ABSTRACT

This paper describes an innovative method to estimate the energy wall losses during the compression and combustion strokes of a gasoline engine using the cylinder pressure measurement. The estimation during the compression and combustion strokes allows to better represent the system during the combustion. A sliding mode observer is derived from a 0-d physical engine model and its convergence and stability are proved. The complete system is validated by comparing simulated cylinder pressure with measurements.

INTRODUCTION

Through the years, the research of convective heat transfer modeling in internal combustion engines has been a hard task due to the difficult prediction of this phenomenon. Most of research has been focused on empirical laws which try to predict the heat transfer coefficient between the gases and the cylinder walls. One of the first laws was proposed by Annand [1], where an empirical law based on the thermal conductivity and the engine temperature is used to calculate the thermal coefficient of the walls heat transfer. Then, in the work of Woschni [2], a new empirical law is proposed, where the heat transfer coefficient depends on the pressure, temperature and engine speed. Woschni’s law has been adopted for almost all the 0d engine models, and many works based on this law have been published, such as Hohenberg [3], Han and al.[4] and more recent Alizon [5]. Alternately, Shayler [6] takes into account the Woshni’s correlation and adds a heat flow distribution depending on the surface of the chamber.

Parallel to those works, the law of the wall works have tried to explain the heat transfer phenomena: in Yang and Martin [7], the unsteadiness of turbulent heat transfer in piston engines depend on the flow properties, the gas conductivity and the viscosity. In Boust [8] a physical approach for wall heat transfer based on the kinetic theory of gases is proposed.

The instantaneous heat flow during the engine cycle is a necessary input for realistic cycle calculations [9], even if the existent empirical laws approximate the wall losses phenomenon, the current heat transfer models are not universally applicable and many tuning parameters remain undefined. This situation opens the perspective to the use of a different strategy than the empirical approximations.

The purpose of this work is to give an alternative method to estimate the heat wall losses. The heat wall transfer makes part of the whole enthalpy flow during the engine cycle. The whole enthalpy flow modeling requires the use of discontinuous terms to represent its dynamics. The use of an sliding mode observer is a suitable strategy for observation in these kind of systems; this technique is based on the choice of a sliding surface of the state space according to the desired dynamical specifications of the closed-loop system. The sliding choices are designed so that the state trajectories reach the surface and remain [10]. The sliding mode technique has been used by many researchers for estimation of non measurable and/or uncertain parameters in space state system. The sliding model method has been popularized thanks to the work of Utkin [11]. The main advantages of this method are its robustness against a large class of perturbations or model uncertainties, the need of a reduced amount of information and the possibility of stabilizing some nonlinear system which are not stabilizable by continuous state feedback laws.
In this work, an innovative method to estimate the wall losses during the compression and combustion stroke of a gasoline engine is proposed. To develop the observer, a 0d one zone thermodynamical model of a gasoline engine has been developed.

In the first part of this paper the one zone thermodynamical model and the combustion model are described. The result of this model is validated with cylinder pressure measurements. This model is reduced to design the sliding model observer. In the next sections, the complete observer system is developed and the estimation is compared to the cylinder wall during the compression and combustion strokes using the cylinder pressure measurement.

**0D ENGINE MODEL**

The thermodynamical model describes the energy balance inside the combustion chamber. The 0d virtual engine model is divided into two main elements: a one zone thermodynamical model that describes the energy balance inside the combustion chamber and the combustion model.

**ONE ZONE THERMODYNAMICAL MODEL**

In the one zone thermodynamical model, the combustion chamber is considered as a unique open system and a uniform in-cylinder pressure is assumed. The mass flow rate in the cylinder is deduced from a balance equation corresponding to the mass transfer through intake and exhaust valves.

