Belief function theory for image and signal processing

« De omni re scibili...et quibusdam aliis ! »

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Outline

1. Introduction
2. Applications of BFT to medical imaging
3. Applications of BFT to color imaging
4. Application of BFT to 3D computer generated images
5. Application of BFT to multi-object tracking (not presented)
6. Conclusions
Introduction

- Why BFT in image (and signal) processing?
- For what?
- Warning: basics on BFT are required (see previous lectures)
Why BFT?

Some words to justify the use of Belief Function Theory in the following applications

- All the applications are considered in a multisource context
- They need a fusion process (data, features)
- The data are imperfect (uncertain and/or imprecise)
- Possibility of missing data
BFT applied to MR Images

**Topics**:
- Processing (segmentation)
- Analysis

**Applications**:
- Brain cancer
- Prostat cancer
Medical imaging

Acquisition of image data

Segmentation
Classification
Registration
Reconstruction
Filtering

Medical imaging techniques:
- MRI

Decision making

+
Medical imaging

Many techniques are nowadays available:

Anatomical imaging:
- US
- CT
- MRI → some technical basics are following

Functional imaging:
- PET
- fMRI
- …
Some basics of MRI

- **Background:**
  - MRI technique exploits the magnetic properties of the atomic nuclei [Bloch-46], [Purcell-46]
  - A patient is submitted to a uniform magnetic field $B_0$
  - Excitation of the hydrogen nuclei (80% of the body mass) with a radio frequency field of weak intensity *
  - Measure of the energy restored by protons to create the images
  - Multi echo imaging sequences are possible. They use a series of echoes acquired as a train following after a single excitation pulse.

* : The excitation of hydrogen nuclei results in a change in the orientation of their magnetic moments (*i.e.* direction of the respective magnetic fields) away from the main magnetic field of the MRI scanner.
Some basics of MRI

A part of your body

The same part of your body in the machine

Proton

Magnetic moment of the proton

Angular speed

$\vec{B}_0$
Some basics of MRI

The excitation of hydrogen nuclei results in a change in the orientation of their magnetic moments (i.e. direction of the respective magnetic fields) away from the main magnetic field of the MRI scanner.

Back to equilibrium involves emission of photons (to capture !)
Why BFT in medical imaging?

- Multimodal imaging, multi-echoes (MRI) → multisource context thus fusion process is an option

- Large amount of data (CT and MRI generate stacks of images)

- Diagnosis made by a human expert needs the examination of a lot of images → computer aided diagnosis.

- Variance intra and inter examiners (well-known problem inducing uncertainty and imprecision too)
First application: detection of brain tumors

Some references:


Context

**Diagnosis of brain tumors**
- Suspicion of tumor
- Validation (or not) by means of imaging
  - CT
  - Scintigraphic imaging (*e.g.* PET)
  - Magnetic Resonance Imaging
- Study of the tumors
  - Localization
  - Number
  - Volume

**Segmentation of MR images**
- Divide the images into regions to obtain the localization of the tumors
- Particularly for not crisp tumors

![MRI images showing tumor and edema](image-url)
MRI: characteristics

- Image contrasts depend on acquisition parameters ($T_R$, $T_E$)
- Possibility to obtain several « views » (observations) of the same organ
MRI: characteristics

- MR images: particularly interesting in the diagnosis framework of brain tumors

Complementary points of view
MRI : characteristics

Defects

- Noise (acquisition, motion of the patients)
- Defect of the radiofrequency field (non uniform in the space)
- Drift of the radiometric levels

Scanner 1.5T : 20 % up to 30 % [Meyer-95]

Methods of correction
- Preprocessings
- Simultaneously with the segmentation process
Summary

Segmenting tridimensionnal MR images to detect brain tumors

- Tumors with no well defined contours, invasive
- Characteristics of MR imaging
  - great sensibility – multi-echoes
  - imperfections
Remarks

« Image » approaches
- recognition/identification of a bi/tridimensional signal
  - Thresholding [Kapouleas-90], [Tsai-95] multi-echoes – tumors of white matter
  - Region growing (homogeneous tumors – position of the germs)
  - Snakes

- Not widely used because require (often) the contribution of prior information (location, form, radiometric level…)

MRI
Remarks

**Pattern recognition approaches**
- Define criteria for clustering for allocating voxels to a class
- Methods based on confidence measures
  - Probability theory (paramétric or not)
    - Gaussian modelling – tumors = outliers
  - Fuzzy set theory and possibility theory (partition by FCMs)
  - Belief function theory (BFT)

- The most used methods
- But necessary to take into account contextual information
Segmentation process

The essential aspects:

1. Adaptation to the diversity of the images, the radiometry, the tumors \(\rightarrow\) less *a priori* as possible, learning for each case
   \(\rightarrow\) Pattern recognition

2. Exploiting all the modalities of acquisition at our disposal
   \(\rightarrow\) Multi-echoes

3. Integrating the *imperfection* of the data and of the tumors
   \(\rightarrow\) Belief function theory
   \(\rightarrow\) Include the *spatial dimension* in the data processing
Segmentation process

