Quantitative analysis of noninvasive diagnostic procedures for induction motor drives

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Abstract

This paper reports quantitative analyses of spectral fault components in five noninvasive diagnostic procedures that use input electric signals to detect different types of abnormalities in induction motors. Besides the traditional one phase current spectrum analysis “SC”, the diagnostic procedures based on spectrum analysis of the instantaneous partial powers “$P_{ab}$”, “$P_{cb}$”, total power “$P_{abc}$”, and the current space vector modulus “$csvm$” are considered. The aim of this comparison study is to improve the diagnosis tools for detection of electromechanical faults in electrical machines by using the best suitable diagnostic procedure knowing some motor and fault characteristics. Defining a severity factor as the increase in amplitude of the fault characteristic frequency, with respect to the healthy condition, enables us to study the sensitivity of the electrical diagnostic tools. As a result, it is shown that the relationship between the angular displacement of the current side-bands components at frequencies $(f \pm f_{osc})$ is directly related to the type of induction motor faults. It is also proved that the total instantaneous power diagnostic procedure was observed to exhibit the highest values of the detection criterion in case of mechanical faults while in case of electrical ones the most reliable diagnostic procedure is tightly related to the value of the motor power factor angle and the group motor-load inertia. Finally, simulation and experimental results show good agreement with the fault modeling theoretical results.

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1. Introduction

During the past 15 years, a substantial amount of research has been conducted for a creation of new condition monitoring techniques for induction motor drives. Vibration monitoring techniques are usually installed on expensive and sensitive machines, where the cost of such systems can be justified. Moreover, the environmental sensitivity of the sensors can provide unreliable indications. In some cases, such as submersible electric pumps, sensor installation is not practical or else prohibitively expensive. Motor current signal analysis methods (“MCSA”) are obtained using only non-invasive sensors, such as current and voltage sensors. This
type of measurement (currents and/or voltages) constitutes an ideally non-invasive method to provide data for the diagnosis system and thus to ensure effective monitoring. A variety of faults can occur in induction machines; these faults can be divided into electrical faults: broken rotor bars, shorted stator turns; and mechanical faults: shaft misalignments, load vibrations, and torque oscillations. Indeed, several electrical signals may contain information about the failure. The spectrum lines at frequencies \( f \pm f_{\text{osc}} \) of the current are commonly used for diagnostic purposes, where \( f_{\text{osc}} \) is the fault characteristic frequency [1–6]. Other research has investigated methods using multiple input voltage signals, as well as, currents. This allows larger potential capabilities, since other quantities beyond one phase line current are accounted. The current signature analysis is replaced by a diagnostic procedure based on analysis of the amplitude at the frequency \( f_{\text{osc}} \) in the spectrum of the space vector current modulus [7,8], the instantaneous powers [9–11], and the electromagnetic torque [12–16]. In order to classify the electrical diagnostic methods, the study in [12] treated a comparison between the three instantaneous powers using a restrictive model. The study in [18] also treated noise immunity of the electrical diagnostic procedures using the same restrictive model. In fact, they consider that the angular displacements of the current side-bands are null which is not accurate because as it will be shown, these angles are directly linked to the fault type. The study in [17] has used a more accurate electrical signal model but only to evaluate electrical rotor failure such as broken rotor bars. This last study shows the superiority of the simple current diagnostic procedure over the other proposed methods to detect this type of fault.

This paper uses the electrical signal model proposed by the authors of the paper [17] by treating different mechanical and electrical types of machine abnormalities, which induce not only stator fundamental current side-bands but also load vibration or torque fluctuation. Section 2 reviews the electrical signal models for both a fault free and a defected motor. Section 3 presents analysis of the signal model and shows that the sum of the angular displacement of the current side-bands components is directly related to the machinery fault type. Section 4 treats quantitative analyses of the spectral amplitude of the fault components by defining a severity factor as the relative increase of the amplitude of the fault characteristic frequency with respect to the healthy condition. Finally, simulation results are presented in Section 5 and experimental results are presented in Section 6.

2. Theoretical background

2.1. Fault free current and voltage modeling

Given a healthy induction motor, powered by a balanced three-phase source of sinusoidal voltages and spinning a constant load, the following waveforms of selected stator voltages and currents are modeled by Eqs. (1)–(8) and presented in Fig. 1:

\[
V_a(t) = \sqrt{2} V_m \cos(wt + \alpha),
\]

\[
V_b(t) = \sqrt{2} V_m \cos\left(wt + \alpha - \frac{2\pi}{3}\right),
\]

\[
V_c(t) = \sqrt{2} V_m \cos\left(wt + \alpha + \frac{2\pi}{3}\right),
\]

\[
V_{ab}(t) = \sqrt{2}\sqrt{3} V_m \cos\left(wt + \alpha + \frac{\pi}{6}\right),
\]

\[
V_{cb}(t) = \sqrt{2}\sqrt{3} V_m \cos\left(wt + \alpha + \frac{\pi}{6} + \frac{\pi}{3}\right),
\]

\[
i_a(t) = \sqrt{2} I \cos(wt),
\]
Where \( V_m \) is the “rms” value of the simple voltage, \( I \) the “rms” value of the simple line currents, \( w \) is the supply frequency in radians per second, and \( \alpha \) the power factor angle. The simple current “\( i_a \)” is taken as the reference of our electrical frame. Since in induction motors the simple voltage “\( v_a \)” has a lead of an angle “\( \alpha \)” over the current, then the power factor angle is seen as the angle between “\( i_a \)” and “\( v_a \)”.

The motor space current vector (Current Park Vector) and the space voltage vector are computed by applying the well-known instantaneous symmetrical components transformation:

\[
\tilde{i}_{a0}(t) = \sqrt{2} I \cos \left( wt - \frac{2\pi}{3} \right),
\]

\[
\tilde{i}_{c0}(t) = \sqrt{2} I \cos \left( wt + \frac{2\pi}{3} \right),
\]

where \( V_m \) is the “rms” value of the simple voltage, \( I \) the “rms” value of the simple line currents, \( w \) is the supply frequency in radians per second, and \( \alpha \) the power factor angle. The simple current “\( i_a \)” is taken as the reference of our electrical frame. Since in induction motors the simple voltage “\( v_a \)” has a lead of an angle “\( \alpha \)” over the current, then the power factor angle is seen as the angle between “\( i_a \)” and “\( v_a \)”.

