Receding Horizon Control: an Effective Methodology for Energy Management in Complex Systems

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Abstract: The optimization and control of complex systems represents a crucial issue in the industry community. The classical approaches make use of the decomposition of the system into several subsystems, each of them being locally optimized. However, this idea can lead to strong suboptimal control laws. In this paper, a generic method based on the receding horizon principles is developed, which allows to take into account the system as a whole and to exploit the interconnection dynamics. Several industrial examples are given, showing the versatility of the method.

Keywords: Receding horizon, predictive control, power systems, optimization methods, metaheuristics.

1. INTRODUCTION

A classical control design approach largely adopted in the industry makes use of simplified models and appropriate linear techniques, as, for examples PID, $H_\infty$ methods, to cite only a few. However, industrial systems become more and more complex, and such standard techniques may be intractable (large scale systems) or even inappropriate (wide operating domain). For such systems, a current trend is the use of optimization techniques to compute open loop solutions. It is well known that such a control law is highly non robust due to modelling errors, parameters uncertainties or disturbances. Thus, a closed loop structure is often required. In the industry community, Proportional-Integral controllers are often used for that purpose. Although effective for the reference tracking and disturbance rejection, such controllers may lead to bad quality results when considering the economical aspects.

In this paper, the main idea is to adopt the receding horizon principle for control design in such a closed-loop framework. The method is the basis of predictive control techniques; see (Clarke, et al., 1987; Maciejowski, 2002). It is able to define a closed-loop control law while explicitly taking into account the economical objectives in the design. Further, the proposed approach is able to consider the complex system as a whole, whereas classical approaches divide the system into several sub-systems and optimize them separately without any guarantee on the overall behaviour. It is worth to mention that this methodology can lead to strongly suboptimal solutions when applied to the initial system.

The remaining paper is organised as follows. The overall methodology is briefly presented in section 2. The proposed solution relies on solving successive on-line optimization problems. Several examples of industrial application are given. Due to the complexity of the optimization problems to be solved, a special attention has to be paid on the optimization methods. An ant colony and a genetic algorithm are used in section 3 for the control of energy production sites (hybrid systems based representation), a particle swarm algorithm is chosen in section 4 for the control of district heating networks (partial differential equations based representation), and a Nelder Mead simplex is used for the control of mining ventilation systems in section 5. These examples, exhibiting various mathematical difficulties, illustrate both the viability and versatility of the proposed approach for the control of complex systems, and its efficiency especially in the case where energy management appears as a crucial issue.

2. OVERALL METHODOLOGY

The core of the proposed model-based feedback control methodology is depicted in figure 1. It is based on an optimization philosophy, exploited through the receding horizon principle as the open loop optimal control problem is first solved over a finite horizon. More precisely, the optimal control is computed over a given time interval $[m,m+N-1]$, based on a model of the system and predicted values for the disturbances. The objective function can be generally expressed by:

$$\min_{\{u_n\}_{n=1,...,N}} \sum_{n=m}^{m+N-1} (\hat{x}_n - \hat{x}_n^{des})^2 + \lambda u_n^2$$

s.t. technical constraints are satisfied

where:
The economical aspects are explicitly considered through the penalization on \( u_n \) variables. Next, the first values of the solution are applied to the real system. The outputs of the system are then measured and, from the available information, the state variables of the system are estimated to provide an update for the optimization model (feedback principle). Finally, the whole procedure is performed again at the next sampling time. Note that numerous results have been reported on the study of stability for the corresponding closed loop, but no solution exists in the general case. The aim of this work is to illustrate the fact that such approach can be efficiently used to control complex systems where energy management appears as a crucial issue.

### 3. CONTROL OF ENERGY PRODUCTION SITES

#### 3.1 Case study

Unit Commitment refers to the optimal scheduling of \( K \) production units, while satisfying a global consumer demand. It can be expressed (see (Sen and Kothari, 1998)) as:

\[
\min_{n=1,\ldots,N; k=1,\ldots,K} u_n^Q \sum_{n=1}^{N} \sum_{k=1}^{K} c_{\text{prod}}^k (Q_n^k, u_n^k) + c_{\text{on/off}}^k (u_n^k, u_{n-1}^k)
\]

\[
\begin{align*}
&c_{\text{prod}}^k (Q_n^k, u_n^k) = a_1^k (Q_n^k)^2 + a_2^k Q_n^k + a_0^k u_n^k \\
&c_{\text{on/off}}^k (u_n^k, u_{n-1}^k) = c_{\text{on}}^k u_n^k (1 - u_{n-1}^k) + c_{\text{off}}^k u_{n-1}^k (1 - u_n^k)
\end{align*}
\]

\( u_n^k \) (resp. \( Q_n^k \)) is the on/off status (resp. produced power) of the production unit \( k \) at time interval \( n \). Production costs and start up and shut down costs are defined by: where \( d_0^k, a_1^k, a_2^k, c_{\text{on}}, c_{\text{off}} \) are technical data of production unit \( k \).

