Experimental validation of flow models for a rigid vocal tract replica

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Flow through the vocal tract is studied through an in vitro rigid replica for different geometrical configurations and steady flow conditions with bulk Reynolds numbers $\text{Re} < 15\,000$. The vocal tract geometry is approximated by two consecutive obstacles, representing “tongue” and “tooth,” in a rectangular channel of fixed length. For the upstream tongue obstacle with fixed constriction degree (81%) the streamwise position is varied and for the downstream obstacle the constriction degree is varied from 0% up to 96%. Different upstream pressures are considered for each geometrical configuration. Point pressure measurements at three fixed locations along the channel are experimentally assessed. In addition, the volume airflow rate is measured. The pressure distribution is estimated with a one-dimensional flow model, and the effects of different corrections to a laminar irrotational flow are assessed. The model outcome is validated against experimental data. Depending on the geometrical configuration, the best model accuracy is obtained by accounting for viscosity (needed for constriction degrees at the tooth that are small, i.e., $\leq 58\%$, or very large, i.e., $\geq 96\%$), a sudden constriction (large gap between both constrictions), or a bending geometry (narrow gap between both constrictions). Best overall model errors vary between 4% and 30% for all assessed geometrical configurations in cases where a tongue obstacle is present.

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I. INTRODUCTION

Research on physical modeling of human speech production is mainly concentrated on voiced sound production. In particular, simplified quasi-one-dimensional flow models are commonly used to describe the glottal flow driving the auto-oscillation of the vocal folds (e.g., Ishizaka and Flanagan, 1972; Louis et al., 1998; Ruty et al., 2007). As a consequence, the validation of quasi-one-dimensional flow models is extensively studied on simplified rigid mechanical models of the glottis containing the vocal folds (e.g., Pelorson et al., 1994; Barney et al., 1999; Cisonni et al., 2008). Simplified mechanical geometries with a limited number of geometrical parameters are used to avoid experimental results which can not be reliably interpreted.

Similarly, a necessary step to describe unvoiced fricative sound production is to characterize the flow through the vocal tract downstream the glottis. Aerodynamic and aeroacoustic principles have been introduced in speech production studies dealing with fricatives since Fant (1960). This pioneering work has been further developed by experimental as well as modeling studies (e.g., Shadle, 1985; Sinder, 1999; Adachi and Honda, 2003; Howe and McGowan, 2005; Krane, 2005; Shadle et al., 2008). An extensive and systematic study of flow through the vocal tract downstream of the glottis for configurations relevant for unvoiced sound production is lacking as recently pointed out (Howe and McGowan, 2005; Bodony, 2005). Therefore, a systematic comparison of measurements on simplified mechanical vocal tract models to physical flow models is necessary to gain insight in the observed flow regime. In addition, it is of interest to determine if the simplified quasi-one-dimensional flow model approach, commonly applied to glottal flow, can be extended to model the mean flow behavior in the vocal tract during fricative production.

The underlying mechanism of sibilant fricative sound production is generally described as noise produced due to the interaction of a turbulent jet, issued from a constriction somewhere in the vocal tract, with a downstream wall or obstacle. Consequently, the position and shape of articulators like tongue and teeth determine the generation and development of the jet as well as its downstream interaction with a wall or obstacle as is indeed observed on human speakers (Narayanan et al., 1995; Runte et al., 2001). It follows that experimental and simulation studies have been performed in order to characterize and quantify the influence of “articulators” position and shape on the sound produced (Shadle, 1985, 1991; Ramsay, 2008; Nozaki et al., 2005). Nevertheless, the previous studies focus on the acoustics of fricative noise production and not on the flow. In fact, there are few studies that use flow data to provide a systematic characterization issuing from configurations relevant to human fricative production, i.e., moderate Reynolds $\text{Re}$ numbers covering the range $2000 < \text{Re} < 10^4$ (Stevens, 1998) and low Mach number $M$ (Howe and McGowan, 2005; Bodony, 2005).

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Recently, single sensor anemometry was used to characterize the spatial velocity distribution issued from an extended conical diffuser (Re = 7350) (Van Hirtum et al., 2009b). The data validated self-similar flow models of jet development, which can be applied to model the jet through the constriction between the tongue and the palatal plane, although no obstacle was considered. In addition, flow development through a rectangular channel with a teeth-shaped obstacle inserted was studied by comparing simulated and measured velocity data in the near field downstream of the teeth edge (Re = 4000) (Van Hirtum et al., 2010), although no jet formation upstream of the obstacle was considered. Besides the limitation of Van Hirtum et al. (2009b, 2010) to one Reynolds number, it is evident that no geometry representing jet formation followed by a downstream obstacle was considered. Therefore, a systematic study of flow data for such a geometry typical for sibilant fricative production is lacking.

A simplified rigid mechanical vocal tract replica, characterized by a few geometrical parameters, is described in Shadle (1985, 1991). In the current study, a rectangular rigid mechanical replica, inspired by the one presented in Shadle (1985, 1991), is proposed in order to mimic the vocal tract geometry combining a constriction followed by a downstream obstacle. The replica consists of a constricted portion between the tongue and the “palatal plane” upstream of an obstacle representing a tooth for which the constriction can be varied. Its dimensions are taken to be relevant to the human physiology of an “average” male adult vocal tract (Danillof et al., 1980; Hirano et al., 1987; Narayanan et al., 1995; Runte et al., 2001; Stevens, 1998; Rudolph et al., 1998; Magne et al., 2003). The gap between the constricted vocal tract portion and the obstacle as well as the constriction degree at the obstacle are systematically varied. In addition, the flow conditions are varied so that the relevant range of Reynolds numbers is experimentally assessed. The flow is characterized by measuring the volume flow rate and performing point pressure measurements at different positions along the replica.

