Stabilization of Networked Controlled Systems

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• Deterministic model of the network
  – allow for non-deterministic behavior: robustness
  ⇒ use system information to increase performance

• Application to secure networks (TCP-SPX-LAN)

• Open-loop unstable system
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1. Overview of Network Problems

1. Quantization, encoding/decoding:
   - related to information theory,
   - control with limited information,
   - time-varying sampling,
   - differential coding - Δ-Modulation.

2. Congestion and packet loss:
   - congestion control,
   - discrete analysis and game theory.

3. Link and bandwidth allocation
   - distributed systems,
   - quality of service,
   - control under communication constraints.

4. Time-delays
   - Passivity: teleoperation,
   - Stability: robustness,
   - Stochastic approach: LQG control.

⇒ Pole-placement: state predictor.
Modern cars:
- multiple safety/comfort devices,
- VAN/CAN,
- high jitter.

SX-29:
- open-loop unstable,
- LAN,
- high performance control.

Global Hawk (UAV):
- local + remote control,
- wideband satellite and Line-Of-Sight data link communications.

ITER:
- large multi-systems device,
- LAN: control and data signals,
- scheduled tasks.
II. Problem Formulation

- The network dynamics is described by a dynamical model,

\[ \dot{z}(t) = f(z(t), u_d(t)), \quad z(t_0) = z_0 \]
\[ \tau(t) = h(z(t), u_d(t)) \]

i.e. for secure networks (one flow) \([Misra \ & \ all\ 00]\): TCP with AQM

\[
\frac{dW_i(t)}{dt} = \frac{1}{R_i(q)} - \frac{W_i(t)W_i(t - R_i(q))}{2R_i(q)}p(t)
\]
\[
\frac{dq(t)}{dt} \approx -C + \sum_{i=1}^{N} \frac{W_i(t)}{R_i(q)}
\]
\[
R_i(q) = \frac{q}{C} + T_{pi}
\]
• The remotely controlled system has the form

\[
\dot{x}(t) = Ax(t) + Bu(t - \tau(t)) \\
y(t) = Cx(t)
\]

• Hypotheses
  – \((A, B)\) and \((A, C)\) controllable and observable
  – the network dynamics is such that (secure network)

\[
0 \leq \tau(t) \leq \tau_{\text{max}}, \quad \forall t \geq 0 \\
\dot{\tau}(t) < 1, \quad \text{for almost all } t \geq 0
\]

NB: \(\tau(t)\) is the delay experienced by the signal, i.e. \(\dot{\tau}(t) = 1 \Leftrightarrow\) the data never gets to its destination.
III. Control design

• based on a state predictor with a time-varying horizon $\delta(t)$ [Artstein 82, Nihtilä 89, Uchida & all. 03] [Springer 2005]

$$x(t + \delta(t)) = e^{A\delta(t)}x(t) + e^{A(t+\delta(t))} \int_{t}^{t+\delta(t)} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta$$

$$u(t) = -Kx(t + \delta(t))$$

• results in the pole placement of the time-shifted closed-loop system

$$\frac{dx(t + \delta(t))}{d(t + \delta(t))} = (A - BK)x(t + \delta(t)) = A_{cl} x(t + \delta(t))$$

⇒ Non-linear time transformation $t \mapsto t + \delta(t)$ but exponential convergence if $A_{cl}$ Hurwitz & hyp. on $\tau(t)$ are satisfied.

• explicit use of the network dynamics: $\delta(t) = \tau(t + \delta(t))$
Dynamic computation of $\delta(t) = \tau(t + \delta(t))$ [IEEE CCA 2004]

- Let

$$S(t) = \hat{\delta}(t) - \tau(t + \hat{\delta}(t))$$

with

$$\dot{S}(t) + \lambda S(t) = 0$$

and $\lambda > 0$, to prevent for the numerical instabilities,

$\Rightarrow$ find $\hat{\delta}(t)$ such that $\hat{\delta}(t)$ reaches asymptotically the manifold $S(t) = 0$.