The constraints of the problem are:

- capacity constraints:
  \[
  Q_{\text{min}}^k u_n^k \leq Q_n^k \leq Q_{\text{max}}^k u_n^k
  \]
- consumer demand satisfaction:
  \[
  \sum_{k=1}^{K} Q_n^k \geq Q_{\text{dem}}^n, \quad \forall n \in \{1, \ldots, N\},
  \]
- time up and time down constraints:
  \[
  \begin{cases}
  u_{n-1}^k = 0, u_n^k = 0 \\
  u_{n+1}^k = 1, u_{n+2}^k = 1, \ldots, u_{n+T_{\text{up}}^k - 1}^k = 1 \\
  u_{n-1}^k = 1, u_n^k = 0
  \end{cases}
  \]
- ramp constraints:
  \[
  Q_{\text{up}}^k - Q_{\text{down}}^k \leq \Delta Q_{\text{max}}^k
  \]

Note that the consumer demand can only be predicted over the whole time horizon. Thus, a closed-loop structure is required for the control of the production site.

#### 3.2 Choice of optimization method

Numerous methods have been applied to solve Unit Commitment and related problems (such as facility location). They are listed for instance in (Sen and Kothari, 1998). Here, an ant colony has been hybridized with a genetic algorithm. Indeed, the exact solution of the problem is intractable due to numerous binary variables. Thus, a stochastic algorithm is often required. The difficulty of such algorithms is the handling of constraints as the algorithm “moves” randomly in the corresponding search space. Ant colony (see, for instance (Dorigo, 1996)) appears to be an efficient way to solve the Unit Commitment, as it is able to find near-optimal solutions with an explicit handling of the whole set of constraints. From this initial population of "medium quality solutions" fast computed by the ant colony, a feasibility criterion is optimized by an appropriate genetic algorithm to intensively explore the search space. Finally, the developed optimization method allows for the simultaneous use of the ant colony interesting properties (explicit handling of the constraints) and of the genetic algorithm (deep exploration of the search space, and related high quality of the solution). For more details, see (Sandou and Olaru, 2007).

#### 3.3 Main results

A "four unit" academic case is considered with the characteristics given in table 1.
The capacity and ramp constraints are set for the 4 units as: $Q_{\text{max}} = 40 \text{MWh}, Q_{\text{min}} = 10 \text{MWh}, \Delta Q = 10 \text{MWh}$. For illustration a worst case study is considered: the consumer load is always underestimated. The prediction error is a random value in the range [-5%, 0%]. The time horizon is $N = 24$ hours and the consumer load has a daily oscillation. Thus, the dynamics of the system is set with periodic events over 24 hours and the time horizon has to be greater: a high value has to be given to $N$. The simulation is performed on a 4-days total horizon. The results, obtained with Matlab 6.5 on a PIV 2GHz, are given in figure 2. It clearly appears that the production is very close to the real demand, except for some peaks that have been underestimated. The optimisation of the 96 binary variable problem is performed in just 25 seconds with the proposed ant colony/genetic algorithm method. Due to the computation of successive economical near-optimal solutions and real time slight updates, the global costs are very close to global optimal costs which can be computed by classical Mixed Integer Linear programming in a few particular cases.

### 4. CONTROL OF ENERGY NETWORKS

#### 4.1 Case study

In this section, a district heating network, represented in figure 3 is considered. The simulation model has been fully defined in previous work (Sandou, et al., 2004), which main results are summarized below.

**Production model.** Production sites are made of several production units (see section 3). In this section, production models are aggregated: production site $k$ can be globally modelled by a non dynamic characteristic, identified from technical data with a least square method. For hour $n$, the production costs can be derived from produced thermal power $Q^k_n$ [W]:

$$
c_{\text{prod}}(Q^k_n, Q^{k-1}_n) = a_2 (Q^k_n)^2 + a_1 Q^k_n + a_0 + \lambda (Q^k_n - Q^{k-1}_n)^2
$$

The thermal power given to primary network is related to the network temperatures by:

$$
Q^*_n = c_p m_s (T^k_x - T^k_r)
$$

where $m_s$ [kg.s$^{-1}$] is the mass flow, $T_s$ [K] the supply temperature and $T_r$ [K] the return temperature in primary network; $c_p$ [J.kg$^{-1}$.K$^{-1}$] is the specific heat of water.

