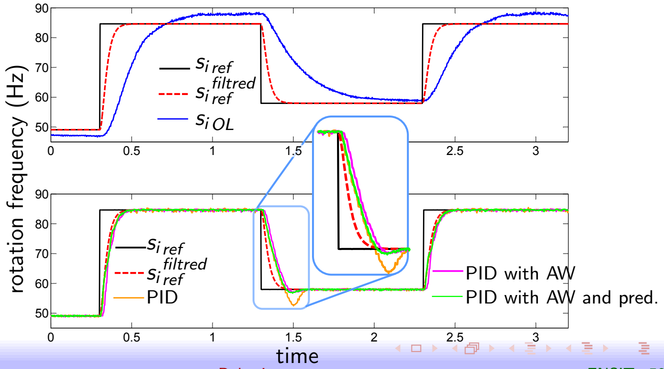

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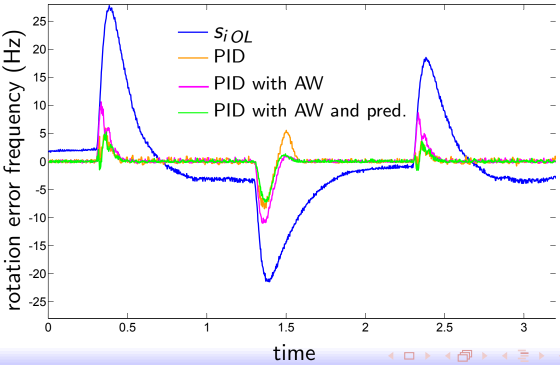

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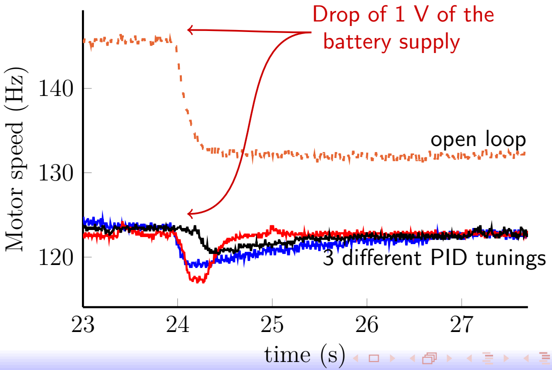

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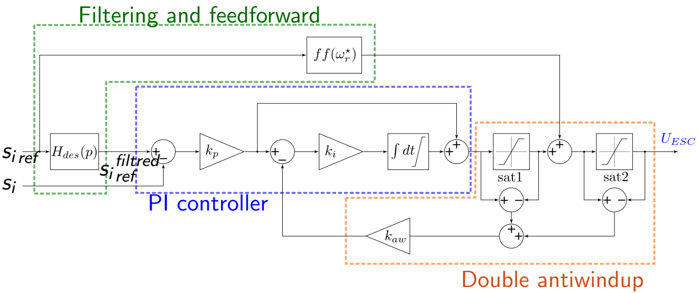

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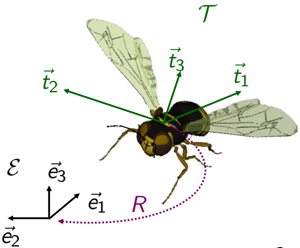
- Valid only *around* zero angles position, but
  - robust
  - easy to tune
  - can handle saturation
  - can be adaptive
  - ...

A saturation based attitude control

Body's dynamics

$$
\begin{aligned}
\dot{\vec{p}} &= \vec{v} \\
\dot{\vec{v}} &= -g\vec{e}_3 + \frac{1}{m}R\vec{f} - c\vec{v} \\
\begin{pmatrix} \dot{q}_0 \\ \dot{q}_v \end{pmatrix} &= \frac{1}{2}\begin{pmatrix} -q_v^T \\ I_3 q_0 - q_v^\times \end{pmatrix}\omega \\
\dot{\omega} &= J^{-1}(\vec{\tau} - \omega \times J\omega)
\end{aligned}
$$

R : Rotation matrix from $\mathcal{T}$ to $\mathcal{E}$



$\mathcal{E}$: Fixed frame

$\mathcal{T}$: Mobile frame

A saturation based attitude control

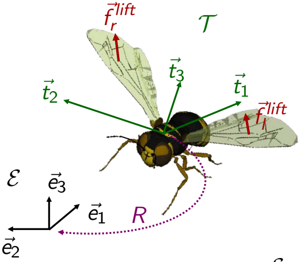
Body's dynamics

$$\dot{\vec{p}} = \vec{v}$$

$$\dot{\vec{v}} = -g\vec{e}_3 + \frac{1}{m}R\vec{f} - c\vec{v}$$

$$\begin{pmatrix}\dot{q}_0 \\ \dot{q}_v\end{pmatrix} = \frac{1}{2}\begin{pmatrix} -q_v^T \\ I_3 q_0 - q_v^{\times}\end{pmatrix}\omega$$

$$\dot{\omega} = J^{-1}(\vec{\tau} - \omega \times J\omega)$$



R : Rotation matrix from $\mathcal{T}$ to $\mathcal{E}$
$\vec{f}, \vec{\tau}$: Aerodynamic force and torque in $\mathcal{T}$
$\vec{p}$, $\vec{v}$: Linear position and velocity in $\mathcal{E}$

$\mathcal{E}$: Fixed frame
$\mathcal{T}$: Mobile frame
m : Body's mass
c : Viscosity coefficient
g : Gravity

A saturation based attitude control

Body's dynamics

$$
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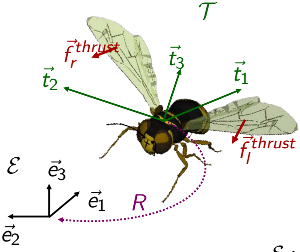

R : Rotation matrix from $\mathcal{T}$ to $\mathcal{E}$

$\vec{f}, \vec{\tau}$: Aerodynamic force and torque in $\mathcal{T}$

$\vec{p}$, $\vec{v}$: Linear position and velocity in $\mathcal{E}$

$q = [q_0 \ q_v]^T$: Quaternion

$q_v^\times$: Skew symmetric matrix associated to q_v

ω : Angular velocity in $\mathcal{T}$

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I_3 : Identity matrix

J : Inertia matrix

A saturation based attitude control

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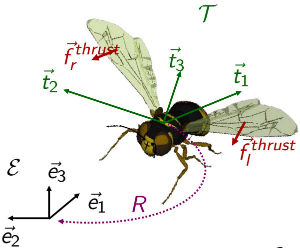