Using the assumption $\dot{\tau} \neq 1$, $\hat{\delta}(t)$ has the following dynamics

$$\dot{\hat{\delta}}(t) = -\frac{\lambda}{1 - d\tau(\zeta)/d\zeta} \hat{\delta} + \frac{d\tau(\zeta)/d\zeta + \lambda \tau(\zeta)}{1 - d\tau(\zeta)/d\zeta}$$

where $\zeta = t + \hat{\delta}$.
Output feedback and two-channels delays [IFAC TdS 2003]

\[ u(t) = -Ke^{A(\delta + \tau_1)}\hat{x}(t) - Ke^{A(t+\delta)}\int_{t-\tau_1}^{t+\delta} e^{-A\theta} Bu(\theta - \tau(\theta))d\theta \]

\[ \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t-\tau_1 - \tau(t-\tau_1)) + H\{y(t-\tau_1) - C\hat{x}(t)\}, \quad \hat{x}(t) \doteq \hat{x}(t - \tau_1(t)) \]
\[
\dot{z}(t) = f(z(t), u_d(t)), \quad z(0) = z_0
\]
\[
\tau(t) = h(z(t), u_d(t))
\]
\[
\dot{x}(t) = Ax(t) + Bu(t - \tau(t))
\]
\[
y(t) = C x(t)
\]
\[
u(t) = -K e^{A\delta(t)} \left[ x(t) + e^{At} \int_{t}^{t+\delta(t)} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta \right]
\]
\[
0 \leq \tau(t) \leq \tau_{max}
\]
\[
\dot{\tau}(t) < 1
\]

Time-Delay

Network Model

Predictor Horizon

Time-Varying Predictive Control

Linear System

\[
\dot{x}(t) = Ax(t) + Bu(t - \tau(t))
\]
\[
y(t) = C x(t)
\]
IV. Robustness with respect to delay uncertainties

Problem description: \( \epsilon(t) = \tau(t) - \bar{\tau}(t), \{ \epsilon_M, \dot{\epsilon}_M \} = \sup_t \{ \epsilon(t), \dot{\epsilon}(t) \} \)?

The estimated delay \( \bar{\tau}(t) \) is modelled by

\[
\begin{align*}
\dot{\bar{z}}(t) &= f_e(\bar{z}(t), u_{meas.}(t), u_{de}(t)), \quad \bar{z}(0) = \bar{z}_0 \\
\bar{\tau}(t) &= h_e(\bar{z}(t), u_{meas.}(t), u_{de}(t))
\end{align*}
\]

The closed-loop system writes as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t - \tau(t)) \\
u(t) &= -Ke^{A\bar{\delta}(t)} \left[ x(t) + e^{At} \int_t^{t+\bar{\delta}(t)} e^{-A\theta} Bu(\theta - \bar{\tau}(\theta))d\theta \right] \\
\bar{\delta}(t) &= \bar{\tau}(t + \bar{\delta}(t)) \quad (\bar{\tau} \neq \tau \Rightarrow \bar{\delta} \neq \delta)
\end{align*}
\]

\( \Rightarrow \) \textit{algebro-differential} system with a delayed state
i.e. differentiating the integral term leads to

\[
\dot{\mathbf{x}}(t) = \begin{bmatrix}
A & 0 \\
-(1 + \dot{\delta})e^{-A\delta} BK e^{A\delta} & A - (1 + \dot{\delta})e^{-A\delta} BK e^{A\delta}
\end{bmatrix} \mathbf{x}(t) \\
+ \begin{bmatrix}
-BKe^{\bar{A}\delta(t-\tau)} & -BKe^{\bar{A}\delta(t-\tau)} \\
0 & 0
\end{bmatrix} \mathbf{x}(t-\tau) \\
+ \begin{bmatrix}
0 & 0 \\
BKe^{\bar{A}\delta(t-\bar{\tau})} & BKe^{\bar{A}\delta(t-\bar{\tau})}
\end{bmatrix} \mathbf{x}(t-\bar{\tau})
\]

\[
= \begin{bmatrix}
A - BK & -BK \\
0 & A
\end{bmatrix} \mathbf{x}(t) \quad \text{when } \tau(t) = \bar{\tau} = 0
\]

⇒ Artstein’s equivalence principle does not work.
Exact method:

• System dynamics

\[
\begin{align*}
\dot{x}(t) &= Ax(t) - BK e^{A\bar{\delta}(t-\tau)} x(t-\tau) - BK I(t-\tau) \\
I(t) &= \int_0^{\bar{\delta}(t)} e^{-A\theta} Bu(t + \theta - \bar{\tau}(t + \theta)) d\theta
\end{align*}
\]

with the control law \( u(t) = -Ke^{A\bar{\delta}(t)} [x(t) + I(t)] \)

• Proposed analysis

1. consider \( I(t) \) as a norm-bounded disturbance on the state :

\[
\Delta(t) \overset{\triangle}{=} -BK I(t-\tau) \quad \text{with} \quad |\Delta(t)| < \infty \quad \text{and} \quad \lim_{t \to \infty} \Delta(t) \neq 0
\]

→ the state remains bounded,

2. show that this perturbation vanishes (stability of the control law).
Solution

- LMI: stability of the state & control law,
- heavy tools inducing conservatism,
⇒ formal approach but far from physical reality:

\[ \vec{\tau}(t), \epsilon_M, \dot{\epsilon}_M \]

distributed IC

\[
\begin{align*}
\dot{w}(t) &= (A - (1 + \dot{\delta})e^{-A\bar{\delta}} BK e^{A\bar{\delta}})w(t) \\
&\quad + BK(e^{A\bar{\delta}(t-\bar{\tau})}w(t - \bar{\tau}) - e^{A\bar{\delta}(t-\tau)}w(t - \tau)) \\
\mathcal{I}(t) &= \int_{0}^{\bar{\delta}(t)} e^{-A\theta} B u(t + \theta - \bar{\tau}(t + \theta))d\theta \\
u(t) &= -Ke^{A\bar{\delta}(t)}w(t)
\end{align*}
\]

Asymptotically stable

\[
\begin{align*}
\dot{x}(t) &= Ax(t) - BK e^{A\bar{\delta}(t-\tau)}x(t - \tau) \\
&\quad - BK\mathcal{I}(t - \tau)
\end{align*}
\]

BIBS
Approximated method: [IEEE TAC 2005]

- **Fundamental facts**
  - a norm-bounded disturbance on a nominally stable time-delayed system leads to a norm-bounded state,
  - the control law quickly exhibits slow variations,
  - the main disturbing effect comes from the fact that we have \( x(t + \bar{\delta}(t)) \) instead of \( x(t + \delta(t)) \).

- **Problem formulation**

  \[
  u(t) = -Ke^{A\bar{\delta}(t)} \left[ x(t) + e^{At} \int_{t}^{t+\bar{\delta}(t)} e^{-A\theta} Bu(\theta - \bar{\tau}(\theta)) d\theta \right]
  \]

  \[
  = -Ke^{A\bar{\delta}(t)} \left[ x(t) + e^{At} \int_{t}^{t+\bar{\delta}(t)} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta \\
  + e^{At} \int_{t}^{t+\bar{\delta}(t)} e^{-A\theta} B[u(\theta - \bar{\tau}(\theta)) - u(\theta - \tau(\theta))] d\theta \right]
  \]

  \[
  \simeq -Kx(t + \bar{\delta}(t))
  \]
\[
x'(t + \delta(t)) = Ax(t + \delta(t)) + Bu(t)
\]

\[
\epsilon_M, \dot{\epsilon}_M \Rightarrow BIBS
\]

\[
-K x(t + \tilde{\delta}(t))
\]

\[
-K e^{A(t+\tilde{\delta}(t))} \int_t^{t+\delta(t)} e^{-A\theta} B[u(\theta - \bar{\tau}(\theta)) - u(\theta - \tau(\theta))]d\theta
\]

\[
\Rightarrow \quad x'(t + \delta(t)) = (A - BK)x(t + \delta(t)) + BK (x(t + \delta(t)) - x(t + \tilde{\delta}(t)))
\]
• Proposed solution [Gu, Kharitonov and Chen 03]

