Remote Output Stabilization Under Two Channels Time-Varying Delays

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I. Background on Time-Delay
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- Lyapunov-Krasovski approaches (Niculescu, Khartitonov, Verriest, Yu...):
- Stochastic approach (Nilsson 98...): LQG control
- Pole-placement (Kwon/Pearson, Manitius/Olbrot...)
- Passivity (Anderson/Spong, Niemeier/Slotine...): Teleoperation
- Stability (Bo Lincoln 03, Meinsma/Zwart, Sename...): Robustness
- Constant time-delays/upper bounds
- Lyapunov-Krasovski approaches (Niculescu, Khartitonov, Verriest, Yu...):
Consider the system:

\[
\begin{align*}
0 & \leq t_A \quad t > |(t)\xi|, \quad \text{s.t.} \quad 0 & \leq t_E \\
0 & \leq t_A \quad 0 < (t)\xi < x_{\text{max}}
\end{align*}
\]

(3)-(4) satisfy

\[
\begin{align*}
(4) & \quad z_i = \tau_i, \quad (t)\xi + (t)\zeta & = (t)\xi \\
(3) & \quad (t)\xi + (t)\zeta & = (t)\zeta \\
(2) & \quad (t)x & = (t)\xi \\
(1) & \quad ((t)\xi - t)B + (t)x & = (t)x
\end{align*}
\]
\((t)g + t)x^pV = ((t)g + t)x(KB - V) = ((t)g + t)x\)

III. Control design

Then the resulting closed-loop equation is

\[
\begin{align*}
\dot{(t)g + t}x &= ((t)g + t)x \\
((t)g + t)x_B + ((t)g + t)xV &= ((t)g + t)x \\
\end{align*}
\]

Assume that \(v(t) = \dot{x}
\]

Define \(n = (t)n\)

or

Assume that the delayed state or output is measurable, i.e., \(x(T - t)\)
Two conditions have to be satisfied:

1. The possibility to predict \( ((t)\varphi + t)x \mathcal{Y} = ((t)\varphi + t)a = (t)n \) and
2. The possibility to assign \( v((t)\varphi + t) \).

From causality:

\[
\theta \mathcal{P}((t)\varphi - \theta) \mathcal{B}_{\theta}v_{\theta}^{\mathcal{E}} \int_{\varphi + t}^{(t)\varphi} (\varphi + t)\mathcal{E} + ((t)\varphi + t)x_{(t)\varphi + t}\mathcal{E} = (\varphi + t)x
\]

Both 1. and 2. are satisfied if \( t = ((t)\varphi + t)\varphi - (t)\varphi + t \) and

\[
\{ t \geq (\theta)\varphi - \theta \quad [\varphi + t, \varphi - t] \ni \theta \mathcal{A} \mid 0 \leq \varphi \} \max \equiv (t)\varphi
\]

I. The possibility to predict

2. The possibility to assign

Both conditions have to be satisfied.
\[
\theta p(\theta)\nu^2 \mathcal{B}_{\theta V} \cdot \varphi \left\{ \int_{(\varphi+\varphi)'V} + (\varphi)z_{\varphi}V \right\}
\]
\[ = (\varphi)\varphi \]

then

\[(\varphi)z_{\varphi}C = (\varphi)\varphi \]

\[0z = (0)z \quad \text{for} \quad (\varphi)\nu^2 + (\varphi)z_{\varphi}V = (\varphi)\varphi \]

\[\theta p((\varphi)\varphi - \theta)\nu \mathcal{B}_{\theta V} \cdot \varphi \left\{ \int_{(\varphi+\varphi)'V} + (\varphi)z_{\varphi}V \right\} - (\varphi)\varphi x_{(\varphi+\varphi)'V} \cdot \lambda - = (\varphi)\varphi \]

The resulting control is
Lemma: Consider the system
\[
\frac{dx}{dt} (t+\pm(t)) = A_{cl} x (t+\pm(t))
\]
for \( t \geq 0 \) and \( \pm(0) = \pm 0 \). If the following conditions hold:

i) the eigenvalues of \( A_{cl} \) are in the open LHP,

ii) \( 1 > \|\phi(t)\| \),

iii) \( 0 < \| \phi(t) \| < \infty \),

then \( \lim_{t \to \infty} \| (\phi(t) + t)x \| = 0 \),

and for all bounded values of \( \phi(0) \), \( 0 = \| (\phi(t) + t)x \| \) as \( t \to \infty \).

\[ ((\phi(t) + t)x)^{\infty} V = ((\phi(t) + t)\frac{\tau p}{x p}) \]

\textbf{Stability analysis}
Theorem 1. Assume that the delay dynamics (3)-(4) is such that the following holds for $\tau_i(t)$, $i = 1, 2, t \geq t_0$:

\[
\theta p((\theta)_{\tau_i} - \theta)nB_{\theta V - e}^{(1-\tau_i)} \int_{\tau_i + T}^{1-\tau_i} (\tau_i - t)X(\tau_i + \tau)Y - = (t)\eta
\]

Then, the feedback control law

\[
((t)\varphi + t)_{\tau_i} = (t)\varphi
\]

ensures that the closed-loop system is bounded, and that the state $x(t)$ converges exponentially to zero.