These data are compared to the results of one-dimensional flow model approach commonly used to describe the mean flow in physical models of phonation in order to validate the degree to which the models are suitable to describe the flow through the entire upper airway from the larynx up to the lips. Since it is obvious that the flow is too complex to be represented by a laminar flow model, assumed in Bernoulli’s equation, several ad hoc corrections are assessed to describe the influence of vorticity and turbulence on the mean flow. The flow model outcome is particularly validated for (1) variation of the distance between jet and obstacle as well as (2) the constriction degree at the obstacle.

II. ONE-DIMENSIONAL FLOW MODELS

Considering a rectangular channel with two constrictions as schematically shown in Fig. 1, the total pressure difference ΔPtot is given as

\[ \Delta P_{\text{tot}} = \Delta P_1 + \Delta P_2 + \Delta P_3 + \Delta P_4 + \Delta P_5, \]  

with

\[ \Delta P_1 = P(x = 0) - P(x = i_1), \]

\[ \Delta P_2 = P(x = i_1) - P(x = s_{i1}), \]

\[ \Delta P_3 = P(x = s_{i1}) - P(x = i_2), \]

\[ \Delta P_4 = P(x = i_2) - P(x = s_{i2}), \]

\[ \Delta P_5 = P(x = s_{i2}) - P(x = i_3). \]

It is assumed that no pressure loss occurs in the uniform inlet portion so that \( P_0 = P(x = 0) = P(x = i_1) \) and \( \Delta P_1 = 0 \). The pressure losses \( \Delta P_i \) in the remaining portions with varying area \( A_{i,s_i} \), with subscript \( i \) denoting the upstream position and subscript \( s_i \) the downstream position, can be modeled by application of a combination of terms from which the pressure distribution \( p(x) \) follows immediately (Blevins, 1992; Kundu, 1990; White, 1991; Cisonni et al., 2008; Van Hirtum et al., 2009a). In the following the different terms are explained and equations are given. Assuming a simplified one-dimensional quasi-stationary incompressible and irrotational flow described by the stationary Bernoulli’s equation given in (2) and denoted \( \Delta P^\text{ber} \),

\[ \Delta P^\text{ber}_i = \frac{1}{2} \rho \left( \frac{1}{A_{i,s_i}^2} - \frac{1}{A_i^2} \right), \]  

with volume flux \( Q \) and mean air density \( \rho \).

Several corrections to (2) can be considered due to flow separation, viscosity, or downstream pressure recovery. Since steady flow conditions are considered no correction for unsteady flow is necessary. For the geometry shown in Fig. 1, flow separation is assumed to occur at locations \( x_{s_{i1}} \) and \( x_{s_{i2}} \) regardless upstream pressure \( P_0 \) or volume flow rate \( Q \) so that the position of flow separation is fixed and no correction for position change is needed. Viscous losses, on the contrary, are known to be important in case of low Reynolds numbers, i.e., low velocity or small height \( h(x) \). Therefore, the Bernoulli equation is corrected for viscosity by adding a viscous pressure loss term (3) denoted \( \Delta P^\text{visc} \) derived from a fully developed viscous Poiseuille profile as outlined in Appendix A,

\[ \Delta P^\text{visc}_i = Q \frac{-12 \mu}{w} \int_{x_i}^{x_{s_i}} \frac{dx}{h(x)^2}, \]  

FIG. 1. Schematic overview of the geometry: rectangular channel with fixed width \( w \) characterized by height variation \( h(x) \). The unobstructed channel height is denoted \( h_0 \) and two constrictions are inserted spanning the intervals \([i_1, s_{i1}] \) and \([i_2, s_{i2}] \). The x-axis indicates the main flow direction.
with dynamic viscosity coefficient \( \mu \).

So far, pressure recovery by flow reattachment upstream the flow separation point is neglected. In Ishizaka and Flanagan (1972) the pressure recovery is estimated by evaluating the quasi steady momentum equation. The resulting expression (4) describes the pressure recovery as a portion of the Bernoulli loss term (2):

\[
\Delta P_{\text{exp}} = \frac{Q^2}{2A_i} \left[ \frac{A_i^2}{A_{i2}^2} \left( 1 - \frac{A_i}{A_{i2}} \right) \right] + \frac{1}{2} \left( 1 - \frac{A_i}{A_{i2}} \right) .
\]

The magnitude of the recovery depends on the area ratio \( A_i/A_{i2} \), at the position of flow separation \( A_i \) and the expanded area \( A_{i2} \) downstream the constriction. It is clear that (4) assumes a uniform flow profile over area \( A_{i2} \), so that the pressure recovery becomes proportional to \( 1 - (A_i/A_{i2})^2 \). On the other hand zero pressure recovery is expected in case a narrow jet flow is assumed to be maintained, so that \( A_{i2} = A_i \) and the term becomes zero since \( (1 - A_i/A_{i2}) = 0 \), corresponding to not taking Eq. (4) into account.

Alternatively, to the extreme cases of no recovery or uniform flow, an intermediate value for the pressure recovery is expected in case of an expanding jet geometry to which (2) can be applied. A geometrical correction for jet expansion is easily obtained by applying an expansion angle \( \theta_{\text{jet}} \) to the uniform narrow jet as

\[
A_{\text{jet}} = [h_i + C_{\text{jet}} \tan(\theta_{\text{jet}})](x_i - x_i)w,
\]

with expansion angle \( \theta_{\text{jet}} \approx 4.2^\circ \) and model constant \( C_{\text{jet}} \) set to 1 or 2 accounting for one-side or two-side geometrical expansion of a two-dimensional jet (Kundu, 1990; White, 1991). The constricted portion indicated \([s_2, s_i2]\) in Fig. 1 can be seen as a thin square-edged contraction for which separation might occur depending on the Reynolds number at the leading edge, \( x = s_i \), instead of the trailing edge, \( x = s_i2 \). In case separation occurs, the flow through the constriction is accelerated and a pressure loss occurs as reported in (7) where \( C_{\text{con}} \) can be seen as an discharge coefficient whose value can be estimated from geometrical considerations (6), (8) or as an \textit{ad hoc} orifice coefficient (7) Blevins (1992).