- **BFT**
  - Consideration of the imperfect nature of the data

- **Multi-echoes**
  - Consideration of the redundancy and the complementarity of the data
  - A point \( s \) in the space is modelised by a vector of dimension \( p \)

- Each point of the volume is assigned to an hypothesis (class) taking into account the spatial neighborhood
  - Segmentation and not only clustering
Preprocessing

Aims:
- To obtain the ROI (brain) in images

Difficulties:
- Tumors have variables characteristics
- Keep these tumors

Proposition:
- To adapt existing processing techniques to brain volumes containing tumors

Principe:
- Thresholdings and mathematical morphology operations
Segmentation process

Data

Preprocessing: brain extraction

Segmentation of the brain (BFT): definition of a set of regions

Detection and extraction of the tumoral zones

Learning step

Credal labelling

Integration of contextual informations

Decision making
Belief mass modelling

- Use of 3 models
- Distance-based model [Denœux-95]
- Likelihood-based models [Shafer-76] and [Appriou-91]

3 tested models
Segmentation process

Summary of needs
1. Number of class
2. Likelihood functions
3. Class prototypes
4. Type of distance

Data

Preprocessing: brain extraction

Segmentation of the brain (BFT): definition of a set of regions

Detection and extraction of the tumoral zones
Segmentation process

Learning step

Background
- EM algorithm under the following hypotheses:
  1. Number of classes fixed
  2. Gaussian modelling of the classes
- Estimates of means and variances
- Allows to compute the likelihood
- Location of prototypes
- Allows the computation of Mahalanobis distances

In practice
Need to define the learning set

Constraint: to have samples for each class (in particular for class « tumor »), without \( \text{priori} \) knowledge about the class for each pattern

- Learning set = set of patterns associated to the volume
- Great amount of points
- Limits the possibility to improve the learning
Segmentation process

*Credal labelling*

(a) Distance-based model  (b) Likelihood-based model  (c) Likelihood-based model (Shafer)
Segmentation process

"Contextual (spatial) information"

**Aims**
- Denoising of the data (evidential filtering)
- Considering the spatial neighborhood (*point to region*)

⇒ A real segmentation process

**Principle**
- Consider every neighbor as an information source able to improve the knowledge
- Combine these informations in the framework of BFT
Segmentation process

**Contextual (spatial) information**

1. One voxel => a mass (issued from the classification process)
2. Consider each voxel relatively to its neighborhood taking into account the distance between the point and each considered neighbor
   - Discounting of the masses associated to the neighbors
     \[ \alpha_q = \phi(d^2(q)) \]
     \[ \text{with } d_q = d(p, q) \]
     \[ \phi(d^2(q)) = \exp(-\beta d^2_q) \]
3. Fuse these masses with Dempster’s rule of combination
   \[ m' = m \bigoplus_{q \in \{1, \ldots, Q\}} m_q^\alpha_q \]

**Brainweb database**
- Free access database
- Synthetic volumes (computer generated images) of the human brain
- Different sequences \{T₁, T₂, DP\}
- Different noise levels (3%, 5%, 7%, 9%)
- Different levels of the radiofrequency field drift (0%, 20%, 40%)
- Known ground truth
Segmentation process

contextual information

Error rate vs. discounting

- \( \beta \) low:
  - Important error rates
  - Too large Influence of the neighbors
  - Neighbors = sources of noise
- Based on the whole set of volumes: \( 0.5 \leq \beta_{\text{opt}} \leq 1.5 \)
- \( \beta > \beta_{\text{opt}} \)
  - New increase of error rates
  - Suboptimal influence of the neighbors

\[
\xi_{\beta \to \infty} = \xi_{\beta=1} = \xi_{\text{classification}}
\]
**Segmentation process**

**Contextual information**

1. One voxel => a mass (issued from the classification process)

2. Consider each voxel relatively to its neighborhood taking into account the distance between the point and each considered neighbor
   - Discounting of the masses associated to the neighbors
     \[ \alpha_q = \phi(d^2(q)) \]
     with \[ d_q = d(p, q) \]
     \[ \phi(d^2(q)) = \exp(-\beta d_q^2) \]

3. Fuse these masses with Dempster’s rule of combination
   \[ m' = m \bigoplus_{q \in \{1, \ldots, Q\}} m' \cdot \alpha_q \]

**Optimal value for \( \beta \)?**

- Heuristically determinated by the use of Brainweb database

\[ \beta_{opt} = \arg \min_{\beta \in [0, +\infty]} \{ \xi(\beta) \} \]
\[ \beta_{opt} = 0.6 \]
Segmentation process

*Contextual information*

(a) Without contextual information

(b) With contextual information

(c) filtering
Segmentation process

Overview

- Distance-based model
- Likelihood-based model
- Likelihood-based model (Shafer)
Segmentation process

Decision making

- Maximization of the pignistic probability
- Maximization of the plausibility

Data

Preprocessing: brain extraction

Integration of contextual information

Segmentation of the brain (BFT): definition of a set of regions

Detection and extraction of the tumoral zones

Learning

Evidential classification

Decision making
Validation on synthetic data (*Brainweb*)

