

Blind Extraction of Smooth Signals based on a Second-Order Frequency Identification Algorithm

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Abstract—We propose a novel blind source separation method tailored for retrieving baseband signals having different bandwidths. Such a configuration is characterized by the existence of inactive bands in the frequency domain. By exploiting the eigenstructure of the mixtures covariance matrix calculated in these inactive bands, we develop a simple yet efficient extraction procedure that works in an ordered fashion, in which the sources are extracted according to their degree of smoothness. Numerical results attest the viability of the proposal.

Index Terms—Blind source separation, source extraction, silent bands, frequency-domain approach, smooth signals.

I. INTRODUCTION

BLIND source separation (BSS) concerns the retrieval of a set of signals (sources) by considering only mixed versions of these original sources. Typically, a linear mixing model is assumed, i.e. the vector $\mathbf{x}(t) = [x_1(t) \dots x_M(t)]^T$ containing the M mixtures is given by

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t), \quad (1)$$

where $\mathbf{s}(t) = [s_1(t) \dots s_N(t)]^T$ represents the N sources, and $\mathbf{A} \in \mathbb{R}^{M \times N}$ is the mixing matrix. The i -th column of \mathbf{A} is represented by \mathbf{a}_i , i.e. $\mathbf{A} = [\mathbf{a}_1 \dots \mathbf{a}_N]$.

Independent component analysis (ICA) [1], [2] has been widely employed to deal with the mixing model (1) when $M \geq N$. In ICA, which works under the assumption that the sources are statistically mutually independent, one searches for a separating matrix \mathbf{W} that makes the retrieved sources $\mathbf{y}(t) = \mathbf{W}\mathbf{x}(t)$ as independent as possible. Alternatively, in second-order methods, BSS is accomplished by exploiting the time structure of the sources. For instance, when the spectral contents of the sources are different, matrix \mathbf{W} can be estimated by jointly diagonalizing several mixtures covariance matrices calculated at different time lags [3], [4]. In this approach, as well as in ICA, neither the scaling of the sources nor their original order can be identified, i.e. the ideal separating matrix is given by $\mathbf{W} = \mathbf{P}\mathbf{D}\mathbf{A}^{-1}$, where \mathbf{P} is a permutation matrix and \mathbf{D} a diagonal matrix.

More recently, much attention has been given to BSS methods that make use of prior information not present in the basic ICA and second-order methods frameworks. For

instance, sparse component analysis (SCA) works under the assumption that the sources can be represented, possibly in a transformed domain, by sparse signals [5]. Another additional prior information, close to the sparsity, that has been exploited comes from the observation that, in applications such as speech separation, the sources may be inactive in some time windows (silent periods) [6], [7], [8].

In this letter, we also consider inactive sources. However, instead of working in the time domain as done in [6], this assumption is exploited in the frequency domain. This gives us a BSS method able to extract baseband sources in an ordered fashion: the signals having smaller bandwidths are first recovered. The proposed method is particularly interesting for extracting one or a few smooth sources from the observations (this may be interesting, for instance, in chemical sensing [9]). As our approach is founded on the eigenstructure of covariance matrices in the frequency domain, it is denoted Second-Order Frequency Identification (SOFI) algorithm.

II. PROPOSED METHOD

A. Assumptions

The following hypotheses are considered:

- H1) $M \geq N$, i.e. the number of mixtures is greater or equal to the number of sources, which is assumed to be known.
- H2) The sources covariance matrix is given by $R_{\mathbf{s}} = E\{\mathbf{s}(t)\mathbf{s}(t)^T\} = \text{diag}(\sigma_{s_1}^2, \dots, \sigma_{s_N}^2)$, where $\sigma_{s_i}^2$ denotes the variance of the i -th source.
- H3) Each source $s_i(t)$ is a baseband signal with maximum frequency given by B_{s_i} , where $B_{s_1} \neq B_{s_2} \neq \dots \neq B_{s_N}$.