### Pressures and mass flows.

**Mechanical losses in pipes:**

$$
H_{\text{out}} = H_{\text{in}} - Z_p m_p^2
$$

where $m_p$ [kg.s$^{-1}$] is the mass flow in the pipe, $H_{\text{in}}$ (resp. $H_{\text{out}}$) [m] the pressure at the beginning (resp. the end) of the pipe, and $Z_p$ [m.kg$^{-2}$.s$^2$] the friction coefficient. For a valve, this coefficient is $Z_p/d$, where $d$ is the opening degree of the valve (0 for a closed valve to 1 for an open one). Pumps are installed in the network, which leads to a pressure increase:

$$
\Delta H = a_2 \left( m \frac{\omega}{\omega_0} \right)^2 + a_1 \left( m \frac{\omega}{\omega_0} \right) + a_0
$$

where $m$ [kg.s$^{-1}$] is the mass flow through the pump, $\omega$ [rad.s$^{-1}$] its rotation speed and $\omega_0$ its nominal rotation speed. For the mass flows computation, nodes are modelled by mass flows balance equations. Finally, mass flows and pressures have to be computed from an important non linear system of algebraic equations obtained from all these static equations.

**Thermal energy propagation.** The thermal energy propagation is associated with the simulation of partial differential equations. Let $R_p$ be the radius of the pipe, $\mu_p$ [J.m$^{-2}$.s$^{-1}$.K$^{-1}$] its thermal loss coefficient, $\rho$ [kg.m$^{-3}$] the relative density of water and $T_0$ [K] the external temperature. The temperature in the pipe $T(x,t)$ can then be modelled as:

![Fig. 3. District heating network.](image-url)
\[
\frac{\partial T}{\partial t}(x,t) + \frac{m_p(t)}{\rho p R_p} \frac{\partial T}{\partial x}(x,t) + \frac{2 \mu_p}{c_p p R_p} (T(x,t) - T_0) = 0
\]  
(12)

From the energy propagation point of view, nodes are modelled thanks to an energy balance equation.

**Consumer model.** Secondary networks of consumers are connected to the primary network through a heat exchanger. The following equation is the classical one for a counter flow heat exchanger with \( S \) [m²] the surface of the heat exchanger and \( e \) [W.K⁻¹.m⁻²] its efficiency \((h \text{ stands for "hot" or primary, and } c \text{ for "cold" or secondary)}:

\[
Q_c = e S \left( T_{h,in} - T_{c,out} \right) - \left( T_{h,out} - T_{c,in} \right)
\]

Assuming no thermal energy loss between primary and secondary networks, the thermal power given by the primary network can also be expressed by:

\[
Q_c = c_p m_h (T_{h,in} - T_{h,out})
\]  
(13)

Finally, the power received by the secondary network is:

\[
Q_c = c_p m_c (T_{c,out} - T_{c,in})
\]  
(14)

Assuming that \( m_h \) and \( T_{c,out} \) are given, and that the mass flow \( m_c \) is determined by the opening ratio of the consumer valve, then \( T_{c,in} Q_c \) and \( T_{h,out} \) can be computed from \( T_{h,in} Q_c \) is an increasing function of \( m_h \) and the maximal thermal power that can be given to a consumer is obtained for \( m_h = m_c \). There is a local regulation, which is not of interest in this study, so that the consumer can choose the value of \( m_h \) in the possible range by controlling the valve opening. Consequently, the given power is finally expressed by:

\[
Q_c = \min(Q_{dem}, Q_{max})
\]  
(16)

where \( Q_{dem} \) is the heat demand of the consumer and \( Q_{max} \) is the maximum power that can be given by the primary network. \( Q_{max} \) is computed by solving the system made of \((13), (14) \) and \((15)\), in the particular case where \( m_h = m_c \). Note that once again the consumer demand \( Q_{dem} \) is only predicted and thus a closed loop is required for the control of the system.