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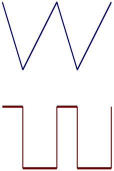
A saturation based attitude control

$\phi^{wing}(\phi_0^{wing}, t)$

$\psi^{wing}(\psi_0^{wing}, t)$

θ^{wing} NUL

A saturation based attitude control



$\phi^{wing}(\phi_0^{wing}, t)$

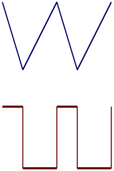
$\psi^{wing}(\psi_0^{wing}, t)$

→ aerodynamic forces and torques

$(\vec{f}, \vec{\tau})$

θ^{wing} NUL

A saturation based attitude control



$\phi^{wing}(\phi_0^{wing}, t)$

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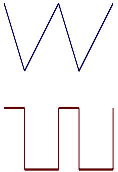
θ^{wing} NUL

A saturation based attitude control



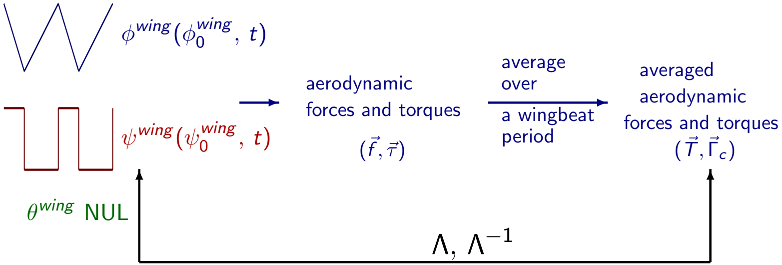
$\phi^{wing}(\phi_0^{wing}, t)$

$\psi^{wing}(\psi_0^{wing}, t)$

θ^{wing} NUL

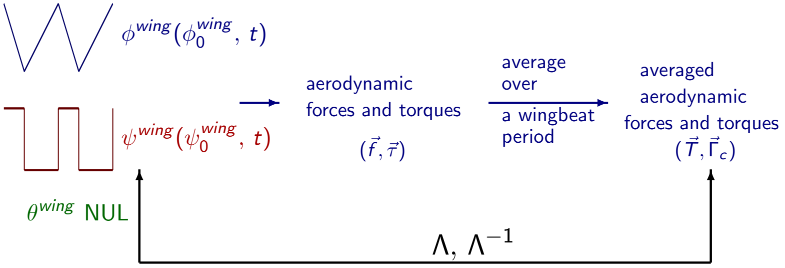
→ aerodynamic forces and torques $(\vec{f}, \vec{\tau})$ —average over a wingbeat period→ averaged aerodynamic forces and torques $(\vec{T}, \vec{\Gamma}_c)$

A saturation based attitude control



$$(\overline{\vec{T}}, \overline{\vec{\Gamma}_c}) = \Lambda(\phi_0^{wing}, \psi_0^{wing})^{d,g}$$

A saturation based attitude control



$$(\overline{\vec{T}}, \overline{\vec{\Gamma}}_c) = \Lambda(\phi_0^{wing}, \psi_0^{wing})^{d,g}$$

Find $\vec{T} = \vec{T}(\overline{p}, \overline{v}, \overline{q}, \overline{\omega})$ and $\vec{\Gamma}_c = \vec{\Gamma}_c(\overline{p}, \overline{v}, \overline{q}, \overline{\omega})$ such that the system has the desired behavior

A saturation based attitude control

Wings angles amplitudes $(\phi_0^{wing}, \psi_0^{wing})^{l,r}$ $\xrightarrow{\Lambda}$ Averaged forces and torques $(\vec{T}, \vec{\Gamma}_c)$

A saturation based attitude control

Wings angles amplitudes $(\phi_0^{wing}, \psi_0^{wing})^{l,r}$ $\xrightarrow{\Lambda}$ Averaged forces and torques $(\vec{T}, \vec{\Gamma}_c)$

↑

Constraints

$$0 \leq \phi_0^{wing} \leq \tilde{\phi}_0^{wing}$$
$$-\tilde{\psi}_0^{wing} \leq \psi_0^{wing} \leq \tilde{\psi}_0^{wing}$$

A saturation based attitude control

Wings angles amplitudes $(\phi_0^{wing}, \psi_0^{wing})^{l,r}$ $\xrightarrow{\Lambda}$ Averaged forces and torques $(\vec{T}, \vec{\Gamma}_c)$

↑ ↕

Constraints $\xrightarrow{?}$ Constraints

$$0 \leq \phi_0^{wing} \leq \tilde{\phi}_0^{wing}$$

$$-\tilde{\psi}_0^{wing} \leq \psi_0^{wing} \leq \tilde{\psi}_0^{wing}$$

Saturations

- Physical limitations: wings angles saturation

$$\begin{array}{rcccl} 0 & \leq & (\phi_0^{wing})^{l,r} & \leq & \tilde{\phi}_0^{wing} \\ -\tilde{\psi}_0^{wing} & \leq & (\psi_0^{wing})^{l,r} & \leq & \tilde{\psi}_0^{wing} \end{array}$$

$$\begin{array}{rcl} \beta x \left[\phi_0^{wing,r^2} \cos \psi_0^d - \phi_0^{wing,l^2} \cos \psi_0^g \right] & = & \Gamma_r^{max} \\ \alpha x \left[\phi_0^{wing,r^2} \sin \psi_0^d - \phi_0^{wing,l^2} \sin \psi_0^g \right] & = & \Gamma_y^{max} \end{array}$$

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- Coupled saturations of the control torques

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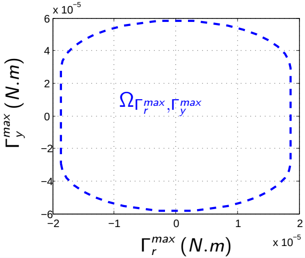

Admissible saturation set

Saturations

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$$\begin{array}{ccccc} 0 & \leq & (\phi_0^{wing})^{l,r} & \leq & \tilde{\phi}_0^{wing} \\ -\tilde{\psi}_0^{wing} & \leq & (\psi_0^{wing})^{l,r} & \leq & \tilde{\psi}_0^{wing} \end{array}$$

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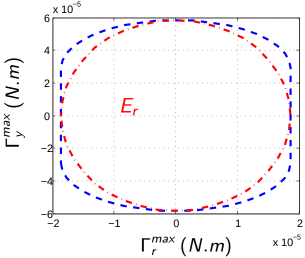

