New adaptive feedforward compensation algorithms for active vibration control with mechanical coupling.

Theory and applications

I.D. Landau – Emeritus Research Director at CNRS, GIPSA-LAB
T.B. Airișitoaie – PhD Student GIPSA Lab, University of Grenoble
M. Alma – Teaching assistant (PhD) GIPSA Lab, University of Grenoble

Attenuation (cancellation) of disturbances

• Adaptive feedforward compensation of disturbances has a long history
• Noise and Vibration cancellation were the driving forces

Let's forget for the moment the history and ask the question:

*Why we need adaptive feedforward compensation?*
Attenuation (cancellation) of disturbances

How about using feedback?

- The performance of disturbance attenuation is limited by the Bode integral
- Only narrow or limited band disturbances can be significantly attenuated by feedback

For “wide band“ disturbances one needs to use:

feedforward compensation

Why it should be adaptive?

Because the characteristics of the disturbance are unknown and time varying

What is the price to pay?

- One needs a correlated measurement with the disturbance.
- This implies:
  1) Hardware (additional transducer)
  2) A good location of the transducer (may require a study).
An “idyllic” scheme for adaptive feedforward disturbance compensation

The reality is different
• The compensator system (secondary path) acts upon the source
• There is an internal positive feedback (physical feedback)
• FIR compensators are not very efficient
Outline

• Why we need “adaptive feedforward compensation”?
• A basic configuration
• The inherent positive internal feedback
• An active vibration control system
• Background
• Problem formulation
• Adaptive IIR feedforward compensators
• Experimental results and comparisons with existing algorithms
• Youla Kucera parametrized adaptive feedforward compensators
• Experimental results and comparison with adaptive IIR
• How about adding feedback?
• Experimental results
• Concluding comments
An active vibration control system using an inertial actuator

New algorithms for adaptive feedforward compensation
New algorithms for adaptive feedforward compensation

**Frequency characteristics**

**Identified Models**

Complexity of the models:

\[
\begin{align*}
 n_{B_D} &= 26, & n_{A_D} &= 26; \\
 n_{B_G} &= 17, & n_{A_G} &= 15; \\
 n_{B_M} &= 16, & n_{A_M} &= 16;
\end{align*}
\]

Rem: secondary path has complex zeros at 108 Hz where primary path has a resonance.

New algorithms for adaptive feedforward compensation
New algorithms for adaptive feedforward compensation

IIR Adaptive feedforward disturbance compensation

Global primary path

\[ s(t) \xrightarrow{W} d(t) \xrightarrow{D} x(t) \xrightarrow{\chi(t)} \]

Measurement of the image of the disturbance

Positive feedback coupling

\[ \hat{\chi}(t) \]

Feedforward filter

\[ \hat{y}^0(t) \]

Parameter Adaptation Algorithm

\[ v^0(t) \]

Residual acceleration measurement

Global primary path
New algorithms for adaptive feedforward compensation

- The adaptive feedforward compensation algorithms have been developed assuming no « internal positive coupling »
- The « internal positive coupling » has been recognized after 1995
- Ad-hoc solutions have been proposed. Not validated by practice

**Directions of research:**
- *Analysis of existing algorithms (Fu-LMS) in presence of internal feedback (Wang-Ren(1999), Fraanje(2003) - convergence but not stability analysis)*
The Problem

The adaptive feedforward compensator should minimize the effect of the disturbance while simultaneously assuring the stability of the internal positive feedback loop.