\[
\Delta P_{\text{con}} = \frac{Q^2}{2A_{i2}} \left[ \frac{A_{i2}^2}{A_{i}^2} \left( 1 - \frac{A_{i2}}{A_i} \right) \right] + \frac{1}{2} \left( 1 - \frac{A_{i2}}{A_i} \right) .
\]

The relative importance of resulting pressure differences \( \Delta P_i \) normalized with respect to a positive real power \( b > 0 \) of volume flow rate \( Q \),

\[
\frac{\Delta P_i}{Q^b} = f(h_i, h_{ij}) \text{ and } b \in R^+_Q,
\]

with \( R^+_Q = \{ x \in R | x > 0 \} \) is illustrated in Fig. 2 for \( b = 2 \) and \( C_{\text{ben}} = 2.2 \). Two different upstream heights \( h_i \) are illustrated, i.e., \( h_i = 16 \) and \( h_i = 3 \) mm. The values of \( h_i \) and \( h_{ij} \) are chosen in the range of magnitudes relevant for the current study. Illustrated pressure losses are normalized with respect to the bending term (10) for \( h_{ij} = 0.6 \) mm, since \( \Delta P_{\text{ben}}^{h_i} (h_i = 0.6) = e_1 \) with \( e_1 \) a constant value independent of \( h_i \). Consequently the applied normalization is independent of \( h_i \) which favors interpretation and comparison between both assessed \( h_i \) values. Figure 2(a) and 2(b) illustrate the resulting pressure losses in case \( h_{ij} \) is varied in the intervals \([0.6, 3] \) and \([3, 16] \) mm, respectively. For a given volume airflow rate and \( h_{ij} \) in the range \([0.6, 3] \) mm, i.e., \( h_{ij} \neq h_i \), the relative pressure losses obtained from (2), (4), (7), and (10) are highly dependent on \( h_{ij} \) as can be seen from Fig. 2(a).
The relative difference between the terms for a given \( h_{si} \) is seen to decrease from 55% to 5% as \( h_{si} \) increases from 0.6 to 3 mm. As expected, application of (4), (7), or (10) increases the pressure loss compared to the Bernoulli term expressed in (2). In order to consider the different contributions to the pressure losses, all terms are presented in Fig. 2(a) regardless their physical relevance. In particular, the expansion term (4) results in an intermediate pressure loss compared to the other loss terms although of course no geometrical expansion is present since \( h_i > h_{si} \) for \( h_i \in [3 \ 16] \) mm. The relative influence of \( h_i \) for \( h_i > h_{si} \) is seen to be less than 10%, except for the unphysical expansion term for which the difference is less than 20%, for the whole range \([0.6 \ 3]\) mm and is decreasing as the ratio \( h_{si}/h_i \) increases. For \( h_{si} = 3 \) mm, the ratio \( h_{si}/h_i \to 1 \) as \( h_i \to 3 \), and consequently, all terms, except the Bending term, become 0 in this limit. For \( h_i = 16 \) mm, all terms remain > 0.

Pressure differences for \( h_{si} \in [3 \ 16] \) mm are illustrated in Fig. 2(b). As in Fig. 2(a), physically meaningless terms are shown in order to illustrate the model behavior completely, i.e., the expansion term (4) for \( h_i = 16 \) mm and the contraction term (7) for \( h_i = 3 \) mm. Pressure values are normalized with respect to the bending term (10) obtained for \( h_{si} = 3 \) mm, i.e., \( \Delta P^\text{ben}(h_{si} = 3) = c_2 \) with \( c_2 \) a constant value independent of \( h_i \). For \( h_i = 16 \) mm the condition \( h_{si}/h_i < 1 \) is still valid and therefore the observations described on Fig. 2(a) can be extended to the relative pressure losses illustrated in Fig. 2(b). The relative difference decreases as \( h_{si} \) approaches \( h_i = 16 \) mm, in which case all terms except the Bending term (10) go to 0. For \( h_i = 3 \) mm, different observations can be made since \( h_{si}/h_i > 1 \) for \( h_i \in [3 \ 16] \) mm. For \( h_{si}/h_i > 1 \) Bernoulli (2) and Expansion (4) result in pressure recovery with respect to the upstream pressure, i.e., \( \Delta P < 0 \). Furthermore, it is easily derived from (4) that a maximum pressure recovery occurs for \( h_{si} = 2h_i \). This is illustrated in Fig. 2(a) where a maximum for the expansion term is indeed observed at \( h_{si} = 6 \) mm for \( h_i = 3 \) mm. From (3) and (5) it is seen that besides a dependence on \( h(x) \), the pressure difference induced by (3) as well as the importance of the geometrical jet expansion (5) increases as the streamwise extent increases.

III. RIGID IN VITRO REPLICA AND EXPERIMENTAL SETUP

The rigid in vitro replica consists of two constrictions, \( C_1 \) and \( C_2 \), inserted in a uniform rectangular channel as schematically depicted in Fig. 1 and detailed in Fig. 3. The unconstricted channel has length \( L_0 = 180 \) mm, height \( h_0 = 16 \) mm, width \( w = 21 \) mm, and aspect ratio \( w/h_0 = 1.3 \). The shape of both constrictions \( C_1 \) and \( C_2 \) is fixed. Their lengths in the \( x \)-direction yield \( l_1 = 30 \) mm for \( C_1 \) and \( l_2 = 3 \) mm for \( C_2 \). The aperture \( h_i \) is fixed to 3 mm, which corresponds to a constriction degree of 81%. The distance of the trailing edge of \( C_2 \) to the channel exit, \( L_2 \), is fixed to 6 mm. The distance of the trailing edge of \( C_1 \) with respect to the channel exit, \( L_1 \), can be varied as well as aperture height \( h_2 \) of constriction \( C_2 \). Therefore, besides the inlet height \( h_0 \), the