1\textsuperscript{st} level: evidential labelling  
(*without contextual information*)
- 3 classes (WM – GM – CF)
- Error rate
- Comparison with C-means, FCM and EM
- Best performances when close to real conditions ($n \geq 5\%$, $rf \geq 20\%$)
Validation on synthetic data (*Brainweb*)

2nd level: segmentation *(with contextual information)*

- Comparison clustering – segmentation
  - Decrease of the error rates
  - Smoothing of the regions while preserving the thin structures
- Comparison with probabilistic methods
  - EM + Markov random fields
  - [Leemput-99] (EM + prior informations) + Markov random fields + drift correction
Segmentation of real data stacks

- Data provided by CHRU of Poitiers \{T_1 – T_1\text{Gado}\} et \{T_1\text{Gado} – T_2\}
- No need of registration between the stacks
- Number of classes fixed (depending of the studied case). In general \{WM, GM, CF, tumor, œdema\}

- Validation: comparison with a hand-made segmentation (expert) ➔ **Expert dependent**!
- Rates:
  1. Good detection
  2. No détection
  3. False alarm
Segmentation of real data stacks

Images of the stacks

Segmentation results
(3D reconstruction)
Spatial combination and conflict

Introduction of contextual information: based on the fusion of neighborhood masses

- Generate conflict
- Give information about the amount of discordance between the sources (neighbors)

2 types of behaviour:
1. Homogenous regions: a priori sources agree → low amount of conflict
2. Borders: discordance between different neighbors → conflict
Spatial combination and conflict

Conflict: gives information about the borders of the regions

But:
- non trivial information
- borders $\Rightarrow$ conflict
- conflict $\not\Rightarrow$ border
Conflict: a border indicator

- **Idea**: use the conflict information to complete the segmentation process

\[ \Pi(x) = \begin{cases} 
0 & \text{si } c(x) < \lambda_c \\
\frac{c(x)}{\lambda_c} & \text{si } c(x) \geq \lambda_c 
\end{cases} \]

- Image of the possible localization of the borders
Cooperation « contour-region »

- Segmentation process provides:
  - Regions
  - « Contour » information

- Towards a cooperation « region-contour » to improve the results

- Experiment: use the borders of the regions to initialize a snake guided by the conflict
Remark

Another approach with BF

Second application : detection of prostat tumors

Reference

N. Makni, « Méthodes d'identification, d'aide au diagnostic et de planification utilisant de l'imagerie multimodalité pour les thérapies focales du cancer de la prostate », *Ph.D. Thesis*, University Lille 1, 2010,
Prostat cancer

- First cancer of men older than 50 years old*
- 80 % of tumors are located in peripheral zone
- Cancer with slow evolution

*: in industrial countries
Prostat cancer

■ Screening:
  □ Digital rectal examination (DRE),
  □ Measure of specific antigens (PSA),
  □ Biopsy,
  □ Medical imaging.

■ Therapeutic options:
  □ Total ablation (prostatectomy),
  □ Active follow-up,
  □ Partial or focused ablation.
Prostat cancer: focused ablation

(A) Véritable ablation focale.
(B) Hémiablation.
(C) Ablation des ¾ de la prostate.
(D) Ablation quasi totale.
Prostat cancer: focused ablation by laser

- Keypoints
  - Treat the tumor and only the tumor
  - Need precision when inserting fibers
  - US monitoring
  - Have a reference map → RMI

- Insertion of laser fibers in the gland
Prostat cancer : diagnosis

- Sensitive techniques (DRE et PSA) : unsufficient.

- Reference : biopsy
  - Invasive,
  - Imprecise : false negatives, …

- Optimization of the biopsy use?
  - Use of RMI is suitable and becomes a « gold standard »
Prostat cancer: diagnosis

- Use of multi-parametric MRI
  - Morphological (anatomical) and functional images,
    - Redundant and complementary informations,
    - High resolution.
Prostat cancer : diagnosis

- Difficulties:
  - Interpretation and analysis of numerous and heterogenous informations,
  - Hand-made contours of the structures of interest: prostat + ZP
  - = computation time (~10 to 50 images/exam)
  - + Lack of reproductibility, risk of errors.

PZ: variability of the human analysis (three different experts)
Extraction of the prostate

T2, T1 DCE, DWI MRI

Partition of the prostate

Detection of the suspected lesions

PZ, CZ (or complement of PZ)

Detection of the lesions

Volume measurement

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Main steps

- Prostat extraction (Extraction from the images)
- Obtaining the different anatomical zones of interest (PZ and CZ)
- Detection of tumors in PZ
Prostat gland extraction from the images

- Use an ASM$^3$ (Active Shape Model),
- Markov Random Fields (MRF).