**4.2 Choice of the optimization method**

The control objective for this system is to minimize the operating costs, given by equation (8) under the technical constraints of the network (pressures, mass flows and temperatures in the acceptable range) and the satisfaction of the consumer’s demands. The model of the system necessitates the simulation of non linear algebraic equations coupled with partial differential equations. Thus, the computation of the constraints can only be made by a simulator. The number of variables to be optimized is relatively high, and consequently a metaheuristic method, the particle swarm optimization (PSO) has been used for the optimisation. This optimization method is inspired by the social behavior of bird flocking or fish schooling. The choice of parameters is very important to ensure the satisfying convergence of the algorithm. Lots of work have been done on the topic; see for instance (Shi and Eberhart, 1998; Eberhart and Shi, 2000). In the following, standard values, which are given in (Kennedy and Clerc, 2006) will be used.

**4.3 Main results**

The receding horizon based control law has been applied for the control of the district heating benchmark depicted in figure 3. Tests have been performed for a total time horizon of 24 or 48 hours, with a sampling time of one hour. The prediction horizon for the optimization problem is 12 hours. Thus, as the benchmark includes 2 producers and 2 valves, the optimization problem is made of 12*(2+2) = 48 optimization variables. The solution of the optimization problem is performed in 120 seconds on a Pentium IV, 2.5 GHz with Matlab 2007, for 50 iterations of the PSO algorithm.

To validate the control law, a worst case experiment has been performed. It is assumed that all consumer demands are always underestimated by a factor of 10%. This represents a worst case experiment as long as in the real world load error predictions can partially compensate each other. Tests of the proposed approach have shown that consumers’ demands are always fulfilled, by using the receding horizon control structure.

In the district heating network, producer 1 is a cogeneration site. Cogeneration refers to the simultaneous production of electric and thermal powers, leading to high global efficiencies. Roughly speaking, the main goal of the producer is to satisfy the thermal power demand. But he has the opportunity to use the exhaust fumes to produce and to sell electric power. Finally, for the thermal power point of view, the higher the price of sold electricity is, the lower the thermal power production costs. The simulation has been performed for different electricity prices, and corresponding total productions over the whole horizon (24 or 48 hours) are given in table 2.

**Table 2. Results of the district heating network control**

<table>
<thead>
<tr>
<th>Electricity price</th>
<th>Production 1 over 24 hours</th>
<th>Production 1 over 48 hours</th>
<th>Production 2 over 24 hours</th>
<th>Production 2 over 48 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 €/MWh</td>
<td>535MWh</td>
<td>947MWh</td>
<td>537MWh</td>
<td>1016MWh</td>
</tr>
<tr>
<td>40 €/MWh</td>
<td>541MWh</td>
<td>963MWh</td>
<td>492MWh</td>
<td>950MWh</td>
</tr>
</tbody>
</table>

The price 40 €/MWh corresponds approximately to the price in France from November 1st to March 31st, whereas the null price corresponds to the price from April 1st to October 31st. Results show that the higher the price is, the higher the

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1 PSO was firstly introduced by Eberhart and Kennedy (Eberhart and Kennedy, 1995)
production of the cogeneration site. The control law uses the interconnection valves to make the extra amount of power to pass from sub-network 1 to sub-network 2. Although obvious, the possibility is not used in classical district heating networks: controls laws only use local information, and the interconnections are often viewed as safety means, and are rarely used. The receding horizon law is able to take into account the whole technological chain "production - distribution - consumption" and the whole system through the solution of the optimization problem. The solution of this problem is made tractable by the use of a stochastic approximated optimization method. Note that in the future, the price of sold electricity may depend on the electricity market. In such a situation, production costs would be predicted, and the closed loop structure is also a good trend to get a robust behaviour against cost uncertainties. For more details, see (Sandou and Olaru, 2008).

5. CONTROL OF MINING VENTILATION SYSTEMS

5.1 Case study

In this section, the focus is on the optimization and control of a mining ventilation system represented in figure 4. The ventilation is achieved by a turbine and a heater connected on the surface to a vertical shaft. The heater is introduced (in winter time at least) to avoid freezing in the upper part of the shaft and air cooling devices are used at high depths (more than 1000 meters) to compensate the geothermal effect (the temperature increases by 1°C every 30m as we go down). From the ventilation shaft, fans located at each extraction level pump fresh air to the extraction rooms via tarpaulin tubes. Bad quality air naturally flows because of the pressure gradient from the extraction rooms back into the decline and to the exhaust ventilation shaft.

The overall objective of the mining ventilation control system is to provide good air quality for the extraction rooms. For a future wireless automation supporting the ventilation control, it is also desirable to increase safety by using the wireless system for personal communication and localization. We specify the objective as the control of air quality (O2, NOx and/or COx) in the extraction rooms at different levels. The objective is then to regulate the ventilation fans based on chemical sensors to ensure air quality in extraction rooms.