Concerning the hypothesis H1, if $M > N$ then a dimensional reduction, e.g. via principal component analysis (PCA), should be done to obtain $M = N$. Hypothesis H2 assures uncorrelated sources. Finally, hypothesis H3 guarantees sources with different spectral contents, which is a necessary condition for identifiability in methods based on second-order statistics. For the sake of clarity, we also assume that $B_{s_1} < B_{s_2} < \dots < B_{s_N}$. In view of the permutation ambiguity, there is no loss of generality in this additional assumption, and our method works no matter the original order of the sources.

B. The covariance matrix eigenstructure in inactive bands

Eq. (1) also holds in the frequency domain¹, i.e.

$$\mathbf{x}(f) = \mathbf{A}\mathbf{s}(f). \quad (2)$$

¹In this work, the discrete cosine transform (DCT) is used to obtain a frequency domain representation. The DCT has the advantage of being a real-valued transform.

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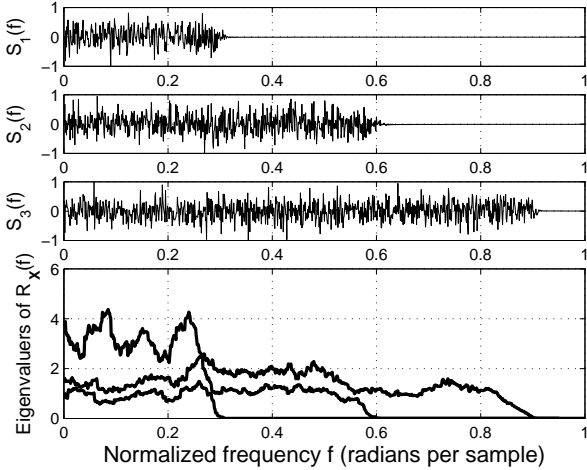


Fig. 1. DCTs of sources (first three signals) and the eigenvalues of $R_{\mathbf{x}}(f)$. The number of null eigenvalues indicates the number of inactive bands.

In view of assumption H3, the baseband sources are inactive in certain frequency bands. For instance, the spectrum of the signal having smaller bandwidth, $s_1(f)$, is inactive beyond the frequency B_{s_1} .

Sources having inactive bands can be detected through second-order statistics [6], [10]. Indeed, let us consider a frequency band, represented by γ , where K signals are inactive, i.e. for $f \in \gamma, \exists \kappa_1, \dots, \kappa_K / s_{\kappa_1}(f) = \dots = s_{\kappa_K}(f) = 0$. In this case, the mixing model becomes overdetermined (N mixtures and $N - K$ sources) and, thus, the covariance matrix of the mixtures in the band² γ , $R_{\mathbf{x}}(\gamma)$, becomes rank deficient. The key point here is that the number of null eigenvalues corresponds to the number of inactive sources, i.e. it is possible to search, in a blind fashion, all the inactive bands by analyzing the profile of the eigenvalues of $R_{\mathbf{x}}(f)$ for all frequencies. This is illustrated in Fig. 1 which shows the DCT of three baseband sources³ and the eigenvalues of $R_{\mathbf{x}}(f)$.

In practice, the identification of inactive bands is done by searching for the eigenvalues that are smaller than a pre-defined threshold ϕ . However, suppose that one of the sources, say s_1 , is much less powerful than the other ones. In this case, even when all the sources are active, the eigenvalue of $R_{\mathbf{x}}(f)$ associated with s_1 is much smaller than the other eigenvalues, which may lead to an incorrect inactive period detection. As shown in [10], [6], this problem can be mitigated by replacing the ordinary eigenvalue decomposition by the generalized eigenvector decomposition (GEVD) of the matrices $(R_{\mathbf{x}}(f), R_{\mathbf{x}}([0, 1]))$. As a corollary of the result shown in [10], the number of null generalized eigenvalues associated with the GEVD of $(R_{\mathbf{x}}(f), R_{\mathbf{x}}([0, 1]))$ also gives the number of inactive sources in the frequency window (size W) centered at f .

²The following notation is adopted: given a vector $\mathbf{x}(f)$ of signals in a frequency representation, $R_{\mathbf{x}}([B_{x_1}, B_{x_2}])$ and $R_{\mathbf{x}}(f)$ denote, respectively, the covariance matrix of $\mathbf{x}(f)$ calculated in the band $[B_{x_1}, B_{x_2}]$ and in a frequency window of size W centered at f .