Two modes of operation

- Self-tuning (adaptation gain vanishes)
- Adaptive (non vanishing adaptation gain)

How to solve the problem:

1) Design the largest family of stable adaptive algorithms for feedforward compensation in the presence of the internal positive feedback loop
2) Find among this family the algorithms which provide the best performance
Hypotheses

Case I (basic case)
H1: disturbance $d(t)$ is bounded
H2: Perfect matching condition

Exists $N(z^{-1}) = \frac{R(z^{-1})}{S(z^{-1})}$ such that

$$\frac{G N}{1 - N M} = \frac{G \cdot A_M R}{A_M S - B_M R} = -D$$

and the characteristic polynomial of the internal feedback loop:

$$P(z^{-1}) = A_M(z^{-1})S(z^{-1}) - B_M(z^{-1})R(z^{-1})$$

is Hurwitz

H3: Measurement noise on the residual error is neglected (deterministic context)
H4: The primary path model $D$ is unknown and constant

Case II: Hypotheses H2 and H3 are removed
IIR Feedforward - Basic equations

Feedforward compensator: \( \hat{N}(t, q^{-1}) = \hat{R}(t, q^{-1}) / \hat{S}(t, q^{-1}) \)

Define:
\[
\hat{\theta}^T(t) = [\hat{s}_1(t),...,\hat{s}_{n_S}(t),\hat{r}_0(t),...,\hat{r}_{n_R}(t)]
\]
\[
\phi^T(t) = [-\hat{y}(t),...,\hat{y}(t-n_S+1),\hat{u}(t+1),...,\hat{u}(t-n_R+1)]
\]

Compensator output:
\[
\hat{y}^\phi(t+1) = \hat{y}(t+1/\hat{\theta}(t)) = \hat{\theta}^T(t)\phi(t)
\]
\[
\hat{y}(t+1) = \hat{y}(t+1/\hat{\theta}(t+1)) = \hat{\theta}^T(t+1)\phi(t)
\]

Measurement (residual acceleration):
\[
\chi^0(t+1)
\]

A priori adaptation error:
\[
v^0(t+1) = v(t+1/\hat{\theta}(t)) = -\chi^0(t+1)
\]

A posteriori adaptation error:
\[
v(t+1) = v(t+1/\hat{\theta}(t+1))
\]

Filtered observation vector:
\[
\phi_f(t) = L(q^{-1})\phi(t)
\]

\[
v(t+1) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})L(q^{-1})} \left[ \theta - \hat{\theta}(t+1) \right]^T \phi_f(t)
\]

New algorithms for adaptive feedforward compensation
Parametric adaptation algorithms

Measurement (residual acceleration): \( \chi^0(t + 1) \)

A priori adaptation error: \( \nu^0(t + 1) = -\chi^0(t + 1) \)

Parametric adaptation algorithm: \( \hat{\theta}(t + 1) = \hat{\theta}(t) + F(t)\varphi_f(t)\nu(t + 1) \)

A posteriori adaptation error: \( \nu(t + 1) = \frac{\nu^0(t + 1)}{1 + \varphi_f^T(t)F(t)\varphi_f(t)} \)

Adaptation gain: \( F(t + 1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\varphi_f(t)\varphi_f(t)^T \)

\( \lambda_1(t) \) and \( \lambda_2(t) \) define the adaptation gain profile

with:

Choice of filter L:
(I): \( L = G \);  (II): \( L = \hat{G} \);  (III): \( L = \frac{\hat{A}_M}{\hat{P}} \hat{G} \)

Best performance

New algorithms for adaptive feedforward compensation
New algorithms for adaptive feedforward compensation

Stability and Convergence condition - IIR

\[ H'(z^{-1}) = H(z^{-1}) - \frac{\lambda_2}{2} = \text{Strictly Positive Real (S.P.R.)}. \tag{\star} \]

for Alg. III: \[ H(z^{-1}) = \frac{\hat{P}A_M G}{P\hat{A}_M \hat{G}} \]

const. adapt. gain: \( \lambda_2 = 0 \) \hspace{1cm} \text{time varying. adapt. gain: } \lambda_2 \leq 2

\[ P(z^{-1}) = A_M(z^{-1})\hat{S}(z^{-1}) - B_M(z^{-1})\hat{R}(z^{-1}) ; \quad \hat{P}(z^{-1}) = \hat{A}_M(z^{-1})\hat{S}(z^{-1}) - \hat{B}_M(z^{-1})\hat{R}(z^{-1}) \]

