Theoretical simulation and experimental validation of inverse quasi-one-dimensional steady and unsteady glottal flow models

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In physical modeling of phonation, the pressure drop along the glottal constriction is classically assessed with the glottal geometry and the subglottal pressure as known input parameters. Application of physical modeling to study phonation abnormalities and pathologies requires input parameters related to in vivo measurable quantities commonly corresponding to the physical model output parameters. Therefore, the current research presents the inversion of some popular simplified flow models in order to estimate the subglottal pressure, the glottal constriction area, or the separation coefficient inherent to the simplified flow modeling for steady and unsteady flow conditions. The inverse models are firstly validated against direct simulations and secondly against in vitro measurements performed for different configurations of rigid vocal fold replicas mounted in a suitable experimental setup. The influence of the pressure corrections related to viscosity and flow unsteadiness on the flow modeling is quantified. The inversion of one-dimensional glottal flow models including the major viscous effects can predict the main flow quantities with respect to the in vitro measurements. However, the inverse model accuracy is strongly dependent on the pertinence of the direct flow modeling. The choice of the separation coefficient is preponderant to obtain pressure predictions relevant to the experimental data.


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I. INTRODUCTION

Physical modeling of phonation or voiced sound production intends to predict the vocal folds oscillatory behavior in terms of relevant physical and physiological quantities. Physical modeling is, in particular, interesting for the study of irregular phonation patterns or vocal folds dysfunction due to pathology (Kreiman and Gerratt, 2005; Mergell et al., 2000; Wong et al., 1991; Wurzbacher et al., 2006; Zhang and Jiang, 2004). Despite the development of clinical in vivo measurement techniques, the observation and quantification of phonation in either normal or disordered conditions remains a difficult task often depending on invasive or indirect measurement methods (Crane and Boves, 1988; Hertegard and Gauffin, 1995; Qi and Schutte, 2006; Sundberg et al., 1999; Svec et al., 2007). Consequently, the subglottal pressure and the glottal aperture, which are the main input parameters for standard physical phonation models, are difficult to be obtained directly from in vivo measurements. The influence of the input parameter set on the model outcome is often assessed following an analysis-by-synthesis approach and is further compared to in vivo measured quantities (Drioli, 2005; Sciamarella and d’Alessandro, 2004; Wurzbacher et al., 2006). Therefore, the interest of inverting classical physical phonation models is multiple and could, besides a purely scientific interest, lead to the development of noninvasive measurement techniques. At long term, inverse physical models might be validated on in vivo data and eventually be applied in pathological conditions or in favor of advanced voice synthesis. Moreover, in a clinical context, inverse models allow one to account for subject-dependent data. Most of the studies considering inversion of phonation models deal with inverse filtering techniques in order to estimate the glottal volume flow rate from which quantitative parameters describing the glottal source can be derived (Alku et al., 1998; Frohlich et al., 2001; Pelorson, 2001; Price, 1989; Rothenberg and Zahorian, 1977; Shadle et al., 1999). Although inverse filtering is successful for phonation quantification purposes, it is limited to a parametrized voice source description in terms of the estimated volume flow rate. Since inverse filtering does not rely on a physical flow model and often adopts the source-filter model (Fant, 1960) neglecting glottal-supraglottal interaction, it is unable to account for the fluid-structure interaction between the vocal fold tissue and the glottal airflow during phonation. Inverting physical phonation models is interesting to obtain biomechanical data and flow properties relevant to the phonation mechanism. A first attempt to tune the parameters of a physical low dimensional glottal model to the volume flow rate obtained from inverse filtering following an analysis-by-

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synthesis approach is presented in Drioli (2005). Considering
the inversion of biomechanical models, an encouraging re-
result is presented for arteries where tissue characteristics
are deduced from blood flow measurements (Lagrée, 2000). The
current research explores the inversion of simplified quasi-
one-dimensional flow models widely applied in classical di-
rect phonation models (Lous et al., 1998; Pelorson et al., 1994; Ruty et al., 2007). The aimed models are derived from
Bernoulli’s one-dimensional flow equation corrected for vis-
cous effects and flow unsteadiness. The use of simplified
flow theories in order to estimate the important phonation
quantities, as phonation onset threshold pressure and oscilla-
tion frequency, obtained in an in vitro context is discussed in
Ruty et al. (2007) and Van Hirtum et al. (2007). At first,
direct flow models are outlined and inverse models are for-
mulated. Then, theoretical simulations are presented in order
to validate the inverse flow model outcome against the direct
model input. Next, inverse flow models are validated on ex-
perimental in vitro data obtained with rigid glottal replicas
for different glottal constriction shapes and flow conditions.
The inverse modeling performance is discussed with respect
to the accuracy of the direct modeling.

II. DIRECT AND INVERSE FLOW MODELS

Low-order physical phonation models exploit simplified
flow models to describe the glottal airflow and the resulting
pressure forces exerted on the vocal fold tissue. The under-
lying assumptions necessary to use simplified flow models
are briefly outlined in Sec. II A. Next, direct simplified flow
models are formulated in Sec. II B and inverse flow models
are described in Sec. II C.