³We consider the normalized frequency where $B = 1$ corresponds, in the analog domain, to $F_s/2$ (F_s being the sampling frequency).

C. SOFI algorithm

In this section, we discuss how the SOFI algorithm makes use of the particular eigenstructure of $R_{\mathbf{x}}(f)$ for sequentially extracting the sources. A first step in this context is to estimate, through a sliding-window of size W , the covariance matrix $R_{\mathbf{x}}(f)$ for all frequencies. Then, as discussed before, the generalized eigenvalues of $(R_{\mathbf{x}}(f), R_{\mathbf{x}}([0, 1]))$ can be used to identify the inactive frequency bands. In practical terms, this is done by comparing the generalized eigenvalues at a given frequency f with a pre-established threshold ϕ .

Based on the information brought by the generalized eigenvalues, one can identify, for instance, the frequency interval $f \in [B_{s_1}, B_{s_2}]$, in which only $s_1(f)$ is inactive. The key point here is that the generalized eigenvector \mathbf{v}_1 associated with the unique null generalized eigenvalue of $(R_{\mathbf{x}}([B_{s_1}, B_{s_2}]), R_{\mathbf{x}}([0, 1]))$ is orthogonal to all the columns of \mathbf{A} except \mathbf{a}_1 . Indeed, in this situation one has

$$R_{\mathbf{x}}([B_{s_1}, B_{s_2}])\mathbf{v}_1 = \mathbf{0}. \quad (3)$$

As the sources are uncorrelated (H2) and $s_1(f)$ is inactive in the interval $[B_{s_1}, B_{s_2}]$ (H3), one can write⁴

$$R_{\mathbf{x}}([B_{s_1}, B_{s_2}]) = \mathbf{A}R_s([B_{s_1}, B_{s_2}])\mathbf{A}^T = [\mathbf{a}_2 \dots \mathbf{a}_N] \text{diag}(\sigma_{s_2}^2(f), \dots, \sigma_{s_N}^2(f)) [\mathbf{a}_2 \dots \mathbf{a}_N]^T. \quad (4)$$

By substituting Eq. (4) into Eq. (3), one readily obtains $\mathbf{v}_1^T \mathbf{A} = [\alpha \ 0 \dots 0]$, where $\alpha \neq 0$, which means that \mathbf{v}_1 can be used to extract the smoothest source $s_1(f)$.

Even though the idea described above aims at the extraction of the smoothest source, i.e. the one having bandlimited in B_{s_1} , it can also be used for recovering the remaining sources. This can be achieved through a deflation procedure (see [11] for instance), in which the goal is to eliminate the contribution of the estimated source $\hat{s}_1(f) = \mathbf{v}_1^T \mathbf{x}(f)$ to $\mathbf{x}(f)$. In mathematical terms, this procedure is given by

$$\mathbf{x}(f) \leftarrow \mathbf{x}(f) - \mathbf{h}_1 \hat{s}_1(f) \quad (5)$$

where $\mathbf{h}_1 = E\{\mathbf{x}(f)\hat{s}_1(f)\}/E\{\hat{s}_1^2(f)\}$ (this vector minimizes $E\{(\mathbf{x}(f) - \mathbf{h}_1 \hat{s}_1(f))^2\}$).

As the outcome of the first deflation is a BSS problem with N mixtures and $N - 1$ sources, we reduce the dimension of $\mathbf{x}(f)$ via PCA to obtain a $(N - 1) \times (N - 1)$ model. After that, we retrieve a similar scenario to the one that we had before the first GEVD, but now $s_2(f)$ appears as the smoothest source. Therefore, this signal can be estimated through the GEVD of $R_{\mathbf{x}}([B_{s_2}, B_{s_3}])$ and $R_{\mathbf{x}}([0, 1])$. Finally the remaining sources can be extracted by repeating the same steps described so far, as detailed in Algorithm 1.