![FIG. 2. (Color online) Relative importance of the pressure difference \( \Delta P/Q = f(h, h_0) \) with \( f \) defined from (2) (\( \times \) Bernoulli), (4) (\( \circ \) expansion), (7) (\( \triangle \) contraction) and (10) (+ bending), respectively, for upstream heights \( h_i = 16 \) mm (no line) and \( h_i = 3 \) mm (full line): a) \( h_{si} \in [0.6 \ 3] \) mm normalized with \( \Delta P^\text{ben}(h_{si} = 0.6) = c_1 \) and b) \( h_{si} \in [3 \ 16] \) mm normalized with \( \Delta P^\text{ben}(h_{si} = 3) = c_2 \).](image)

![FIG. 3. Two dimensional schema of rigid in vitro replica: rectangular channel with fixed unconstricted height \( h_0 = 16 \) mm, uniform width \( w = 21 \) mm and total length \( L_0 = 180 \) mm containing two constrictions \( C_1 \) and \( C_2 \). The main streamwise direction corresponds to the \( x \) axis. The constricted portions \( C_1 \) and \( C_2 \) has streamwise lengths \( l_1 = 30 \) mm and \( l_2 = 3 \) mm, respectively. The minimum aperture height of \( C_1 \) is indicated \( h_1 \) and of \( C_2 \) is denoted \( h_2 \). The distance between the channel exit and the trailing edge of \( C_1 \) and \( C_2 \) is denoted \( L_1 \) and \( L_2 \), respectively. The distance between the trailing edge of \( C_1 \) and the leading edge of \( C_2 \) is denoted \( L \). The geometrical parameters yield \( L_2 = 6 \) mm and \( h_1 = 3 \) mm. The geometrical parameters \( L_1 \) and \( h_2 \) can be varied. Three pressure taps are present at downstream positions \( p_0 = 30 \) mm, \( p_1 = 160 \) mm, and \( p_2 = 173 \) mm.](image)


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TABLE I. Summary of 37 experimentally assessed geometrical conditions illustrated in Fig. 4. All used combinations of \( (L_1, h_2) \) are accounted for by combining 6 values listed for \( L_1 \) in (a) with 6 aperture values \( h_2 \) at the obstacle given in (b). Resulting values for the gap between both constrictions \( L = L_1 - 9 \text{ mm} \) and associated constriction degrees \( [\%] \) due to \( L, 1 - L/h_0, \) and the obstacle aperture \( h_2, 1 - h_2/h_0, \) are indicated. Note that the constriction degree at \( h_1 \) due to \( C_1 \) is fixed to 81% whenever \( C_1 \) is present. Absence of \( C_1 \) is denoted ‘–’ in subtable (a). In addition, to the 36 \( (L_1, h_2) \)-combinations of (a) and (b), a no-front cavity case without obstacle \( (L_1 = 1 \text{ mm}, h_3 = 9 \text{ mm}) \) is considered.

(a) \( L_1 \) and derived variables

<table>
<thead>
<tr>
<th>( L_1 ) [mm]</th>
<th>- (no ( C_1 ))</th>
<th>33</th>
<th>25</th>
<th>19</th>
<th>14</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - ( L/h_0 ) [%]</td>
<td>0 (no ( C_1 ))</td>
<td>0</td>
<td>0</td>
<td>38</td>
<td>69</td>
<td>81</td>
</tr>
</tbody>
</table>

(b) \( h_2 \) and derived variables

<table>
<thead>
<tr>
<th>( h_2 ) [mm]</th>
<th>- (no ( C_2 ))</th>
<th>16</th>
<th>6.8</th>
<th>5.5</th>
<th>2.6</th>
<th>1.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - ( h_2/h_0 ) [%]</td>
<td>0 (no ( C_2 ))</td>
<td>58</td>
<td>66</td>
<td>84</td>
<td>91</td>
<td>96</td>
<td></td>
</tr>
</tbody>
</table>

pressure distribution is determined by the set of geometrical parameters \( \{ h_1, L_1, h_2 \} \) among which \( L_1 \) and \( h_2 \) can be varied. In order to validate the pressure drop, three pressure taps are assessed at positions \( p_0 = 30 \text{ mm}, p_1 = 160 \text{ mm}, \) and \( p_2 = 173 \text{ mm} \) from the channel inlet. The position of the pressure taps is fixed to prevent leakage.

All combinations of \( \{L_1, h_2\} \), for which the values are summarized in Table I, are experimentally assessed. The cases \( h_2 = 16 \text{ mm} \) and \( L_1 = 1 \text{ mm} \) correspond to “no-tooth” configurations. Since the position of the pressure tap \( p_1 \) is fixed, \( p_1 \) is either situated in the gap between \( C_1 \) and \( C_2 \) for large \( L_1 \) or along \( C_1 \) in case \( L_1 \) is small. The successive different downstream positions of \( C_1 \), given in Table I, respectively, correspond to a trailing edge position situated either at 13 and 5 \text{ mm} downstream of \( p_1 \) or at 1, 6, 8, or 19 \text{ mm} upstream of \( p_1 \). An overview of the different trailing edge positions is depicted in Fig. 4. A comparison between experimental replica values and typical values observed on \( \text{in vitro} \) speakers is given in Table II.

Next, the \( \text{in vitro} \) replica is mounted into a suitable experimental setup. Air is supplied by a compressor (Copco GA7). The compressor is connected by a tube with diameter 1 \text{ cm} to a pressure regulator (Norgren type 11-818-987) and a downstream manual valve in order to provide steady flow. A tube with diameter 1 \text{ cm} connects the manual valve to a massflowmeter (TSI 4001), which is connected further downstream to a pressure tank. The upstream pressure \( P_0 \) is connected further downstream to a pressure tank. The upstream pressure \( P_0 \) is 4000 \text{ Pa}. The associated bulk Reynolds numbers, defined as \( \text{Re} = Q/(\mu v) \), are \( 0 < \text{Re} < 15000 \). Figure 5 shows the measured values of tapered with acoustical foam (SE50AL-ML). The \( \text{in vitro} \) replica described before is directly mounted to the pressure tank. The upstream pressure \( P_0 \), the intermediate pressure \( P_1 \), and pressure \( P_2 \) at constriction \( C_2 \), are measured with pressure transducers (Kulite XCS-093) at the pressure taps \( p_0, p_1, \) and \( p_2 \) shown in Figs. 3 and 4. The volume flow rate \( Q \) is measured using the massflowmeter (TSI 4001) upstream of the pressure tank.