A. Assumptions and nondimensional numbers

The flow models described in Ruty et al. (2007) account
for severe assumptions on the flow behavior through the
glottal constriction. The assumptions are motivated by a non-
dimensional analysis of the governing Navier–Stokes equa-
tions while accounting for typical values of physiological
geometrical and flow characteristics in case of normal pho-
nation by a male adult (Deverge et al., 2003; Pelorson et al.,
1994; Ruty et al., 2007; Vilain et al., 2004). The main non-
dimensional numbers considered are the geometrical aspect
ratio $r_g = h_g/l_g$, Mach number $M = v/c_0$, Reynolds number
$Re = v h/v$, and Strouhal number $Sr = f L/v$, where $h_g$ denotes
the minimum aperture, $l_g$ is a typical width normal to the
flow direction and to the constriction, $v$ is a characteristic
flow velocity, $c_0 = 350$ m/s is the speed of sound, $h$ is a typi-
cal dimension, $v = 1.5 \times 10^{-5}$ m$^2$/s is the kinematic air vis-
cosity, $L$ is the constriction length in the flow direction, and
$f$ is a characteristic frequency. Typical physiological values
for glottal flow during voice production yields the following
order of magnitudes for the nondimensional numbers:
$r_g \sim O(10^{-1})$, $M^2 \sim O(10^{-4})$, $Re \sim O(10^4)$, $h_g/2
\sim O(10^{-2})$ with $v_g$ as the flow velocity at the minimum
aperture. These typical values allow one to assume the flow as
one dimensional, incompressible, laminar and quasi-steady.
The glottal area, $A(x) = l_g h(x)$, normal to the flow direction $x$,
is assumed to be rectangular with fixed width $l_g$ and glottal

height $h(x)$ which only varies along the flow direction, as
shown in Fig. 1. Although viscous effects can be negligible
for the main flow, viscosity is expected to be important near
the walls and the resulting effects are discussed in Sec. II B 1.

B. Direct simplified flow models

1. Direct flow models description

Under the assumptions of one-dimensional, laminar,
fully inviscid, steady and incompressible flow, the one-
dimensional Bernoulli’s equation can be used to estimate the
pressure distribution along the glottal constriction, where $\rho
= 1.2$ kg/m$^3$ indicates the mean air density. Therefore,
the pressure difference $\Delta P_b(a, b, t) = P_b(t) - P_a(t)$ between
two positions, $a$ and $b$, along the constriction in the $x$ di-

\[
\Delta P_b(a, b, t) = \frac{\Phi^2}{1} \left( \frac{1}{h_b(t)^2} - \frac{1}{h_a(t)^2} \right)
\]

(1)

for a rectangular glottal geometry with area $A(x, t) = l_g h(x, t)$. The quantities $h_a(t)$ and $h_b(t)$ correspond to the
constriction heights at positions $a$ and $b$ in the flow direc-
tion, while $\Phi(t) = v(x, t) A(x, t) = v(x, t) h(x, t) l_g = \text{const}$
denotes the volume flow rate which is assumed to be constant along the
constriction.

In order to predict a pressure drop across the constriction
and hence to be useful, Eq. (1) needs to be corrected to
account for the flow separation and jet formation in the di-

\begin{align*}
&\Delta P_b(a, b, t) = \frac{\Phi^2}{1} \left( \frac{1}{h_b(t)^2} - \frac{1}{h_a(t)^2} \right) \\
&\Delta P_b(a, b, t) = 1 \frac{\Phi^2}{1} \left( \frac{1}{h_b(t)^2} - \frac{1}{h_a(t)^2} \right) \\
&\Delta P_b(a, b, t) = 1 \frac{\Phi^2}{1} \left( \frac{1}{h_b(t)^2} - \frac{1}{h_a(t)^2} \right) \\
&\Delta P_b(a, b, t) = 1 \frac{\Phi^2}{1} \left( \frac{1}{h_b(t)^2} - \frac{1}{h_a(t)^2} \right) \\
&\Delta P_b(a, b, t) = 1 \frac{\Phi^2}{1} \left( \frac{1}{h_b(t)^2} - \frac{1}{h_a(t)^2} \right) \\
&\Delta P_b(a, b, t) = 1 \frac{\Phi^2}{1} \left( \frac{1}{h_b(t)^2} - \frac{1}{h_a(t)^2} \right)
\end{align*}

(1)

ferring part of the constriction downstream of the minimum
aperture $h_s(t)$ (Pelorson et al., 1994). The turbulent jet for-
mation downstream of the separation point is due to very
strong viscous pressure losses and thus cannot be predicted
by the Bernoulli law (Alipour and Scherer, 2006; Grand-
champ et al., 2007). In literature, the area associated with
flow separation $A_s(t)$ is empirically ad hoc chosen as 1.1, 1.2,
or 1.3 times the minimum glottal constriction area $A_g(t)$, i.e.,
$A_s(t) = c_s A_g(t)$ with $c_s = A_s(t)/A_g(t) \geq 1$ the ad hoc separation
coefficient (Deverge et al., 2003; Hofmans et al., 2003; Luc-
ereo, 1999; Pelorson et al., 1994). Accounting for a rectangu-
lar glottal area, the separation criterion becomes $c_s
= h_s(t)/h_s(t) \geq 1$. The separation position and corresponding
height $h_s(t)$ are indicated with $s$ and $h_s(t)$, respectively, in
Fig. 1. Consequently, Eq. (1) only holds down to the separa-
tion point and, therefore, $c_s$ is an important parameter in the
flow model. The pressure in the constriction after the sepa-
ration point is considered to be equal to the downstream
pressure. In addition to the occurrence of flow separation, the

FIG. 1. Schematic representation of the glottal geometry. The $x$ dimension
indicates the flow direction. $0$, $g$, and $s$ indicate the positions of the origin,
minimum aperture, and flow separation along the channel. The corresponding
heights are indicated.
preceding assumption of inviscid flow is also not valid for low Reynolds numbers. This is the case for small constriction heights where viscous effects cannot be neglected (Blevins, 1992; Kundu, 1990). To account for the pressure drop induced by viscous friction along the walls, an additional Poiseuille term \( \Delta P_p(a,b,t) \) can be added to the Bernoulli term (1)

\[
\Delta P_p(a,b,t) = 12 \mu \frac{\Phi}{l_g} \int_a^b \frac{dx}{h(x)^2},
\]