The energy equation for the cylinder is inferred from the first thermodynamical principle:

\[
dU = -\delta Q_{th} - pdV + \left( \sum_j h_j dm_j \right) \tag{1}
\]

where the subscript \( j \) denotes energy getting into or out of the combustion chamber, \( U \) is the internal energy of the cylinder gas mixture, \( \delta Q_{th} \) expresses the heat transfer of the cylinder contents to the surroundings, \( pdV \) corresponds to the work delivered by the piston, \( \sum_j h_j dm_j \) is the total energy flowing into or out of the cylinder and \( h \) is the specific enthalpy.

Assuming that the specific heat constant \( c_v \) is constant, the left hand side of (1) can be written as:

\[
dU = T(t)c_v dm(t) + m(t)c_v dT(t) \tag{2}
\]

where \( m(t) \) is the total mass of all the species in the cylinder and \( T(t) \) corresponds to the temperature of the gases.

Solving (1) and (2) for \( dT(t) \), an ordinary differential equation is implemented as the governing equation for the system temperature dynamics:

\[
dT(t) = \frac{1}{m(t)c_v} \left( -p(t) dV(t) - \delta Q_{th}(t) + \left( \sum_j h_j(t) dm_j(t) \right) - T(t)c_v dm(t) \right) \tag{3}
\]

where \( V(t) \) is the gases volume. The heat losses from the gases in the combustion chamber to the cylinder walls are given by:

\[
\delta Q_{th}(t) = h_c(t) A_w(t)(T(t) - T_w) \tag{4}
\]

where \( A_w(t) \) is the wall transfer area, \( T(t) - T_w \) is the temperature difference between the gases and the cylinder walls, and \( h_c \) is the heat transfer coefficient computed from Woschni's empirical law [2]:

\[
h_c(t) = \alpha D^{-0.2} p(t)^{0.8} T(t)^{-0.53} \left( C_1 V_p + C_2 \frac{V_t T_1}{p_1 V_1} (p(t) - p_0(t)) \right) \tag{5}
\]

where \( D \) is the cylinder bore, \( C_1 \) and \( C_2 \) are calibration constants, \( p_1 \) and \( T_1 \) represent the known state of the working gas related to the instantaneous cylinder volume \( V_1 \), i.e. at IVC, and \( p_0 \) is the pressure reference in the absence of combustion.

The total energy flowing into or out of the cylinder is considered as the enthalpy flow from the breathing process and the combustion:

\[
\sum_j h_j(t) dm_j(t) = Q_{h_{in}}(t) - Q_{h_{out}}(t) + Q_{h_{comb}}(t) \tag{6}
\]

To find the dynamics of \( p(t) \), the ideal gases law is used:

\[
p(t)V(t) = rm(t)T(t) \tag{7}
\]

Taking the derivative of (7) and solving for \( dp(t) \):
\[ dp(t) = \frac{rT(t)dm(t)}{V(t)} + \frac{r m(t) dT(t)}{V(t)} - \frac{rT(t)m(t)dV(t)}{V(t)} \]  

(8)

where \( dm(t) = dm_{in} - dm_{out} \) is the mass balance through the cylinder valves. Finally, putting together equations (3), (6) and (8) and solving for \( dp(t) \), the in-cylinder pressure dynamics is modeled as:

\[ dp(t) = \frac{r V(t) c_v}{c_v} Q_{h_{inj}}(t) - \frac{r V(t)}{c_v} Q_{h_{out}}(t) - \frac{r V(t)}{c_v} p(t) dV(t) - \frac{p(t)}{V(t)} dV(t) + \frac{r V(t) c_v}{c_v} Q_{h_{inj}}(t) + \frac{r V(t) c_v}{c_v} LHV Q_{m_{comb}}(t) - \frac{r V(t) c_v}{c_v} Q_{th}(t) \]  

(9)

where \( Q_{h_{comb}}(t) \) has been replaced by \( Q_{h_{inj}} + LHV Q_{m_{comb}}(t) \), the sum of the enthalpy supplied by the injection and the combustion process. \( LHV \) is the lower heat value: for gasoline engines it can be approximated to \( 4.15 \times 10^7 \text{MJkg}^{-1} \). \( Q_{h_{inj}} \) supplies a small amount of energy during few time (less than 3 CAD), in this model, its value is obtained from a map.