IV. DATA AND OPTIMAL DATA APPROXIMATION

Measured point pressures and volume flow rates for imposed flow and geometrical conditions are discussed in Sec. IV A. In Sec. IV B, measured data are analyzed by means of an \( \text{ad hoc} \) estimation of the coefficients for a parametrical function derived from (2). A data-based optimization approach enables us to determine if such a simple mathematical expression is able to explain the data. The resulting error indicates a minimum value and is therefore a reference for the model validation presented in Sec. V. In particular, the influence of the geometrical parameters on the accuracy of predictions with the parametrical function is assessed.

A. Experimental volume flow and pressure data

The geometrical configurations depicted in Fig. 3 and Fig. 4 are assessed for upstream pressures \( P_0 \leq 4000 \text{ Pa} \). The associated bulk Reynolds numbers, defined as \( \text{Re} = Q/(\mu v) \), are \( 0 < \text{Re} < 15000 \). Figure 5 shows the measured values of...
Re and downstream pressures ($P_1, P_2$) as a function of $P_0$ and the geometrical parameters ($L_1, h_2$).

Figure 5(a) illustrates the measured relationship $\text{Re}(P_0)$. For $h_2 = 0.6$ mm corresponding to a constriction degree of 96% the relationship $\text{Re}(P_0)$ is seen to be nearly independent of $L_1$ since the relative difference is less than 5% for all assessed volume airflows. For $h_2 = 1.5$ mm the presence of both constrictions becomes notable since increasing $L_1$ from 12 to 33 mm slightly decreases $P_0$ with 8% and further to 12% in absence of $C_1$. This dependence on $L_1$ is even more important as the aperture $h_2$ is further increased up to 6.8 mm. The relative pressure decrease with increasing $L_1$ from 12 to 33 mm yields 25%, 44%, and 51% for $h_2 = 2.6$, $h_2 = 5.5$, and $h_2 = 6.8$ mm and decreases further to 38%, 78%, and 87% in absence of $C_1$. Consequently, the pressure drop increases when the gap between both constrictions is narrowed indicating that pressure recovery is favored in case of a wide gap between both constrictions.
In absence of $C_2$, i.e., $h_2 = h_0 = 16$ mm, pressure recovery is mainly determined by constriction degree of 81% due to the fixed aperture of $h_1 = 3$ mm. Consequently, varying $L_1$ from 33 to 1 mm results in a fairly constant pressure drop $P_0$, regardless the volume airflow rate. The slight pressure increase, less than 4%, for increasing $L_1$ is the result of a small pressure recovery in the channel. Figure 5(b) illustrates the pressure measured at $p_1$ normalized by the upstream pressure, $r_1/P_0$. As illustrated in Fig. 4, the relative position of the pressure tap $p_1$ with respect to the trailing edge of constriction $C_1$ depends on $L_1$. For $L_1 = 33$ and 25 mm the pressure is measured in the gap between both constrictions at 15 and 7 mm, respectively, downstream the trailing edge. For $L_1 = 19$ mm the pressure is measured 1 mm upstream of the trailing edge, whereas for $L_1 = 14$ and 12 mm the pressure tap corresponds to 4 and 6 mm upstream of the trailing edge. From Fig. 5(b) it is seen that in absence of $C_1$, the pressure ratio $r_1/P_0$ collapses to a single curve, which is independent of both $h_2$ and the volume airflow velocity $Q$. Nevertheless, the pressure loss increases with input pressure up to 30% due to friction since the friction factor is Reynolds number dependent and due to the development of entry flow in the uniform inlet portion of the channel (Van Dyke, 1970; Wilson, 1971; Kapila et al., 1973). In addition, since the aspect ratio $h_0/w = 1.3$ is significantly smaller than 4, three-dimensional flow development is likely to occur (White, 1991; Schlichting and Gersten, 2000). Note that the one-dimensional models given in Sec. II are unable to account for such entry flow effects since all terms become 0, including the range of Reynolds numbers under study. Changing the geometry at the entry of the channel is likely to reduce this pressure loss. Nevertheless, since the loss is independent of $h_2$, it is not an important issue for the present study.

Inserting constriction $C_1$ in absence of constriction $C_2$, i.e., $h_2 = h_0 = 16$ mm, increases the pressure drop compared to the unconfined channel. For $L_1 = 1$ to $L_1 = 33$ mm the pressure tap $p_1$ is situated consecutively along the converging portion of $C_1$, at the minimum constriction and finally downstream $C_1$, so that the associated pressure drop is seen to increase from about 40% up to about 100%, i.e., $r_1/P_0 \approx 0$. The pressure drop, $r_1/P_0$, measured in presence of both constrictions $C_1$ and $C_2$ is intermediate to the previous configurations: a lower limit is reached in absence of $C_1$ and an upper limit in absence of $C_2$. As for $Re(P_0)$ shown in Fig. 5(a), the influence of $L_1$ on pressure $P_1$ is most noticeable for large $h_2 > h_1 = 3$ mm, i.e., 6.8 and 5.5 mm, for which the pressure loss is seen to decrease with 12% or more as the gap $L_1$ becomes wider. In addition, the pressure loss $r_1/P_0$ measured for $h_2 > h_1 = 3$ mm is more pronounced than for smaller $h_2$, i.e., $h_2 \leq h_1 = 3$ mm, for which the pressure loss $r_1/P_0 \geq 0.5$. Consequently, the relative pressure drop $r_1/P_0$ reduces as $h_2$ decreases since the pressure drop across $C_2$ is increasing.