where \( \mu \) is the dynamic viscosity of the fluid and again \( a < b \). The Poiseuille term \( \Delta P_p(a,b,t) \) assumes a nonuniform parabolic two-dimensional velocity profile and therefore presents a viscosity related correction to the one-dimensionality assumed in Eq. (1). Although several in vitro experimental studies confirm the quasisteady approximation made in Eq. (1) (Deverge et al., 2003; Hofmans et al., 2003; Vilain et al., 2004; Zhang et al., 2002), pressure differences induced by flow unsteadiness due to the fluctuations of \( h(x,t) \), related to wall movements and/or \( p_0(t) \) in time, are important for high frequency variations and/or vocal folds wall vibrations involving collision or glottal closure (Deverge et al., 2003; Vilain et al., 2004). The additional pressure loss \( \Delta P_p(a,b,t) \) due to the resulting unsteadiness in the volume airflow \( \Phi(t) \) is expressed as

\[
\Delta P_v(a,b,t) = \frac{\rho}{l_g} \int_a^b \frac{d}{dt} \left( \frac{\Phi(t)}{h(x,t)} \right) dx.
\]

### 2. Direct flow models, input and output

From the pressure terms outlined in Sec. II B 1, several classical direct flow models can be considered. The physical variables defining their input and output, and the inherent model parameters are pointed out. For all direct flow models, the pressure upstream of the constriction is the main driving control parameter. This pressure is labeled \( p_0(t) \) in correspondence with the 0 position indicated in Fig. 1. The downstream pressure at flow separation and beyond is assumed to be equal to the atmospheric pressure \( p_{atm} \) which corresponds to zero since all pressures are expressed relatively to \( p_{atm} \) i.e., \( p(t) = 0 \) Pa. The flow separation position \( x=s \) is determined by the value of the separation coefficient \( c_s \). Since the flow separation position determines the flow model outcome to a large extent, the coefficient \( c_s \) is an important inherent model parameter. Other known inherent model parameters are the required physical constants \( \mu, \rho \), and the glottal width \( l_g \). The constriction geometry, which is characterized by the channel height \( h(x,t) \) illustrated in Fig. 1, is assumed to be known. The pressure distribution along the constriction, i.e., \( p(x,t) \) with \( 0 \leq x \leq s \), is estimated from the pressure difference \( \Delta P(0,x,t) = p_0(t) - p(x,t) \) which can be defined by the addition of the pressure terms (1)–(3). Thus, four direct flow models based on the pressure term combinations presented in Table I are considered: steady Bernoulli model (BS-d), steady Poiseuille model (PS-d), unsteady Bernoulli model (BU-d), and unsteady Poiseuille model (PU-d).

For each of the four flow models, the input parameters are the set \( (p_0(t), h(x,t), c_s(t)) \), where \( p_0(t) \) and \( h(x,t) \) characterize the physical problem and \( c_s(t) \) is a parameter inherent to the chosen modeling approach. The direct flow model output \( (p(x,t), \Phi(t)) \) is obtained in two steps. First, the volume flow rate \( \Phi(t) \) is estimated from the total pressure difference across the constriction \( \Delta P(0,s,t) \). Next, the pressure profile \( p(x,t) \) along the constriction is obtained from the upstream pressure \( p_0(t) \) and the retrieved \( \Phi(t) \) value.

### C. Inverse simplified flow models

In this section, inverse flow models derived from each of the four direct flow models detailed in Sec. II B 2 are formulated. The assumptions discussed in Secs. II A and II B 1 extend naturally to the proposed inverse models. Therefore, the estimation of the physical quantities obtained with the proposed inverse flow models are a priori subjected to the same limitations as their direct counterparts. Further approximations might be introduced due to the applied inversion strategies outlined in Sec. II C 2. The inverse model variables and parameters are discussed in the following section.

### 1. Inverse flow models, input and output

The pressure distribution \( p(x,t) \) is the main output quantity of the direct flow models. In the presented inverse flow models, the role of direct flow model input and output variables are interchanged. The pressure at the minimum constriction height \( p_c(t) \) is therefore considered as the known input quantity of the inverse models. From the direct flow models previously described, three different cases of inverse problems are defined. Firstly, the upstream pressure \( p_0(t) \) is the unknown quantity (inverse model 1). Secondly, the minimum constriction height \( h_g(t) \) is searched (inverse model 2). Thirdly, the flow separation coefficient \( c_s \) is estimated (inverse model 3). Each of the four direct models outlined in Sec. II B 2 can be exploited to resolve the three inversion problems in order to retrieve the quantities of interest, as shown in Table II.

The interest of the first inversion problem, i.e., to retrieve the upstream pressure \( p_0(t) \), is obvious since it is the main driving parameter of the direct phonation flow models determining the total pressure difference across the glottal

### TABLE I. Pressure terms used in the four direct flow models.

<table>
<thead>
<tr>
<th>Flow model</th>
<th>( \Delta P_g )</th>
<th>( \Delta P_p )</th>
<th>( \Delta P_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS-d</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>PS-d</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>BU-d</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>PU-d</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

### TABLE II. Input and output variables of the three inverse problems.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct model (d)</td>
<td>( p_0(t), h_g(t), c_s(t) )</td>
</tr>
<tr>
<td>Inverse problem 1 (inv1)</td>
<td>( p_0(t), h_g(t), c_s(t) )</td>
</tr>
<tr>
<td>Inverse problem 2 (inv2)</td>
<td>( p_0(t), p_c(t), c_s(t) )</td>
</tr>
<tr>
<td>Inverse problem 3 (inv3)</td>
<td>( p_0(t), h_g(t), p_c(t) )</td>
</tr>
</tbody>
</table>
constriction. In human phonation $p_0$ corresponds to the sub-
glottal pressure which is, as outlined in the introduction, very
hard to retrieve from in vivo measurements. The glottal con-
striction geometry $h(x,t)$ is also a main known input param-
eter in direct flow modeling. Assuming a known geometrical
shape for the constriction, the geometry is fully characterized
by the minimum constriction height $h_s(t)$. Therefore, the sec-
ond inversion problem consists in estimating $h_s(t)$. Although $c_s$
is not a physical quantity, this parameter allows to account
for major physical phenomena, i.e., flow separation and jet
formation. Since the value of $c_s$ acts to a large extent on the
accuracy of the direct flow models, it is quite an issue in the
literature (Alipour and Scherer, 2004; Deverge et al., 2003;
Hofmans et al., 2003; Lucero, 1999; Pelorson et al., 1994) and
the third inverse problem consists in estimating this param-
eter. The resulting inverse models and the related inversion
strategies are discussed in the next section.

