Combining Stepwise Uncertainty Reduction and Functional Data Reduction for robust inversion

M.R. El Amri[1,3], **C. Prieur**[1], C. Helbert[2], D. Sinoquet[3], M. Munoz Zuniga[3], and O. Lepreux[3]

[1]Université Grenoble Alpes, Laboratoire Jean Kuntzmann, Inria project/team AIRSEA
[2]Université Lyon 1, Ecole Centrale de Lyon
[3]IFP Energies nouvelles

In the present talk (see also [2]), we consider a system evolving in an uncertain environment. That system is modeled by a numerical simulator f, whose inputs are of two types: a set of control variables $x \in \mathbb{X}$, and a set of uncertain variables $v \in \mathcal{V}$. More precisely, $f : \mathbb{X} \times \mathcal{V} \to \mathbb{R}_+$. Robust inversion consists in seeking the set of control variables $x \in \mathbb{X}$ such that $\sup_{v \in \mathcal{V}} f(x,v)$ is bounded by a prescribed threshold $c > 0$. Then, the difficulty of solving the robust inversion problem strongly depends on the uncertainty set $\mathcal{V}$. In our framework, the uncertainty set $\mathcal{V}$ is a functional space. We also assume that the uncertainty has a probabilistic description: the uncertainty is modeled by a random variable V valued in $\mathcal{V}$. We then consider the following stochastic inversion problem: we are seeking the set $\Gamma^* := \{x \in \mathbb{X},\ g(x) := \mathbb{E}_V[f(x,V)] \leq c\}$. In our setting, the probability distribution of V is only known from a set of M realizations $\{v_1, \ldots, v_M\}$.

A Stepwise Uncertainty Reduction (SUR) strategy aims at constructing a sequence $x_1, x_2, \ldots$ of evaluation points of g in such a way that the residual uncertainty about Γ^* given the information provided by the evaluation results is small. More precisely, SUR strategies are based on three main ideas [1]. The first (Bayesian) idea is to consider g as a sample path of a random process, which is assumed Gaussian for the sake of tractability. Doing so entails that any quantity depending on g is formally a random variable. The second idea is to introduce a measure of the uncertainty about the quantity of interest conditioned on the σ-algebra $\mathcal{A}_n$ generated by $\{(x_i, g(x_i)), 1 \leq i \leq n\}$. We will denote by $H_n(g)$ such a measure of uncertainty, which is an $\mathcal{A}_n$-measurable random variable. In the context of robust inversion, its choice is based on the theory of random closed sets [3]. The third idea is to choose evaluation points sequentially in order to minimize at each step n the expected value of the future uncertainty $H_{n+1}(g)$ with respect to the random outcome of the new evaluation of g:

$$x_{n+1} = \arg\min_{x \in \mathbb{X}} J_n(x) := \mathbb{E}_n\left(H_{n+1}(g) \,|\, x_{n+1} = x\right)$$

where $\mathbb{E}_n(\cdot)$ stands for the conditional expectation $\mathbb{E}(\cdot|\mathcal{A}_n)$.

The key contribution of the present work is to adapt the aforementioned SUR strategy to our setting where g is defined as $g(x) = \mathbb{E}_V[f(x,V)]$ with V a random variable taking its values in a functional space $\mathcal{V}$. The expectation is estimated by an average on a set of m realizations of V among the M realizations $\{v_1, \ldots, v_M\}$. We propose an adaptive strategy, sampling alternatively one point x in the control set $\mathbb{X}$ with the SUR strategy and m realizations of V in the set $\{v_1, \ldots, v_M\}$ using an innovative space filling strategy.

Our new procedure will be tested on an analytical test case, as far as on an industrial application from the French Oil Institute (IFP Energies nouvelles).

References

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