Event-Based Sampling Algorithm for State Feedback Tracking Controllers

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Objective

- Reducing energy consumption,
- Relieving the load on the communication channels,
- Reducing the computational load on the CPU.
Consider the Linear Time-Invariant system

\[
\dot{x}(t) = Ax(t) + Bu(t), \\
y(t) = Cx(t),
\]

(1)

\(x(t) \in \mathbb{R}^n, y(t) \in \mathbb{R}^p, u(t) \in \mathbb{R}^m.\)

Continuous control

\[
u_r(t) = -Kx(t) + Gr(t),
\]

(2)

Event-Based control

if \(C(x(t))\) true, \(\bar{u}(t_k) = -Kx(t_k) + Gr(t_k).\)  

(3)

if \(C(x(t))\) false, \(\bar{u}(t) = \bar{u}(t_k) \quad \forall t \in (t_k, t_{k+1}).\)  

(4)
**Problem Definition**

\[ \dot{x}_r = (A - BK)x_r + BGr \]
\[ y_r = Cx_r \]

Mux

\[ r(t) \]
\[ x(t) \]

Event Generator
\[ e = x - x_r \]
\[ V(e(t)) = \delta \]

\[ \dot{u}(t) = 0 \]
\[ \bar{u}(t_k) = -Kx + Gr \]

\[ \dot{x} = Ax + B\bar{u} \]
\[ y = Cx \]
Event-triggering Conditions

**Definition**

We define the time-instant $t_{k+1}$ ($k \in \mathbb{N}$) at which the control-law $\bar{u}(t)$ is updated as the minimum time instant $t > t_k$ for which $V(e(t)) = \delta$:

$$t_{k+1} = \inf\{t > t_k, V(e(t)) = \delta\}.$$  \hfill (5)
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(5)
Theorem (Error Boundedness)

If the event-based control $\bar{u}(t)$ is updated according to the event-triggering condition defined previously, then the tracking error $e(t)$ remains confined in the ball of radius $\epsilon$, i.e.

$$\|e(t)\| \leq \epsilon,$$  \hspace{1cm} (6)

for $\epsilon > 0$. 
Results

Theorem (Error Boundedness)

*If the event-based control $\tilde{u}(t)$ is updated according to the event-triggering condition defined previously, then the tracking error $e(t)$ remains confined in the ball of radius $\epsilon$, i.e.*

$$\|e(t)\| \leq \epsilon,$$

(6)

*for $\epsilon > 0$.***

Theorem (Minimum Delay)

*Let $r(t)$ be a Lipschitz input signal. Then, there exists a minimum time $\tau_{\text{min}} > 0$, independent of $k$, such that*

$$\forall k \in \mathbb{N}, \quad t_{k+1} - t_k > \tau_{\text{min}},$$

*where the $t_k$, $k \in \mathbb{N}$ are defined in Theorem 1.*
Simulation Results

A simplified model of a jet aircraft during cruise flight, with 2 inputs and 2 outputs.
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A simplified model of a jet aircraft during cruise flight, with 2 inputs and 2 outputs.

For $\epsilon = 0.5 \; \delta = 0.1233$
The Lyapunov-like function $V(e)$

Simulation Results (Cont’d)
Simulation Results (Cont’d)

The Lyapunov-like function $V(e)$

For $\epsilon = 0.05$

304 updates for 300,000 simulation instants.
Added Value and Submission

Difference with the existing literature

- Event-based works have been focused on stability rather than tracking,
- A few works on tracking which used external systems as reference systems.

This work has been submitted to the EBCCSP conference.
Conclusion and Further Work

We have managed to achieve fairly good tracking of a reference input signal through an event-based approach. We are able to reduce the number of updates and therefore calls to the controller and communication between the system and controller considerably.

Further Work

- Vary $\epsilon$ with respect to $r$,
- Self-triggered scheme,
- Network systems.