2. Inverse flow models description and inversion
strategies

The applied inversion strategies are adapted with respect
to the direct quasi-one-dimensional flow model under con-
consideration. In the following, the quantities estimated by
the models are designated by $\hat{\rho}_0(t)$, $\hat{p}_0(t)$, $\hat{h}_s(t)$, $\hat{c}_s(t)$, and $\hat{F}(t)$.

a. Inverse steady Bernoulli model. Inversion of the
steady Bernoulli model (BS-d) given by Eq. (1) is easily
obtained analytically. Firstly, the upstream pressure $\hat{\rho}_0(t)$ is
retrieved as

$$\hat{\rho}_0(p_g(t),h(x,t),c_s(t),t) = \hat{p}_0(t) c_s^2(t) h_s^2(t) - h_0^2 \over h_0^2(c_s^2(t) - 1) . \tag{4}$$

Secondly, the minimum constriction height $\hat{h}_s(t)$ becomes

$$\hat{h}_s(p_g(t),p_0(t),c_s(t),t) = h_0 c_s(t) \sqrt{1 + \frac{p_0(t)}{p_g(t)}(c_s^2(t) - 1)} . \tag{5}$$

Thirdly, the separation coefficient $\hat{c}_s \geq 1$ is given as

$$\hat{c}_s(p_g(t),p_0(t),h_0(t),h_g(t)) = h_0 \sqrt{\frac{p_0(t) - p_g(t)}{p_0(t) h_0^2 - p_g(t) h_g^2(t)}} . \tag{6}$$

Applying the assumption $h_0 \gg h_g$, the expression for $\hat{c}_s$
simplifies to

$$\hat{c}_s(t) = \sqrt{1 - \frac{p_g(t)}{p_0(t)} \over \frac{p_0(t)}{p_g(t)}} = 1 - \hat{c}_s^2(t) , \tag{7}$$

which illustrates that the parameter $c_s$ determines the impor-
tance of the pressure drop at the minimum constriction height relatively to the upstream pressure $p_0$. Equation (7)
also indicates that the steady Bernoulli model can only pre-
dict $\hat{p}_g \leq 0$ since $p_0 \geq 0$ and $c_s \geq 1$.

b. Inverse steady Poiseuille model. Inversion of the
steady Poiseuille model (PS-d) defined by the sum of pres-
sure terms (1) and (2) can not be achieved analytically.
Therefore, a numerical iterative method is applied in order to
invert the model (Kelley, 1995). This way, each of the three
inversion problems becomes a classical minimization prob-
lem. By considering $\hat{p}_g(t)$ as a function of $p_0(t)$, $h_0(t)$, and $c_s(t)$, i.e., $\hat{p}_g(t) = f(p_0(t), h_0(t), c_s(t))$, the three inversion prob-
lems can be rewritten as the solution of the following three
minimization problems:

$$\|p_g(t) - f(\hat{\rho}_0(t), h_0(t), c_s(t))\|_2 < \epsilon , \tag{8}$$

$$\|p_g(t) - f(p_0(t), \hat{h}_s(t), c_s(t))\|_2 < \epsilon , \tag{9}$$

$$\|p_g(t) - f(p_0(t), h_0(t), \hat{c}_s(t))\|_2 < \epsilon , \tag{10}$$

where $\epsilon$ denotes the tolerance of the convergence process.
The minimization problem is solved with the Newton algo-
rithm. The inversion process of the steady Poiseuille model
can be repeated for each time instant independently or ap-
plied to a whole signal in order to obtain the searched quan-
tity at all time instants. The position of the separation point
$x=s$ and, consequently, the constriction height at separation
$h_s$ depends on the minimum constriction height $h_g$ and the
separation coefficient $c_s$ due to the separation criterion $h_s$
$= c_s h_g$. Therefore, the position of the separation point varies
during the convergence process of minimization problems
(9) and (10). The moving separation point introduces nu-
merical discontinuities in the minimization function when a
spatial discretization of the geometry is used. This is avoided
by approximating the integral in Eq. (2) with a Gauss–
Chebychev quadrature (Kincaid and Cheney, 1996)

c. Inverse unsteady Bernoulli model. The three inversion
problems for the unsteady Bernoulli model (BU-d) defined by
the sum of the pressure terms (1) and (3) are solved follow-
ing the same approach as for the PS-d model, i.e., using
numerical iterative methods. As for the steady Poiseuille
model, the spatial discretization of the geometry is again
avoided by approximating the integral in Eq. (3) with a
Gauss–Chebychev quadrature. Contrary to the steady mod-
els, the inversion process is applied to an entire time range in
order to reduce the error propagation on the inverted values,
since due to Eq. (3), the predictions at instant $t=i$ are de-
pendent on the predictions at previous instants $t<i$.

d. Inverse unsteady Poiseuille model. The strategy used
for the inversion of the unsteady Poiseuille model (PU-d) is
similar to the approach outlined for the inversion of the un-
steady Bernoulli model.