Admissible saturation set

Reduced to an ellipse in order to simplify the computations

Saturations

- Physical limitations: wings angles saturation

$$
\begin{array}{ccccc}
0 & \leq & (\phi_0^{wing})^{l,r} & \leq & \tilde{\phi}_0^{wing} \\
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$$

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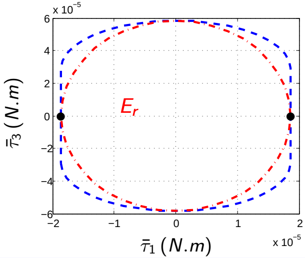

Admissible saturation set

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Nul yaw torque for a maximal roll torque

Saturations

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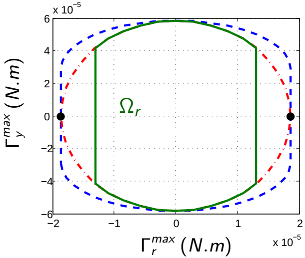


Admissible saturation set

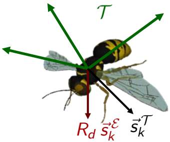
Reduced to an ellipse in order to simplify the computations

Nul yaw torque for a maximal roll torque

Roll stabilization is preferred in order to bring the body to a horizontal position

Attitude stabilization

- Attitude with reference vectors:

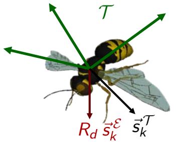


Current vector: $s_k^{\mathcal{T}} = R\, s_k^{\mathcal{E}}$

Desired vector: $s_{k_d}^{\mathcal{T}} = R_d\, s_k^{\mathcal{E}}$

Attitude stabilization

- Attitude with reference vectors:



Current vector: $s_k^{\mathcal{T}} = R\, s_k^{\mathcal{E}}$
Desired vector: $s_{k_d}^{\mathcal{T}} = R_d\, s_k^{\mathcal{E}}$

  - Yaw control is impossible
  - Add another sensor giving a non collinear measurement (magnetometer for example)

Attitude stabilization

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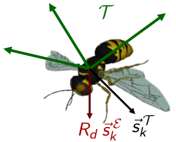

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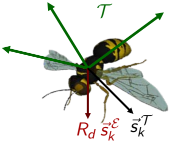
- Yaw control is impossible
- Add another sensor giving a non collinear measurement (magnetometer for example)
- Number of non collinear sensors $n \geq 2$

Attitude stabilization

- Attitude with reference vectors:



Current vector: $s_k^{\mathcal{T}} = R\, s_k^{\mathcal{E}}$

Desired vector: $s_{k_d}^{\mathcal{T}} = R_d\, s_k^{\mathcal{E}}$

- Attitude error:

$$\vec{\zeta} = \frac{\Delta^{-1}}{n} \sum_{k=1}^{n} \vec{s}_k^{\mathcal{T}} \times R_d \vec{s}_k^{\mathcal{E}}$$

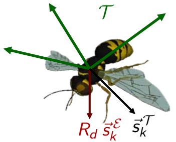
Δ : positive diagonal matrix

n : sensors number

R_d : desired orientation of $\mathcal{T}$ relatively to $\mathcal{E}$)

Attitude stabilization

- Attitude with reference vectors:



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- If $\vec{s}_k^{\mathcal{T}}$ and $R_d\vec{s}_k^{\mathcal{E}}$ are collinear, then $\vec{\zeta} = 0$

A saturation based attitude control

- Sensors used: Rate gyros and reference sensors

A saturation based attitude control

- Sensors used: Rate gyros and reference sensors
- **Control torques**

$$\bar{\tau}_j = -\operatorname{sat}_{\tilde{\tau}_j}(\lambda_j[\bar{\omega}_{G_j} + \rho_j \bar{\zeta}_j)]) \qquad j = \{1,2,3\}$$

λ_j, ρ_j : positive tuning parameters

$\operatorname{sat}_{\tilde{\tau}_j}$: saturation function

$\bar{\omega}_{G_j}$: averaged angular velocity (measured by the rate gyros)

$\bar{\zeta}_j$: averaged attitude error

A saturation based attitude control

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$\operatorname{sat}_{\tilde{\tau}_j}$	:	saturation function
$\bar{\omega}_{G_j}$	:	averaged angular velocity (measured by the rate gyros)
$\bar{\zeta}_j$	:	averaged attitude error

- Stability proved (rigid body) using Lyapunov function:

$$V = \frac{1}{2}\vec{\omega}^T J \vec{\omega} + \frac{1}{n}\sum_{k=1}^{n}(1 - \vec{s}_k^{\,m\,T} R_d\, \vec{s}_k^{\,f})$$

A saturation based attitude control

- Sensors used: Rate gyros and reference sensors
- **Control torques**

$$\bar{\tau}_j = -\operatorname{sat}_{\tilde{\tau}_j}(\lambda_j[\bar{\omega}_{G_j} + \rho_j\bar{\zeta}_j)]) \qquad j = \{1,2,3\}$$

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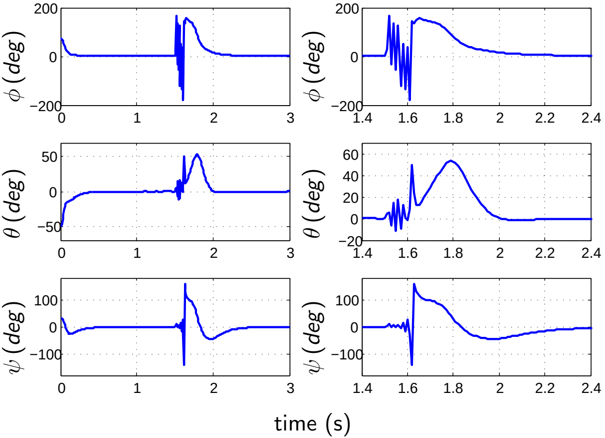
- **Generalized PID controller**, *almost* global stability, simpler version using quaternion exists, stability independent from the knowledge of J, robust to velocity sensor saturation.