The motor space current vector (Current Park Vector) and the space voltage vector are computed by applying the well-known instantaneous symmetrical components transformation:

\[
\tilde{i}_{a0} = \frac{\sqrt{2}}{\sqrt{3}} \left( i_{a0} + i_{b0} e^{i(2\pi/3)} + i_{c0} e^{-i(2\pi/3)} \right) = \sqrt{3} I e^{iw} t,
\]

\[
\tilde{v}_a = \frac{\sqrt{2}}{\sqrt{3}} \left( v_a + v_b e^{i(2\pi/3)} + v_c e^{-i(2\pi/3)} \right) = \sqrt{3} V_m e^{i(wt+\alpha)}.
\]

Given a balanced three-phase source, the current space vector modulus can be computed as follows:

\[
|\tilde{i}_{a0}| = \sqrt{3} I.
\]

Partial instantaneous input powers “\( P_{ab0} \)” and “\( P_{cb0} \)” are computed by multiplying “\( V_{ab} \)” by “\( i_{a0} \)” and “\( V_{cb} \)” by “\( i_{c0} \)”:

\[
P_{ab0}(t) = \sqrt{3} V_m I \left( \cos \left( \alpha + \frac{\pi}{6} \right) + \cos \left( 2wt + \alpha + \frac{\pi}{6} \right) \right),
\]

\[
P_{cb0}(t) = \sqrt{3} V_m I \left( \cos \left( \alpha - \frac{\pi}{6} \right) - \cos \left( 2wt + \alpha + \frac{\pi}{6} \right) \right).
\]
The total instantaneous input power \( P_{ab0} \) is then computed by adding the two partial powers:

\[
P_{ab0} = P_{ab0}(t) + P_{c0}(t) = 3V_mI \cos(\omega t).
\]

(14)

Note that all the instantaneous powers contain a DC component function of the power factor angle. On the other hand, the two partial powers enclose a component at twice the fundamental frequency with constant amplitude of \( \sqrt{3}V_mI \).

As a final point, the instantaneous electromagnetic torque, \( T_{emb} \), is computed from the stator flux \( \Psi_s \) and the stator space current vector \( i_{l0} \) as presented by Eqs. (15) and (16):

\[
T_{emb} = P \text{Im}[\bar{\Psi}_s^*i_{l0}] = 3PP' \sin(-\varphi_P),
\]

(15)

\[
\bar{\Psi}_s = \frac{\sqrt{2}}{\sqrt{3}}(\Psi_a + \Psi_be^{i(2\pi/3)} + \Psi_ce^{-i(2\pi/3)}) = \sqrt{3}\Psi e^{i(wt-\varphi_P)},
\]

(16)

where \( \Psi \) is the “rms” value of the stator flux, \( P \) the number of pair poles of the induction motor and \( \varphi_P \) the displacement angle between the stator flux space vector and the simple current \( i_s \) taken as the reference. \( \Psi_a, \Psi_b, \Psi_c \) are the three stator flux components computed by the equation below, where \( R_s \) is the stator resistor:

\[
\Psi_x = \int (v_x - R_s i_x) dt, \quad x = a \text{ or } b \text{ or } c.
\]

(17)

In this paper, we will not treat diagnostic methods using the electromagnetic torque Since the torque estimation requires the knowledge of the stator resistor and the use of complex estimators for the motor flux components and finally because it is roughly proportional to the total instantaneous power \( P_{abc} \).

2.2. Fault detection procedures using current signature

If within a drive system an electrical or mechanical abnormality develops (such as a rotor cage fault, motor-load shaft misalignment, broken teeth in the load gearbox, or vibration), harmonic torques are generated in the motor, accompanied by oscillations in speed and modulations in the stator currents, which are typically periodic \([4,17]\). In the case of periodic disturbances, all three line currents “\( i_a \)”, “\( i_b \)”, and “\( i_c \)” are simultaneously affected by the fundamental frequency “\( f_{osc} \)” of the fault-induced oscillation of motor variables. This effect will cause a chain of spectral components in the input currents that still represent a multi-frequency three phase symmetrical system. If only the first side-band components around the supply component “\( w \)” were considered, the three input currents could then be expressed as

\[
i_A(t) = \sqrt{2}I \cos(\omega t) + \sqrt{2}I_I \cos[(\omega - \omega_{osc})t - \varphi_I] + \sqrt{2}I_r \cos[(\omega + \omega_{osc})t - \varphi_r],
\]

(18)

\[
i_B(t) = \sqrt{2}I \cos\left(\omega t - \frac{2\pi}{3}\right) + \sqrt{2}I_I \cos\left[(\omega - \omega_{osc})t - \varphi_I - \frac{2\pi}{3}\right] + \sqrt{2}I_r \cos\left[(\omega + \omega_{osc})t - \varphi_r - \frac{2\pi}{3}\right],
\]

(19)

\[
i_C(t) = \sqrt{2}I \cos\left(\omega t + \frac{2\pi}{3}\right) + \sqrt{2}I_I \cos\left[(\omega - \omega_{osc})t - \varphi_I + \frac{2\pi}{3}\right] + \sqrt{2}I_r \cos\left[(\omega + \omega_{osc})t - \varphi_r + \frac{2\pi}{3}\right],
\]

(20)

where \( I \) is the “rms” value of the fundamental current component, \( I_I \) the “rms” value of the lower side-band current component, \( I_r \) is the “rms” value of the upper side-band current component, \( \omega_{osc} \) is the modulating radian frequency (“\( \omega_{osc} = 2\pi f_{osc} \)”), \( \varphi_I \) is the current left component displacement angle and \( \varphi_r \) the right component displacement angle.