- Approximation of the time-varying delay: $\epsilon(t)$ is characterized by its average $\epsilon_a$, max. deviation $\epsilon_d$ and max. variation $\dot{\epsilon}_M$

$$x'(\zeta) = Ax(\zeta) - BKx(\zeta - \epsilon_a) + BK \int_{\zeta - \epsilon(t)}^{\zeta - \epsilon_a} [Ax(\theta) - BKx(\theta - \epsilon(\theta))] \, d\theta$$

\[ G \begin{cases} 
  x'(\zeta) & = Ax(\zeta) - BKx(\zeta - \epsilon_a) + \epsilon_d BK u_2(\zeta) \\
  y_1(t) & = \frac{1}{\sqrt{1 - \dot{\epsilon}_M}} x(t) \\
  y_2(t) & = Ax(t) - BKu_1(t) \\
  u_1(t) & = \Delta_1 y_1(t) = \sqrt{1 - \dot{\epsilon}_M} y_1(t - \epsilon(t)) \\
  u_2(t) & = \Delta_2 y_2(t) = \frac{1}{\epsilon_d} \int_{t - \epsilon(t)}^{t - \epsilon_a} y_2(\theta) \, d\theta \\
\end{cases} \]

\[ \Delta : \begin{cases} 
  u_1(t) & = \Delta_1 y_1(t) = \sqrt{1 - \dot{\epsilon}_M} y_1(t - \epsilon(t)) \\
  u_2(t) & = \Delta_2 y_2(t) = \frac{1}{\epsilon_d} \int_{t - \epsilon(t)}^{t - \epsilon_a} y_2(\theta) \, d\theta \\
\end{cases} \]

[Gu, Kharitonov and Chen 03]: $\gamma(\Delta) < 1$
- Scaled small gain: show that $\gamma_0(G_X) < 1$ for $X = \text{diag}(X_1 \, X_2)$, $X_1, X_2 \in \mathbb{R}^{n \times n}$ non-singular, with

$$G \subset \begin{cases} \dot{x}(t) &= A_0 x(t) + A_1 x(t - r) + Eu(t) \\ y(t) &= G_0 x(t) + G_1 x(t - r) + Du(t), \end{cases}$$

analyzed with a parameterized Lyapunov-Krasovskii functional

$$V(t, \phi) = \phi^T(0) P \phi(0) + 2 \phi^T(0) \int_{-r}^{0} Q(\xi) \phi(\xi) d\xi$$

$$+ \int_{-r}^{0} \left[ \int_{-r}^{0} \phi^T(\xi) R(\xi, \eta) \phi(\eta) d\eta \right] d\xi + \int_{-r}^{0} \phi^T(\xi) S(\xi) \phi(\xi) d\xi$$

which is then discretized for LMI synthesis.

$\Rightarrow$ more realistic result:

* full use of the predictor’s properties,
* Gu’s approach is far less conservative.
Example

\[ \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -21.54 & 0 & 14.96 & 0 \\ 0 & 0 & 0 & 1 \\ 65.28 & 0 & -15.59 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 8.10 \\ 0 \\ -10.31 \end{bmatrix} u(t - \tau) \]

<table>
<thead>
<tr>
<th>$\dot{\epsilon}_M$</th>
<th>0.00</th>
<th>0.09</th>
<th>0.18</th>
<th>0.27</th>
<th>0.36</th>
<th>0.45</th>
<th>0.54</th>
<th>0.63</th>
<th>0.72</th>
<th>0.81</th>
<th>0.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_a$ (ms)</td>
<td>8.1</td>
<td>7.8</td>
<td>7.6</td>
<td>7.3</td>
<td>7.0</td>
<td>6.7</td>
<td>6.3</td>
<td>5.8</td>
<td>5.2</td>
<td>4.4</td>
<td>3.4</td>
</tr>
</tbody>
</table>

i.e. $\dot{\epsilon}_M = 0.6167, \epsilon_d = 2\epsilon_a \Rightarrow \epsilon_a = 5.9 ms$

\[ \epsilon(t) = \epsilon_a + \epsilon_a \sin \left( \frac{\dot{\epsilon}_M}{\epsilon_a} t \right) \]
\[ \tau(t) = \bar{\tau}(t) + \epsilon(t) \]

\[ \bar{\tau}(t) = \theta(t) - z(t) \]
V. Application: control of an inverted pendulum through a TCP network

T-shape ECP Inverted Pendulum:

- Dynamics: 4\(^{th}\) order, OL unstable, nonminimum phase, coupled nonlinearities...