(not given in this lecture)
Segmentation: background

- Automatic clustering:
  - PZ: Peripheral zone (80% of the tumors are found there),
  - CZ: gland centre (complement of ZP).
- A unique reference in the literature: Allen et al., IEEE ISBI, 2006
  - Method limited to axial slices of the center of the gland.
Segmentation : method

- Data : multiparametric RMI
- Sources = redundant and complementary,
- +/- sensitive with respect to the prostatic zones,
- Discrimination between some tissues = difficult (e.g. PZ and kystes) : necessity of an *a priori* in the process
Segmentation: method

Each voxel $v_i \leftrightarrow \left[ att_{T2} , att_{T1} , att_{DWI} , att_{apriori} \right]_i$

Credal segmentation
Segmentation : method

- PZ (axial slices) ≈ catenary

- Catenary = form taken by a cable because of its own mass.
Segmentation : method

- Mathematical model : \((x, y)\) cartesian coordinates

\[ y = \frac{\theta}{2} \left( e^{x/\theta} + e^{-x/\theta} \right) + \text{cst} \]

- Feature (attribute) based on parameter \(\theta\)

\[
\text{Cat}: \mathbb{R}^3 \rightarrow \mathbb{R} \\
(x, y, z) \mapsto \begin{cases} 
+ \frac{1}{\theta^2} & \text{si } y \leq G_y \\
- \frac{1}{\theta^2} & \text{sinon}
\end{cases}
\]
Segmentation : method

- Imperfect data
- Ambiguity and « anomalies »
- Automatic segmentation = a difficult (non trivial) task
Segmentation: method

- Modify Evidential C-means (ECM) proposed by Denœux and Masson\(^6\)
- ECM principle: similar to FCM (or PCM) with belief functions
- Credal partition
  - \( n \) data (pattern) \( x \) to classify in \( C \) clusters: \( \mathcal{X} = \{x_1, \ldots, x_n\}, \Omega = \{\omega_1, \ldots, \omega_K\} \)
  - Each \( x_i \rightarrow \text{bba} m_i(A_j) \) with \( A_j \in 2^\Omega \)
  - Credal partition described by a matrix \( M = (M_{ij}) = m_i(A_j) \)
    \[ 1 \leq i \leq n, 1 \leq j \leq 2^C \]
  - \( \overline{v_j} \) barycenter associated to \( A_j \)
  - Threshold distance to the empty set (\( \emptyset \))
    \[ \delta^2 = \lambda \frac{1}{n.C} \left( \sum_{i=1}^{n} \sum_{j=1}^{C} d_{ij}^2 \right) \]

outliers, noisy data
[Davé, 1991]
Segmentation: method

- Optimization step
  - Find a credal partition $M$ and the set $V$ of barycenters which minimize the following objective function

$$J_{ECM} = \sum_{i=1}^{n} \sum_{A_j \subseteq \Omega, A_j \neq \emptyset} |A_j|^{\alpha} m_{ij}^b d_{ij}^2 + \sum_{i=1}^{n} \delta^2 m_i(\emptyset)^b$$

$$\forall A_j \subseteq \Omega, A_j \neq \emptyset$$
$$\forall 1 \leq i \leq n$$

$$m_{ij} = \frac{|A_j|^{-\alpha/(b-1)} d_{ij}^{-2/(b-1)}}{\sum_{A_k \subseteq \Omega, A_k \neq \emptyset} (|A_k|^{-\alpha/(b-1)} d_{ik}^{-2/(b-1)}) + \delta^{-2/(b-1)}}$$

$$m_i(\emptyset) = 1 - \sum_{A_j \neq \emptyset} m_i(A_j)$$

And:

$$HV = B$$

$$H_{ik} = \sum_{i} \sum_{A_j \supseteq \{\omega_i, \omega_k\}} |A_j|^{a-1} m_i^b(A_j)$$

$$B_{iq} = \sum_{i} x_{iq} \sum_{\omega_l \in A_j} |A_j|^{a-1} m_i^b(A_j)$$

Iterative optimization (like FCM) until convergence of $V$

6: Deneux et Masson, Pattern Recognition, 2008
Segmentation: method

- Adapt Evidential C-means: ECM proposed by Denœux and Masson\textsuperscript{6}
- Segmentation = contextual information (spatial neighborhood)

\[ v_i \leftrightarrow \left[ att_{T2}, att_{T1}, att_{DWI}, att_{apriori} \right]_i \]

\( t=0 \)

- Extract the « beliefs »
- Correct the beliefs

\( t=t+1 \)

- Convergence?
  - no
  - yes

Computation of probabilities

Modified ECM (MECM)

Decision step

63 : Denœux et Masson, Pattern Recognition, 2008
Segmentation : method

- « Back » to image,
- Correct the « belief » about the membership of a voxel according to the beliefs of its neighbors,
- A neighbor influence is inversely proportional to the distance.

\[ m'_i = m_i \oplus m^\alpha_{k_1} \oplus \ldots \oplus m^\alpha_{k_{nc}} \]

: discounted belief masses of the neighbors.
\( \oplus \): combination rule
Segmentation: method

- Construction of a « morphological » feature
  - In line computation, no time consuming,
  - Independent of the learning bias
Segmentation : results