Obviously, these expressions must refer to a quasi-stationary state condition which means the mechanical speed has a sinusoidal variation with a radial frequency much lower than that of current components. The angles “\( \varphi_I \)” and “\( \varphi_r \)” depend on the time range considered, on the stationary rotor position, and on the rotor position evolution during the transient state to reach the mean speed value. Thus, it is not possible to assign a physical meaning to “\( \varphi_I \)” and “\( \varphi_r \)”, but, as demonstrated by the authors of [17], when the upper side-band
component results from the lower side-band component, the angular displacements “φ₁” and “φᵣ” are linked. Specifically, in the case of broken rotor bars, the two components are shifted by approximately “π”:

φᵣ = π − φ₁.

The expressions for the modulated instantaneous powers, obtained by multiplying the voltages by their corresponding currents, can be expressed as follows:

\[ P_{ab}(t) = V_{ab}(t)i_a(t) = \sqrt{3}V_m \left[ I \cos \left( \frac{\alpha + \pi}{6} \right) + I \cos \left( 2\omega t + \frac{\pi}{6} \right) \right. \]

\[ + I_I \cos \left( \omega_{osc}t + \alpha + \frac{\pi}{3} \right) + I_I \cos \left( (2w - w_{osc})t + \alpha - \phi_I + \frac{\pi}{3} \right) \]

\[ + I_r \sin \left( \omega_{osc}t - \alpha - \phi_r + \frac{\pi}{3} \right) + I_r \cos \left( (2w + w_{osc})t + \alpha - \phi_I + \frac{\pi}{6} \right) \],

\[ (22) \]

\[ P_{cb}(t) = V_{cb}(t)i_c(t) = \sqrt{3}V_m \left[ I \sin \left( \frac{\alpha + \pi}{6} \right) - I \cos \left( 2\omega t + \frac{\pi}{6} \right) \right. \]

\[ + I_I \sin \left( \omega_{osc}t + \alpha + \frac{\pi}{3} \right) - I_I \cos \left( (2w - w_{osc})t + \alpha - \phi_I + \frac{\pi}{6} \right) \]

\[ + I_r \cos \left( \omega_{osc}t - \alpha - \phi_r + \frac{\pi}{3} \right) - I_r \cos \left( (2w + w_{osc})t + \alpha - \phi_I + \frac{\pi}{6} \right) \],

\[ (23) \]

\[ P_{abc}(t) = P_{ab}(t) + P_{cb}(t) = 3V_mI \cos \alpha + I_I \cos(\omega_{osc}t + \alpha + \frac{\pi}{3}) + I_r \cos(\omega_{osc}t - \alpha - \frac{\pi}{3}) \].

\[ (24) \]

Clearly, in the two partial power spectrums, in addition to the DC component, two side-band components will appear at frequencies “f₁ = (2w + w_{osc})/2π” and “f₂ = (2w - w_{osc})/2π”.

With these assumptions, the space current vector, referred to the stator frame, is shown below:

\[ i_s = \frac{\sqrt{3}}{\sqrt{3}} \left( i_a + i_b e^{j(2\pi/3)} + i_c e^{-j(2\pi/3)} \right) = \sqrt{3} \left( Ia^\omega t + I_I e^{j(w - w_{osc})t - \phi_I} + I_r e^{j(w + w_{osc})t + \phi_I} \right). \]

\[ (25) \]

The vector modulus |i_s| can be computed from Eq. (25) and is given by Eq. (26)

\[ |i_s| = \frac{1}{2} \sqrt{6I^2 + 6I_I^2 + 12II_I \cos(\omega_{osc}t + \phi_I) + 12II_r \cos(\omega_{osc}t - \phi_I) + 12II_r \cos(2\omega_{osc}t + \phi_I - \phi_r)} \]

\[ (26) \]

and with “Iᵣ” and “Iₗ” having small values with respect to “I”. The result is the following equation:

\[ |i_s| \cong \sqrt{3}[I + I_I \cos(\omega_{osc}t + \phi_I) + I_r \cos(\omega_{osc}t - \phi_I)]. \]

\[ (27) \]

The Fourier transform of the Park Vector Modulus will show, besides the constant component \( \sqrt{3}I \), only a component at frequency “w_{osc}” whose amplitude can be obtained as the sum of the vector \( \vec{I}_r = \sqrt{3}e^{-j\phi_I} \) and the complex conjugate of \( \vec{I}_r \).

\[ |\vec{i}_{s,\omega_{osc}}| = \sqrt{3} \left( I_I^2 + I_r^2 + 2II_r \cos(\phi_r + \phi_I) \right). \]

\[ (28) \]

In Section 3, using simplified relationships, it will be shown that, considering both the upper and lower side-band components, their angular displacements are linked.

3. Model analysis

The quantitative model is based on the amplitude analysis of the spectral fault components in the five diagnostic procedures. Authors in [17] have proposed a procedure to calculate the relation between the two side-band components angle displacement (φ_I and φ_r) when electrical faults, such as broken rotor bars, occur. By this way, the current side-bands components first appear leading to a torque and a speed ripple. Thus, in our paper we not only study electrical faults but also other types of faults such as bearing defects, motor load shaft misalignments, broken teeth in the load gear box, or mechanical vibrations which develop torque ripple and then lead to the creation of current side-bands components. This effect will lead to a new correlation between the two fault component angular displacements (φ_I and φ_r).
Let us start with the instantaneous electromagnetic torque. We have seen that, in the case of no abnormalities, the electromagnetic torque is constant; refer to Eq. (15). If mechanical abnormalities at a fault characteristic frequency \(w_{\text{osc}}\) and a torque component \(\Delta T_{em}\) at frequency \(w_{\text{osc}}\) will appear, the following equation results:

\[
\Delta T_{em} = M(3\Psi \Pi) \Im \left[ e^{j(w_{\text{osc}}t - \varphi_{\Pi} + \varphi_{\Psi})} \right] = M(3\Psi \Pi) \sin(w_{\text{osc}}t - \varphi_{\Pi} + \varphi_{\Psi}),
\]

where \(M\) is the modulation index and \(\varphi_{\Pi}\) is the angular displacement of the new additive fault torque component. The total electromagnetic torque can then be expressed as

\[
T_{em} = T_{em0} + \Delta T_{em},
\]

where \(T_{em0}\) is the nominal torque for a fault-free motor. As a consequence of torque ripple, a speed ripple occurs which may be expressed as

\[
\Delta w_s(t) = \frac{1}{J} \int \Delta T(t) \, dt = -\frac{3\Psi \Pi M I}{J w_{\text{osc}}} \cos(w_{\text{osc}}t - \varphi_{\Pi} + \varphi_{\Psi}).
\]