Some remarks on the SOFI algorithm. First, if the extraction of only a few smooth sources is envisaged, then there is no need to estimate the eigenvalues profile for all frequencies: one may stop when the number of inactive bands is equal to the number of sources to be extracted. Second, even when H3 is only approximated, the SOFI algorithm can recover all the sources except those having the same bandwidth.

⁴Without loss of generality, $s_2(f), \dots, s_N(f)$ are supposed centered here.

Algorithm 1 SOFI algorithm

- 1: **if** $M > N$, reduce the dimension of $\mathbf{x}(t)$ to N via PCA
end if
 - 2: Calculate a frequency representation of the mixtures $\Rightarrow x_i(f) = DCT(x_i(t))$, for $i = 1, \dots, N$
 - 3: For all frequencies f , estimate $R_{\mathbf{x}}(f)$ (covariance matrix calculated in a frequency window of size W and centered in f) and compute the GEVD of $(R_{\mathbf{x}}(f), R_{\mathbf{x}}([0, 1]))$
 - 4: Based on the number of generalized eigenvalues smaller than ϕ for each frequency, identify the frequency bands $[0, B_{s_1}], [B_{s_1}, B_{s_2}], \dots, [B_{s_N}, 1]$
 - 5: **for** $i = 1$ to $N - 1$ **do**
 - 6: Compute GEVD($R_{\mathbf{x}}([B_{s_i}, B_{s_{i+1}}]), R_{\mathbf{x}}([0, 1])$) $\Rightarrow \mathbf{v}_i$ is the generalized eigenvector associated with the smallest generalized eigenvalue.
 - 7: Estimation of $s_i(f) \Rightarrow \hat{s}_i(f) = \mathbf{v}_i^T \mathbf{x}(f)$
 - 8: Deflation step \Rightarrow Eq. (5)
 - 9: Reduce the dimension of $\mathbf{x}(f)$ to $N - i$ through PCA
 - 10: **end for**
-

III. EXPERIMENTS

Before describing the experiments⁵, let us discuss the selection of W and ϕ . W acts as a sort of frequency resolution in the sense that a small W allows the separation of sources having close bandwidths. Of course, too small a W means that only few samples are used in the estimation of $R_{\mathbf{x}}(f)$, i.e. there is a tradeoff between frequency resolution and estimation accuracy. In the experiments provided in this section, we checked that a good empirical strategy to set W is to consider about 5% of the total number of samples.

Ideally, the smaller the threshold ϕ , the better the silent periods detection is and, consequently, the better the performance is. This was actually observed in experiments with noiseless models. For example, in Fig. 2, we plot the index SIR_1 as a function of ϕ (we considered here the extraction of one source from three mixtures). Note, however, that the influence of ϕ is minimal when $\phi < 0.1$. In noisy scenarios, the definition of ϕ is more tricky and requires a visual inspection of the eigenvalues profile (see Sec. III-C)

A. Source Separation

We here consider the separation of the sources shown in the first column of Fig. 3: an exponential signal and three signals obtained from low-pass filtering of white Gaussian noise; the respective bandwidths are given by $B_{s_1} = 0.2$, $B_{s_2} = 0.5$, and $B_{s_3} = 0.8$. The mixtures are shown in the second column of Fig. 3 (2000 samples were considered). The third column of Fig. 3 presents the signals recovered by the SOFI algorithm ($W = 71$ and $\phi = 0.001$). The original order of extraction is kept in Fig. 3; note that SOFI indeed ranks the components according to their smoothness. The performance indices in this case were: $SIR_1 = 63$ dB,

⁵The performance is assessed by the signal-to-interference ratio: $SIR_i = 10 \log \left(\frac{E\{s_i(t)^2\}}{E\{(s_i(t) - \hat{s}_{p(i)}(t))^2\}} \right)$, where $s_i(t)$ and $\hat{s}_{p(i)}(t)$ are, respectively, the original source and its corresponding estimation after scale normalization.