Theorem 1. Assume that the delay dynamics (3)-(4) is such that the delay dynamics (3)-(4) is such that the
**IV. Observer-Based Control**

Luenberger state-observer:

\[ \dot{x}(t) = Ax(t) + Bu(t) + Ky(t) - Cx(t) \]

The resulting observation error is:

\[ e(t) = x(t) - \hat{x}(t) \]

The complete closed-loop dynamics is:

\[ \dot{x}(t) = (A - \hat{A})x(t) + Bu(t) + Ky(t) - Cx(t) \]

Exponentially stable if \( \gamma > 0 \) and the error dynamics are:

\[ \dot{e}(t) = (A - \hat{A})e(t) \]

- Luenberger state-observer:
Example: Remote output stabilization

\[
\begin{align*}
(\gamma \frac{\tau}{I}) & \frac{\tau}{I} \tau \frac{\tau}{I} - \tau \frac{\tau}{I} = \tau \frac{\tau}{I} \\
\phi &= (0) \frac{\tau}{I} + \frac{\tau}{I} \frac{\tau}{I} = \tau \frac{\tau}{I} \\
\phi &= (0) \frac{\tau}{I} + \frac{\tau}{I} \frac{\tau}{I} = \tau \frac{\tau}{I}
\end{align*}
\]

\[
J[0, 0] = 0 \text{ due } [I - \tau - I] = Y \\
\frac{x[0, I]}{\tau} = \phi
\]

Delay, simulation with \( t = 10 \text{ ms} \)

\[
(\tau \frac{\tau}{I} - \tau) n \begin{bmatrix} I \\ 0 \end{bmatrix} + x \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} = \tau
\]
Conclusions

- Computation of $\vartheta$.  
- Use of state-dependent models.

Open problems:

- Applied to remote output stabilization.
- Placement on the time-shifted system.
- Results in an exponentially converging closed-loop system and pole placement.

The proposed controller:

- Stabilizing an open-loop unstable system with $T(t)$.

Remote stabilization via communication networks.

Conclusions
Introduce \( t = t + \pm(t) \), then

\[
 \frac{dx(\cdot)}{dt} = \theta(t) A x(\cdot); \quad \theta(t) = 1 + d\pm(t) dt
\]

Consider \( V(t) = x(t) T x(t) \).

\[
\frac{dp(t)}{q(t)q} + I = (t) \wedge (5) x^p V(t) = \frac{5p}{(5)x^p}
\]

Introduce \( (t)q + \tau = (t)5 \).

Proof of Stability Analysis
\[ \infty + = (\mathcal{S}, 0)_{\Phi} \]

and from \( \infty \leftarrow \mathcal{S} \) implies \( \infty \leftarrow \mathcal{T} \).

\[ (0, \mathcal{S} - \mathcal{S}) \frac{\mathcal{C}(d)_{\mathcal{W} \mathcal{Y}}}{\mathcal{I}(\mathcal{O})_{\mathcal{W} \mathcal{Y}}} \leq \theta P(\theta) \int_{\mathcal{S}} \frac{(d)_{\mathcal{W} \mathcal{Y}}}{(\mathcal{O})_{\mathcal{W} \mathcal{Y}}} = (\mathcal{S}, 0)_{\Phi} \]

where

\[ (\mathcal{S}0, \mathcal{S})_{\Phi} - \mathcal{C} ||(0, \mathcal{S})_{x}|| \frac{(d)_{\mathcal{W} \mathcal{Y}}}{(d)_{\mathcal{W} \mathcal{Y}}} \leq \mathcal{C} ||(\mathcal{S})_{x}|| \leq (\mathcal{S}, 0)_{\Phi} - \mathcal{C} \text{ and integrating from the bounds of (3) } \leq (\mathcal{S})_{\Lambda} \]

\[ : (4) \mathcal{S} \mathcal{S} = (0) \mathcal{S} \text{ to } (\mathcal{S})_\Lambda \text{ and integrating from the bounds of (3) } \]
Existence of $\{ \tau \geq (\theta) \tau - \theta, \theta + [0, \tau] \cap \theta A \mid 0 < \theta \} \max \{ \sigma \} \approx (\tau) \sigma$
Corollary:
The control law applied to the system (1)-(2), has a bounded solution and exponentially converges to zero. For all $t \geq 0$. From the previous lemma, the state then exponentially converges to zero.

Proof:

- Linear: its states cannot diverge in finite time.
- $A^0 \phi(0)$ bounded.
- $x(0)$ bounded.
- From the previous lemma, the state then exponentially converges to zero.
Theorem 2.
Assume that the delay dynamics (3)-(4) is such that the following holds for $\theta^i(t)$,

$$
\forall i \in \{1, 2\}, \quad \theta^i(t) \geq (t) + T - t \geq 0.
$$

Then, the observer-based feedback control law

$$
u(t) = K_1 \dot{x}(t) + K_2 x(t) + H f(x(t) - C x(t))
$$

with $0 < \Theta < |\frac{\nu}{A}|$ (A2)

$$
\forall \Theta > 0 \geq (t) + T - t < \infty \quad \forall i = 1, 2.
$$

Following holds for $\theta^i(t)$,

$$
\forall i \in \{1, 2\}, \quad \theta^i(t) = 1.
$$

Assume that the delay dynamics (3)-(4) is such that the