**COMBUSTION MODEL**

The combustion process is commonly defined with a burned mass fraction curve, provided by a Wiebe’s law [12]:

\[ Q_{m_{comb}}(\theta, u) = m_o a e^{-a\left(\frac{\theta}{\Delta \theta}\right)}\left(\frac{\theta}{\Delta \theta}\right)^m \frac{2N\pi}{60\Delta \theta} \]  

(10)

where \( N \) is the engine speed in \text{rev/s}, \( m_o \) is the injected fuel and \( a, m \) and \( \Delta \theta \) are calibration parameters.

**ENGINE MODEL VALIDATION**

The engine model is tested taking as reference the measurements of a 1.2 liters engine. The data to fit the model is the cylinder pressure. The results presented in this paper correspond to a test performed at \( N = 2000 \text{rpm} \) and BMEP = 10 bar.

Results of the validated model are shown in Figure 1. Complementary results are shown in Figure 2, where the percentage of error between the measurements and the model carried out in a data base of 90 operating points are shown. The model has less than 10% of error in the cylinder pressure prediction. Those results are accurate enough for the purpose of this work.

**MODEL REDUCTION**

The cylinder pressure dynamics depends on the energy balance in the combustion chamber. The enthalpy flows represent a different physical phenomenon depending on the the crank angle position (engine stroke). Figure 3 shows how are given the energy exchanges on the combustion chamber depending on the combustion stroke. In the figure:

- Inlet valve opening (IVO) - Inlet valve closure (IVC): Admission stroke.

![Figure 1: Cylinder pressure. BMEP=10 bar, N=2000rpm.](image)

![Figure 2: Cylinder pressure error for 90 operating points. BMEP=1-20 bar, N=1200-5500rpm.](image)

![Figure 3: Energy exchanges on the combustion chamber.](image)
As it was presented in the last sections, four main enthalpy flows are considered in the system: The enthalpy due to the valves flows $Q_{h_{in/out}}(t)$, the enthalpy due to combustion $Q_{comb}(t)$ and the enthalpy due to the wall losses $Q_{th}(t)$, the remaining energy component is the work delivered by the piston and the gases moving which depend on the cylinder pressure. Table 1 explains how the enthalpy dynamics are taken into account depending on the engine stroke (from here the time dependence notation is omitted to simplify the notations).

| Admission: $Q_{h_{comb}} = 0$, $(Q_{h_{in}}, Q_{h_{out}}, \delta Q_{th}) \neq 0$ |
| Exhaust: $Q_{h_{comb}} = 0$, $(Q_{h_{in}}, Q_{h_{out}}, \delta Q_{th}) \neq 0$ |
| Compression: $(Q_{h_{in}}, Q_{h_{out}}, Q_{h_{comb}}) = 0$, $\delta Q_{th} \neq 0$ |
| Combustion: $Q_{h_{in}} = 0, Q_{h_{out}} = 0$, $\delta Q_{th} \neq 0$ |

The state $x_2$ groups all the enthalpy flows in the system, different from the heat supplied by the combustion and the work delivered by the piston. The dynamics of $x_2$ is assumed to be unknown. With this model, during the compression and combustion strokes, the state $x_2$ represents only the wall losses, according to Table 1.

### SLIDING MODE OBSERVER

Consider a system in additive triangular nonlinearity form:

$$\dot{z} = A_0 x + \phi(z, u)$$
$$y = C_0 z$$

where

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \ldots & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The nonlinear observer is designed according to the following theorem:

**Theorem 1** [13]. If $\phi(z, u)$ is globally Lipschitz in $z$ and $u$ and such that $\partial \phi_j(z, u)/\partial z_j = 0$, for $j \geq i + 1$, $1 \leq i, j \leq n$, then the system (12) admits an observer of the form [10]:

$$\dot{\hat{z}}_1 = \hat{\dot{z}}_2 + \phi_1(z_1, u) + \lambda_1 \text{sgn}(z_1 - \hat{z}_1)$$
$$\dot{\hat{z}}_2 = \hat{\dot{z}}_3 + \phi_2(z_1, \hat{z}_2, u) + \lambda_2 \text{sgn}(\hat{z}_2 - \hat{z}_2_2)$$
$$\dot{\hat{z}}_n = f(z_1, \hat{z}_2, .., \hat{z}_n) + \lambda_n \text{sgn}(\hat{z}_n - \hat{z}_n)$$

where

$$\hat{z} = \hat{z} + \lambda \text{sgn}(z - \hat{z})$$

To analyze the observer stability, the first state space variable is analyzed:

Considering $e_1 = z_1 - \hat{z}_1 \neq 0$ and the Lyapunov function $V_1 = \frac{1}{2}e_1^2$. To guaranty the stability, the condition $V_1(x) < 0$ must be full filled. Considering the dynamics of $e_1$:

$$\dot{e}_1 = e_2 - \lambda_1 \text{sgn}(e_1)$$

and

$$\dot{\hat{e}}_1 = e_1 e_2 - \lambda_1 \text{sgn}(e_1)$$

The state $x_2$ groups all the enthalpy flows in the system, different from the heat supplied by the combustion and the work delivered by the piston. The dynamics of $x_2$ is assumed to be unknown. With this model, during the compression and combustion strokes, the state $x_2$ represents only the wall losses, according to Table 1.
which verifies $V < 0$ when $\lambda > |e_2|_{\max}$. As the function $sgn$ is used, and the Lyapunov function decreases, the convergence to the sliding surface $S = e_1$ in a finite time $t_0$ is obtained. Thus, for $\lambda_1 > |e_2|_{\max}$, $\dot{z}_1$ converges to $z_1$ in finite time $t_0$ and remains equal to $z_1$ for $t > t_0$.

Moreover, $\dot{e}_1 = 0$ for $t > t_0$, so from (16):

$$e_2 = \lambda_1 sgn(e_1)$$  \hspace{1cm} (18)

and

$$\dot{z}_2 = \dot{z}_2 + \lambda_1 sgn(e_1)$$  \hspace{1cm} (19)

is equal to $z_2$ for $t > t_0$.

The same procedure if followed for the states $z_2, \ldots, z_n$ and the stability is shown. Refer to [10] for the complete demonstration.

SLIDING MODE OBSERVER APPLICATION

Using the sliding mode observer (14) in system (11) it yields:

$$\dot{x}_1 = -\left( \frac{r}{c_v} + 1 \right) \frac{dV}{V} \dot{\hat{x}}_1 + \frac{r}{c_v V} Q_{h,\text{amb}} + \frac{r}{c_v V} \lambda_1 sgn(x_1 - \dot{\hat{x}}_1)$$

$$\dot{x}_2 = \frac{r}{c_v V} \lambda_2 sgn(x_2 - \dot{\hat{x}}_2)$$

$$\dot{x}_2 = \dot{\hat{x}}_2 + \frac{r}{c_v V} \lambda_1 sgn(x_1 - \dot{\hat{x}}_1)$$  \hspace{1cm} (20)

Where $\lambda_1$ and $\lambda_2$ are the observer gains to be chosen to ensure the system stability. Differently from (12), in System (20) the first non zero component in matrix $A_0$ is different than 1. It is possible to use an equivalent transformation through a diphemorfism to obtain the exact form, however, in this work the effect of this component has been added to the observer inputs multiplying it by $r/(c_v V)$.