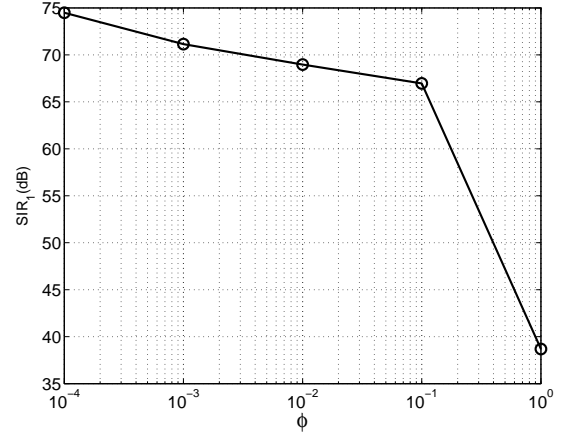


Fig. 2. Performance as a function of the threshold ϕ .

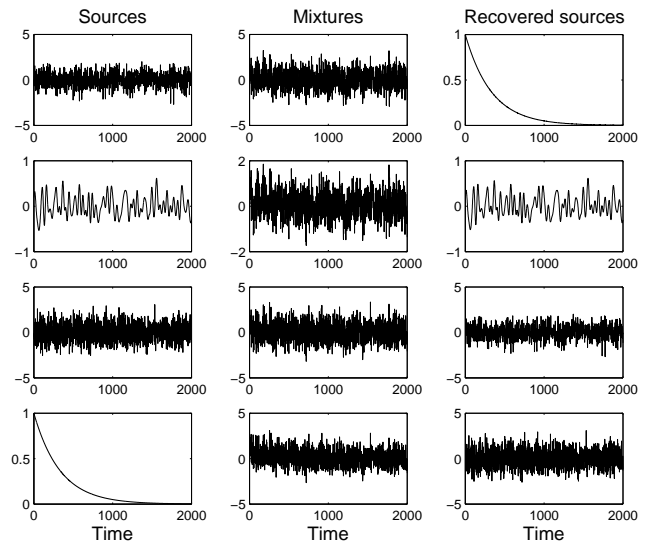


Fig. 3. Example of source separation: sources (first column), mixtures (second column) and retrieved signals (third column).

$SIR_2 = 30$ dB, $SIR_3 = 30$ dB, and $SIR_4 = 27$ dB. Note that the SIR decreases as the extraction procedure progresses; this is typical in deflation-based approaches and is due to the accumulation of errors from the precedent iterations.

B. Extraction of a smooth signal from a large number of mixtures

We consider the extraction of an exponential signal from N mixtures of N sources having bandwidths between 0.4 and 0.9. As shown in Tab. I, the SOFI algorithm (with $\phi = 0.0001$ and $W = 101$) has led to high SIRs, even for a large N . We also show in this table the performances achieved by the SOBI [3] and FastICA [2] algorithms⁶. As these two methods do not rank the sources, the exponential source was obtained by analyzing each retrieved signal⁷.

⁶The FastICA did not converge for $N = 50$ and $N = 70$.

⁷[12] proposed a method to extract smooth sources based on the FastICA. As the idea was to force the first component to be the smoothest one, the performance of this constrained FastICA approach is equivalent to that of the ordinary FastICA (symmetric orthogonalization version).

TABLE I

EXTRACTION OF THE SMOOTHEST SIGNAL: SIR (dB) FOR DIFFERENT NUMBER OF SOURCES N (AVERAGE OVER 50 EXPERIMENTS).

	$N = 2$	$N = 10$	$N = 20$	$N = 50$	$N = 70$
SOFI	86.7	72.5	68.7	63.2	55.5
SOBI	61.0	43.7	40.4	36.3	34.9
FastICA	39.2	23.7	19.6	—	—

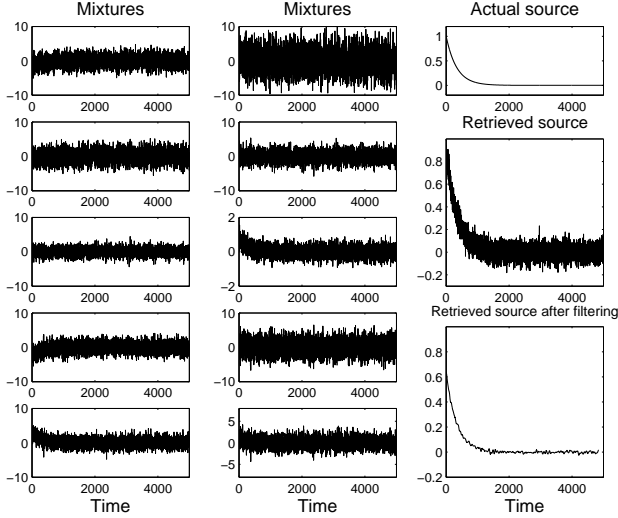


Fig. 4. Source extraction: mixtures (first and second columns). Third column contains: actual source, its estimation, and a filtered version of this estimation.