Then the mechanical angular variation is computed as follows:

\[
\Delta \theta = \int \Delta w_s(t) \, dt = -\frac{3\Psi \Pi M I}{J(w_{\text{osc}})^2} \sin(w_{\text{osc}}t - \varphi_{\Pi} + \varphi_{\Psi}).
\]

Let \(K = 3\Psi \Pi / J(w_{\text{osc}})^2\), then the linkage flux expression becomes

\[
\Psi_s = \sqrt{3} \psi e^{j(wt - \varphi_{\Psi} - \Delta \theta)} = \sqrt{3} \psi e^{j(wt - \varphi_{\Pi})} e^{jK\Pi \sin(w_{\text{osc}}t - \varphi_{\Pi} + \varphi_{\Psi})},
\]

where \(\varphi_{\Psi} \equiv \pi/2\).

By replacing the second exponential by its Taylor series and truncating it to the second term, the equation becomes

\[
\Psi_s = \sqrt{3} \psi e^{j(wt - \varphi_{\Pi})} \left[ 1 + jK\Pi \sin(w_{\text{osc}}t - \varphi_{\Pi} + \varphi_{\Psi}) \right].
\]

This approximation is true only if we assume that the value of the “\(K\)” index is low. This low value means we impose high values of the inertia. Using the Euler formula for the sine function yields

\[
\Psi_s = \sqrt{3} \psi e^{j(wt - \varphi_{\Pi})} + \frac{1}{2} \sqrt{3} \psi KM\Pi e^{j(wt - \varphi_{\Pi} - 2\varphi_{\Pi} + \varphi_{\Psi})} - \frac{1}{2} \sqrt{3} \psi KM\Pi e^{j(w - \varphi_{\Pi} - \varphi_{\Psi})}.
\]

The derivative of the first term of the flux is the fundamental electromagnetic force (EMF), while the derivative of the other two terms produces the following electromagnetic forces:

\[
\vec{e}_l = \frac{2}{3} \sqrt{3} \psi KM \Pi (w - w_{\text{osc}}) e^{j(w - \varphi_{\Pi} - \varphi_{\Psi})},
\]

\[
\vec{e}_r = \frac{2}{3} \sqrt{3} \psi KM \Pi (w + w_{\text{osc}}) e^{j(w + \varphi_{\Pi} - \varphi_{\Psi})}.
\]

The consequent space vector currents are then

\[
\vec{i}_l = -\frac{\sqrt{3}}{2} \psi KM \Pi (w - w_{\text{osc}}) Z_s e^{j\varphi_{\Psi}},
\]

\[
\vec{i}_r = \frac{\sqrt{3}}{2} \psi KM \Pi (w + w_{\text{osc}}) Z_s e^{j\varphi_{\Psi}},
\]

where \(Z_s e^{j\varphi_{\Psi}} = R_s + jX_s\) is the stator winding impedance seen by the two electromagnetic forces.

For medium and large machines, it can be assumed that \((R_s \ll X_s)\) thus \(\varphi_{\Psi} = \pi/2\). Hence, we can extract the amplitude and the angular displacement of the two current side-band components:

\[
I_l = \sqrt{3} \psi KM \Pi (w - w_{\text{osc}}) / Z_s,
\]

\[
I_r = \sqrt{3} \psi KM \Pi (w + w_{\text{osc}}) / Z_s.
\]
Using Eqs. (25), (38) and (39), a relation between angular displacements is calculated:

\[-\varphi_r = \frac{\pi}{2} - 2\varphi_T + \varphi_T - \varphi_s = -\pi + \varphi_T,\]  

\[-\varphi_l = -\frac{\pi}{2} - \varphi_T - \varphi_s = -\pi - \varphi_T.\]  

Therefore, we can eliminate \(\varphi_T\) from Eqs. (42) and (43), and we obtain

\(\varphi_r = 2\pi - \varphi_l.\)

Note that the angular displacements have a physical meaning only when they are referred to the same frequency \(w_{osc}\). Therefore, the sum of the angular displacements \((\varphi_r + \varphi_l)\) could represent the type of machine fault. Specifically, in the case of pure electrical faults such as broken bars, this sum is approximately \(\pi\) \cite{17}. In the case of pure mechanical abnormalities, this sum takes the approximate value of \(2\pi\) as demonstrated before. Therefore, generally in experimental tests, mechanical and electrical faults may occur simultaneously so the value of \((\varphi_r + \varphi_l)\) will differ from the theoretical values and move toward \(\pi\) if the fault severity of electrical faults increases and will be much closer to \(2\pi\) if mechanical faults become dominant.

4. Quantitative analysis of the spectral fault components

In summary, the most effective diagnostic information can be detected in the simple current spectrum at frequencies \(f \pm f_{osc}\). This information can be extracted from the instantaneous power spectrums at frequencies \(f \pm 2f_{osc}\) or at the fault characteristic frequency \(f_{osc}\). Finally, it can also be detected from the current space vector modulus spectrum at the frequency \(f_{osc}\). In order to quantify the increase of the fault characteristic frequencies with respect to the healthy condition, a severity factor is calculated for each diagnostic method. The severity factor is taken as the normalized value of the fault characteristic amplitude with respect to a reference of the healthy motor.