From the previous discussion of measured $r_1/P_0$ values and from the model terms presented in Sec. II, accounting for pressure recovery in the gap between both constrictions is expected to be important for $h_2$ in the range $h_0 > h_2 > h_1$ and much less for $h_2 < h_1$ when regarding the limited influence of $L_1$ for $h_2 < h_1$. As a consequence, the geometrical correction, i.e., interchanging physical height and gap width in the model geometry, in order to describe the flow direction is expected to be relevant for $h_2 < h_1$ and much less for $h_0 > h_2 > h_1$.

Figure 5(c) reports measured pressure losses $P_2/P_0$ observed at pressure tap $p_2$. The pressure drop for $C_2$ is most important for small apertures $h_2$ resulting in negative pressures for $h_2 \leq 1.5$ mm with an order of magnitude about 10% of $P_0$. Nevertheless, the pressure drop is more pronounced for $h_2 = 1.5$ mm than for $h_2 = 0.6$ mm. This might be due to (1) viscosity as seen from (3), (2) the strong asymmetry resulting in a downstream shift of the minimum pressure (Lagré et al., 2007), or (3) a small recirculation zone at the position $p_2$. Varying $L_1$ is seen to influence $P_2/P_0$ in particular for small apertures $h_2 \leq 1.5$ mm for which the presence of $C_1$ is seen to decrease the pressure drop for the assessed flow conditions.

**B. Optimal data approximation**

In this section, the aim is to verify the extent to which the mathematical expression relative to the physical model described in (2) explains the experimental observations. Therefore, the mathematical expression (2) is approximated under the assumptions $A_{ii} \ll A_i$ and $\Delta P = P_{0,E}$ as the following parametrical function:

$$P_{0,E}[z_k(j)] = \sum_{k=1}^{n} a_k z_k(j) \quad x_k \leq 0, \quad a_k \geq 0,$$

with parameters $a_k$ and $x_k$ to be estimated by minimizing the mean square error (see Appendix B). For a fixed value of volume airflow rate $Q$, variables $z_k(j)$ are defined from the $j$th measured values of geometrical experimental parameters as: $z_k(j) = h_2(j)$, $z_2(j) = h_1(j)$, and $z_3(j) = L(j)$. The number of terms $n$ included in the summation varies from 1 to 3 as function of the number of geometrical experimental parameters taken into account. As one can see, for each individual geometrical variable, expression $a_k z_k(j)^{x_k}$ is similar to model (2) under the assumptions $A_{ii} \ll A_i$ and $\Delta P = P_{0,E}$.

From the estimated parameter set $\{a_k, x_k\}_{1 \leq k \leq n}$ obtained by minimizing the mean square error (Appendix B), an estimation $\hat{Q}$ of the measured volume airflow rate $Q$ is obtained as

$$\hat{Q} = \left(\frac{2\bar{a}_1}{\rho}\right)^{-1/\bar{z}_1}.$$

The relative errors of the estimated volume airflow rate $\hat{Q}$, expressed as 100$(\hat{Q}/Q) - 1$, are illustrated in Fig. 6 for $n$ set to 1, 2, and 3. Accounting for geometrical variable $h_1$ in addition to $h_2$ decreases the error percentage below 100% [Fig. 6(b)]. The error percentage is seen to reduce further by about 40% [Fig. 6(e)] when including $L (L = L_1 - 9$ mm) as a geometrical variable. Therefore, accounting for the 3 geometrical variables ($h_2, h_1, L$) enables to get a satisfactory estimation of the volume airflow rate $Q$ by (14).
\[ a_k = -2, \quad \alpha_k = \frac{Q^2 \rho}{2w^2}, \quad 1 \leq k \leq n, \]  

reflect the need to take into account pressure losses or recovery due to, e.g., viscosity, jet formation or reattachment, which are not dealt with in Bernoulli term (2). Including additional pressure losses or recovery terms as expressed in (3)–(10) is validated in Sec. V.

V. VALIDATION OF ONE-DIMENSIONAL FLOW MODELS

The one-dimensional flow models introduced in Sec. II are validated on the measured data. It is sought to determine the model accuracy in terms of the geometrical parameters \((L_1, h_2)\). Therefore, the pressure distribution is estimated from models taking into account different terms, (2)–(10), as discussed in Sec. II. Resulting models \(q\) and their principal features are summarized in Table III. The assessed geometry and the total pressure difference corresponding to the measured upstream pressure, i.e., assuming \(AP = P_0\), are model input parameters from which the volume airflow velocity and pressure distribution along the in vitro replica geometry, parameterized by \((L_1, h_2)\), are estimated.

Model estimations of the volume airflow velocity and of the pressures at the positions of the pressure taps, i.e., \(P_1, P_2\) and \(\dot{Q}\), can be quantitatively compared to experimentally observed values for each set of input parameters \((P_0, L_1, h_2)\) in order to determine the model accuracy. Consequently, the accuracy of the model estimations for \(P_1, P_2\), and \(\dot{Q}\) is sought as function of \((P_0, L_1, h_2)\) for each model \(q\). Relative error functions \(\varepsilon_{1,q}(\dot{P}_m, P_0, L_1, h_2)\), \(\varepsilon_{2,q}(\dot{P}_m, P_0, L_1, h_2)\), \(\varepsilon_{1,q}(\dot{Q}, P_0, L_1, h_2)\), and \(\varepsilon_{2,q}(\dot{Q}, P_0, L_1, h_2)\) are obtained for each model, denoted by superscript \(q\), as

\[ \varepsilon_{1,q}(\dot{P}_m, P_0, L_1, h_2) = \frac{|\dot{P}_m - P_m|}{P_0}, \quad m \in \{1, 2\}, \]  

\[ \varepsilon_{2,q}(\dot{Q}, P_0, L_1, h_2) = \frac{|\dot{Q}_m - \dot{Q}|}{\dot{Q}}, \]  

where as before \(P_0, P_m\), and \(\dot{Q}\) indicate the measured values. An error function \(\varepsilon_{k,q}\) for all \(N_q(L_1, h_2)\) assessed \(P_0\)-values is defined as

\[ \varepsilon_{k,q}(\cdot, L_1, h_2) = \frac{1}{N_q} \sum_{m=1}^{N_q} \left[ \varepsilon_{k,q}(\cdot, P_0, L_1, h_2) \right], \]  

