Adaptive Control

Part 5: Direct and Indirect Adaptive Control
Adaptive Control – A Basic Scheme

- Indirect adaptive control
- Direct adaptive control (*the controller is directly estimated*)
Outline

• Digital control systems

• Tracking and regulation with independent objectives (known parameters)

• Adaptive tracking and regulation with independent objectives (direct adaptive control)

• Pole placement (known parameters)

• Adaptive pole placement (indirect adaptive control)
The control law is implemented on a digital computer

ADC: analog to digital converter
DAC: digital to analog converter
ZOH: zero order hold
- Sampling time depends on the system bandwidth
- Efficient use of computer resources
The R-S-T Digital Controller

\[ q^{-1} y(t) = y(t-1) \]
Discrete time model – General form

\[(*) \ y(t) = - \sum_{i=1}^{n_A} a_i y(t - i) + \sum_{i=1}^{n_B} b_i u(t - d - i)\]

d – delay (integer multiple of the sampling period)

\[
1 + \sum_{i=1}^{n_A} a_i q^{-i} = A(q^{-1}) = 1 + q^{-1} A^*(q^{-1}) ; \quad A^*(q^{-1}) = a_1 + a_2 q^{-1} + \ldots + a_{n_A} q^{-n_A+1}
\]

\[
\sum_{i=1}^{n_B} b_i q^{-i} = B(q^{-1}) = q^{-1} B^*(q^{-1}) ; \quad B^*(q^{-1}) = b_1 + b_2 q^{-1} + \ldots + b_{n_B} q^{-n_B+1}
\]

\[(*) \ A(q^{-1}) y(t) = q^{-d} B(q^{-1}) u(t)\]

\[(*) \ A(q^{-1}) y(t + d) = B(q^{-1}) u(t)\] (Predictive form)

\[(*) \ y(t) = H(q^{-1}) u(t) ; \quad H(q^{-1}) = \frac{q^{-d} B(q^{-1})}{A(q^{-1})} \] - pulse transfer operator

\[q^{-1} \rightarrow z^{-1} \quad H(z^{-1}) = \frac{q^{-z} B(z^{-1})}{A(z^{-1})} \] - transfer function
First order systems with delay

Continuous time model

\[ H(s) = \frac{Ge^{-s\tau}}{1 + Ts} \]

\[ \tau = dTs + L; \quad 0 < L < Ts \]

Discrete time model

\[ H(z^{-1}) = \frac{z^{-d}(b_1z^{-1} + b_2z^{-2})}{1 + a_1z^{-1}} = \frac{z^{-d-1}(b_1 + b_2z^{-1})}{1 + a_1z^{-1}} \]

\[ a_1 = -e^{\frac{T_s}{\tau}} \]

\[ b_1 = G(1 - e^{\frac{L-T_s}{T_s}}) \]

\[ b_2 = Ge^{\frac{-T_s}{T_s}}(e^{\frac{L}{T_s}} - 1) \]

Remark: For \( L > 0.5Ts \Rightarrow b_2 > b_1 \Rightarrow \text{unstable zero} \ (\left| \frac{b_2}{b_1} \right| > 1) \)

\[ \frac{b_2}{b_1} < 1 \]

\[ \frac{b_2}{b_1} > 1 \]

\[ a_1 \]

\[ T_s \]

\[ z \]

\[ o \ - \ zero \]

\[ x \ - \ pole \]

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Tracking and regulation with independent objectives

It is a particular case of pole placement
(the closed loop poles contain the plant zeros))

It is a method which simplifies the plant zeros
Allows exact achievement of imposed performances

Allows to design a RST controller for:
• stable or unstable systems
• without restrictions upon the degrees of the polynomials $A$ et $B$
• without restriction upon the integer delay $d$ of the plant model
• discrete-time plant models with stable zeros!!!