A saturation based attitude control

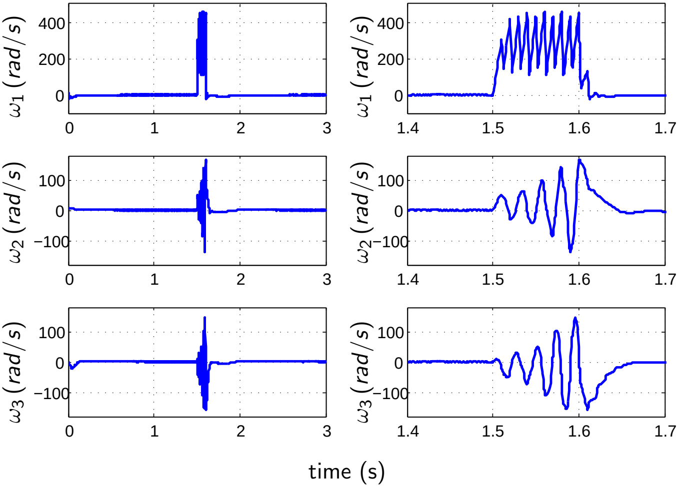
Initial orientation: $(\phi, \theta, \psi) = (70, -50, 30)^\circ$

Angles (Roll, pitch, yaw)



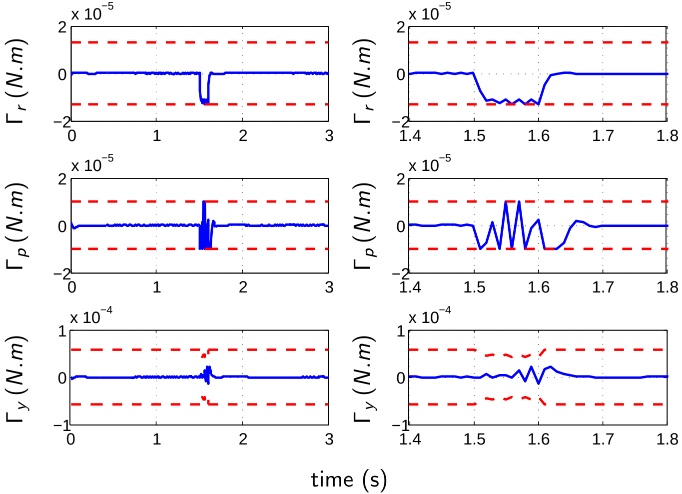
A saturation based attitude control

Angular velocities



A saturation based attitude control

Control torques



Position control

Linearization

- First thing people want to try ?
- Many possible approaches
- Taking

$$x := (\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, p^T, v^T)^T$$

the linearization of the linear and angular dynamics around some reference x^r of the form $(0, 0, \psi^r, 0, 0, 0, {p^r}^T, 0)^T$ is given by :

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

with the following matrices A and B:

$$A := \begin{pmatrix} 0_{3\times 3} & I_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & I_{3\times 3} \\ \begin{matrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \end{pmatrix}, \quad B := \begin{pmatrix} 0_{3\times 3} & 0_{3\times 1} \\ J^{-1} & 0_{3\times 1} \\ 0_{3\times 3} & 0_{3\times 1} \\ 0_{3\times 3} & \begin{matrix} 0 \\ 0 \\ \dfrac{1}{m} \end{matrix} \end{pmatrix}$$

- Linear control is always possible but not very suitable

Position control

- **A robotic oriented nonlinear approach**
  - Based on fast attitude and actuator loops
  - Angle tracking is assumed to be perfect
  - The aim is to bring the UAV to $\vec{p}^\star = \begin{pmatrix} p_1^\star & p_2^\star & p_3^\star \end{pmatrix}^T$
  - Filter the position target $\vec{p}_f^\star = \dfrac{\vec{p}^\star}{(\tau_f s + 1)^3}$, $\vec{p}_f^\star$ must be C^3
  - Let the tilde denote the error, for instance $\tilde{x} = x_f^\star - x$
  - With PIDs controllers, define an acceleration target on the two first direction ($i = 1, 2$):

$$[\ddot{p}_{i_f}]^\star = k_P \tilde{p}_i + k_I \int_0^t \tilde{p}_i dt + k_D \left(\dot{p}_{i_f}^\star - v_i\right) + \ddot{p}_{i_f}^\star$$

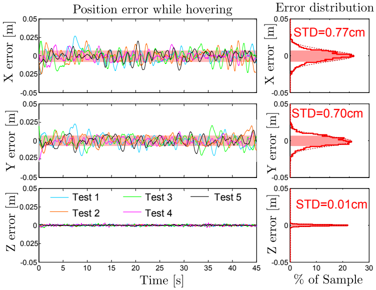
  - With a PID controller, compute the thrust control:

$$T^\star = \frac{k_P \tilde{p}_3 + k_I \int_0^t \tilde{p}_3 dt + k_D \left(\dot{p}_{3_f}^\star - v_3\right)}{c_\phi c_\theta} + \frac{m}{c_\phi c_\theta}\left(g + \ddot{p}_{3_f}^\star\right)$$

  - Yaw angle ψ can be stabilized to any direction independently
  - Compute the roll and pitch control:

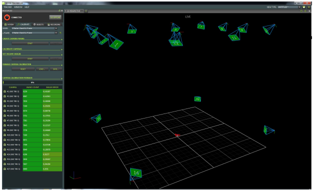
$$\begin{aligned} \phi^\star &= \sin^{-1}\left(\frac{m}{T^\star}([\ddot{p}_{1_f}]^\star s_\psi - [\ddot{p}_{2_f}]^\star c_\psi)\right) \\ \theta^\star &= \sin^{-1}\left(\frac{m}{T^\star}([\ddot{p}_{1_f}]^\star c_\psi + [\ddot{p}_{2_f}]^\star s_\psi)\right) \end{aligned}$$

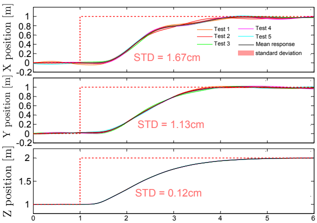
Position control

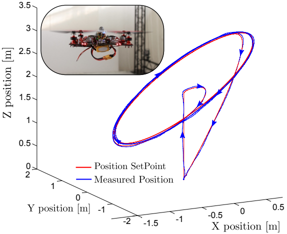


x, y error: $\pm 1.5cm$

z error: milimetric







Outline

Jointed-arm robots

- Characteristic variables:

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- Actuator dynamics
- Robot dynamics

Geometrical model of robots

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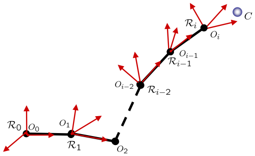
- Despite the limitations, this approach is widely used (oversized robots)