III. THEORETICAL STUDY AND DISCUSSION

The accuracy of the inverse models with respect to the-
thoretical simulations of the corresponding direct models is
discussed in this section. The outcome of the direct models
is used as the input of the inverse models in order to re-
estimate the original input variables of the direct models. The
influence of the pressure terms, which are expressed in Eqs.
(1)–(3), on the simulated results is discussed. The glottal
geometry is approximated by the varying channel height $h(x)$
two between two half circles with 1 cm radius, depicted in Fig.
5(d) and the upstream channel height of $h_0=23.6$ mm. The
importance of the flow model parameter $c_s$ related to flow
separation in the diverging downstream part of the con-
striction can be assessed.
flow circulation is assumed to be more difficult so that the volume flow rate predicted by PS-d is slightly below the BS-d estimation. Moreover, even if the volume flow rates predicted by both models are very close, their relative difference increases as the constriction height decreases. Inversion of PS-d is obtained numerically, following the minimization procedure outlined in Sec. II C 2 b. The inverse model accuracy and the required number of iterations rely on the convergence parameter ε in Eqs. (8)–(10). The choice of ε relies on a trade-off between the precision of the inverse model results and the required computation time. By setting $\epsilon = 10^{-8}$ in PS-inv1, PS-inv2, and PS-inv3, the error introduced by the inversion process becomes negligible and the convergence can be obtained within 20 iterations regardless of the searched variable.

B. Unsteady flow conditions

Variations in time of the upstream pressure $p_0(t)$ and/or the minimum constriction height $h_g(t)$ can affect the flow characteristics. Introducing time dependency in the direct and inverse flow model descriptions allows one to account for flow effects due to unsteadiness. As mentioned in Secs. II A and II B 1, the glottal flow is assumed to be quasisteady during normal phonation since typically $Sr = 10^{-2}$ holds (Zhang et al., 2002). During singing, abnormal phonation or in case of vocal fold pathologies, the glottal flow might be characterized by a greater Strouhal number. This can result from an increase in the fundamental frequency $f$ or the vocal folds length $L$ so that unsteadiness effects can become important. Moreover, unsteadiness effects are known to be important during glottal closure (Deverge et al., 2003). In Secs. III B 1 and III B 2, the unsteady Bernoulli and Poiseuille models are considered. The reported validation passes the limits of the physical values encountered during in vivo phonation in order to fully validate the inversion process. Due to unsteadiness, the occurrence of flow separation and jet formation is known to depend on the Reynolds number and the Strouhal number (Sobey, 1983). Classical phonation models assume a constant flow separation coefficient. In the following, the separation coefficient $c_s(t)$ is assumed to be able to vary in time.

1. Unsteady Bernoulli

The discrepancies between $\hat{p}_g(t)$ and $\hat{\Phi}(t)$ predictions obtained with BS-d and BU-d are illustrated in Fig. 3. The simulations are obtained for the round constriction shape depicted in Fig. 5(d). The input signals $p_0(t)$, $h_g(t)$, and $c_s(t)$ are varying in time with frequencies of 300, 500, and 100 Hz, respectively. These frequencies are chosen arbitrarily different in order that effects related to each parameter can be individually identified in the output waveforms. The input signals cannot be compared to those observed in vivo. The variation of $h_g(t)$ corresponds to $Sr = 0.3$ so that unsteadiness has a visible influence on the models predictions. Comparing $\hat{p}_g(t)$ signals, it can be observed that the BS-d estimation is fully determined by $c_s(t)$ contrary to the BU-d estimation which presents amplitude variations due to the dependency on $h_g(t)$ expressed in Eq. (3). Thus, BU-d predicts a greater
pressure drop at the minimum constriction height during the constriction closure. On the contrary, the pressure drop predicted by BU-d when the constriction is more widely open is less important than the one predicted by BS-d. The addition of Eq. (3) in the direct modeling introduces magnitude and phase differences between the volume flow rate signals generated by BS-d and BU-d, as shown in Fig. 3. Inversion of BU-d is obtained numerically as outlined in Sec. II C 2 c. The approximations introduced by the convergence process in the estimated signals \( \hat{p}_0(t), \hat{h}_g(t), \) and \( \hat{c}_g(t) \) can be considered as negligible, as discussed in Sec. III A 2. However, despite the application of the inversion process to an entire signal, the error propagation in time can lead to inaccurate estimations at the end of the signal.

2. Unsteady Poiseuille

In PU-d, two additional pressure terms are in competition for the prediction of the flow characteristics. In Fig. 3, it can be seen that the \( \hat{p}_g(t) \) signals generated by BU-d and PU-d are similar when the constriction is open. However, when the constriction is closing, viscous effects become predominant so that the \( \hat{p}_g(t) \) predicted by PU-d is closer to the prediction of PS-d. Besides, viscous effects seem to be emphasized by the unsteady flow conditions since the maximum pressure peaks observed for PU-d at the constriction closure instant are higher than those observed for PS-d whereas BU-d predicts negative pressures. Concerning the volume flow rate predictions, the Poiseuille pressure term (2) has a minor influence. Thus, in Fig. 3, the \( \Phi(t) \) signals predicted by BS-d and PS-d are quasideposited. As noticed in Sec. III B 1, the addition of Eq. (3) in the direct modeling intro-

FIG. 3. (Color online) Theoretical simulations for unsteady flow conditions with a round constriction. (Three top figures) Input signals of the upstream pressure \( p_0 \), the ratio between the minimum constriction height and the upstream height \( h_g/h_0 \) and the separation coefficient \( c_g \). (Middle) Simulated signals of the ratio between the pressure at the minimum constriction height and the upstream pressure \( p_t/p_0 \). (Bottom) Simulated signals of volume flow rate \( \Phi \). Frequency of \( h_g \) vibration is 500 Hz corresponding to \( Sr \sim 0.3 \). Output signals are simulated by BS-d (thin dashed line), PS-d (thin solid line), BU-d (thick solid line) and PU-d (thick dashed line).


produces magnitude and phase differences between the \( \Phi(t) \) signals predicted by steady and unsteady models so that the BU-d and PU-d predictions are also quasideposited. The considerations about the inversion of BU-d made in Sec. III B 1 hold for the inversion of PU-d.