C. Source extraction in a noisy situation

We here consider the extraction of the exponential depicted in Fig. 4 (first row third column) from $M = 10$ mixtures (first two columns of Fig. 4) of $N = 5$ sources (signals of bandwidths $B_{s_2} = 0.4$, $B_{s_3} = 0.6$, $B_{s_4} = 0.8$ and $B_{s_5} = 0.9$). Each mixture was corrupted by additive white Gaussian noise of signal-to-noise ratio (SNR) equal to 15 dB. Regardless of noise, it is still possible⁸ to identify the inactive bands; by inspecting Fig. 5 it is clear that the inactive bands can be identified by looking at the eigenvalues lower than approximately 0.1 (this value is thus attributed to the threshold ϕ). The first inactive band is thus the one in which there is only one eigenvalue lower than ϕ ($\hat{B}_{s_1} = 0.03$ and $\hat{B}_{s_2} = 0.43$ in this case). The SOFI algorithm ($W = 301$) provided the second signal (SIR₁ = 10.4 dB) of the third column in Fig. 4. Note that the estimated bandwidths can be used to improve the extracted source. Indeed, after low-pass filtering (stopband at $\hat{B}_{s_1} = 0.03$), we obtained SIR₁ = 23.9 dB (see the third column, last row of Fig. 4). For matter of comparison, the performance of the SOBI algorithm was SIR₁ = 10.4 dB.

Finally, we consider the same situation described above but now with a noise of SNR = 10 dB. Again, based on a visual inspection of the eigenvalues profile, we set $\phi = 0.4$. The performance obtained by the SOFI algorithm was SIR₁ = 6.0 dB, and SIR₁ = 22.6 dB after low-pass filtering at the estimated $\hat{B}_{s_1} = 0.013$. A similar performance of the SOBI algorithm was obtained (SIR₁ = 6.0 dB).

⁸This is due to the GEVD; if the ordinary EVD were considered, the noise terms would be taken as inactive sources.

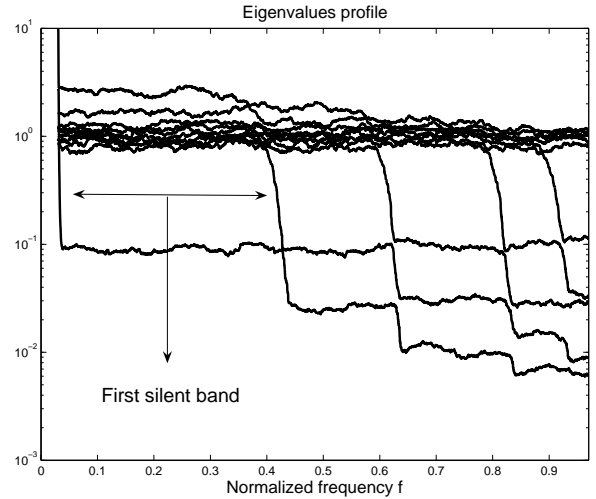


Fig. 5. Generalized eigenvalues of $(R_x(f), R_x([0, 1]))$ in a noisy scenario.

IV. CONCLUSION

We introduced in this letter the SOFI algorithm, a BSS method tailored for separating baseband signals. By exploiting the existence of inactive bands, we developed a simple algorithm that is based on second-order statistics and on a deflation procedure. As it could be checked in simulations, the proposed method performs well even in the presence of noise; moreover, it outperformed standard BSS algorithms in the problem of extracting a smooth signal from a great number of mixtures. Future works include the derivation of an automatic strategy for adjusting the parameter ϕ in noisy scenarios.

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