Remarks:
• Does not tolerate fractional delay $> 0.5 T_s$ (unstable zero)
• High sampling frequency generates unstable discrete time zeros!
Tracking and regulation with independent objectives

The model zeros should be stable and enough damped

Admissibility domain for the zeros of the discrete time model
Tracking and regulation with independent objectives

Reference signal (tracking): \( y^*(t + d + 1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t) \)

Controller: \( S(q^{-1})u(t) = T(q^{-1})y^*(t + d + 1) - R(q^{-1})y(t) \)

\[
P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})
\]
Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

T.F. of the closed loop without $T$:

$$H_{CL}(q^{-1}) = \frac{q^{-d+1} B^*(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d+1} B^*(q^{-1}) R(q^{-1})} = \frac{q^{-d+1}}{P(q^{-1})} = \frac{q^{-d+1} B^*(q^{-1})}{B^*(q^{-1})P(q^{-1})}$$

The following equation has to be solved:

$$A(q^{-1})S(q^{-1}) + q^{-d+1} B^*(q^{-1}) R(q^{-1}) = B^*(q^{-1})P(q^{-1}) \quad (*)$$

$S$ should be in the form:

$$S(q^{-1}) = s_0 + s_1 q^{-1} + \ldots + s_{n_s} q^{-n_s} = B^*(q^{-1}) S'(q^{-1})$$

After simplification by $B^*$, (*) becomes:

$$A(q^{-1})S'(q^{-1}) + q^{-d+1} R(q^{-1}) = P(q^{-1}) \quad (**)$$

Unique solution if:

$$n_p = \text{deg } P(q^{-1}) = n_A + d \quad \boxed{\text{deg } S'(q^{-1}) = d} \quad \text{deg } R(q^{-1}) = n_A - 1$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots r_{n_A-1} q^{-n_A-1} \quad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_d q^{-d}$$
Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

(**) is written as: $Mx = p \rightarrow x = M^{-1}p$

\[
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
a_1 & 1 & & & \\
a_2 & a_1 & 0 & & \\
\vdots & \vdots & 1 & \ddots & \\
a_d & a_{d-1} & \cdots & a_1 & 1 \\
a_{d+1} & a_d & & & \\
a_{d+2} & a_{d+1} & & & \\
0 & 0 & \cdots & 0 & a_{nA} \\
\end{bmatrix} \begin{bmatrix}
\vdots \\
d + 1 \\
nA + d + 1 \\
\end{bmatrix}
\]

$x^T = [1, s_1', \ldots, s_d', r_0, r_1, \ldots, r_{n-1}]$

$p^T = [1, p_1, p_2, \ldots, p_{nA}, p_{nA+1}, \ldots, p_{nA+d}]$

Use of WinReg or `predisol.sci(.m)` for solving (**)

Insertion of pre specified parts in R and S is possible
Tracking (computation of $T(q^{-1})$)

Closed loop T.F.: $r \rightarrow y$

$$H_{BF}(q^{-1}) = \frac{q^{-(d+1)}B_m(q^{-1})}{A_m(q^{-1})} = \frac{B_m(q^{-1})T(q^{-1})q^{-(d+1)}}{A_m(q^{-1})P(q^{-1})}$$

Desired T.F.

It results: $T(q^{-1}) = P(q^{-1})$

Controller equation:

$$S(q^{-1})u(t) = P(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)$$

$$u(t) = \frac{P(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$u(t) = \frac{1}{b_1} \left[ P(q^{-1})y^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t) \right] \quad (s_0 = b_1)$$

Reference signal (tracking): $y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$
Tracking and regulation with independent objectives
A time domain interpretation

\[ y(t) = \frac{q^{-(d+1)}}{P(q^{-1})} y^*(t + d + 1) \]

\[ P(q^{-1})y(t) = q^{-(d+1)} y^*(t + d + 1) \]

\[ P(q^{-1})y(t) - q^{-(d+1)} y^*(t + d + 1) = 0 \]

\[ \varepsilon^0(t) \equiv 0 \]

(in case of correct tuning)

Reformulation of the “design problem”:
Find a controller which generate \( u(t) \) such that:
\[ \varepsilon^0(t + d + 1) = P[y(t + d + 1) - y^*(t + d + 1)] = 0 \]
Tracking and regulation with independent objectives

Synthesis in the time domain – an example

For $d=0$ ($S'=1$)

$S(q^{-1}) = B^*(q^{-1})$

$A(q^{-1}) + q^{-1}R(q^{-1}) = P(q^{-1})$

$R(q^{-1}) = P^*(q^{-1}) - A^*(q^{-1})$

$P(q^{-1}) = 1 + q^{-1}P^*(q^{-1})$

$A(q^{-1}) = 1 + q^{-1}A^*(q^{-1})$

Example:

$y(t+1) = -a_1y(t) + b_1u(t) + b_2u(t-1); \quad P(q^{-1}) = 1 + p_1q^{-1}$

$\epsilon^0(t) \equiv 0$

$\epsilon^o(t+1) = P[y(t+1) - y^*(t+1)] = y(t+1) + p_1y(t) - Py^*(t+1) =$

$= [-a_1y(t) + b_1u(t) + b_2u(t-1) + p_1y(t) - Py^*(t+1)] = 0 \quad \text{Solve for } u(t)$

$u(t) = \frac{Py^*(t+1) - b_2u(t-1) - r_0y(t)}{b_1}; \quad r_0 = p_1 - a_1$

Controller satisfies:

$P(q^{-1})y^*(t+1) = b_1u(t) + b_2u(t-1) + r_0y(t) = \theta^T \phi(t)$

$\theta^T = [b_1, b_2, r_0] \quad \phi^T(t) = [u(t), u(t-1), y(t)]$
Adaptive tracking and regulation with independent objectives

Three techniques:

• Model reference adaptive control (direct)
• Plant model estimation + computation of the controller (indirect)
• Re-parametrized plant model estimation (direct)
Model Reference Adaptive Control

Objective: \[ \lim_{t \to \infty} \varepsilon^0(t + 1) = \lim_{t \to \infty} P(q^{-1}) [y(t + 1) - y^*(t + 1)] = 0 \]

Adjustable controller:
\[ u(t) = \frac{Py^*(t + 1) - \hat{b}_2(t)u(t - 1) - \hat{r}_0(t)y(t)}{\hat{b}_1(t)} \]

\[ P(q^{-1})y^*(t + 1) = \hat{\theta}^T(t)\phi(t) \]

\[ \hat{\theta}^T(t) = [\hat{b}_1(t), \hat{b}_2(t), \hat{r}_0(t)]; \phi^T(t) = [u(t), u(t - 1), y(t)] \]

But for the correct values of controller parameters one has:
\[ P(q^{-1})y(t + 1) = P(q^{-1})y^*(t + 1) = \theta^T\phi(t) \]

And therefore one has:
\[ \varepsilon^0(t + 1) = \left[ \theta - \hat{\theta}(t) \right]^T\phi(t) \]

Define the a posteriori adaptation error: \[ \varepsilon(t + 1) = \left[ \theta - \hat{\theta}(t + 1) \right]^T\phi(t) \]

Use P.A.A.

However one should show in addition that \( \|\phi(t)\| \) is bounded (i.e. plant input and output are bounded)
**Plant model estimation + computation of the controller (indirect)**

**Step 1: Plant model estimation**

Plant model (unknown):

\[ y(t + 1) = -a_1y(t) + b_1u(t) + b_2u(t - 1) = \theta^T_p \phi(t) \]

Adjustable predictor:

\[ \hat{y}^0(t + 1) = -\hat{a}_1(t)y(t) + \hat{b}_1(t)u(t) + \hat{b}_2(t)u(t - 1) = \hat{\theta}^T_p(t)\phi(t) \]

A priori prediction error:

\[ \varepsilon^0(t + 1) = y(t + 1) - \hat{y}^0(t + 1) = [\theta_p - \hat{\theta}_p(t)]^T \phi(t) \]

A posteriori prediction error:

\[ \varepsilon(t + 1) = y(t + 1) - \hat{y}(t + 1) = [\theta_p - \hat{\theta}_p(t + 1)]^T \phi(t) \]

Use PAA

**Step 2: Computation of the controller**

Compute at each instant \( t \):

\[ \hat{r}_0(t) = p_1 - \hat{a}_1(t) \]

Adjustable controller:

\[ P(q^{-1})y^*(t + 1) = \hat{\theta}^T(t)\phi(t) \]

\[ \phi^T(t) = [u(t), u(t - 1), y(t)]; \hat{\theta}^T(t) = [\hat{b}_1(t), \hat{b}_2(t), \hat{r}_0(t)] \]

In the general case \( d > 0 \) one will have to solve equation (***)
Re-parametrized plant model estimation (direct)