Computation of the geometrical model

Combination of rotations and translations



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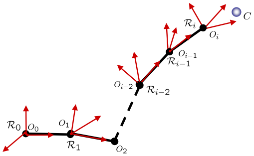

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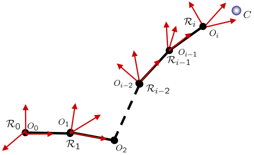

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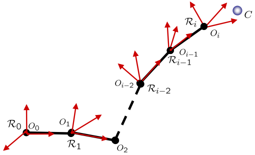

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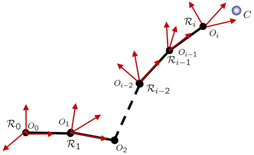

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Combination of rotations and translations

Robotics

N. Marchand

Introduction

Modeling
Cartesian coordinates
Orientation
Frames
Newton

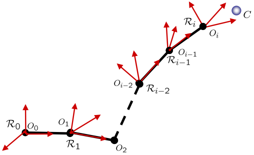
X4

Kinematics
Arm robots
Inner-loop
Geometrical model
Kinematic model
Dynamical model
Conclusion

Path planning
Workspace and obstacles
path planning

Mobile robotics

Visual servoing



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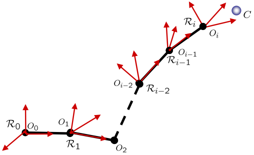

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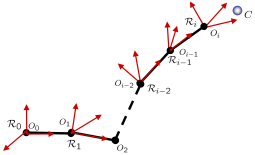

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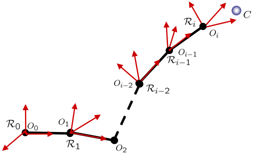

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- where R_i^{i+1} is the rotation matrix from $\mathcal{R}_i$ to $\mathcal{R}_{i+1}$:
  - $R_i^{i+1} = {R_{i+1}^i}^T$, $\det R_i^{i+1} = 1$

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Combination of rotations and translations

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- Let $\vec{v} := \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$, then it is equivalent to the 4-dimension vector $\vec{V}$ with $\omega = 1$:

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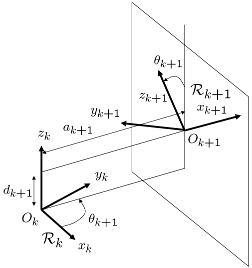
- **Rotation**: a rotation of matrix R is given by:

$$\text{Rot} = \begin{pmatrix} R & 0_{3\times 1} \\ 0_{1\times 3} & 1 \end{pmatrix}$$

Note that still $R^{-1} = R^T$ and $det(R) = 1$

Computation of the geometrical model

Denavit-Hartenberg's convention



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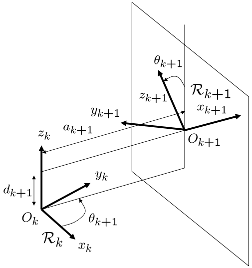

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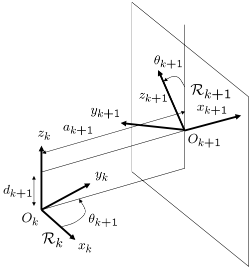

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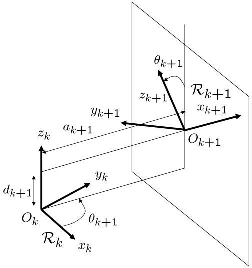

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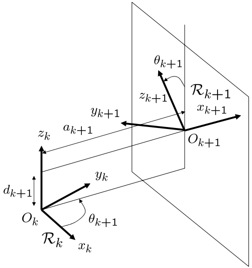

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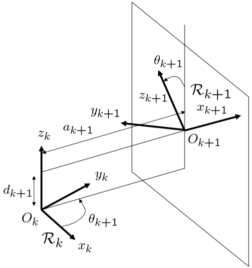

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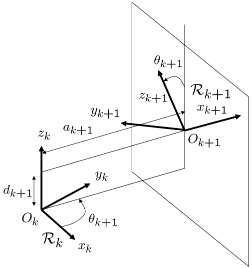

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- The DH parametrization **always exists and is unique**

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  - The matrix transformation of the Denavit-Hartenberg's convention is: $R_2 \cdot T_2 \cdot T_1 \cdot R_1$

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- Fastidious in many cases but systematic ! See books for that

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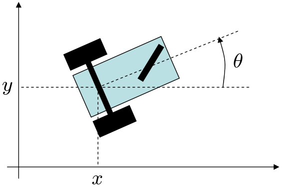
- **The kinematic model is a state space representation of a controlled system**

Example of kinematic model

- Example: the car in the plane



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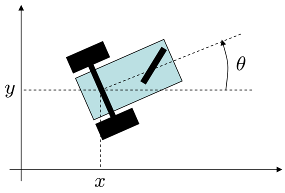

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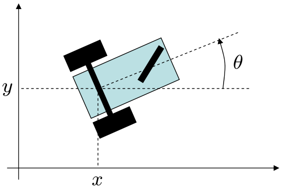

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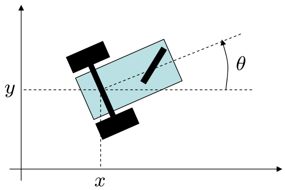

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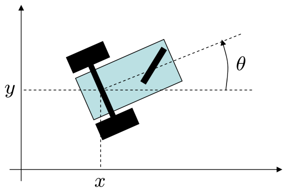

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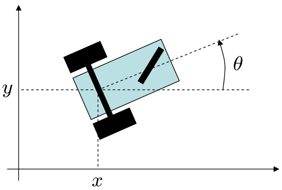

$$\dot{x} = \frac{V_l + V_r}{2} \cos\theta$$

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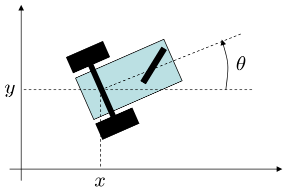
  - Is this system underactuated or overactuated ? Explain why



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$$J = \frac{1}{2}\begin{pmatrix}\cos\theta & \cos\theta\\ \sin\theta & \sin\theta\\ -\frac{2}{d} & \frac{2}{d}\end{pmatrix}$$

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- the Jacobian must be derived at each origin O_i of each link frame

Kinematic redundancy

When a robot is given by its kinematic model $\dot{X} = J\dot{\theta}$

- J is usually $n \times p$ with $X \in \mathbb{R}^n$ and $\theta \in \mathbb{R}^p$
- $r = p - n$ is called the **kinematic redundancy number**
- When $r < 0$, the robot is underactuated, usually the case with mobile robots $\Rightarrow$ advanced control
- When $r > 0$, the robot is overactuated. It has redundancy.