IV. IN VITRO VALIDATION AND DISCUSSION

The high accuracy of the inverse flow models, described in Sec. II C with respect to data obtained by simulations with the corresponding direct flow models, is pointed out in the previous Sec. III. This section assesses the validation of the inverse flow models against in vitro experimental data. The validity of the inverse flow models for real world flow data is likely to depend on the accuracy of the direct flow models. Therefore, the in vitro validation of the inverse steady and unsteady flow models, presented in, respectively, Secs. IV A and IV B, is inspired on the in vitro validation of flow models with glottal constriction rigid replicas reported in a.o. Deverge et al. (2003); Hofmans et al. (2003); Pelorson et al. (1994); Ruty et al. (2007); Van Hirtum et al. (2007); Vilain et al. (2004).

A. Steady flow conditions

The inverse model validation for steady flow conditions aims to quantify the influence of taking into account the viscosity and of the choice of the separation coefficient on the inverse model performance.

1. Setup for steady flow measurements

The experimental setup is schematically depicted in Fig. 4. Steady flow is provided by a valve controlled air supply [A] connected to a pressure tank of 0.75 m\(^3\) [B] enabling to impose an airflow through the rigid vocal fold replica [D,E]. An upstream pipe [C] of 95 cm is used to prevent from turbulent flow at the replica position. Pressure transducers (Endevco 8507C or Kulite XCS-093) are positioned in pressure taps upstream of the replica [F] and at the minimum constriction height of the constriction [G] allowing to measure the upstream pressure \( p_0 \) and the pressure at the minimum constriction height \( p_x \). The volume flow rate \( \Phi \) is measured (TSI 4000) upstream of the constriction [H]. The in vitro constriction is formed by two vocal fold metal replicas in a fixed position. The minimum constriction height \( h_g \) between the two rigid vocal folds can be changed by means of two adjustment screws. Different minimum constriction heights are studied: \( h_g = 0.2 \) mm, \( h_g = 0.5 \) mm, and \( h_g = 1.0 \) mm. Two different constriction shapes depicted in Fig. 5 are considered, (a) uniform (with a rounded entrance) and
2. Uniform Constriction

A uniform channel is particularly interesting to evaluate the flow modeling with respect to viscous wall effects without interference of the flow separation position. For a uniform constriction, flow separation always occurs at the constriction end so that $h_s = h_c$. This implies that $c_s = 1$ so that $c_s$ is no further considered in this section. Therefore, two inversion problems are maintained in order to retrieve respectively the upstream pressure $\hat{p}_0$ (inv1) and the minimum constriction height $\hat{h}_g$ (inv2). Measurements of $p_0$, $p_g$, and $\Phi$ are presented in Fig. 6. The upstream pressure $p_0$ covers the range of interest from 100 up to 1000 Pa and $h_g$ is set to 0.2 and 0.5 mm. Predictions of $\hat{p}_g$ and $\hat{\Phi}$ given by BS-d and PS-d are shown. Following Eq. (7), BS-d predicts $\hat{p}_g$ to be equal to the downstream pressure, i.e., $\hat{p}_g = p_0 = 0$, regardless of $p_0$ and $h_g$. Therefore, BS-inv1 and BS-inv2 are not applicable and are not further illustrated in this section. It can be observed from Fig. 6(a) that PS-d predicts the measured pressure data with 1% for $h_g = 0.2$ mm. In this case, the gap between the vocal fold replicas is very narrow compared to the upstream height ($h_g / h_0 \approx 10\%$), which indicates that viscous effects are predominant in the pressure determination and allow one to explain the very accurate pressure predictions of PS-d. For $h_g = 0.5$ mm, the prediction error increases from 10% to 40% for $p_0$ increasing from 100 to 1000 Pa. In this case, viscous effects are less important and their approximation by the Poiseuille pressure term (2) does not allow to obtain accurate pressure predictions compared to in vitro measurements. Concerning the volume flow rate, Fig. 6(b) shows that BS-d predicts the measured data more accurately than PS-d for $h_g = 0.2$ mm and $h_g = 0.5$ mm. For the inverse modeling, Fig. 7 presents the relative errors between the measurements and the predictions made by PS-d, PS-inv1, and PS-inv2. This figure illustrates, in a quantitative way, the link between the predictions errors of the direct and inverse models. For $h_g = 0.2$ mm [Fig. 7(a)], PS-inv1 predicts the measured upstream pressure $p_0$ within 1%, which is similar to the PS-d predictions accuracy. On the other hand, the inverse estimation of $h_g$ with PS-inv2 appears to be more sensitive to the error made with PS-d since an error of 1% for $\hat{p}_g$ can yield up to an error of 10% for $\hat{h}_g$; in particular, for low

![FIG. 5. Geometries of the rigid vocal fold replicas. Uniform (a) and round (b) constriction for steady flow measurements. Uniform (c) and round (d) constriction for unsteady flow measurements.](image)

![FIG. 6. (Color online) (a) Ratio between pressure at the minimum constriction height and upstream pressure $p_g/p_0$ and (b) volume flow rate $\Phi$ as function of the upstream pressure $p_0$ measured for steady flow conditions with the uniform constriction, for the minimum constriction heights $h_g = 0.2$ mm (×) and $h_g = 0.5$ mm (+). The corresponding predictions of the Bernoulli (□, ○) and Poiseuille (▶, ◦) models are shown.](image)

![FIG. 7. (Color online) Relative errors of the steady Poiseuille models predictions compared to the steady experimental measurements performed with the uniform constriction for (a) $h_g = 0.2$ mm and (b) $h_g = 0.5$ mm: pressure at the minimum constriction $p_g$ (PS-d, ○), upstream pressure $p_0$ (PS-inv1, ◦) and minimum constriction height $h_g$ (PS-inv2, □).](image)
Reynolds numbers corresponding to low upstream pressures. For $h_g=0.5 \text{ mm}$ [Fig. 7(b)], it can be noticed that the inverse model error rates are proportional to the error rate of the direct model. In this case, the inverted upstream pressure $p_0$ is largely overestimated and the corresponding error reaches more than 100%. The error rate of the PS-inv2 predictions is similar to the one observed for PS-d predictions. Thus, steady Poiseuille models predictions become less accurate as the experimental minimum constriction height and the upstream pressure increase.

