Human Friendly Control : an application to Drive by Wire

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Control Objectives

– safety of the driver and performance $\Rightarrow$ passivity and transparency,
– linear approximation of the plant,
– optimal multi-objective synthesis using Linear Matrix Inequalities (LMI).
Outline

1. System Presentation and objectives
2. Control problem expression
3. Control Synthesis
4. Simulations and testing ground
1. System Presentation and objectives

**Fig. 1:** Schematic principle of the system
The Plant

**Fig. 2:** Typical Structure of a manipulator
Control objectives

The model of Hu-Salcudean-Loewen, 95:

\[ Z_d = f_e(N_f, N_v) \]

\[ v_e = v_h / n_v \]

FIG. 3: 2 ports representation of an ideal manipulator idéal

⇒ Use \( K \) and \( n_f, n_v \) to shape the desired impedance of the system satisfying the control objectives.
Coupled Transparency (impedance)

Defines a desired mapping between force and speed.

$$v_{h}^{d} = y^{d} (f_{h} + n_{f} f_{e})$$  \hspace{1cm} (1)$$

$$v_{e}^{d} = y^{d} \left( \frac{1}{n_{v}} f_{h} + \frac{n_{f}}{n_{v}} f_{e} \right)$$  \hspace{1cm} (2)$$

$$\Rightarrow \text{minimize} \; \tilde{v}_{h} = v_{h}^{d} - v_{h} \; \text{and} \; \tilde{v}_{e} = v_{e}^{d} - v_{e}$$

with adequate filters to set the frequency domain of minimization.
Passivity

*Bilateral coupled passivity* :

\[
<v_h, F_h > = \int_0^\infty v_h^T F_h dt > - \beta
\]

(3)

\[
<v_e, F_e > = \int_0^\infty v_e^T F_e dt > - \beta
\]

(4)

That is, \( Z_{th} \) and \( Z_{te} \) have to be passive.
2. Control problem expression

Given a desired admittance $Y_d(s)$, the goal is to find a control $K$ such that:

i. Closed loop transparency, $Y_t(K) = Y_d$.

ii. Closed loop passivity of $Z_{th}(K) = Z_t(K) + \frac{n_f}{n_v} Z_e$ and $Z_{te}(K) = \frac{n_v}{n_f} Z_t(K) + \frac{n_v}{n_f} Z_h$, where $Z_t(K)$ is the actual impedance of the system.

⇒ but it may not have a solution and $Z_h, Z_e$ are unknown, 
⇒ relax i. and extend ii.:

iii. $\min_K \|Y_t(K) - Y_d\|$, with

iv. $Y_t(K) : F \rightarrow v$ ESPR.
3. Control Synthesis

State-space representation of the plant $P$ and the control $K$ :

$$
\Sigma_P : \begin{cases} 
\dot{x} = Ax + B_w w + Bu \\
z = C_z x + D_{zw} w + D_z u \\
y = C x + D_w w 
\end{cases} \\
\Sigma_K : \begin{cases} 
\dot{\zeta} = A_K \zeta + B_K y \\
u = C_K \zeta + D_K y 
\end{cases} 
$$

(5)

Criterion specification :

$$z = T_{zw} w = \begin{pmatrix} T_{vw} \\ T_{\tilde{v}w} \end{pmatrix} w \hspace{1cm} et \hspace{1cm} \begin{bmatrix} A \\ C_j \end{bmatrix} \begin{bmatrix} B_j \\ D_j \end{bmatrix} = \begin{bmatrix} A + BD_K C \\ B_K C_j \\ C_j + E_j D_K C \end{bmatrix} \begin{bmatrix} B_j + BD_K F_j \\ B_K F_j \\ E_j D_K F_j \end{bmatrix}$$

with $B_j = B_w R_j$, $C_j = L_j C_z$, $D_j = L_j D_{zw}$, $E_j = L_j D_z$, $F_j = D_w R_j$.