Since the simple mathematical expression (13) is derived on a simple model structure presented in Sec. II, it is expected that discussed models are able to explain the measured data. In addition, this optimization approach provides a minimum mean square error and is therefore a reference for the error range that results from applying the physical model terms described in Sec. II. The differences between estimated parameters \(\alpha_k\) and \(\beta_k\) and the corresponding values identified from Bernoulli term (2) as
whereas (19) and (20): assessed models $q$ is then straightforwardly quantified as the model $q$ minimizing the cost function $J(q)$ as expressed in (19) and

$$J(q, L_1, h_2) = \frac{1}{2} (\bar{\varepsilon}_1(q)(\hat{P}_1, L_1, h_2) + \bar{\varepsilon}_1(q)(\hat{P}_2, L_1, h_2) + \bar{\varepsilon}_2(q)(\hat{Q}_1, L_1, h_2), \text{ (19)}$$

The overall best mean model errors $J(\hat{q}(L_1, h_2))$ are plotted in Fig. 7. Figure 8 depicts the corresponding averaged errors $\bar{\varepsilon}_k(L_1, h_2)$ (18) for $\hat{P}_1, \hat{P}_2$, and $\hat{Q}$. In addition to the error values (18), the error bars in Fig. 8 illustrate the sensitivity of the model accuracy for variations of the upstream pressure $P_0$. In general, the error sensitivity increases as the error values $\bar{\varepsilon}_k(L_1, h_2)$ increases. The overall best mean model error yields $J(\hat{q}(L_1, h_2)) \leq 30\%$ for all $(L_1, h_2)$ except in absence of $C_1$, denoted $L_1 = \text{none}$. In absence of $C_1$, the errors for $h_2 > 1.5$ are significantly larger than in presence of $C_1$, so that the upper limit for the overall mean model error increases to $J(\hat{q}(L_1, h_2)) \leq 50\%$. From Fig. 8 it is observed that in presence of $C_1$ large overall errors $J(\hat{q}(L_1, h_2))$, e.g., $h_2 = 5.5$ compared to $h_2 = 1.5$ mm in Fig. 7, are due to large errors of $\bar{\varepsilon}_1(\hat{P}_1, L_1, h_2)$, and/or $\bar{\varepsilon}_2(\hat{Q}_1, L_1, h_2)$. In absence of $C_1$, the error $\bar{\varepsilon}_1(\hat{P}_1, L_1, h_2)$ is seen to increase as well explaining the increased overall best mean model upper limit of $J(\hat{q}(L_1, h_2)) \leq 50\%$ instead of $J(\hat{q}(L_1, h_2)) \leq 30\%$.

The models resulting in the overall best mean model error $J(\hat{q}(L_1, h_2))$ (19), illustrated in Fig. 7, are summarized in Table IV. From Table IV it is seen that for $h_2 = 16$ mm as well as $h_2 = 0.6$ mm accounting for viscous effects, i.e., $\hat{q} = \text{Visc}$, results in minimal errors $J(\hat{q}(L_1, h_2))$ regardless the value of $L_1$. For intermediate values, $0.6 < h_2 < 16$, the overall best mean model errors $J(\hat{q}(L_1, h_2))$ are obtained for models $\hat{q} = \text{Con}$ or $\hat{q} = \text{Ben}$ depending on $(L_1, h_2)$. It is observed that inserting $L_1$ upstream from $h_2$ and moving it further downstream, i.e., decreasing $L_1$, causes a model shift from $\hat{q} = \text{Con}$ to $\hat{q} = \text{Ben}$. So, in case of a large gap $L_1$ between both constrictions $C_1$ and $C_2$, the narrowed passage at $C_2$ can be modeled as a sudden constriction whereas for smaller $L_1$ the narrowed passage $C_2$ can be approximated as a bend in the geometry. The transition between both model approaches, i.e., constriction $\rightarrow$ bending, is seen to depend on the value of the aperture $h_2$. From Table IV it is seen that in presence of $L_1$ both small ($h_2 \leq 1.5$) and large ($6.8 \leq h_2$)
TABLE IV. Overview of the selected models $\tilde{q}(L_1, L_2)$ resulting in the overall best mean error $J(\tilde{q}, L_1, h_2)$ (19) whose value is plotted in Fig. 7. Models are referred to as outlined in Table III. For completeness also the constriction degree due to $h_2$ [mm], i.e., $\vartheta(L_2) = 1 - h_2/h_0$ [%], and the constriction degree of the gap between both constrictions due to $L_1$ [mm], i.e., $\vartheta(L_1) = 1 - (L_1 - 9)/h_0$ [%], are indicated as well.

<table>
<thead>
<tr>
<th>$\vartheta(L_1)$</th>
<th>$\vartheta(L_2)$</th>
<th>$h_2$</th>
<th>$L_1 = \text{none}$</th>
<th>$L_1 = 33$</th>
<th>$L_1 = 25$</th>
<th>$L_1 = 19$</th>
<th>$L_1 = 14$</th>
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<td>Visc</td>
<td>Visc</td>
<td>Visc</td>
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<td>Ben</td>
<td>Ben</td>
<td>Ben</td>
<td>Ben</td>
<td>Ben</td>
<td>Ben</td>
<td>constriction $\rightarrow$ bending</td>
</tr>
<tr>
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<td>Con</td>
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<td>Con</td>
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<tr>
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<tr>
<td>91</td>
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<td>constriction $\rightarrow$ bending</td>
</tr>
<tr>
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<td>Visc</td>
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<td>Visc</td>
<td>viscosity</td>
</tr>
</tbody>
</table>

$h_2$ values favor $\tilde{q} = \text{Ben}$. For intermediate $h_2$ values ($1.5 < h_2 < 6.8$) decreasing $h_2$ extend the range of $\tilde{q} = \text{Con}$ in terms of decreasing $L_1$. Finally, we note that altering the geometry, e.g., assuming jet expansion (5), does not improve the model accuracy derived as $J(\tilde{q}, L_1, h_2)$.  