4.1. Simple current diagnostic method

Obviously, the amplitude of the two current fault characteristic frequencies are \(I_l\) and \(I_r\). The chosen reference for the current spectrum is the amplitude of the fundamental frequency for a fault free motor (50 Hz). Thus, the severity factor denoting the increase in percentage of the left fault characteristic frequency is given by Eq. (45):

\[A_{cl} = 100 \frac{I_l}{\sqrt{2I^2}}.\]  

Accordingly, the severity factor of the right fault characteristic frequency \((f + f_{osc})\) is given by the equation below:

\[A_{cr} = 100 \frac{I_r}{\sqrt{2I^2}}.\]

4.2. Current space vector modulus diagnostic method

As shown in Section 2, the vector modulus \(|\vec{i}_{s,wosc}|\) is given by Eq. (26). With \(I_l\) and \(I_r\) having small values with respect to \(I\), the amplitude of the fault characteristic frequency is given by

\[|\vec{i}_{s,wosc}| = \sqrt{3} \sqrt{I_l^2 + I_r^2 + 2I_lI_r \cos(\varphi_r + \varphi_l)}.\]
In order to quantify the increase of this fault characteristic frequency with respect to the healthy condition, we choose the amplitude of the direct DC component “$\sqrt{3}I$” of the current space vector modulus spectrum for a fault free motor as the reference. Thus, the increase of the fault characteristic amplitude in percentage of its reference at the frequency “$f_{osc}$” is given by

$$A_{csvm} = 100 \frac{|I_{t,osc}|}{\sqrt{3}I} = 100\sqrt{I_t^2 + I_r^2 + 2I_tI_r \cos(\varphi_r + \varphi_l)}.$$ (48)

As shown in Section 3, the value of $(\varphi_r + \varphi_l)$ is directly linked to the type of the fault and varies between “$\pi$” and “$2\pi$”; therefore, the severity factor “$A_{csvm}$” will vary between $A_{csvm\min}$ and $A_{csvm\max}$ given that

$$A_{csvm\min} = 100 \frac{|I_t - I_r|}{I},$$ (49)

$$A_{csvm\max} = 100 \frac{|I_t + I_r|}{I}.$$ (50)

Clearly, we can notice that the severity factor “$A_{csvm}$” is directly affected by the fault type. In other words, the “$A_{csvm}$” is proportional to the amplitude difference of the two simple current fault characteristic frequencies in the case of broken rotor bars while this severity factor is proportional to the sum of the two simple current fault characteristic frequency amplitudes in the case of purely mechanical faults. We also notice, as Eqs. (40) and (41) reveal that this diagnostic procedure has low effectiveness in detecting broken rotor bars while this severity factor is proportional to the sum of the two simple current fault characteristic frequency amplitudes in the case of purely mechanical faults. The increase in effectiveness occurs to the summing effect of the two amplitudes of the current side-band components.

4.3. Instantaneous powers diagnostic methods

The instantaneous partial powers “$P_{ab}$”, “$P_{cb}$” for a motor with abnormalities are given by Eqs. (22) and (23). The total power “$P_{abc}$” is given by Eq. (24). The component of the partial powers at $(2w = 100$ Hz) for a fault free motor is chosen as the reference. The fault characteristic increase in amplitude components in percentage of the chosen reference can be obtained as in the case of space vector modulus:

$$AP_{ab} = 100 \frac{\sqrt{3}I_t V_m \cos(\omega_{osc}t + \alpha + \varphi_l + (\pi/6)) + \sqrt{3}V_m I_r \sin(\omega_{osc}t - \alpha - \varphi_r + (\pi/3))}{\sqrt{3}IV_m}.$$ (51)

After computing, the value of this severity factor becomes

$$AP_{ab} = 100 \frac{[I_t^2 + I_r^2 + 2I_tI_r \cos(2\alpha + \varphi_r + \varphi_l + (\pi/3))]^{1/2}}{I}.$$ (52)

Also, we can calculate the two other severity factors:

$$AP_{cb} = 100 \frac{[I_t^2 + I_r^2 + 2I_tI_r \cos(2\alpha + \varphi_r + \varphi_l - (\pi/3))]^{1/2}}{I},$$ (53)

$$AP_{abc} = 100 \sqrt{3} \frac{[I_t^2 + I_r^2 + 2I_tI_r \cos(2\alpha + \varphi_r + \varphi_l)]^{1/2}}{I}.$$ (54)

We can see clearly that the severity factors of the three diagnostic procedures $AP_{ab}$, $AP_{cb}$ and $AP_{abc}$ depend on the value of the power factor angle and on the value of the sum of the angular displacement $(\varphi_r + \varphi_l)$.

In deed, we define the power diagnosis factor “$PDF$” as the sum of the two displacement angles $(\varphi_r + \varphi_l)$ and twice the power factor angle $\alpha$:

$$PDF = 2\alpha + \varphi_r + \varphi_l.$$ (55)

As Eqs. (52)–(54) reveal, the three severity factors vary periodically with the value of the PDF. In Section 5, we will present a simulation in order to validate the relationships reported in the previous sections.
5. Simulation setup

Signals are simulated with respect to the model of electrical signals within both a fault-free motor and a defected motor.

5.1. Electric signals generation

In the simulation, Eqs. (1)–(8) model the electrical signals for a fault-free motor, while Eqs. (18)–(20) model the electric signals for a motor with abnormalities. A simulated 3 kW, 380 V, 50 Hz two-pole induction motor is used. The nominal current of the motor is $I_f = 10 \text{ A}$ as an rms value, the amplitude values of the current side-bands are $I_r = 7I_f/100$, $I_l = 5I_f/100$ and the fault characteristic frequency value is 5 Hz. Finally, for each generated electrical signal, 4995 values are generated with a sampling frequency of 1 kHz, which correspond to 4.995 s for each signal length.

In order to better simulate the electrical signals, additional white noise is added to the simulated currents and motor voltages.

5.2. Simulation analysis of the fault characteristic amplitudes

In our first simulation approach, we generate the theoretical computed equations (45)–(54) presented in Section 4. Fig. 2 shows the amplitude variation of the severity factors $A_{csrm}$, $P_{abc}$, $A_{Pab}$, and $A_{Pcb}$. The values

![Graphs showing theoretical severity factors as a function of the power factor angle and the fault type.](image)

Fig. 2. Theoretical severity factors as a function of the power factor angle $\phi$ and of the fault type $(\phi_r + \phi_l)$: (a) $A_{csrm}$; (b) $A_{Pabc}$; (c) $A_{Pab}$; (d) $A_{Pcb}$.
of the severity factors are analyzed with respect to the power factor variation and with respect to the fault type represented by the variation of the value of \((\varphi_r + \varphi_l)\). Indeed, the values of severity factors \(A_{cr}\) and \(A_{cl}\) do not depend on the values of \((\varphi_r + \varphi_l)\) nor on the power factor angle “\(z\)”; thus, we will not present the variation of these two values versus \((\varphi_r + \varphi_l)\) and “\(z\)”.