Plant model (unknown):
\[ y(t + 1) = -a_1 y(t) + b_1 u(t) + b_2 u(t - 1) \pm p_1 y(t) \]
\[ = -p_1 y(t) + (p_1 - a_1) y(t) + b_1 u(t) + b_2 u(t - 1) = -p_1 y(t) + \theta^T \phi(t) \]
\[ \hat{y}^0(t + 1) = -p_1 y(t) + \hat{r}_0^T(t) y(t) + \hat{b}_1(t) u(t) + \hat{b}_2(t) u(t - 1) \]

Re-parametrized adjustable predictor:
\[ \hat{y}^0(t + 1) = -p_1 y(t) + \theta^T(t) \phi(t) \]
\[ \hat{\theta}^T(t) = [\hat{b}_1(t), \hat{b}_2(t), r_0(t)]; \phi^T(t) = [u(t), u(t - 1), y(t)] \]

A priori prediction error:
\[ \varepsilon^0(t + 1) = y(t + 1) - \hat{y}^0(t + 1) = [\theta - \hat{\theta}(t)]^T \phi(t) \]

A posteriori prediction error:
\[ \varepsilon(t + 1) = y(t + 1) - \hat{y}(t + 1) = [\theta - \hat{\theta}(t + 1)]^T \phi(t) \]

Use PAA

One estimates directly the parameters of the controller

One has:
\[ \lim_{t \to \infty} \varepsilon^0(t + 1) = \lim_{t \to \infty} P(q^{-1}) [y(t + 1) - y^*(t + 1)] = 0 \]
Adaptive tracking and regulation with independent objectives

- Easy generalization for the case $d > 0$
- Elegant and simple solution for adaptation (direct)
- Unfortunately restricted use in practice because it requires that the plant zeros remain always stable and well damped
Direct Adaptive Control – Simulations results

Tracking

\[ P(q^{-1}) = 1 \]

\[ P(q^{-1}) = (1 - 0.4q^{-1}) \]

The choice of the poles for the closed loop (regulation) has a great influence upon adaptation transient behavior!
The choice of the poles for the closed loop (regulation) has a great influence upon adaption transient behavior!

\[ P(q^{-1}) = 1 \]

\[ P(q^{-1}) = (1 - 0.4q^{-1}) \]
Indirect Adaptive Control
of non-necessarily zeros-stable plants
Pole placement

The pole placement allows to design a R-S-T controller for:
- stable or unstable systems
- without restriction upon the degrees of $A$ and $B$ polynomials
- without restrictions upon the plant model zeros (stable or unstable)
- but $A$ and $B$ polynomials should not have common factors
(controllable/observable model for design)

*It is a method that does not simplify the plant model zeros*
Plant: 

\[ H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \]

\[ A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} \quad B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n_B} q^{-n_B} = q^{-1}B^*(q^{-1}) \]
Pole placement

Closed loop T.F. (r → y) (*reference tracking*)

\[ H_{BF}(q^{-1}) = \frac{q^{-d}T(q^{-1})B(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{q^{-d}T(q^{-1})B(q^{-1})}{P(q^{-1})} \]

\[ P(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = 1 + p_1q^{-1} + p_2q^{-2} + \ldots \]

Defines the (desired) closed loop poles

Closed loop T.F. (p → y) (*disturbance rejection*)

\[ S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \]

Output sensitivity function

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Choice of desired closed loop poles (polynomial $P$)

$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Dominant poles  Auxiliary poles

**Choice of $P_D(q^{-1})$ (dominant poles)**

Specification in continuous time $\rightarrow$ 2nd order $(\omega_0, \zeta) \rightarrow$ discretization

$$T_e \quad P_D(q^{-1})$$

$$0.25 \leq \omega_0 T_e \leq 1.5$$

$$0.7 \leq \zeta \leq 1$$

**Auxiliary poles**

- Auxiliary poles are introduced for robustness purposes
- They usually are selected to be faster than the dominant poles
Regulation (computation of \( R(q^{-1}) \) and \( S(q^{-1}) \))

(\text{Bezout}) \quad A(q^{-1})S(q^{-1}) + q^{-d} B(q^{-1})R(q^{-1}) = P(q^{-1}) \quad (*)

\[ n_A = \deg A(q^{-1}) \quad n_B = \deg B(q^{-1}) \]