\( A_M, B_M \) and \( G \) are constant and very good estimations are available

Condition (\( \star \)) for \( \lambda_2=1 \) becomes:

\[ \left| \left( \frac{A_M \cdot \hat{P} \cdot G}{\hat{A}_M \cdot P \cdot \hat{G}} \right)^{-1} \right| < 1 \quad \text{for all } \omega \]

Rem: Alg III needs initialization with Alg II in order to get a first estimation of \( \hat{P} \)
An interpretation of the SPR condition

For constant adaptation gain the SPR condition on \( H' (=H \text{ in this case}) \) implies that the angle between the inverse of the true gradient (which can not be computed) and the direction of correction (\( \text{defined by } \phi_f \)) is less than 90° in all directions. (see Appendix)

For time varying adaptation gains a similar interpretation holds
Case II : non perfect matching + measurement noise

Measurement noise

In the presence of measurement noise independent of the disturbance, convergence occurs under same conditions as for stability in the deterministic context

Non perfect matching

- Boundedness of the variables can be guaranteed under mild hypotheses
- The approximation in the frequency domain is given by:

\[
\hat{\theta}^* = \arg \min_{\theta} \int_{-\pi}^{\pi} \left[ |S_{NM}|^2 |N - \hat{N}|^2 |S_{\hat{NM}}|^2 |G|^2 \Phi_d(\omega) + \Phi_w(\omega) \right] d\omega
\]

With:

\[
S_{NM} = \frac{1}{1 - NM} ; \quad S_{\hat{NM}} = \frac{1}{1 - \hat{NM}} \quad \Phi_d = PSD \ of \ d \\
\Phi_w = PSD \ of \ meas.\ noise
\]

- Good approximation of the optimal compensator \( N \) will be obtained in the frequency regions where \( \Phi_d \) is significant and \( G \) has high gain.
- Further weighting is introduced by the sensitivity function of the internal loop
New algorithms for adaptive feedforward compensation

Time varying matrix adaptation gain (decreasing gain + constant trace)

<table>
<thead>
<tr>
<th>Number of parameters</th>
<th>20</th>
<th>32</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global attenuation (db)</td>
<td>16.23</td>
<td>16.49</td>
<td>16.89</td>
</tr>
</tbody>
</table>

Wide band + narrow band

Adaptive IIR feedforward compensator – Experimental results
Adaptive IIR feedforward compensator – Experimental results

Scalar adaptation gain – comparison with FULMS

Adaptation transient for FULMS (adaptation starts at t=50s)

Instability occurs on a long run

Adaptation transient for Alg.III scalar (adaptation starts at t=50s)

Power Spectral Density Estimates

- Open loop
- Alg. III with scalar gain: -16.08dB
- Alg. II with scalar gain: -14.35dB
- \( H_\infty \) controller: -12.37dB
- FULMS: -11.77dB
New algorithms for adaptive feedforward compensation

IIR Filter + Adaptive YK_IIR filter feedforward compensation

Global primary path

Primary path

Positive feedback coupling

Secondary path

PAA

Parameter Adaptation Algorithm

Residual acceleration measurement

Measurement of the image of the disturbance

\[ \hat{A}_Q = 1, \hat{B}_Q = \hat{Q} \]

New algorithms for adaptive feedforward compensation
IIR Filter + Adaptive Youla Kucera (IIR) – Basic equations

Optimal Q filter:

\[ Q(z^{-1}) = \frac{B_Q(z^{-1})}{A_Q(z^{-1})} = \frac{b^Q_0 + b^Q_1 z^{-1} + \ldots + b^Q_{nB_Q} z^{-nB_Q}}{1 + a^Q_1 z^{-1} + \ldots + a^Q_{nA_Q} z^{-nA_Q}} \]

\[ \hat{R}(q^{-1}) = \hat{A}_Q(q^{-1})R_0(q^{-1}) - A_M(q^{-1})\hat{B}_Q(t,q^{-1}); \quad \hat{S}(q^{-1}) = \hat{A}_Q(q^{-1})S_0(q^{-1}) - B_M(q^{-1})\hat{B}_Q(t,q^{-1}) \]