From Fig. 2a, we notice that the value of the severity factor \(A_{csvm}\) does not depend on the power factor angle. In contrast, the \(A_{csvm}\) increases when \((\varphi_r + \varphi_l)\) increases. The \(A_{csvm}\) is minimal when \((\varphi_r + \varphi_l) = \pi\), and it is maximal when \((\varphi_r + \varphi_l) = 2\pi\). From Fig. 2b to d, we notice that the severity factor \(AP_{abc}\) and \(AP_{ab}\) increase when the power factor angle increases if \((\varphi_r + \varphi_l) = \pi\) and decrease if \((\varphi_r + \varphi_l) = 2\pi\). In the case of \(AP_{cb}\), we notice that for \(x = \pi/6\), we have a minimum if \((\varphi_r + \varphi_l) = \pi\) and a maximum if \((\varphi_r + \varphi_l) = 2\pi\). Figs. 3–5 present a comparison between the five severity factors. Fig. 3 shows variation of \(A_{ch}, A_{csvm}, AP_{abc}, AP_{cb},\) and \(AP_{abc}\) for electrical faults \((\varphi_r + \varphi_l) = \pi\). Fig. 4 shows variation of \(A_{ch}, A_{csvm}, AP_{abc}, AP_{cb},\) and \(AP_{abc}\) for mechanical faults \((\varphi_r + \varphi_l) = 2\pi\). Finally, Fig. 5 shows variation of the five theoretical severity factor for abnormalities with \((\varphi_r + \varphi_l) = \pi\). We can observe that in the case of electrical faults \((\varphi_r + \varphi_l)\) equal to “\(\pi\)” with “\(z\)” less then 4°, the simple current diagnostic procedure shows the greatest fault characteristic amplitude. In the range of 4–28°, it is the partial power amplitude \(AP_{ab}\) that has the most pronounced amplitude of the severity factor. The total power amplitude, \(AP_{abc}\), shows the greatest amplitude when the value of “\(z\)” is greater than 28°. Note the periodicity of these signals. Referring to Fig. 4, the case of mechanical abnormalities, we see that the total power severity factor, \(AP_{abc}\), shows the greatest amplitude when \(x\) has values less then 60°. These comparisons are made for a wide range of the power factor angle, but in general, motors are designed so that within a wide range of load torque, the power factor

![Fig. 3. Theoretical severity factors variation as a function of the power factor angle. Case of \((\varphi_r + \varphi_l) = \pi\).](image-url)

![Fig. 4. Theoretical severity factors variation as a function of the power factor angle. Case of \((\varphi_r + \varphi_l) = 2\pi\).](image-url)
angle does not change significantly but remains low in order to maintain a high power factor. Thus, this diagnostic procedure will be the finest to deal with these types of faults. Finally, we can notice that signals in Fig. 4 are the same signals as in Fig. 3, but are shifted by $90^\circ$. In the case of Fig. 5, signals are shifted by $25^\circ$.

In our second simulation procedure, we generate the current and the voltage signals. Then we generate the instantaneous powers by multiplying the corresponding voltages and currents. Also, we generate the current space modulus by using the well known Park Transformation as seen in Eq. (9). For each computed signal, the fast Fourier transformation is applied then the spectrum is normalized according to its reference. As seen before the fundamental frequency amplitude is taken as the reference for the current spectrum. For instantaneous powers, the reference is the component amplitude of the partial powers at twice the fundamental line frequency $2w$ for a fault free motor. Finally, in the case of the current space vector modulus spectrum, the reference is the amplitude of the DC component for a fault free motor.

Fig. 6 presents a comparison between the six severity factors $A_{cr}, A_{cl}, A_{csvm}, AP_{ab}, AP_{cb}$, and $AP_{abc}$ as a function of the power factor angle for $(\varphi_r + \varphi_l) = \pi$. Fig. 7 presents for $(\varphi_r + \varphi_l) = 2\pi$ the six severity factors variation as a function of power factor variation. The only difference that we notice between the theoretical

Fig. 5. Theoretical severity factors variation as a function of the power factor angle. Case of $(\varphi_r + \varphi_l) = (180 + 100)^\circ$.

Fig. 6. Severity factor variation obtained by simulation setup and in the case of $\varphi_r + \varphi_l = \pi$. 
values (Figs. 3 and 4) and the simulated amplitudes shown in Figs. 6 and 7 is the value of the severity factor corresponding to the current space modulus procedure. Indeed, the theoretical value of the fault characteristic frequency given by Eq. (48) is an approximation, assuming that "I_l" and "I_r" have very small values with respect to "I_f".

Then, we investigate the severity factor amplitude variation \( (A_{csvm}, A_{P_{ab}}, A_{P_{cb}}, A_{P_{abc}}) \) of the five diagnostic procedures as a function of the fault type \((\varphi_r + \varphi_l)\) and the increase in one of the current side-band component. In fact, the inertia of the group motor-load and the fault severity has a direct and high influence over the amplitude of the simple current fault components. Thus, an increase in the amplitude of the current side-band components (left or side band component amplitude) can either be caused by an increase in the motor inertia or an increase in the fault severity. Consequently, we have taken the left side-band component to vary between \((I_l = 0(I_f/100))\) and \((I_l = 14(I_f/100))\). Note that both "I_r" is taken as constant with the value of 7% of \(I_f\). The power factor angle is taken as 18°. Thus, Fig. 8 presents the variation of the sensitivity of several diagnostic procedures as a function of the fault type and the motor inertia or the fault severity. Fig. 8a presents the amplitude of the severity factors, "ACr", as a function of the current left side band amplitude increase and as a function of the fault type (value of the sum of the two angular displacements \((\varphi_r + \varphi_l)\)). Fig. 8b presents the amplitude of the severity factor, "Acsvm". We notice that the "Acsvm" value presents a minimum if \(I_l = I_r\) as in the case of pure electrical faults. In fact, Eq. (49) clearly explains this phenomenon. Fig. 8c and d shows the amplitude of the severity factors of the partial powers "\(P_{ab}\)" and "\(P_{cb}\)". Fig. 9a shows simultaneously all the five severity factors relative to the five diagnostic procedures, as a function of the motor group inertia for pure electrical faults. Fig. 9b shows simultaneously all five severity factors relative to the five diagnostic procedures, as a function of for pure mechanical faults.