3. Round constriction

The influence of the flow separation coefficient can be studied with a round constriction geometry since flow separation occurs in the diverging downstream part. In the quasi-one-dimensional models under study, the flow separation position is determined by the choice of the flow separation coefficient $c_s$. In order to limit the influence of viscous wall effects, the minimum constriction heights of $h_g=0.5 \text{ mm}$ and $h_g=1.0 \text{ mm}$ are experimentally assessed. The BS-d and PS-d predictions of the pressure $p_g$ and the volume flow rate $\Phi$ computed with $c_s=1.2$ are presented in Fig. 8 as well as the in vitro measurements. The value $c_s=1.2$ is commonly found in literature. Therefore, $h_g=c_s h_g$ yields 0.6 and 1.2 mm, respectively. As for the uniform constriction, the PS-d predictions are closer to the experimental pressure data, as shown in Fig. 8(a), even if both direct models overestimate (by a factor of 3–4) the pressure drop at the minimum constriction height. Likewise, the volume flow rate predictions given by BS-d are closer to the experimental measurements than the predictions given by PS-d. However, the difference between the BS-d and PS-d predictions decreases with increasing $h_g$.

FIG. 8. (Color online) (a) Ratio between pressure at the minimum constriction height and upstream pressure $p_g/p_0$ and (b) volume flow rate $\Phi$ as function of the upstream pressure $p_0$ measured for steady flow conditions with the round constriction, for the minimum constriction heights $h_g=0.5 \text{ mm}$ (×) and $h_g=1.0 \text{ mm}$ (+). The corresponding predictions of the Bernoulli (●, ○) and Poiseuille (▵, △) models using $c_s=1.2$ are shown.

BS-d are closer to the experimental measurements than the predictions given by PS-d. However, the difference between the BS-d and PS-d predictions decreases with increasing $h_g$.

Regarding the pressure determination, the model’s accuracy appears to depend mainly on the choice of $c_s$ since the measured pressure ratios $p_g/p_0$ are about −10% and corresponds to a smaller separation coefficient. This is illustrated in Fig. 9, where BS-inv3 and PS-inv3 are applied in order to estimate the separation coefficient $\hat{c}_s$ which best fits the experimental data for both assessed minimum apertures. The separation coefficient estimations are clearly smaller than 1.2 and moreover are found to vary as function of the Reynolds number (Sobey, 1983). Therefore, from Fig. 9, the mean value $c_s=1.06$ seems to be more adapted to the experimental conditions. Figure 10 shows the predictions $\hat{p}_g$ (a) and $\Phi$ (b) given by BS-d and PS-d using this value of $c_s$. Thus, the model’s accuracy is improved for the estimation of $p_g$, but the change in $c_s$ increases the discrepancy between the estimated and measured flow rates.

B. Unsteady flow conditions

The unsteady Bernoulli and Poiseuille models include the unsteadiness pressure term (3). In this section, the inverse model’s validity is tested against in vitro measurements performed on the setup outlined in Sec. IV B 1 for unsteady flow conditions. The results obtained for respectively uniform and round replicas are discussed in Secs. IV B 2 and IV B 3 illustrating both the steady and unsteady model’s predictions.

1. Setup for unsteady flow measurements

The experimental setup used to perform steady flow measurements is maintained to perform unsteady flow measurements. The setup is schematically depicted in Fig. 4 and is described in Sec. IV A 1. The unsteady flow conditions are obtained thanks to a moving rigid constriction replica previously used and described in Deverge et al. (2003); Vilain et al. (2004). Flow variations are generated by the driven movement of one of the rigid vocal fold replicas ([E] in Fig. 4). The frequencies $f$ under consideration are included between 3 and 30 Hz, corresponding to Strouhal numbers in the range of $10^{-3}–10^{-2}$. The resulting time varying constriction height $h_g(t)$ is measured by means of an optical sensor.
(OPB700). Two different constriction shapes depicted in Fig. 5 are considered: (c) uniform (with a rounded entrance) and (d) round.

2. Uniform constriction

As for steady flow conditions, a uniform constriction is assessed in order to rule out the influence of the choice of the separation coefficient on the model outcome since in this case, $c_s = 1$. Therefore, the direct and inverse Bernoulli and Poiseuille models are validated with respect to unsteadiness. Three periods of measured and modeled signals obtained for a driving frequency of 25 Hz are shown in Fig. 11. It can be observed that direct Bernoulli models, BS-d and BU-d, are unable to predict the measured $p_{\lambda}(t)$ so that the corresponding inverse models are not considered in the following. Besides, for the Strouhal number under consideration, $Sr = 10^{-2}$, unsteadiness has a minor impact on the pressure determination so that the $\hat{p}_{\lambda}(t)$ signals given by steady and unsteady models appear quasisuperposed in Fig. 11. Thus, both direct Poiseuille models, PS-d and PU-d, predict the experimental pressure $p_{\lambda}(t)$ within 20%. These two models provide a good approximation of the timing and the amplitude of the $p_{\lambda}(t)$ signal. Therefore, PS-inv2 and PU-inv2 also give accurate results estimating the imposed minimum constriction height $h_{\lambda}(t)$ with a mean error less than 15%. On the contrary, it can be observed that PS-inv1 and PU-inv1 are not able to estimate correctly the input upstream pressure $p_0(t)$. Indeed, the maximum error can reach 100% even if the mean error is less than 40%.