- **Data**
  - Synthetic → Ground truth available
  - Real data
    - Multi-observer variations: Simultaneous Truth And Performance Level Estimator (STAPLE)
    - Groups of patients: criteria = age, prostat volume, tumor

- **Evaluation**
  - ECM,
  - ECM-Cat: ECM with an *a priori* information,
  - MECM: Modified ECM, with *a priori* information.
Segmentation: results

- Truth
- Simulated data
- ECM
- ECM-Cat
- MECM
Segmentation : results

- T1 with diffusion
- DWI à B0
- T2
- inter-expert variation
- STAPLE
- Results with MECM

- DSC > 0.8
- Good precision

8 : Zijdenbos et al., IEEE TMI, 1994
Segmentation : discussion

- Pioneer work: segmentation PZ, CZ in multispectral RMI
- Precision = satisfactory and reproducible
- Performances = no sensitivity with respect to variations of the conditions (groups): p-value > 0.05

- Open questions
  - Other *a priori* model (Probabily map → ProstAtlas©)
  - Reliabilty of RMI sources ?
Tumor detection in the PZ

- Add features associated to the texture\textsuperscript{9}
  - Tumor ~ heterogeneities,
  - Fractal geometry: characterization of the tumors,
  - Need to select texture features.

\textsuperscript{9} Lopes, Betrouni et al., Med. Phy. 2010
Tumor detection in the PZ: method

Texture features issued from fractal geometry
Tumor detection in the PZ : method

- Classical approach: labelling voxels (MECM)
  - Tumor,
  - Normal tissue.

- How many classes\textsuperscript{9,10} of tissues ?
  - 1 class = PZ homogeneous (no tumor)
  - 2 classes = anomalies (suspicious)
  - > 2 classes = analyze each tissue OR decrease the number of classes

\textsuperscript{9} Vannoorenberghe, E. Lefebvre, and O. Colot, 2003
\textsuperscript{10} Capelle-Laize, C. Fernandez-Maloigne, and O. Colot, 2006
Tumor detection in the PZ: method

\[ \text{voxels}\{v_i\} \leftrightarrow \left\{ \text{att}_{DWF}, \text{att}_{DF}, \text{att}_{opt-mBm}, \text{att}_{T2} \right\}_i \]

Initialization: number of classes as much as number of voxels

Evidential k-NN

\begin{align*}
\text{fuse k-NN} & \quad \text{t} = \text{t} + 1 \\
\text{Compute again} & \\
\text{Convergence ?} & \\
\text{yes} & \\
\text{no} & \\
\end{align*}

Analyze the number of classes \( N_c \)

\[ N_c > 2 \quad \text{Reduce (or not!)} \]
\[ N_c = 2 \quad \text{Tumor (?)} \]
\[ N_c = 1 \quad \text{Homogeneous tissue} \]
Tumor detection in the PZ: results

- Data
  - 27 exams of RMI T2-weighted
  - Correlation RMI-histopathology

\[ Sens = \frac{VP}{VP + FN} \quad \text{Spec} = \frac{VN}{VN + FP} \]

- Measures of precision: (specificity, sensitivity)
  - ROC curves:
    - Sens = f(1-Spec)

* : Service d'urologie, Villers et al., Hop. Claude Huriez CHRU Lille
Tumor detection in the PZ: results

MRI

T2 Images

Truth

MECM
Detection of the tumors in the PZ: results with k-NN

- Better specificity, but need to «force» for obtaining two regions
3D view

Transition Zone
Peripheral Zone
Prostate Gland
BFT
applied to color images

Topics:
- Segmentation
- Quantization
BFT applied to color image segmentation

References


Problem

Original image
(subject to be corrupted by noise)
Unsupervised algorithm

- Patterns (vectors): \( \mathcal{X} = \{X_1, \ldots, X_n\} \)
- Initial condition: one vector = one cluster
  \( X_i \in \omega_i \) with \( i = 1, \ldots, n \);
- Initial framework:
  \( \Omega_0 = \{\omega_1, \ldots, \omega_n\} \)
- Initial credal partition:
  \( \mathcal{P}_0 = \{m_1, \ldots, m_n\} \)

Model based distance
Mass modelling

\(X_i\) of \(X\): a pattern

\(m_i\) : associated bba

\(m_i\) is computed with the information given by its \(k\) neighbors

\[
m_i = m_{i1} + \cdots + m_{ik}
\]

Cluster of the neighbors \(j\)

\[
\begin{align*}
    m_{ij}(\{\omega_c\}) &= \alpha \phi(d_{ij}^c) \\
    m_{ij}(\Omega) &= 1 - \alpha \phi(d_{ij}^c)
\end{align*}
\]

masses of the neighbor \(k\)

distance between \(X_i\) and the neighbors \(j\)

cf. T. Denœux
Mass modelling

\[
\begin{align*}
  m_{ij}(\{\omega_c\}) &= \alpha \phi(d_{ij}^c) \\
  m_{ij}(\Omega) &= 1 - \alpha \phi(d_{ij}^c)
\end{align*}
\]

\[m_i = m_{i1} \oplus \cdots \oplus m_{ik}\]

\[
\begin{align*}
  \{m_4(\{\omega_1\}) \\
  m_4(\{\omega_2\}) \\
  m_4(\{\omega_3\}) \\
  m_4(\{\omega_4\}) \\
  m_4(\Omega)
\end{align*}
\]