For the five diagnostic procedures, Fig. 9a presents the severity factor variation as a function of the increase in the amplitude of the left current side-band component, (increase of the inertia of the group motor-load or the increase of the fault severity), for pure electrical faults. We can notice that, for pure electrical faults, the sensitivity of the three partial powers diagnostic procedures vary nonlinearly with respect to the motor inertia variation. Notably, the fault diagnostic procedure "\(P_{ab}\)" gains in fault detection sensitivity over the total power diagnostic procedure. In case of mechanical faults, see Fig. 9b: we can notice that the severity factors of the five diagnostic procedures vary linearly with respect to the motor inertia variation. We can also notice that the slopes of the five severity factor are approximately equal so, in case of pure mechanical faults, the sensitivity of all the five diagnostic procedures are affected equally by the machinery inertia variation.

Figs. 10a and b present the variation of the five severity factor as a function of the fault type, respectively, for \((I_l = 1\%I_f)\) and \((I_l = 14\%I_f)\) taking the value of the power factor angle to be 18°. We can see that, in most
cases, the diagnostic procedure using the total power exhibit the greatest amplitude of the severity factor when
the type of the fault is purely electrical or purely mechanical ($j + jl = \pi$ or $2\pi$). We can also notice that the
two amplitudes $AP_{ab}$ and $AP_{cb}$, relative to partial powers, act in a completely opposite way when the fault type
varies from a pure electric fault to pure mechanical fault.

Fig. 8. Severity factor variation as a function of the current left side band amplitude increase and as a function of the fault type (value of $(\varphi + \varphi)$) for $x = 18$: (a) $A_{cb}$, (b) $A_{csm}$, (c) $AP_{ab}$, (d) $AP_{cb}$.

Fig. 9. Severity factor variation as a function of the current left side band amplitude increase. For $x = 18$ and $I_r = 7\%I_f$: (a) case of $(\varphi + \varphi) = \pi$, (b) case of $(\varphi + \varphi) = 2\pi$. 
6. Experimental setup

The experimental tests are carried out using data from two sources. The first source is the LAII (Laboratoire d'Automatique et Informatique Industrielle) Laboratory of Poitiers-France. The second source is the Laboratory of Images and Signals “LIS” (Laboratoire des Images et des Signaux) of Grenoble-France. Data from the LAII laboratory treat broken rotor bars in induction motors. The purpose of the benchmark “GOTIX” of the LIS Laboratory is to detect mechanical abnormalities in rotating machinery, especially defects in gear boxes using vibration, acoustical and or electrical supply signals. The benchmark “GOTIX” is formed by a 55 kW “Leroy Somer” asynchronous motor by a gear box with a multiplicative ratio of (57/15) (see Fig. 11), and by a speed variable controller. The asynchronous motor drives a DC current generator in order to simulate variable loads. The “GOTIX” is equipped with three voltage sensors, three current sensors, eight accelerometers, torque sensor, several temperature probes and a speed sensor. The benchmark is equipped by a data acquisition system used to acquire synchronously twenty instantaneous signals with a 100 kHz maximum sampling frequency per channel.

The detection tests are performed with the equipment described above for a healthy mode without mechanical or electrical abnormalities. This mode is taken as the reference time (time = 0). The second mode corresponds to 15 h of continuous work driving a load of 172 N m (mode 1) and finally after 300 h of continuous work (mode 2). Obviously, after 300 h of continuous effort the gear box is exhausted and contains defects.
For every mode of the three operation modes, a number of eight signals, of 10 s each, are acquired. These signals are re-sampled at a frequency of 2.5 kHz in order to reduce computation time. It is important to note that this re-sampling does not affect the detection procedures because all the fault characteristic frequencies for the five diagnostic procedures are situated in the low-frequency range.

The spectrums of the simple current, the Current Park Vector Modulus, and the three instantaneous powers are computed and then averaged over the eight experimental sets of electrical signals for the three operation modes. The simple Current spectrum is normalized with respect to the amplitude of the fundamental supply frequency. The spectrum of the Current Park Vector Modulus is normalized with respect to the direct component (0 Hz) for a fault-free motor. Finally, the three instantaneous powers are normalized with respect to the amplitude of twice the fundamental frequency (100 Hz) of the partial power “Pab”.

Fig. 12a shows a comparison between the spectrum of the Current Space Vector Modulus for a fault-free machine and for a defected gear. The spectral analysis clearly shows that when a mechanical fault and gear fatigue are present, several components appear at the fault’s characteristic frequencies in the spectrum “fosc”. These fault characteristic frequencies are directly related to the shaft rotating frequency and its sub-harmonics. We can see three characteristic frequencies: (3.68 Hz), (12.2 Hz) and (19.6 Hz) in Fig. 12a.

![Signal spectrums for a fault free machine (dotted line) and for a defected machine (straight line): (a) current space vector, (b) total power, (c) partial power “Pcb” spectrum in the frequency range [0. 30 Hz], (d) Partial power “Pcb” spectrum in the frequency range [70–130 Hz].](image-url)
Fig. 12b shows a comparison between spectrums of the total instantaneous power for a fault free machine and for a defected gear. The spectral analysis clearly shows that when a mechanical fault is present, a component appears at several frequencies. Fig. 12c shows the spectrum of the partial power “Pcb” for a fault free machine and for a defected gear in the low band frequencies [0.30] Hz. Fig. 12d shows spectrum of the partial power “Pcb” for the two functioning modes in the band around twice the fundamental supply frequency [70.130] Hz.