VI. CONCLUSION

A rigid in vitro replica is proposed in order to study airflow through the human vocal tract during sibilant fricative production. Two geometrical parameters are studied experimentally: the position of an upstream tongue shaped constriction in the main flow direction ($L_1$) and the constriction degree of a tooth shaped downstream obstacle ($h_2$). The shape of both obstacles is extremely simplified in order to limit the number of geometrical and flow parameters to be taken into account. Obviously, the proposed rigid replica is a severe simplification of real life physiology and several improvements can be proposed concerning (1) the shape of tongue, tooth, or/and tract and (2) extending the number of geometrical parameters.

Point pressure was measured at several streamwise locations and appeared to vary significantly over the range of imposed $L_1$ and $h_2$. In addition, varying $L_1$ while maintaining $h_2$ fixed is seen to influence the pressure at the tooth constriction. Consequently, besides $h_2$ (transverse tooth aperture), $L_1$ (streamwise tongue position) influences the resulting airflow. This is confirmed by fitting the measured pressure drop as function of imposed geometrical parameters since the accuracy of the volume flow rate estimation increases by 40% when the geometrical parameter $L_1$ is taken into account.

Measured pressures and volume airflow rates are compared to the outcome of one-dimensional flow models assuming a laminar incompressible irrotational and one-dimensional flow governed by Bernoulli’s equation to which corrections are applied for viscosity, note that the viscosity corrections are based on Poiseuille’s formula, sudden geometrical expansion, sudden geometrical constriction, and bending. In presence of the tongue shaped constriction, the accuracy for each set of geometrical parameters ($L_1$, $h_2$) expressed as a mean error for all predicted quantities and all imposed upstream pressures yields <30%. The relevance of additional corrections (resulting in the smallest errors) varies as function of ($L_1$, $h_2$). For very small ($<58\%$) or very large ($>96\%$) constriction degrees at the tooth the most accurate model is obtained by accounting for viscosity regardless the value of $L_1$. For intermediate constriction degrees, in the interval [58 96]%, narrowing the gap between both constrictions, i.e., decreasing $L_1$, causes the most accurate model to shift from constriction to bending. Therefore, the geometrical parameter $L_1$, although not explicitly appearing as a parameter in the validated one-dimensional models, does determine the appropriate corrective term for the applied cost function. In addition, it is interesting to note that the model are least accurate for tooth constriction degrees ($\approx 60\%$) for which the influence of $L_1$ on the measured pressures is most significant.

Consequently, one-dimensional flow models can be applied to describe the flow through the vocal tract when accounting for the relevant corrections in order to compensate, based on geometrical considerations, for the non realistic assumption of a laminar and irrotational flow. This way the approach of one-dimensional flow modeling, commonly used in physical phonation models, can be extended to the vocal tract.

Several topics for further research can be formulated. With respect to modeling, more complex flow modeling is motivated in order to describe the influence of the geometrical parameter $L_1$. In addition, further flow and acoustic experimental characterization needs to be assessed either qualitatively (flow visualization) or/and quantitatively (Particle Image Velocimetry, anemometry, microphone).

APPENDIX A: VISCOSITY CORRECTION TERM

Under the assumptions of two-dimensional, steady and parallel flow, i.e., Poiseuille flow between two parallel plates (White, 1991), the viscid Euler equation, describing the relationship between local velocity $u$ and driving pressure $P$, reduces to

$$\mu \frac{d^2 u}{dy^2} = \frac{dP}{dx}. \quad (A1)$$

The local velocity $u(y)$ is then easily derived as function of the pressure difference $dP/dx$ and the height between the...
parallel plates $h(x)$ so that after some calculus the mean velocity $U(x)$ follows

$$U(x) = -\frac{1}{12\mu} \frac{dP}{dx} h(x)^2. \quad (A2)$$

Introducing volume flow rate $Q = U(x)A(x)$ and assuming a rectangular area with constant width $w$, $A(x) = h(x)w$, leads straightforwardly to

$$dP = \frac{12\mu Q}{w} \frac{1}{h(x)^3} dx, \quad (A3)$$

so that Eq. (3) follows immediately:

$$\Delta P = -\frac{12\mu Q}{w} \int_{x_1}^{x_2} \frac{1}{h(x)^3} dx. \quad (A4)$$

**APPENDIX B: PARAMETER ESTIMATION**

The parameters $a_k$ and $z_k$ defined in Sec. IV B are estimated in a least square sense from the experimentally measured $P_0(j)$ data described in Sec. IV A and geometrical variables ($h_2$, $h_1$, $L$) as shown in Sec. III and Fig. 3. Therefore, the criterion is defined as

$$J(\{a_k, z_k\}_{1 \leq k \leq 5}) = \frac{1}{N} \sum_{j=1}^{N} \left| P_0(j) - P_{0,E}(j) \right|^2,$$

with $N$ the number of measured upstream pressures $P_0(j)$ for fixed volume flow rate $Q$. Parameters estimation is thus obtained by

$$\{\hat{a}_k, \hat{z}_k\}_k = \text{arg} \min_{\{a_k, z_k\}} J(\{a_k, z_k\}_{1 \leq k \leq 5}). \quad (B1)$$

The steepest gradient method (Avriel, 2003) is applied to solve the optimization problem resulting in the sought parameter estimation $\{\hat{a}_k, \hat{z}_k\}_{1 \leq k \leq 5} = f[\hat{P}_0(j), \hat{z}_k(j)]_{1 \leq j \leq N}$.

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