For a robot with redundancy, one can write:

- $J = \begin{pmatrix} J_n & J_{p-n} \end{pmatrix}$ with J_n invertible

Control through the kinematic equation

Control with J^t Take a robot given by its kinematic model $\dot{X} = J\dot{\theta}$

- Control with J^t
  - Apply a fictive force $F = K(X - X_d)$ with K positive and symmetric
  - Take $\dot{\theta} = J^t F = J^t K(X - X_d) = J^t K e$
  - Then the elastic potential $\Phi(e) = \frac{1}{2} e^t K e$ is such that

$$\dot{\Phi}(e) = -e^t K J J^t K e < 0$$

  - $e \rightharpoonup 0$, $X \rightharpoonup X_d$
  - Automatically handles redundancy

Control through the kinematic equation

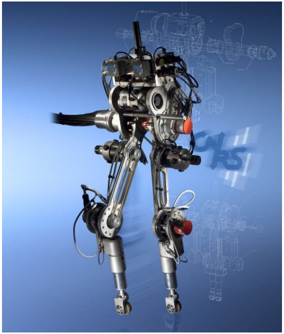
Control with J^+ Take a robot given by its kinematic model $\dot{X} = J\dot{\theta}$

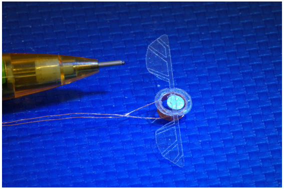
- Control with $J^+ := J^t(JJ^t)^{-1}$
  - J^+ is the Moore-Penrose pseudo-inverse (`pinv` in Matlab)
  - Can be obtained through SVD decomposition. $J = U\Delta V^t$, Δ diagonal $\Longrightarrow J^+ = V\Delta^+ U^t$, Δ^+ is the inverse of the nonzero coefficient of Δ
  - Taking $\dot{\theta} = J^+\dot{X}$ minimizes the energy $\dot{\theta}^t\dot{\theta}$
  - Taking $\dot{\theta} = J_M^+\dot{X}$ with $J_M^+ := M^{-1}J^t(JM^{-1}J^t)^{-1}$ minimizes the kinetic energy $T = \frac{1}{2}\dot{\theta}^t M(\theta)\dot{\theta}$

Outline

Dynamical model of robots

- Express the accelerations of movement as a function of the actuation variables





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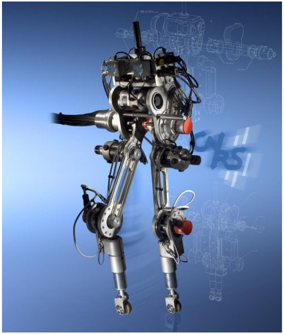

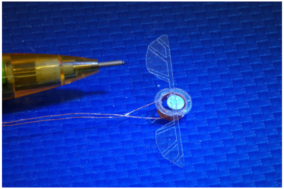

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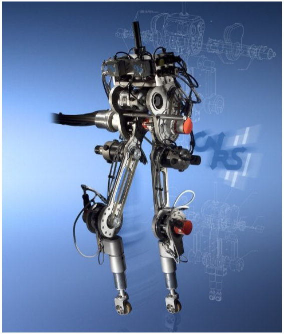

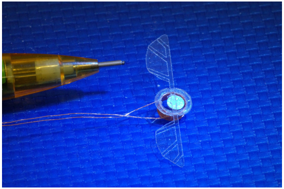

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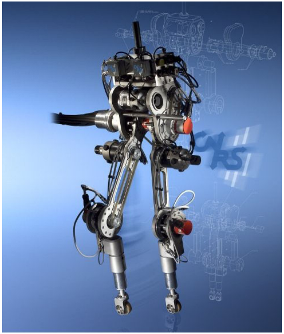

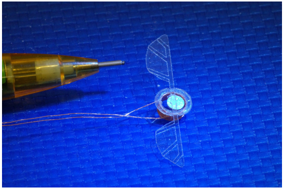

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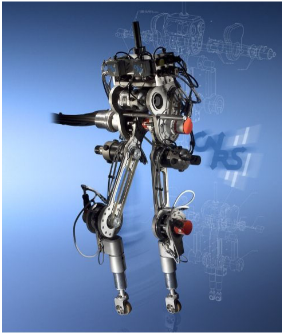

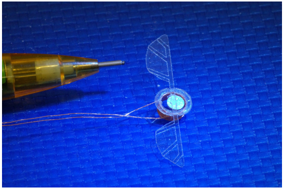

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- Very complex and most of the time impossible to control (too complex to design a control)
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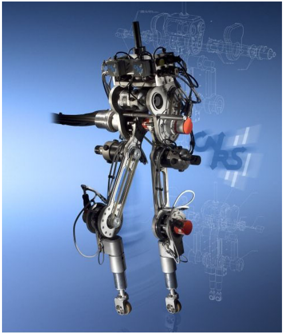

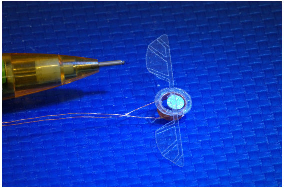

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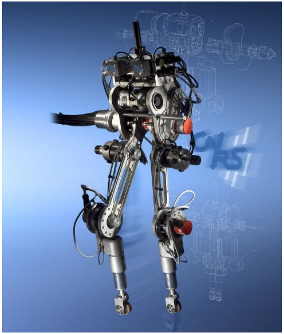

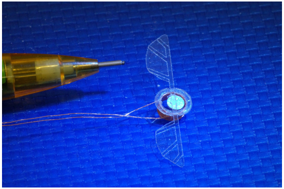

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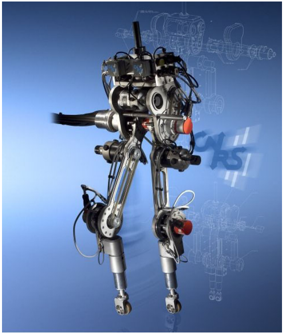