Fig. 13a shows the severity factor of the five diagnostic procedures relative to the frequency ($f_{osc1} = 3.68$ Hz) as a function of the power factor angle “$\alpha$” at mode 1. Fig. 13b shows the amplitude of severity factor of the five diagnostic procedures relative to the frequency ($f_{osc1} = 3.68$ Hz) as a function of the power factor angle “$\alpha$” for mode 2. Notice that we shift the voltage signals with respect to the three currents in order to vary the power factor angle “$\alpha$”. Fig. 14 shows the severity factor of the five diagnostic procedures relative to the frequency ($f_{osc1} = 12.2$ Hz) as a function of the power factor angle “$\alpha$” for mode 1 and Fig. 15 shows the severity factor of the five diagnostic procedures relative to the same fault characteristic frequency as a function of the power factor angle “$\alpha$” for mode 2. Comparing the experimental results presented in Figs. 13–15 with the theoretical computation presented in Fig. 5, we can deduce the value of the sum of the two angular displacements ($\varphi_r + \varphi_l$). In our case, Figs. 13–15 show that this value is approximately $280^\circ$ for both the two

![Figure 13](image-url)

Fig. 13. Severity factor of the five diagnostic procedures relative to the frequency “3.68 Hz” using the “GOTIX” experimental setup (a) after 15 working hours; (b) after 300 working hours.

![Figure 14](image-url)

Fig. 14. Severity factors relative to the fault characteristic frequency “12.27 Hz” using the “GOTIX” experimental setup. Motor after 15 working hours.
characteristic frequencies \( f_{osc1} \) and \( f_{osc2} \). The value of the sum of the two angular displacements \((\phi_r + \phi_l)\) reveal that the fault type is not purely mechanical but it is more mechanical than electrical.

Concerning the benchmark of Poitier, the motor used has the following properties: voltage supply of 220/380 V; supply frequency of 50 Hz; power consumption of 1.1 kW; number of pair poles equal 2 and the power factor angle is given by its cosine \((\cos(\alpha) = 0.65)\). The motor is equipped with three voltage and three current sensors. The acquired signals are then filtered with low-pass fifth-order Butterworth filters. The measured signal-sampling period is 0.7 ms. The detection tests are performed with the equipment described above, at the motor nominal load, first using an undamaged motor, a subsequent motor with one broken bar, and finally a motor with two broken bars. For every of the three operation modes, a number of ten signals of 10 s each are acquired. Notice that we shift the voltage signals with respect to the three currents in order to vary the power factor angle \(\alpha\). Fig. 16 shows a comparison of severity factors relative to the fault characteristic frequency \((f_{osc1} = 2f)\) which, in our case, is twice the slip frequency as a function of the power factor angle \(\alpha\) for a motor with one broken bar. Fig. 17 shows a comparison of the five severity factors relative to the same fault characteristic frequency as a function of the power factor angle \(\alpha\) for a motor with two broken bars. Comparing the theoretical computation presented by Figs. 3 and 4, we can extract the value of \((\phi_r + \phi_l)\). This value is approximately 110° for one broken bar and approximately 145° for two broken bars. Obviously, electrical and mechanical faults may occur simultaneously so the value of \((\phi_r + \phi_l)\) will differ from the theoretical values and move toward \(\pi\) if

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**Fig. 15.** Severity factors relative to the fault characteristic frequency 12.27 Hz using the “GOTIX” experimental setup. Motor with 300 h.

**Fig. 16.** Severity factors relative to the broken rotor bar characteristic frequency using the LAII experimental setup. Motor with one broken bar.
the fault severity of electrical faults increases and will be much closer to “2π” if mechanical faults become dominant.

7. Conclusion

This paper reports quantitative analysis of spectral fault components in five noninvasive diagnostic procedures that use input electric signals to detect different types of abnormalities in induction motors. Stator voltages and stator currents are simulated first for a fault-free motor, and then for a defected motor. Besides the traditional one phase current spectrum analysis “SC”, the diagnostic procedures based on spectrum analysis of the instantaneous partial powers “$P_{ab}$”, “$P_{cb}$”, total power “$P_{abc}$”, and finally the current space vector modulus “$csvm$” are considered. As a result, it is shown that the relationship between the angular displacement of the current side-bands components at frequencies ($f\pm f_{osc}$) are directly related to the type of induction motor faults. It is proved that the fault component amplitude of the “$csvm$” method are only function of the fault severity and the inertia of the group motor-load and do not depend on the power factor angle. In case of electrical faults, the normalized fault component amplitude of the “$csvm$” method is directly linked to the amplitude difference of the simple current side-bands components and linked to the sum of these amplitudes in case of mechanical faults. Regarding the three instantaneous powers, the fault characteristic frequency amplitudes are related in a periodic manner to the sum of the power factor angle and the two current side-band angular displacements. Assuming that motors are designed that within a wide range of load torque the power factor angle remains less than 60°, it is proved that the total instantaneous power diagnostic procedure was observed to exhibit the highest values of the detection criterion in case of mechanical faults while in case of electrical faults it is shown that the most reliable diagnostic procedure is tightly related to the value of the motor power factor angle and the inertia of the group motor-load. Therefore, in case of electrical faults, when the power factor angle is less then 4°, the simple current diagnostic procedure shows the greater detection criterion. In the range of 4–28°, it is the partial power “$P_{ab}$” that has the most pronounced amplitude of the severity factor. The total power severity factor shows the greatest amplitude when the value of the power factor angle is greater than 28°. Also, simulation results show that the sensitivity of the three partial powers diagnostic procedures vary nonlinearly with respect to the motor inertia variation and the fault diagnostic procedure “$P_{ab}$” gains in fault detection sensitivity over all the diagnostic procedure when the inertia of the group motor load increases. Finally, experimental result using data from the “LIS” Laboratory and the “LAII” laboratory, both in France, concerning detection of mechanical abnormalities in rotating machinery and particularly defects in a gear boxes and broken rotor bars, show good agreement with the fault modeling theoretical and simulation results.
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