Decision:

\[
\text{BetP}(\{\omega_c\}) = m_i(\{\omega_c\}) + \frac{m_i(\Omega)}{|S2|}
\]

\[\omega = \arg \max_c \text{BetP}(\{\omega_c\})\]

cf. T. Denœux
Unsupervised algorithm

Initial conditions:
\( \mathcal{X} = \{X_1, \ldots, X_n\} \quad \Omega_0 = \{\omega_1, \ldots, \omega_n\} \quad t = 0 \quad \mathcal{P}_0 \)

\( X_i \in \mathcal{X}; \omega_i \in \Omega_0; |\Omega_0| = n \)

Repeat

for all pattern vector \( X_i \) of \( \mathcal{X} \) do

Compute \( m_i \) using distance-based model

Compute the pignistic function \( BetP_i \) from \( m_i \)

Take a decision using \( R_{BetP} \) on \( \Omega_t \)

end for

Updates:
\( t \leftarrow t + 1 \);
\( \mathcal{P}_t \leftarrow \mathcal{P}_{t+1} \)
\( \Omega_t \leftarrow \Omega_{t+1} \) with \( |\Omega_{t+1}| \leq |\Omega_t| \)

until partition \( \mathcal{P}_t \) is stable

Pignistic decision rule

No changes in the decision step
Unsupervised algorithm

For each iteration → 2 steps

Step #1
- Estimate the credal partition $P$

Step #2
- Decision making considering the present partition $P$ with the pignistic decision rule.
- At the end of the step, destroy the empty clusters.
- Updating the membership to clusters which make the new learning set for the next iteration.

What does the process ?
From the initial partition with a great number of cluster, the algorithm estimates iteratively a new partition of the color space. The number of clusters decreases until it reaches a stable partition.
Taking into account the spatial neighborhood

Each neighbors $q$ of a pixel $P(i,j)$ is considered as an information source $S_q$

$$m_{\alpha_q S_q} (H_n) = \alpha_q m_{S_q} (H_n)$$

$$m \oplus = m \oplus m_{\alpha S_1} \oplus m_{\alpha S_2} \oplus ... \oplus m_{\alpha S_Q}$$

$$\alpha_q = \exp(- (d^{S_q})^2)$$

Discounting the masses according to the distance to the central pixel
Results

(a) Labeled pixels at the initial iteration step. (b) Labeled pixels at the first iteration step. (c) Labeled pixels at the second iteration step. (d) Labeled pixels at the final iteration step.

(a) Doubt degree at the initial iteration step. (b) Doubt degree at the first iteration step. (c) Doubt degree at the second iteration step. (d) Doubt degree at the final iteration step.
Results
BFT
applied to color image quantization

Reference

Color image quantization: why?

- **Color images**
  - Size, format, color space,…
  - Numerous colors

Is it necessary to preserve all the colors from a perceptual point of view?
→ Human eye and noticeable color differences

- **Aim:**
  - Reduce the number of colors of an image
  - Find the most important colors in a color image while preserving the perceptual quality of an image

- **Application**
  - Image compression

Original image of a beautiful girl (148279 colors)

Image of the beautiful girl with 256 colors
Color image quantization: how?

- **Objective:**
  - Reduce the number of colors of an image
  - Find the most important colors in a color image while preserving the perceptual quality of an image

- **The main approaches:**
  1. Splitting approaches

**Splitting algorithms**

- The simplest splitting algorithm: the regular quantization
  - Divide the RGB cube in regular sub-cubes (bins)

- More elaborated methods:
  - estimate the best axis to divide the RGB cube
  - Median-Cut, Octree…
Color image quantization: how?

**Aim:**
- Reduce the number of colors of an image
- Find the **most important** colors in a color image while **preserving** the perceptual quality of an image

**The main approaches:**
1. Splitting approaches
2. Clustering-based approaches

**Clustering-based algorithms**

RGB cube = space of characteristics

1. Find the best partition of the color space
   - C-Means algorithms
   - Fuzzy-C-Means algorithms
   - Genetic algorithms
2. Reduced color palette=
   \{ representants of each cluster\}
Color image quantization: how?