- Linearized model $\rightarrow A, B$

- LQR synthesis $\rightarrow K$

TCP network:
From the fluid flow model developed by [Misra & all 00] and assuming that $N(\zeta)$ is known at $t$, $\delta(t)$ is obtained from

$$
\tau(\zeta) = \frac{1}{2} \left[ \frac{q(\zeta)}{C_r} + T_{pcs} \right], \quad \frac{d\tau}{d\zeta}(\zeta) = \frac{1}{2C_r} \left[ \sum_{i=1}^{N(\zeta)} \frac{W_i(\zeta)}{R_i(\zeta)} - C_r \right] \rightarrow \delta(t)
$$
Experimental setup

Network model (simulated):

\[
\begin{align*}
\frac{dW_1(t)}{dt} &= \frac{1}{R_1(t)} - \frac{W_1(t)W_1(t - R_1(t))}{2R_1(t - R_1(t))}p_1(t), \\
\frac{dW_2(t)}{dt} &= \frac{1}{R_2(t)} - \frac{W_2(t)W_2(t - R_2(t))}{2R_2(t - R_2(t))}p_2(t), \\
\frac{dq(t)}{dt} &= -300 + \sum_{i=1}^{2} \frac{W_i(t)}{R_i(t)}, \quad q(0) = 5 \\
\tau(t) &= R_1(t)/2
\end{align*}
\]

Inverted Pendulum:

With

\[
\begin{align*}
R_1(t) &\doteq \frac{q(t)}{300} + 0.001 \\
R_2(t) &\doteq \frac{q(t)}{300} + 0.0015 \\
p_{1,2}(t) &= 0.005q(t - R_{1,2}(t)) \\
W_1(0) &= W_2(10) = 10 \text{ packets}.
\end{align*}
\]
Experimental results [video]

Network Behavior

Induced Delay (s)

Queue length and average TCP window size (packets)

State Predictor with Time–Varying Delay

x (m)

force (N)

theta (deg)
Comparison with other methods

Fixed horizon predictor:

\[
    u(t) = -K \left[ e^{A\tau_{\text{max}}} x(t) \\
    + e^{A(t+\tau_{\text{max})}} \int_{t}^{t+\tau_{\text{max}}} e^{-A\theta} B u(\theta - \tau_{\text{max}}) d\theta \right]
\]

- better when $\tau(t)$ close to $\tau_{\text{max}}$,
- HF disturbance on the control signal,
- deteriorated system response.
Buffer strategy:

\[
-u(t) = -K \left[ e^{A\tau_{max}}x(t) + e^{A(t+\tau_{max})} \int_{t}^{t+\tau_{max}} e^{-A\theta} Bu(\theta - \tau_{max}) d\theta \right]
\]

- a buffer with delay \( \tau_{max} - \tau(t) \) is set at the system’s input,
- HF disturbance on the control signal,
- deteriorated system response.
Robustness issues

Robustness wrt. model uncertainties

Sensor offset and impulses

- Angle of 5° nominal impulse
- Sensor offset and impulses:
  - 65 mm
  - 69 mm
  - 71 mm
  - 73 mm (model)
  - 60 mm

Robustness wrt. model uncertainties:
- 73 mm (model)
- 71 mm
- 69 mm
- 65 mm
- 60 mm
Robustness issues (2)
Conclusions and Perspectives

• Remote stabilization via communication networks
  ⇒ stabilizing an open-loop unstable system with a time-varying delay.

• The proposed controller:
  – based on a $\delta(t)$-step ahead predictor,
  – results in an exponentially converging (non uniform) closed-loop system and pole placement on the time-shifted system,
  – applied to remote output stabilization and observer-based control,
  – robust with respect to time-delay uncertainties.

• Perspectives:
  – experiments: faster system or longer delay (wireless),
  – extension to the nonlinear case,
  – investigate the network delay estimation and the dedicated network control [Briot05],
  – coupling between the system controller and the dedicated network controller.