Perfect matching →

\[
\frac{G \cdot A_M \left( R_0 A_Q - A_M B_Q \right)}{A_Q \left( A_M S_0 - B_M R_0 \right)} = -D
\]

Characteristic polynomial of the internal feedback loop:

\[ P(z^{-1}) = A_Q \left[ A_M(z^{-1})S_0(z^{-1}) - B_M(z^{-1})R_0(z^{-1}) \right] \]

Define:

\[ \hat{\theta}^T(t) = \left[ \hat{b}^Q_0(t), \ldots, \hat{b}^Q_{nB_Q}(t), \hat{a}^Q_1(t), \ldots, \hat{a}^Q_{nA_Q}(t) \right] \]

\[ \hat{\alpha}(t+1) = B_M \hat{y}(t+1) - A_M \hat{u}(t+1); \quad \hat{\beta}(t) = S_0 \hat{y}(t) - R_0 \hat{u}(t) \]

with

\[ \varphi^T(t) = [\hat{\alpha}(t+1), \ldots, \hat{\alpha}(t-n_{B_Q}+1), -\hat{\beta}(t), \ldots, -\hat{\beta}(t-n_{A_Q})] \]

Compensator output:

\[ \hat{y}^0(t+1) = -S_0^* \hat{y}(t) + R_0 \hat{u}(t+1) + \hat{\theta}^T(t) \varphi(t) \]

\[ \hat{y}(t+1) = -S_0^* \hat{y}(t) + R_0 \hat{u}(t+1) + \hat{\theta}^T(t+1) \varphi(t) \]
Parametric adaptation algorithms (PAA)

Measurement (residual acceleration): $\chi^0(t+1)$

A priori adaptation error: $\nu^0(t+1) = -\chi^0(t+1)$

Parametric adaptation algorithm: $\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\varphi_f(t)\nu(t+1)$

A posteriori adaptation error: $\nu(t+1) = \frac{\nu^0(t+1)}{1 + \varphi_f^T(t)F(t)\varphi_f(t)}$

Adaptation gain: $F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\varphi_f(t)\varphi_f(t)^T$

$\lambda_1(t)$ and $\lambda_2(t)$ define the adaptation gain profile

with:

$\varphi_f(t) = L(q^{-1})\varphi(t)$

Choice of filter $L$:
(1) $L = G$; (II) $L = \hat{G}$; (III) $L = \frac{\hat{A}_M}{\hat{P}}\hat{G}$

Best performance

$\nu(t+1) = \frac{A_M(q^{-1})}{P(q^{-1})L(q^{-1})}G(q^{-1})[\theta - \hat{\theta}(t+1)]^T\varphi_f(t)$

New algorithms for adaptive feedforward compensation
Stability and Convergence condition

\[ H(z^{-1}) - \frac{\lambda_2}{2} = \text{Strictly Positive Real (S.P.R.).} \]

for Alg. III: \[ H(z^{-1}) = \frac{\hat{P}A_M G}{P\hat{A}_M \hat{G}} \] (for IIR, YK_FIR, YK_IIR)

const. adapt. gain: \( \lambda_2 = 0 \)  
time varying. adapt. gain: \( \lambda_2 \leq 2 \)

Adaptive IIR
\[ P(z^{-1}) = A_M(z^{-1})\hat{S}(z^{-1}) - B_M(z^{-1})\hat{R}(z^{-1}) \]

IIR + adaptiveYK_FIR
\[ P(z^{-1}) = A_M(z^{-1})S_0(z^{-1}) - B_M(z^{-1})R_0(z^{-1}) \]

constant and known

IIR + adaptiveYK_IIR
\[ P(z^{-1}) = \hat{A}_Q[A_M(z^{-1})S_0(z^{-1}) - B_M(z^{-1})\hat{R}_0(z^{-1})] \]

\( A_M, B_M \) and G are constant and very good estimations are available

New algorithms for adaptive feedforward compensation
Theoretical comparative analysis

Adaptive IIR, Fix IIR+adaptive YK_FIR, Fix IIR+adaptive YK_IIR, same Strictly Positive Real condition for stability and convergence