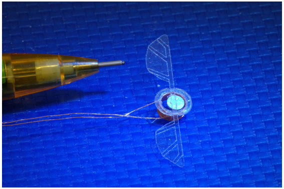

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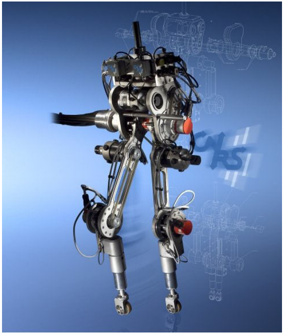
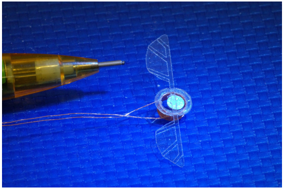
- Widely used for flying or diving robots (UAVs, AUVs, etc.) or walking robots





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n-link manipulator

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  - An important literature on the control of this type of systems can be found

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- The number of available controls depends upon the system

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- The set of θ_i's corresponding to a possible configuration is noted Q

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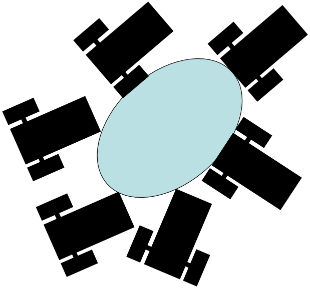
Example: the car



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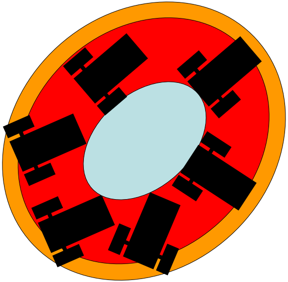

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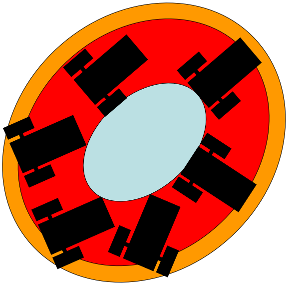

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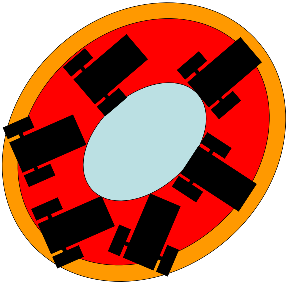

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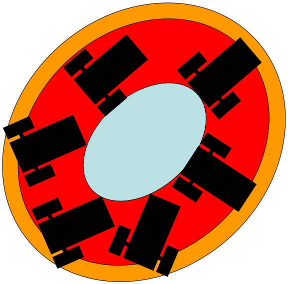

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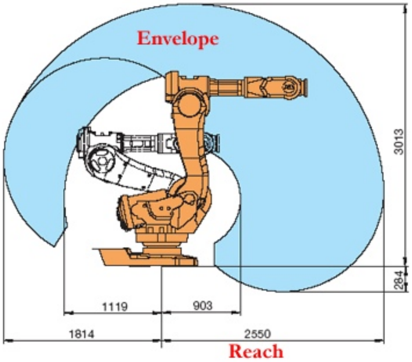
- Numerical simulation can easily solve this problem (systematic simulation)

Example: arm robot



Outline

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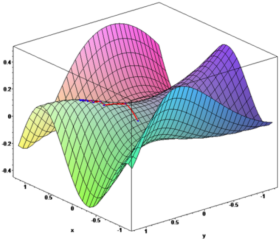

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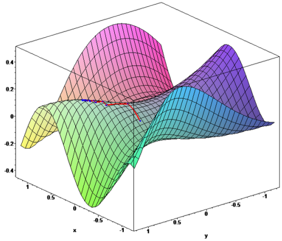

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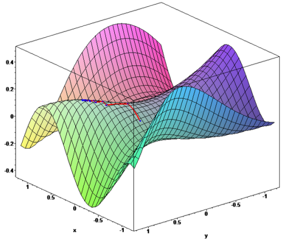

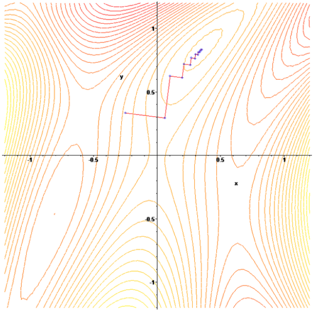

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- No more slope (almost the same as previous condition)
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- Quasi-Newton method:

$$z_{k+1} = z_k - \rho_k B_k \nabla F(z_k)$$

B_k: approximation of the Hessian

http://en.wikipedia.org/wiki/Quasi-Newton_method)

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Simple exemple of objective function

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- **The main problem consist in finding a criterium that will be convex** (or close to)

Manipulator robots

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- We define a potential field for each articulation
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Manipulator robots

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- The repulsive field will create a fictitious force that will prevent collisions by repelling the robot from the obstacles

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From attractive/repulsive forces to actuator torques

- The total joint torques acting on a robot is the sum of the torques from all attractive and repulsive potentials:

$$\tau(\theta) = \sum_i J_{O_i}^T(\theta)\left(F_{att_i}(\theta) + F_{rep_i}(\theta)\right)$$

Gradient descent

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  4. Sum the joint torques in the configuration space
  5. Use gradient descent to reach your target configuration

Gradient descent

1. $i = 0,\ \theta[0] = \theta_0$
2. if $||\theta[i] - \theta_f|| > \varepsilon$, then:
   - $\theta[i+1] = \theta[i] + \alpha[i] \dfrac{\tau(\theta[i])}{||\tau(\theta[i])||}$
   - $i = i + 1$
   - goto 2

   else:
   - return $\theta[0], \dots, \theta[i]$

- Many other algorithm are possible
  - steepest descent (gradient) (Euler)
  - Newton
  - ... see optimization books
- the $\theta[0], \dots, \theta[i]$ are the successive configuration to track = path
- It is possible to add random to escape local minima

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- Chose the path in the connected space

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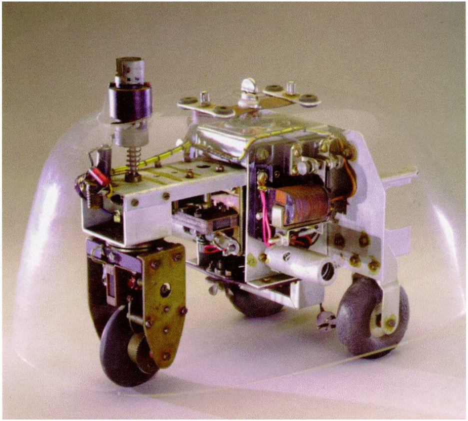
- Predictive control can also be used to handle constraints "on line"
- Adding fictive force is a very power tool also widely used in formation control or robotics with communication constraints (mainly range)