The concentration of pollutant \( j \) at height \( z \) is modelled by:

\[
c_j(z,t) = \frac{\alpha_j(t)}{1 + e^{-\beta_j(t)(z - \gamma_j(t))}}
\]

where \( \alpha_j(t) \) is the amplitude, \( \beta_j(t) \) is the dilatation, \( \gamma_j(t) \) and is the inflection of the distribution. The state space representation of the problem with \( n \) pollutants is given by:

\[
\begin{bmatrix}
\dot{\alpha}_1(t) \\
\dot{\beta}_1(t) \\
\dot{\gamma}_1(t) \\
\vdots \\
\dot{\alpha}_n(t) \\
\dot{\beta}_n(t) \\
\dot{\gamma}_n(t) \\
\end{bmatrix}
= f
\begin{bmatrix}
\alpha_1(t),\beta_1(t),\gamma_1(t),\ldots, \alpha_n(t),\beta_n(t),\gamma_n(t),u(t),\dot{m}_{\text{in}}(t)
\end{bmatrix}
\]

where \( u(t) \) is the control input of the fan, and \( \dot{m}_{\text{in}}(t) \) refers to the incoming pollutant. This value is only predicted as it depends on the number of trucks in the room, and so a closed loop framework is required. The control objective is to minimize the fan energy consumption while ensuring an acceptable air quality. Due to the height-dependent model, the air quality objective is rephrased as guaranteeing a maximum pollutant concentration at a given height \( z_r \):

\[
y_j(t) = c_j(z_r,t) \leq y_j^\text{max}
\]

where \( y_j^\text{max} \) is the threshold value on pollutant \( j \). The idea is now to solve the following optimization problem and to use the receding horizon principle:

\[
\min_{\{u_i\}_{i=1}^{N}} \int_{\tau}^{\tau+N} \left[ \sum_{j} \left( \dot{y}_j(\tau) - y_{j,\text{des}}(\tau) \right)^2 \right] d\tau + \lambda u^2(\tau) 
\]

with \( u(\tau) = u_i \)

for \( \tau \in \{kT+(i-1)T/N_u,kT+iN/N_u\} \)

with \( N_u \) the number of degrees of freedom in the control law, \( N \) : the prediction horizon, \( T \) the sampling time in the receding horizon strategy, \( \dot{y}_j(\tau) \) the predicted concentration of pollutant based on predictions of incoming pollutant rate, \( u(\tau) \): the fan control input, \( y_{j,\text{des}}(\tau) \): the desired concentration of pollutant and \( \lambda \) : the weighting factor.

5.2 Choice of optimization method

The cost function given in equation (20) can only be computed by a simulator. However, the number of optimization variables remains relatively small (typically \( N_u = 2 \) to \( 5 \)). Thus, it has been decided to choose a Nelder Mead Simplex method to solve the problem.

5.3 Main results

We consider two pollutants, namely COx and NOx. The thresholds are defined as:

Fig. 4. Mining ventilation system.
With $y_{CO}^{\max} = 0.000234 \text{kg.m}^{-3}$, and $y_{NO}^{\max} = 0.0059 \text{kg.m}^{-3}$.

The pollutant sources are depicted in figure 5. For the prediction model, we consider a number of trucks equal to 2. The tuning parameters are set to $N=100$, $N_u=2$ and $\lambda=10^{-7}$.

The corresponding results are given in figure 6. The level of pollutant, control inputs, power and energy consumed by the electric motor of the fan are given. Results are much than satisfactory with a smooth regulation satisfying the constraints over the whole time interval. As we have only one actuator to control two outputs, the regulator mostly takes into account the hardest constraint of CO. Computation times for the simulation of the 1500s (real time) is performed in 10 minutes with Matlab 2007a on a Pentium IV, 2.80 GHz. For more details, see (Witrant, et al., 2009).

6. CONCLUSIONS

In this paper, a generic methodology for the control of complex industrial systems has been presented. The method is based on the successive solutions of optimization problems. The main interest of this approach is the possibility to capture the whole system instead of controlling several subsystems. Thus, the quality of the solutions is better and can take into account the economical aspects.

The main point in the methodology is to solve an optimization problem at each sampling time. Thus, a special attention has to be paid on that topic to get a tractable control law. To overcome this difficulty, forthcoming works will deal with the use of hierarchical predictive control laws.

REFERENCES