3. Round constriction

Figure 12 shows three periods of the measured and predicted signals for a round constriction vibrating at 25 Hz. Two different constriction shapes depicted in Fig. 5 are considered: (a) uniform (with a rounded entrance) and (d) round.

FIG. 10. (Color online) (a) Ratio between pressure at the minimum constriction height and upstream pressure $p_{\lambda}/p_0$ and (b) volume flow rate $\Phi$ as function of the upstream pressure $p_0$ measured for steady flow conditions with the round constriction, for the minimum constriction heights $h_{\gamma} = 0.5$ mm (×) and $h_{\gamma} = 1.0$ mm (+). The corresponding predictions of the Bernoulli (□, ○) and Poiseuille (✓, Δ) models using $c_s = 1.06$ are shown.

FIG. 11. (Color online) Measurements (●) and models predictions for the uniform vocal folds vibrating at 25 Hz ($S = 10^{-2}$): (top) minimum constriction height $h_{\gamma}$, (middle) upstream pressure $p_0$ and (bottom) pressure at the minimum constriction height $p_{\lambda}$. Predictions are given by the steady Bernoulli (thin dashed line), steady Poiseuille (thin solid line), unsteady Bernoulli (thick solid line) and unsteady Poiseuille (thick dashed line) models.

FIG. 12. (Color online) Measurements (●) and models predictions for the round vocal folds vibrating at 25 Hz ($S = 10^{-2}$): (top) minimum constriction height $h_{\gamma}$, (middle) pressure at the minimum constriction height $p_{\lambda}$ (computed with $c_s = 1.2$) and (bottom) inverted separation coefficient $c_s$. Predictions are given by the steady Bernoulli (thin dashed line), steady Poiseuille (thin solid line), unsteady Bernoulli (thick solid line) and unsteady Poiseuille (thick dashed line) models.
overestimated, resulting in a mean error exceeding 100%. Previously, the severe impact of the choice of the separation coefficient $c_s$ on the predictions accuracy is extensively shown. Therefore, the separation coefficient is estimated from the measured data with the inverse models (inv3). The estimated $c_s$ is illustrated at the bottom of Fig. 12. PS-inv3 and PU-inv3 yield $c_s = 1.08$, whereas for BS-inv3 and BU-inv3, $c_s$ varies between 1.02 and 1.07. Due to the poor qualitative accuracy of BS-d and BU-d and the inconsistent values of $c_s$ given by BS-inv3 and BU-inv3, the value of $c_s = 1.08$ obtained from PS-inv3 and PU-inv3 is used to re-estimate the direct and inverse models. The results are presented in Fig. 13 and show that the accuracy of the four direct models is largely improved. Since BS-d and BU-d are still unable to qualitatively predict $p_g(t)$, only the inverse Poiseuille models are considered to assess $\hat{p}_g(t)$ and $\hat{h}_g(t)$. As the PS-d and PU-d signals are very accurate predicting the measured $p_g(t)$ signal within 5%, the mean error of the PS-inv1 and PU-inv1 predictions is about 5% when $c_s$ is set to 1.08. In this case, the mean error noticed for PS-inv2 and PU-inv2 is about 30%. This increased mean error is indeed due to a severe overestimation (greater than 50%) of the minimum constriction height when this one is large. The error amplification relative to direct Poiseuille models occurs when the viscosity related correction is the least effective, which shows the inaccuracy of the inviscid Bernoulli modeling. As for the uniform constriction, the steady model predictions match the unsteady model predictions to a fair extent in accordance with the low Strouhal number under consideration, $Sr = 10^{-2}$.

V. CONCLUSION

Inverse models derived from quasi-one-dimensional flow models commonly applied in simplified physical phonation models are formulated in order to retrieve the main physical variable and model parameters, i.e., the upstream pressure, the minimum constriction height, or the flow separation coefficient. The accuracy of the inverse models is validated firstly against theoretical simulations obtained with the corresponding direct models and secondly against in vitro experimental data. The proposed inverse models allow to retrieve quasiexactly the original direct model input with a minimum of computational effort. Moreover, the severe influence of flow separation, viscosity, and unsteadiness due to wall movement on the model predictions is pointed out for, respectively, a divergent round constriction shape, small apertures, and high Strouhal numbers. In vitro experimental validation in steady and unsteady flow conditions is achieved on rigid vocal folds replicas with uniform and round constriction shapes in order to study the impact of viscosity and flow separation on the inverse model performance. It appears that both viscosity and the flow separation position determine the relevance of the inverse quasi-one-dimensional models. A mean prediction accuracy of 20% for the searched physical variables can be achieved for a divergent round constriction shape when the flow modeling includes a viscosity related correction and uses a suitable separation coefficient. Remark that the necessity of a corrective term related to viscosity outlines the limitations of the one-dimensional inviscid Bernoulli model. Moreover, the prediction errors increase when the contribution of the viscosity related term to the pressure determination is less important. The performance of the inverse models is seen to reflect the accuracy of the direct models. Therefore, it seems interesting, on the one hand, to include the sensitivity to the input variables errors in the minimization problem and, on the other hand, to use more advanced flow models in order to validate the value of the separation coefficient. Indeed, this parameter is often chosen as a constant but this study confirms that it depends on the Reynolds number so that it should be adapted in accordance with the aimed range of flow conditions relevant to phonation.

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FIG. 13. (Color online) Measurements (•) and models predictions for the round vocal folds vibrating at 25 Hz (Sr = 10^{-2}): (top) minimum constriction height $h_g$, (middle) upstream pressure $p_0$ and (bottom) pressure at the minimum constriction height $p_g$. Predictions are computed with $c_s = 1.08$ and given by the steady Bernoulli (thin dashed line), steady Poiseuille (thin solid line), unsteady Bernoulli (thick solid line) and unsteady Poiseuille (thick dashed line) models.

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