$\Rightarrow$ Express the system into two transfer functions; one for each objective.
LMI expression : (S. Boyd et al. 94)

(iii. : equivalent to $\|T_{\tilde{\nu}w}\|_\infty < \gamma$, $\Rightarrow$ Bounded Real lemma) :

$$
\begin{pmatrix}
\mathcal{A}^T \mathcal{P} + \mathcal{P} \mathcal{A} & \mathcal{P} \mathcal{B}_2 & \mathcal{C}_2^T \\
\mathcal{B}_2^T \mathcal{P} & -\gamma I & \mathcal{D}_2^T \\
\mathcal{C}_2 & \mathcal{D}_2 & -\gamma I
\end{pmatrix} > 0, \quad \mathcal{P} > 0
$$

(6)

(iv. : Positive Real Lemma :

$$
\begin{pmatrix}
\mathcal{A}^T \mathcal{P} + \mathcal{P} \mathcal{A} & \mathcal{P} \mathcal{B}_1 - \mathcal{C}_1^T \\
\mathcal{B}_1^T \mathcal{P} - \mathcal{C}_1 & -\mathcal{D}_1^T - \mathcal{D}_1
\end{pmatrix} > 0, \quad \mathcal{P} > 0
$$

(7)
Singularity problem

\[ \mathcal{D}_1 \text{ singular } \Rightarrow \begin{pmatrix} A^T \mathcal{P} + \mathcal{P}A & \mathcal{P}B_1 - C_1^T \\ B_1^T \mathcal{P} - C_1 & -D_1^T - D_1 \end{pmatrix} > 0, \quad \mathcal{P} > 0 \]

implies to combine a LMI with a LME.

To solve this problem (Khalil, 96):

– Introduce a fictitious sector bound non-linearity simulating some uncertainties,
– use the circle criterion to design \( K \) such that the closed-loop fictitious system is SPR,
– conclude that the original system is SPR.
Where $\psi$ is a sector bound non-linearity defined by:

$$\left[\psi(t)z - Q_{\min}z\right]^T\left[\psi(t)z - Q_{\max}z\right] \leq 0, \quad \forall \ t \geq 0, \ \forall \ z \in \Gamma \subset \mathbb{IR}^p \quad (8)$$
The resulting fictitious system is:

$$\begin{pmatrix}
\bar{A} & \bar{B}_w & \bar{B} \\
C_z & D_{zw} & D_z \\
\bar{C} & \bar{D}_w & \bar{D}
\end{pmatrix} = \begin{pmatrix}
A - B_w Q_{\text{min}} C_z & B_w & B \\
(Q_{\text{max}} - Q_{\text{min}}) C_z & I & 0 \\
C - D_w Q_{\text{min}} C_z & D_w & 0
\end{pmatrix}$$

(9)

The identity matrix in “D” allows for the use of the positive real lemma.
Optimal multi-objective synthesis using LMI (Scherer-Gahinet-Chilali, 97)

- Two LMIs to solve simultaneously, with $\mathcal{P} > 0$,
- a linearizing change of variables to include the closed-loop expression,
- $A$, $B$ et $C$ real and fictitious have to be the same $\Rightarrow Q_{\text{min}} = 0$,
- a unique control $K$ can be found, its order is the same as the system,
- two parameters still need to be fixed: $\gamma$ et $Q_{\text{max}}$.

Analysis tools
- passivity: Nyquist plot (SPR),
- transparency: $\mathcal{H}_\infty$ norm and Bode plot,
4. Simulations and testing ground

$G_m(s)$ was given by:

$$G_m(s) = \frac{1}{a_m s + b_m} = \frac{1}{0.0222 s + 0.0042}$$  \hspace{1cm} (10)

The desired admittance was chosen as:

$$y^d = \frac{1}{0.1s + 1}$$  \hspace{1cm} (11)

Some ‘good’ values for the remaining parameters:

$$\gamma = 0.7, \: qh_{max} = 0.892 \: \text{and} \: qe_{max} = 0.874.$$
The Control obtained has the form:

\[ K = \frac{1}{D} \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix} \]  

(12)

with \( D \) and \( K_{ii} \) some 4\(^{th}\) order polynomials.
Fig. 5: Passivity of the closed-loop system
Fig. 6: Minimized criterion $\tilde{v}$
FIG. 7: $v_h$ and $v_e$ with a speed factor $n_v = 1.5$ (input force = sinusoïde + noise)
Fig. 8: Testing ground
Conclusions

– Different approaches were explored for the transparency,
– the control has been obtained with some tuning parameters, allowing for the freedom of the user,
– the simulations results are satisfying and the procedure is validated,
– experimental results were also obtained.