- Another way?
  - A clustering approach based on the TBM use
The unsupervised algorithm

The same as for color image segmentation but with some differences (explained hereafter)
The unsupervised algorithm (Credal-Q)

Initial conditions:
\[ \mathcal{X} = \{X_1, \ldots, X_n\}, \quad \Omega_0 \]
\[ X_i \in \mathcal{X}; \quad \omega_i \in \Omega_0; \quad |\Omega_0| = m \]

Repeat

for all pattern vector \( X_i \)

\begin{align*}
\text{Compute } m_i & \text{ using decision indices } \omega_i \\
\text{Compute the pignistic probability } & m_i^* \\
\text{Take a decision using } & m_i^* \\
\end{align*}

end for

Updates:
\[ t \leftarrow t + 1 : \]
\[ \mathcal{P}_t \leftarrow \mathcal{P}_{t+1} \]
\[ \Omega_t \leftarrow \Omega_{t+1} \text{ with } |\Omega_{t+1}| \leq |\Omega_t| \]

until partition \( \mathcal{P}_t \) is stable

**Problem:**
The final number of clusters is \textit{a priori} unknown

**Our proposition:**
Add a new criterion to converge toward a defined number \( P_d \) of clusters

\( \rightarrow \) Evidential quantization algorithm

No change during decision step
Evidential quantization algorithm

- **Aim**: we want to obtain $P$ clusters
- 4 possible cases at the end of the decision step:
  1. The number of clusters $> P$ $\Rightarrow$ continue
  2. The number of clusters $= P$ $\Rightarrow$ end
  3. The number of clusters $< P$
  4. The partition is stable but the number of clusters $> P$ \{Problem !!\}

- **Case 3 : proposition**
  - Backward step: number of clusters $< P$
    ($|\Omega_{t-1}| = |\Omega|$= the previous number of clusters)
  - Find the best $P$ clusters among the $|\Omega|$ clusters: the **R. rule**
The R* decision rule

- **Basic idea**: select the $P$ most relevant clusters by using the pignistic function:
  - Mean pignistic value associated with a cluster $\omega_j$
    \[
    \overline{BetP}(\omega_j) = \frac{1}{n} \sum_{i=1}^{n} BetP_{X_i}(\omega_j), \quad X_i \in X
    \]
  - Mean pignistic vector
    \[
    \overline{BetP} = \left[ \overline{BetP}(\omega_1), \ldots, \overline{BetP}(\omega_{|\Omega|}) \right]
    \]
The $R_\star$ decision rule

- Order the mean pignistic values

\[
\text{Bet}_P(\omega_{q_1}) \geq \cdots \geq \text{Bet}_P(\omega_{q_P}) \geq \cdots \geq \text{Bet}_P(\omega_{q_{|\Omega|}})
\]

- Select the $P$ clusters associated with the $P$ highest pignistic values

\[\rightarrow \text{New frame of discernment } \Omega' \subset \Omega \text{ where } |\Omega'| = P\]

\[\Omega' = \{\omega_{q_i}, \text{with } i = 1, \ldots, P \text{ and } \omega_{q_i} \in \Omega\}\]

- New decision rule

\[
R_\star(X_i) = \arg \max_{\omega_j \in \Omega'} \text{Bet}_{P_{X_i}}(\omega_j)
\]
Evidential quantization algorithm

- **Aim**: to obtain \( P \) clusters
- 4 possible cases at the end of the decision step
  1. The number of clusters > \( P \) → we continue
  2. The number of clusters = \( P \) → we finish
  3. The number of clusters < \( P \)
  4. The partition is stable but the number of clusters > \( P \)

- **Case 4**: our proposition
  - Idem than case 3 without backward step
  - Find the best \( P \) clusters among the \(|\Omega|\) clusters: the R. rule
Evaluation of the proposed algorithm

- Qualitative evaluation: the quantized image should preserve the visual perception of the original one

- Quantitative criterion: Mean Square Error

\[
MSE = \frac{1}{n} \sum_{i=1}^{n} ((r_i - \hat{r}_i)^2 + (g_i - \hat{g}_i)^2 + (b_i - \hat{b}_i)^2)
\]

where: \( \hat{r}_i, \hat{g}_i, \hat{b}_i \) are the color components of the cluster prototype associated to pixel having components \( r_i, g_i, b_i \)
Results

Original image
183525 colors

(a) original image
(b) $K = 256$
(c) $K = 128$

(d) $K = 64$
(e) $K = 32$
(f) $K = 16$
Results

Original image
230427 colors

(a) original image
(b) $K = 256$
(c) $K = 128$

(d) $K = 64$
(e) $K = 32$
(f) $K = 16$
Quantitative criterion: MSE

<table>
<thead>
<tr>
<th>Table 1: Quantization results - MSE criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>------</td>
</tr>
<tr>
<td>Baboon</td>
</tr>
<tr>
<td>16</td>
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<td>64</td>
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<tr>
<td>128</td>
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<td>256</td>
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</tbody>
</table>

Credal quantization:
- best MSE values
- performance increases when the number of colors is small

Adaptative clustering algorithm

Splitting algorithm
BFT applied to 3D computer generated images

Topics:
- Shape matching
- Indexing
3D-matching method of non-rigid and partially similar models using curve analysis

References


Outline

- Motivation
- Feature point extraction
- Local patch description
- 3D-shape matching using belief function
- Results on Shrec07, Tosca and Sumner Datasets
- Conclusion
Motivation
Objective: Content-based search Analyzing deformations of surfaces