\text{unique minimal solution for :}
\[ n_P = \deg P(q^{-1}) \leq n_A + n_B + d - 1 \]
\[ n_S = \deg S(q^{-1}) = n_B + d - 1 \quad n_R = \deg R(q^{-1}) = n_A - 1 \]

\[ S(q^{-1}) = 1 + s_1 q^{-1} + \ldots + s_{n_S} q^{-n_S} = 1 + q^{-1} S^*(q^{-1}) \]

\[ R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots + r_{n_R} q^{-n_R} \]

A and \( B \) do not have common factors
Computation of $R(q-1)$ and $S(q-1)$

Equation (*) is written as:

$$Mx = p \quad \rightarrow \quad x = M^{-1}p$$

$$x^T = [1, s_1, \ldots, s_n, r_0, \ldots, r_n]$$

$$p^T = [1, p_1, \ldots, p_i, \ldots, p_{n_p}, 0, \ldots, 0]$$

$$\begin{bmatrix}
1 & 0 & \ldots & 0 & 0 & \ldots & \ldots & 0 \\
a_1 & 1 & & & b'_1 \\
a_2 & 0 & b'_2 & b'_1 \\
& 1 & \cdot & b'_2 \\
a_n & a_2 & b'_{n_B} & \cdot \\
0 & \cdot & 0 & \cdot & \cdot \\
0 & \ldots & 0 & a_n & 0 & 0 & 0 & b'_{n_B}
\end{bmatrix}$$

$$n_A + n_B + d$$

$$b'_i = 0 \quad \text{pour} \quad i = 0, 1 \ldots d \quad ; \quad b'_i = b_i - d \quad \text{pour} \quad i > d$$

Use of WinReg or bezoutd.sci(.m) for solving (*)
Structure of $R(q^{-1})$ and $S(q^{-1})$

R and S may include pre-specified fixed parts (ex: integrator)

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \quad \quad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

$H_R, H_S$ - pre-specified polynomials

$$R'(q^{-1}) = r'_0 + r'_1 q^{-1} + ... r'_{n_R} q^{-n_R} \quad S'(q^{-1}) = 1 + s'_1 q^{-1} + ... s'_{n_S} q^{-n_S}.$$

- The pre specified filters $H_R$ and $H_S$ will allow to impose certain properties of the closed loop.
- They can influence performance and/or robustness

$$A(q^{-1})H_S(q^{-1})S'(q^{-1}) + q^{-d} B(q^{-1})H_R(q^{-1})R'(q^{-1}) = P(q^{-1})$$

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Fixed parts ($H_R, H_S$). Examples

Zero steady state error ($S_{yp}$ should be null at certain frequencies)

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})}{P(q^{-1})}$$

**Step disturbance**:  
$$H_S(q^{-1}) = 1 - q^{-1}$$

**Sinusoidal disturbance**:  
$$H_S = 1 + \alpha q^{-1} + q^{-2} ; \alpha = -2 \cos \omega T_s$$

**Signal blocking** ($S_{up}$ should be null at certain frequencies)

$$S_{up}(q^{-1}) = -\frac{A(q^{-1})H_R(q^{-1})R'(q^{-1})}{P(q^{-1})}$$

**Sinusoidal signal**:  
$$H_R = 1 + \beta q^{-1} + q^{-2} ; \beta = -2 \cos \omega T_s$$

**Blocking at $0.5f_S$**:  
$$H_R = (1 + q^{-1})^n ; n = 1, 2$$
Tracking (computation of $T(q^{-1})$)

Ideal case

\[ r(t) \xrightarrow{q^{-1}B_m} y^*(t) \]

Tracking reference model ($H_m$)

- Desired trajectory for $y(t)$

Specification in continuous time ($t_M, M$) \(\rightarrow\) 2\textsuperscript{nd} order ($\omega_0$, $\zeta$) \(\xrightarrow{T_s} H_m(q^{-1})\)

\[ 0.25 \leq \omega_0 T_s \leq 1.5 \]
\[ 0.7 \leq \zeta \leq 1 \]

The ideal case can not be obtained (delay, plant zeros)

Objective: to approach $y^*(t)$

\[ y^*(t) = \frac{q^{-(d+1)} B_m(q^{-1})}{A_m(q^{-1})} r(t) \]

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Tracking (computation of $T(q^{-1})$)