The Lyapunov stability theorem [14] is used to bound the choice of $\lambda_1$ and $\lambda_1$. In a first place, the bounds for $\lambda_1$ are chosen:

Consider the Lyapunov function $V_1 = \frac{1}{2}e_1^2$, where $e_1 = x_1 - \dot{\hat{x}}_1$. The condition $\dot{V}_1(x) < 0$ must be full filled:

$$\dot{V}_1(x) = e_1(x)e_1(x)$$  \hspace{1cm} (21)

$r/(c_v V) > 0$, then keeping $\lambda_1 > |e_2|$ ensures $\dot{V}_1(x) < 0$. The same procedure is applied to chose the second parameter $\lambda_2$.

Consider the second Lyapunov function $V_2 = \frac{1}{2}e_2^2$, where

$$e_2 = x_2 - \dot{\hat{x}}_2$$  \hspace{1cm} (23)

The conditions $\dot{V}_2(x) < 0$ must be fulfilled:

$$\dot{V}_2(x) < 0 = e_2(x)e_2(x)$$  \hspace{1cm} (24)

$$\dot{V}_2(x) < 0 = -e_2 r/(c_v V) \lambda_2 sgn(e_2)$$  \hspace{1cm} (25)

Keeping $\lambda_2 > 0$ ensures $\dot{V}_2(x) < 0$. The results of the implemented observer are presented next.

OBSERVER RESULTS

Figures 4a and 4b show the observation results of the system in equations (20), compared to the model in equations (4) and (9). For the cylinder pressure ($x_1$), the initial condition is the measured pressure at IVC, for the enthalpy flows, the initial condition is taken as $-0.4 J/s$. The observer is efficient and effective in successive engine cycles.

Using the measured pressure $p = x_1$, the observer is able to estimate the second state $x_2$ that represents the enthalpy flow. When the observation is made overall the whole engine cycle, the estimated enthalpy flow represents different physical phenomena depending on the engine stroke, as it is presented in Table 1. Taking the portion corresponding to the compression and combustion strokes of one engine cycle from Figures 4a and 4b, Figure 5 is obtained. In this figure, the estimation during the admission stroke represents the enthalpy flow due to the valves flows and the wall losses, similarly during the exhaust stroke.

The remain portion of the engine cycle is extracted in Figure 6, which corresponds to the compression and the combustion strokes. In this stage of the engine cycle, besides the enthalpy flow supplied by the combustion which is represented in the model by the Wiebe’s law and the work delivered by the piston, the only remaining enthalpy flow is the heat transfer to the walls. Then, the enthalpy flow shown in Figure 6b corresponds to the heat wall losses.
CONCLUSIONS

A sliding mode observer to estimate the heat wall losses during the compression and combustion strokes has been implemented. The observer is able to estimate the wall losses even if its dynamics is unknown for the system.

The model has been validated using the validated 0d model of a park ignited engine against experimental measurements. The observer performs accurately in a large data base of operating points.

Using a sliding mode observer allows to have a simple design and implementation structure, which is robust against modeling error and perturbations due to parametric variations [15].

REFERENCES


Figure 6: Cylinder pressure estimation and heat flow estimation during compression and combustion strokes. BMEP=10 bar, N=1200rpm.


**NOMENCLATURE**

All variables are in S.I Metric Units.

\( \alpha \) Calibration constant
\( \omega_e \) Engine speed (rad/s)
\( IT \) Ignition timing
\( A_w \) Heat transfer wall area
\( CAD \) Crank angle degrees
\( c_v \) Specific heat at constant volume
\( C_1 \) Calibration constant
\( C_2 \) Calibration constant
\( D \) Cylinder bore
\( EVC \) Exhaust Valve Closure
\( EVO \) Exhaust Valve Opening
\( h \) Enthalpy
\( h_c \) Heat transfer coefficient for wall looses
\( H_p \) Piston height
\( IVC \) Inlet Valve Closure
\( IVO \) Inlet Valve Opening
\( k_0 \) Calibration constant
\( k_1 \) Calibration constant
\( N \) Engine speed (rpm)
\( m \) Total mass in the combustion chamber
\( p \) Pressure
\( r \) Specific gases constant
\( T \) Temperature
\( T_w \) Wall temperature
\( U \) Energy
\( V \) Cylinder volume