Levels, peaks, slopes... which sampling for which purpose?

Brigitte Bidégaray-Fesquet and Laurent Fesquet

Laboratoire Jean Kuntzmann and TIMA
Univ. Grenoble Alpes, France

Special Session ”Mathematical Modeling of Event-Based Systems”

Preliminary version
Aim

- Make the best of nonuniformity
- Explore pros and cons of various sampling techniques
  - Level-crossing sampling
  - Peak sampling
  - Level and peak sampling
  - Slope sampling
Figure: Principle and notations for level crossing sampling.
Level-crossing – features

**Pros** ability to adapt to signal variations, no sample during inactive parts, filtering of high-frequency components

**Cons** over-sampling in large amplitude active parts

Figure: Level crossing sampling on a small ECG fragment.
Level-crossing – going further

- Decimation of equal amplitude successive samples, in view of further processing where the signal is at most linearly interpolated
- Almost the same as send on delta sampling
- Application-driven choice of levels
Peak sampling – principle and features

- A sample each time a local minimum or maximum is detected.

Figure: Peak sampling on a small ECG fragment.

**Pros** does not depend on extra parameters inactive parts, only relevant data in peaky parts

**Cons** not adapted to noisy signals, no sample taken if monotonous
Level and peak sampling – principle

- Level crossing
- Detection of local extrema

Figure: Demonstration of level and peak sampling on a small ECG fragment.
Level and peak sampling – features

**Pros**
- less samples as levelcrossing sampling
- no oversampling in active parts: competitive with Nyquist sampling
- possible to add levels, with the same number of final samples
- less peak clipping

**Cons**
- as peak sampling: no sample taken if monotonous
Slope sampling – principle

Figure: Principle and notations for "slope sampling".
Slope sampling – features

**Pros** filters small amplitude noise

eventually takes samples if the signal has only small variations, since small errors accumulate

**Cons** as peak sampling: no sample taken if monotonous

Figure: Slope sampling on a small ECG fragment.
Number of samples – ECG example

- ECG, regularly sampled with $t_s = 5 \times 10^{-4}$ s period, leading to 2801 samples
- Noisy biological signal

<table>
<thead>
<tr>
<th>signal</th>
<th>number of samples</th>
<th>compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_n$</td>
<td>141</td>
<td>95%</td>
</tr>
<tr>
<td>$y_n$</td>
<td>1179</td>
<td>58%</td>
</tr>
<tr>
<td>$z_n$</td>
<td>14</td>
<td>99.5%</td>
</tr>
<tr>
<td>$w_n$</td>
<td>86</td>
<td>97%</td>
</tr>
</tbody>
</table>

**Table:** Number of samples and compression for an ECG fragment.
Theoretical results for level crossing sampling obtained under prescribed Hölder regularity, worst case study

Analogue impossible for peak sampling

Level and peak sampling always better in terms of compression than simple level crossing sampling.

To study: impact of extra parameters: levels, $L^1$-norm
Impact of noise – Number of samples

Input signal: \( f(t) = \frac{1}{2} + \frac{1}{2} \sin(2\pi t) \).

- Not a sporadic signal
- initially: 100 periods, uniformly sampled at 100 Hz, thus yielding 10000 samples
- additive noise with zero mean and standard deviation \( \sigma \): \( \sigma = \alpha q \) for \( \alpha \in [0, 1] \)

Figure: Number of samples in terms of \( \alpha \).

- Slope sampling: \( N \) constant
- Level-based sampling: \( N \) linear for large \( \alpha \)
- Peak sampling: \( N \) saturates for large \( \alpha \).
Impact of noise – Reconstruction

- Same test case
- $L^1$-error between the noisy input signal and the nonuniform signal
  - Methods equivalent for large values of $\alpha$
  - Level crossing, peak and slope sampling: linear behavior away from $\alpha = 0$.
  - Expect slope sampling to saturate for larger $\alpha$
  - A little noise is quite favorable for peak sampling
  - Level and peak sampling bad for small $\alpha$.

Figure: $L^1$ error in terms of $\alpha$. 
Applications

Pattern recognition  Correct pattern analysis vs. good reconstruction.
   Main issue: good placement of the levels.

System activity detection  Time delays in nonuniform signal allow to detect activity regions.
   The regions are regularly resampled, and treated with usual algorithms.

Applications of slope sampling  Introduced to test for the main frequency of a signal in a specific industry collaboration.
   Very costly procedure for nonuniform samples since there is no FFT-like algorithm at hand.
   High compression with slope sampling needed.

Embedded systems  Low number of samples needed
   Capability of implementing the sampling and the subsequent processing.
Conclusion

- Comparison from the point of view of the number of samples and the robustness of this criterion when noise is added, as well as the reconstruction error.
- Possibility to integrate in electronic systems
- Positioning of levels
- Security issues
Event-driven community in Grenoble

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e-BaCCuSS action-team: Event-Based Control, Circuits and Processing towards Ultra-Low Power Consumption

Goal: use the less possible samples

- to process signal and images in asynchronous systems,
- to control event-based systems.

Applications: embedded systems (medical implants, bee-robots...)

B. Bidégaray and L. Fesquet (LJK/TIMA)

Which sampling for which purpose? EBCCSP 2015, 15/06/2016
Thanks for your attention