Mobile robotics

- Born in the 50s, aiming to *autonomously moving* robots



Grey Walter's "Turtle" (machina speculatrix): attracted by light

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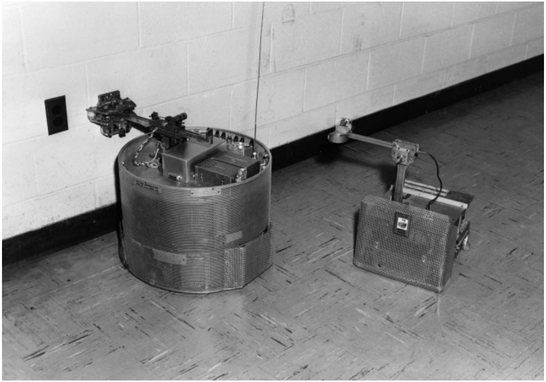

John Hopkins Univ. "Beast" robot: first use of transistor based sensing (ultrasound and photodiodes)

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Shakey robot from Stanford Univ.
Platform used to show first results on AI (1969)

Mobile robotics

- Bio inspired locomotion: first biped robot

Honda E0 first biped robot (1986)

Mobile robotics

- Bio inspired locomotion: first biped walk

Rabbit robot CNRS-Grenoble (2004)

Mobile robotics

- Bio inspired locomotion: more about mobility

Boston Dynamics (SoftBank)

Mobile robotics

- SLAM: Simultaneous localization and mapping

https://github.com/erik-nelson/blam

Mobile robotics

- SLAM: Simultaneous localization and mapping

Mobile robotics

- Aerial robotics

Mobile robotics

- Bionics

Basis of navigation

Navigation gathers different problems

Approach of a given visible target, going to the target. Each new sensing produces an action. Typically what some insects do.
Usually based on a gradient approach

Guidance Ability to go to some position characterized by a visible environnement.
Usually based on a gradient approach

To goal navigation In that case, the target don't need to be visible but the robot has a representation of the world.
Graph or gradient approach

Topological navigation Same as previous one with a memory of the possible the spatial relationship between positions: the robot can go back)
Graph or gradient approach

Metric navigation Same as above but the robot is capable to memorize the metric positions: the robot can go back to a point without taking the same path.

- The 3 first strategies: reactiv navigation
- The 2 last enable trajectory planification also called path planning

Basis of navigation

Three key words of navigation

- Navigation relies on

Perception	where am i ?
Planification	where should i go ?
Action	how can i move ?

- The order of Perception/Planification/Action is not trivial
- Sometimes it may be necessary to move to see where to go: perception depends upon control
- Sometimes it may be necessary see to know where to go: navigation depends upon perception

Basis of navigation

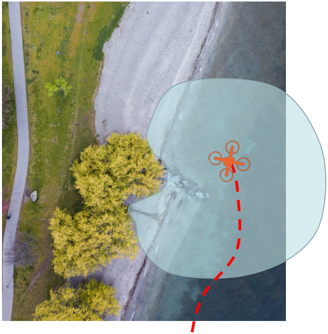
the perception

- Two kind of perceptions:
  - **Proprioceptive information** Everything that the robot can measure independently from the environnement, typically the rotation of its wheels, accelerometers, gyrometers, etc.
  - **Exterioceptive information** Everything that the robot sense in the outside world, typically distance to obstacles. Sensors are cameras, infrared/laser/ultra sound sensors, etc.
- Two type of problems
  - **Perception variability** The perception of the same place can vary (e.g. because of the sun)
  - **Perceptual aliasing** The same perception signals can correspond to 2 different places
- Perception is merged via a fusion step

Basis of navigation

the perception

- Different kind of usage of perception information
  - Direct



Basis of navigation

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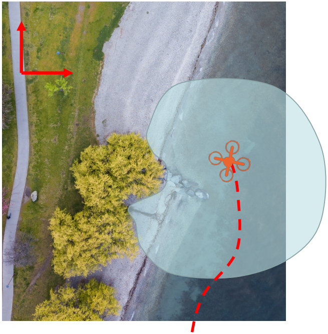

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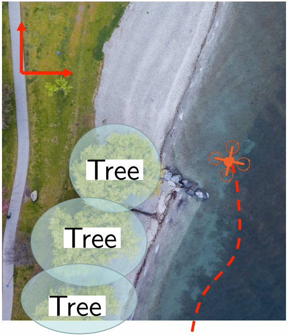
  - To built a metric map with objects
  - To built a metric map with objects of known typology



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- An arm robot equipped with a camera

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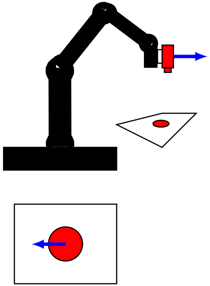

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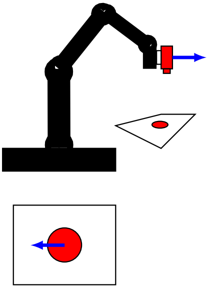

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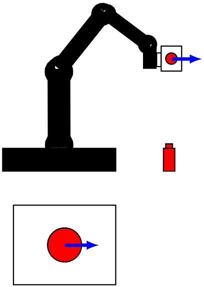


Introduction to visual servoing

The key points

- Being able to extract feature from the image: "recognize" points of the object
- Being able to characterize the relation between the robot movement and the image changes

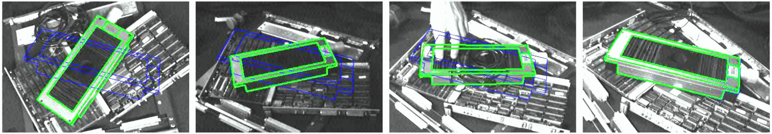
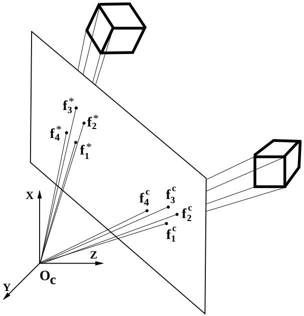


Image based visual servoing

The interaction matrix



The interaction matrix links the mouvement of O_c (lateral and rotational) to the movement of the feature points (f_i^c)

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A short mathematical background

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Introduction to visual servoing

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  - $L_s \in \mathbb{R}^{k\times 6}$: interaction matrix (Jacobian)

Control in visual servoing

A simple control approach

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- Practically, L_s is never known perfectly and we use an approximation

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- Camera parameters can be obtained by calibration

Image-based visual stereo servoing

- We assume now that we have two cameras