Differences

• The poles of the internal loop
  IIR: unknown, can be very close to the unit circle
  IIR+YK_FIR: assigned and fixed
  IIR+YK_IIR: partly assigned and fixed, the remaining can be bounded

• The SPR condition (to be achieved by filtering)
  IIR: one needs an estimation of the poles to build the filter III
  IIR+YK_FIR: the poles for the filter III are known from the beginning
  IIR+YK_IIR: some poles are known from the beginning, the other are directly obtained from the PAA

• Use of a model based designed compensator for initialization
  IIR: difficult (dimension problem)
  IIR+ YK_FIR or +YK_IIR: easy (it is the central stabilizing controller)
Summary of experimental results

Disturbance: wide band disturbance

Comparable performance both in frequency and in time domains

Global attenuation

<table>
<thead>
<tr>
<th>Number of adjustable parameters</th>
<th>0</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global attenuation-IIR (db)</td>
<td>-</td>
<td></td>
<td></td>
<td><strong>16.49</strong></td>
<td>16.89</td>
</tr>
<tr>
<td>Global att.-YK_FIR/H_∞ (db)</td>
<td>14.70</td>
<td>15.4</td>
<td>15.6</td>
<td><strong>16.52</strong></td>
<td>16.03</td>
</tr>
<tr>
<td>Global att.-YK_FIR/PP (db)</td>
<td>4.61</td>
<td>14.69</td>
<td><strong>15.89</strong></td>
<td>15.7</td>
<td>15.33</td>
</tr>
<tr>
<td>Global attenuation-IIR/H_∞ (db)</td>
<td>14.70</td>
<td></td>
<td></td>
<td><strong>16.53</strong></td>
<td>16.47</td>
</tr>
<tr>
<td>Global att.-YK_IIR/PP (db)</td>
<td>4.61</td>
<td>15.53</td>
<td><strong>16.21</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( H_∞ = H_∞ \text{ MBD central controller} \quad \text{PP = Pole Placement central controller} \)

Model Based Design requires identification of the disturbance and of the primary path

*For IIR+YK_F(I)IR, performance depends upon the central controller*

New algorithms for adaptive feedforward compensation
New algorithms for adaptive feedforward compensation

Time domain results (32 parameters)
New algorithms for adaptive feedforward compensation

Frequency domain results (32 parameters)

Adaptive IIR

IIR + Adaptive YK(FIR)

IIR + Adaptive YK(IIR)

YKIIR4/4(\(H_\infty\))

YKIIR4/4(PP)
Concluding Remarks

• Adaptive IIR, Fix IIR + adaptive YK_FIR and Fix IIR + adaptive YK_FIR have close performances.

• The performances of the Fix IIR + adaptive YK_FIR depends upon the performance of the central controller.

• The performance of the Fix IIR + adaptive YK_IIR is less dependent upon the performance of the central controller.

• Fix IIR + adaptive YK_FIR allows the easiest implementation of the algorithm (the filter for SPR condition).

• Fix IIR + adaptive YK_IIR offers the best ratio performance/nb. of adaptive parameters.
The “logical” approach in active vibration control (AVC):
Do as much as possible by feedback (limitations due to Bode Integral) and then add feedforward compensation of disturbance

The “reality”:
The developments in the field of AVC started by using feedforward compensation and only recently the interest of adding feedback has been recognized (Eyzmailzadeh et al 2002 Ray et al 2006, de Callafon 2010)

No reference available for the analysis of the algorithms for adaptive feedforward compensation:
• in the presence of a feedback controller
• in the joint presence of a feedback controller and of an internal positive feedback coupling
New algorithms for adaptive feedforward compensation

Adaptive IIR feedforward compensation + feedback controller

Global primary path

Primary path

Positive feedback coupling

Secondary path

Measurement of the image of the disturbance

Feedforward filter

Parameter Adaptation Algorithm

Feedback controller:

\[ K(q^{-1}) = \frac{B_K(q^{-1})}{A_K(q^{-1})} \]
Stability and Convergence condition