- Which parametrization and representation of surfaces?
- How to compute the dissimilarity between shapes?
SHREC07, Tosca\(^1\) and Sumner\(^2\) Datasets

- Examples of 3D objects

1. [http://tosca.cs.technion.ac.il/](http://tosca.cs.technion.ac.il/)
3. [http://partial.ge.imati.cnr.it/](http://partial.ge.imati.cnr.it/)

○ = topology change
Previous works on 3D object retrieval

Most of the works on 3D retrieval can be categorized in 2 categories:

- **Non-rigid**: view based methods, global descriptors…
- **Rigid**: Graphs, local descriptors…
- **Bag-of-Features (BoF)**
  - Ohbuchi *et al.* SMI 2008,
    - 2D approach, SIFT,
    - MSB, McGill Benchmarks
  - Lian *et al.* SMI 2010,
    - 2D approach, SIFT
    - PSB, NSB, ESB, McGill Benchmarks
  - Bronstein and Kokkinos CVPR 2010,
    - Heat Kernel Signature
    - Shape Google Dataset, Tosca
Proposed approach

1) Feature Detection

2) Patch representation

3) Matching using Belief Function techniques

<table>
<thead>
<tr>
<th>( P_1 )</th>
<th>( m_{P_1}(O_1) )</th>
<th>( m_{P_1}(O_2) )</th>
<th>( \cdots )</th>
<th>( m_{P_1}(O_M) )</th>
<th>( \Omega )</th>
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<tbody>
<tr>
<td>( P_2 )</td>
<td>( m_{P_2}(O_1) )</td>
<td>( m_{P_2}(O_2) )</td>
<td>( \cdots )</td>
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<td>( \cdots )</td>
<td>( m_{P_N}(O_M) )</td>
<td>( \Omega )</td>
</tr>
</tbody>
</table>
Feature point extraction

\[ f_2(v) = d(v, v_2) \]

\[ f_1(v) = d(v, v_1) \]

Example

[Tierny et al., CGF2009]
Feature point extraction
Local patch description

- Surfaces = represented by 3D-curves extracted around feature points.
- 3D object : set of patches
- Patches = a collection of closed curves
- Patches are compared by analyzing their curves.
Local patch description
Distance between Patches

The shapes of any two parts are compared by using their corresponding curves:

\[ D(P_1, P_2) = \int_0^L d(c^1_\lambda, c^2_\lambda) d\lambda. \]
3D-Shape matching using belief functions
Belief functions

Confidence coefficient (based on partial similarity of object [Bronstein, 2008])

\[
m_{P_i}(O_j) = \mu(P_i) \cdot \frac{1 - D_1(P_i, O_j)}{\sum_{l=1}^{M} (1 - D_1(P_i, O_l))}
\]

\[
m_{P_i}(\Omega) = 1 - \mu(P_i)
\]

With:

\[
D_1(P_i, O_j) = \min_{k, P_k \subset O_j} (D(P, P_k))
\]

\[
\mu(P_i) = \exp(-n\text{partiality}(P_i)^2)
\]

Where the \text{partiality} depends on the area of the 3D part

\[
n\text{partiality}(P_i) = \frac{\text{area}(Q) - \text{area}(P_i)}{\text{area}(Q)}
\]
Algorithm

Algorithm 1. 3D-shape search algorithm
Given a 3D-object query $Q$.
1: Extract a set of $N$ feature points $V_i$ from $Q$.
2: for each feature point $V_i$ where $i = 1, \ldots, N$ do
3: Extract a set of closed curves (i.e., part $P_i$).
4: Compute the distance between $P_i$ and all objects in $\Omega$.
5: Set a BBA value for all the objects $m_{P_i}(O_j)$.
6: end for
7: Combine all BBAs $m_{P_i}$ into a new BBA $m_Q$.
8: Compute the pignistic probability induced by $m_Q$.
9: Display 3D objects in the order according to the pignistic probability.
Final matching

- When all patches $P_i$ of the query $Q$ are modeled by their corresponding mass functions, Dempster’s rule of combination is applied in order to get a mass function which measures the amount of belief committed to the assumption: « $Q$ is similar to $O_j$ ».

- Decision making: Pignistic probability
- Retrieval: objects are sorted according to the pignistic probability values.
Shape retrieval results

Matrix of pairwise distances between seven 3D objects

Low (objects are similar)  High (objects are different)
Shape retrieval results

<table>
<thead>
<tr>
<th>Query</th>
<th>Retrieved results</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Query Image" /></td>
<td><img src="image2" alt="Retrieved Results" /></td>
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<tr>
<td><img src="image3" alt="Query Image" /></td>
<td><img src="image4" alt="Retrieved Results" /></td>
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<tr>
<td><img src="image5" alt="Query Image" /></td>
<td><img src="image6" alt="Retrieved Results" /></td>
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</table>

3D images
### Shape retrieval results

<table>
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<tbody>
<tr>
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Results: comparison with some well-known methods

Comparison on TOSCA database
Results: Robustness to query noises
Remark

- Limitation (topological changes)
That’s all folks!