Same structure of the adaptation algorithm but different filtering

Choice of filter L:

(I): \( L = G \);  
(II): \( L = \frac{\hat{G}}{1 + \hat{G}K} \);  
(III): \( L = \frac{\hat{A}_M \hat{A}_G \hat{A}_K \hat{G}}{\hat{P}_{fb-ff}} \)

\[
H\left(z^{-1}\right) - \frac{\lambda_2}{2} = \text{Strictly Positive Real (S.P.R.).}
\]

const. adapt. gain: \( \lambda_2 = 0 \)  
time varying. adapt. gain: \( \lambda_2 \leq 2 \)

for Alg. III:  
\[
H\left(z^{-1}\right) = \frac{\hat{P}_{fb-ff} A_M A_G G}{P_{fb-ff} \hat{A}_M \hat{A}_G \hat{G}} = \frac{\hat{P}_{fb-ff} A_M B_G}{P_{fb-ff} \hat{A}_M \hat{B}_G}
\]

\[
\hat{P}_{fb-ff}\left(z^{-1}\right) = \hat{A}_M \hat{S} \left[ \hat{A}_G A_K + \hat{B}_G B_K \right] - \hat{B}_M \hat{R} A_K \hat{A}_G
\]

- \( M(A_M, B_M) \) and \( G(A_G, B_G) \) are constant and very good estimations are available
- Implementation of alg. III requires an estimation of S and R (one runs alg. II during an initialization horizon)
- It is possible to continuously update \( \hat{S} \) and \( \hat{R} \) in the filter

New algorithms for adaptive feedforward compensation
Summary of experimental results

Disturbance: wide band disturbance

Global attenuation

<table>
<thead>
<tr>
<th></th>
<th>Feedback only</th>
<th>Feedforward only (H_{inf})</th>
<th>Adaptive Feedforward only</th>
<th>Feedback &amp; Adaptive Feedforward</th>
<th>Feedforward (H_{inf}) &amp; Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attenuation</td>
<td>-14.40 dB</td>
<td>-14.70 dB</td>
<td>-16.23 dB</td>
<td>-20.53 dB</td>
<td>-18.42 dB</td>
</tr>
</tbody>
</table>

Feedback controller: nB_{K}=16, nA_{K}=18 (Pole placement with sensitivity functions shaping)

Adaptive IIR feedforward compensator: 20 adjustable parameters (n_{R}=10, n_{S}=9)

Attention:
The design of the fixed H_{inf} feedforward compensator (last column) requires the knowledge of the disturbance characteristics (unknown and time varying in practice) and of the model of the primary path (not required by the adaptive algorithms)
Concluding Remarks

- Adding feedback to adaptive feedforward compensation improves significantly the performance of AVC
- The stability conditions for the adaptive feedforward algorithms are drastically changed by adding feedback
- The filter used in the adaptive feedforward algorithms depends upon the parameters of the feedback controller

Future work:
Combining adaptive feedforward compensation with adaptive feedback regulation
New algorithms for adaptive feedforward compensation

Bibliography

• Landau, I. D., Alma, M., Airimitoaie, T. B., Adaptive feedforward compensation algorithms for active vibration control with mechanical coupling, Automatica 47 (2011) 2185–2196
• Landau, I. D., Airimitoaie, T. B., Alma, M., A Youla-Kucera Parametrized Adaptive Feedforward Compensator for Active Vibration Control, IFAC World Congress (2011), Volume # 18 | Part# 1
• Landau, I. D., Airimitoaie, T. B., Alma, M., An IIR Youla-Kucera parametrized adaptive feedforward compensator for active vibration control with mechanical coupling, CDC 2011
• Alma, M., Landau, I. D., Martinez, J. J., Airimitoaie, T. B., Hybrid adaptive feedforward-feedback compensation algorithms for active vibration control systems, CDC 2011
New algorithms for adaptive feedforward compensation
Appendix
Influence of the physical feedback upon the source $d(t)$

The influence is significant and can not be neglected

New algorithms for adaptive feedforward compensation
New algorithms for adaptive feedforward compensation
IIR Filter + Adaptive Youla Kucera (FIR) – Basic equations

\[ \hat{R}(q^{-1}) = R_0(q^{-1}) - A_M(q^{-1})\hat{Q}(t,q^{-1}); \quad \hat{S}(q^{-1}) = S_0(q^{-1}) - B_M(q^{-1})\hat{Q}(t,q^{-1}) \]

Optimal Q filter: \[ Q(z^{-1}) = q_0 + q_1z^{-1} + \ldots + q_{n_Q}z^{-n_Q} \]

Perfect matching \[ \frac{G \cdot A_M \cdot (R_0 - A_M Q)}{A_M S_0 - B_M R_0} = -D \]

Characteristic polynomial of the internal feedback loop:
\[ P(z^{-1}) = A_M(z^{-1})S_0(z^{-1}) - B_M(z^{-1})R_0(z^{-1}) \] (remains unchanged)

Define:
\[ \theta^T(t) = [\hat{q}_0(t) \ldots \hat{q}_{n_Q}(t)]; \quad \varphi^T(t) = [\hat{\alpha}(t+1) \ldots \hat{\alpha}(t-n_Q+1)] \]

with \[ \hat{\alpha}(t+1) = B_M \hat{y}(t+1) - A_M \hat{u}(t+1) \]

Compensator output:
\[ \hat{y}^0(t+1) = -S_0^* \hat{y}(t) + R_0 \hat{u}(t+1) + \hat{\theta}^T(t)\varphi(t) \]
\[ \hat{y}(t+1) = -S^* \hat{y}(t) + R_0 \hat{u}(t+1) + \hat{\theta}^T(t+1)\varphi(t) \]

New algorithms for adaptive feedforward compensation
IIR Feedforward + Feedback - Basic equations (1)

Feedforward compensator:
$$\hat{N}(t, q^{-1}) = \frac{\hat{R}(t, q^{-1})}{\hat{S}(t, q^{-1})}$$

Perfect matching (optimal) \(\Rightarrow\)
$$\frac{\text{GN}}{1 - \text{NM}} = \frac{G \cdot A_M R}{A_M S - B_M R} = -D$$

The algorithms have been developed under “perfect matching” assumption and analyzed in the context of non perfect matching.

Characteristic polynomial of the internal “positive” feedback loop:
$$P(z^{-1}) = A_M(z^{-1})\hat{S}(z^{-1}) - B_M(z^{-1})\hat{R}(z^{-1})$$

Characteristic polynomial of the feedback loop:
$$P_{cl}(z^{-1}) = A_G(z^{-1})A_K(z^{-1}) + B_G(z^{-1})B_K(z^{-1})$$

Characteristic polynomial of the coupled feedforward-feedback loop:
$$P_{f_{b-f_{b}}}(z^{-1}) = A_M S[A_GA_K + B_GB_K] - B_M R A_K A_G$$

New algorithms for adaptive feedforward compensation
Define:

\[ \hat{\theta}^T(t) = [\hat{s}_1(t), \ldots, \hat{s}_{n_s}(t), \hat{s}_0(t), \ldots, \hat{s}_{n_R}(t)] \]

\[ \varphi^T(t) = [-\hat{y}_1(t), \ldots, -\hat{y}_1(t-n_s+1), \hat{u}(t+1), \ldots, \hat{u}(t-n_R+1)] \]

Feedforward Compensator output:

\[ \hat{y}^o(t+1) = \hat{y}(t+1/\hat{\theta}(t)) = \hat{\theta}^T(t)\varphi(t) \]

\[ \hat{y}(t+1) = \hat{y}(t+1/\hat{\theta}(t+1)) = \hat{\theta}^T(t+1)\varphi(t) \]

Measurement (residual acceleration):

\[ \chi^0(t+1) \]

A priori adaptation error:

\[ \nu^0(t+1) = \nu(t+1/\hat{\theta}(t)) = -\chi^0(t+1) \]

A posteriori adaptation error:

\[ \nu(t+1) = \nu(t+1/\hat{\theta}(t+1)) \]

Filtered observation vector:

\[ \varphi_f(t) = L(q^{-1})\varphi(t) \]

\[ \nu(t+1) = \frac{A_M(q^{-1})A_G(q^{-1})A_K(q^{-1})}{P_{fb-ff}(q^{-1})L(q^{-1})} G(q^{-1})[\theta - \hat{\theta}(t+1)]^T \varphi_f(t) \]
Parametric adaptation algorithms (PAA)

Measurement (residual acceleration): $\chi^0(t+1)$

A priori adaptation error: $\nu^0(t+1) = -\chi^0(t+1)$

Parametric adaptation algorithm: $\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi_f(t)\nu(t+1)$

A posteriori adaptation error: $\nu(t+1) = \frac{\nu^0(t+1)}{1 + \phi_f^T(t)F(t)\phi_f(t)}$

Adaptation gain: $F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\phi_f(t)\phi_f(t)^T$

$\lambda_1(t)$ and $\lambda_2(t)$ define the adaptation gain profile

$0 < \lambda_1(t) \leq 1$ ; $0 \leq \lambda_2(t) < 2$ ; $F(0) > 0$

with:

$\phi_f(t) = L(q^{-1})\phi(t)$

Choice of (I): $L = G$; (II): $L = \frac{\hat{G}}{1 + \hat{G}K}$; (III): $L = \frac{\hat{A}_M\hat{A}_G\hat{A}_K}{\hat{P}_{fb-ff}}\hat{G}$

Best performance

New algorithms for adaptive feedforward compensation
New algorithms for adaptive feedforward compensation

Time domain and frequency domain experimental results

Feedback & Ad. Feedforward IIR20

Adaptive IIR20

Can not react to unknown disturb.
An interpretation of the SPR condition

Without filtering and neglecting the non commutativity of time arrying operators

\[ \nu(t+1) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})} \left[ \theta - \hat{\theta}(t+1) \right]^T \varphi(t) = \left[ \theta - \hat{\theta}(t+1) \right]^T \varphi_f'(t) \]

\[ \varphi_f'(t) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})} \varphi(t) \]

Let's minimize \( J(t+1) = \nu^2(t+1) \) using gradient technique

\[ \frac{1}{2} \frac{\delta J(t+1)}{\delta \hat{\theta}(t+1)} = -\varphi_f'(t) \nu(t+1) \]

\[ \hat{\theta}(t+1) = \hat{\theta}(t) + F \varphi_f'(t) \nu(t+1); \ F = \alpha I, \ \alpha > 0 \]

In fact one uses (alg III):

\[ \varphi_f(t) = \frac{\hat{A}_M}{\hat{P}} \hat{G} \varphi(t) \]

\[ \hat{\theta}(t+1) = \hat{\theta}(t) + F \varphi_f(t) \nu(t+1) \]

SPR condition implies that the angle between the true gradient and the direction of correction is less than 90°
Interpretation of the SPR condition

Consider minimization of \( J(t+1) = \nu^2(t+1) \) using gradient technique

\[
\hat{\theta}(t+1) = \hat{\theta}(t) + F\varphi_f(t)\nu(t+1); \ F = \alpha I, \ \alpha > 0 \quad \varphi_f(t) = \frac{A_M G}{P} \phi(t)
\]

In fact one uses (alg III):

\[
\hat{\theta}(t+1) = \hat{\theta}(t) + F\varphi_f(t)\nu(t+1) \quad \varphi_f(t) = \frac{A_M \hat{G}}{\hat{P}} \phi(t)
\]

SPR condition on \( H'(=H\text{ in this case}) \) implies that the angle between the true gradient and the direction of correction is less than 90° in all directions

The “phase” of \( H \) is given by:

\[
-90^\circ < \angle \frac{A_M (e^{-j\omega}) G(e^{-j\omega})}{P(e^{-j\omega})} - \angle \frac{\hat{A}_M (e^{-j\omega}) \hat{G}(e^{-j\omega})}{\hat{P}(e^{-j\omega})} < 90^\circ \quad \text{at all frequencies}
\]