Build:

$$y^*(t + d + 1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Choice of $T(q^{-1})$:

- Imposing unit static gain between $y^*$ and $y$
- Compensation of regulation dynamics $P(q^{-1})$

$$T(q^{-1}) = G P(q^{-1}) \quad G = \begin{cases} 1/B(1) & \text{if } B(1) \neq 0 \\ 1 & \text{if } B(1) = 0 \end{cases}$$

F.T. $r \rightarrow y$: $H_{BF}(q^{-1}) = \frac{q^{-(d+1)} B_m(q^{-1})}{A_m(q^{-1})} \cdot \frac{B^*(q^{-1})}{B(1)}$

Particular case: $P = A_m$ 

$$T(q^{-1}) = G = \begin{cases} P(1)/B(1) & \text{if } B(1) \neq 0 \\ 1 & \text{if } B(1) = 0 \end{cases}$$
Pole placement. Tracking and regulation

\[ S(q^{-1})u(t) + R(q^{-1})y(t) = T(q^{-1})y^*(t + d + 1) \]
Pole placement. Control law

\[ u(t) = \frac{T(q^{-1})y^*(t + d + 1) - R(q^{-1})y(t)}{S(q^{-1})} \]

\[ S(q^{-1})u(t) + R(q^{-1})y(t) = GP(q^{-1})y^*(t + d + 1) = T(q^{-1})y^*(t + d + 1) \]

\[ S(q^{-1}) = 1 + q^{-1}S^*(q^{-1}) \]

\[ u(t) = P(q^{-1})Gy^*(t + d + 1) - S^*(q^{-1})u(t - 1) - R(q^{-1})y(t) \]

\[ y^*(t + d + 1) = \frac{B_m(q^{-1})}{A_m(q^{-1})}r(t) \]

\[ A_m(q^{-1}) = 1 + q^{-1}A_m^*(q^{-1}) \]

\[ y^*(t + d + 1) = -A_m^*(q^{-1})y(t + d) + B_m(q^{-1})r(t) \]

\[ B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots \quad A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots \]

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Indirect adaptive control

At each sampling instant:

_Step I_: Estimation of the plant model \((\hat{A}, \hat{B})\)
ARX identification (Recursive Least Squares)

_Step II_: Computation of the controller

Solving Bezout equation (for \(S'\) and \(R'\))

\[
\hat{A}H_S S' + q^{-d} \hat{B}H_R R' = P
\]

Compute:

\[
R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \quad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})
\]

\[
T = \hat{G}P = \begin{cases} 
\frac{1}{\hat{B}(1)} & \text{if } \hat{B}(1) \neq 0 \\
\hat{G} = 1 & \text{if } \hat{B}(1) = 0 
\end{cases}
\]

Remark:

It is time consuming for large dimension of the plant model
Supervision

Estimation:
• Check if input is enough “persistently exciting”
  (if not, do not take in account the estimations)
• Check if $\hat{A}$ and $\hat{B}$ are numerically “sound” (condition number)
  (no close poles/zeros)
• If necessary, add external excitation (testing signal)

Control:
• Check if desired dominant closed loop poles are compatible
  with estimated plant poles
• Check robustness margins

Additional problem:
• How to deal with neglected dynamics?
  (filtering of the data, robustification of PAA)
Adaptive Control

Part 6: Robust Control Design for Adaptive Control
Notations

\[ G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \]
\[ K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} \]

Sensitivity functions:
\[ S_{yp}(z^{-1}) = \frac{1}{1 + KG} \]
\[ S_{up}(z^{-1}) = -\frac{KG}{1 + KG} \]
\[ S_{yy}(z^{-1}) = \frac{G}{1 + KG} \]
\[ S_{yr}(z^{-1}) = \frac{KG}{1 + KG} \]

Closed loop poles:
\[ P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \]

True closed loop system: \((K,G), P, S_{xy}\)
Nominal simulated(estimated) closed loop: \((K, \hat{G}), \hat{P}, \hat{S}_{xy}\)
Templates for the Sensitivity Functions

Output Sensitivity Function

Critical frequency region for control

Input Sensitivity Function

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Robust Control Design for Adaptive Control

Parameter variations (low frequency) → Adaptation → Unstructured uncertainties (high frequency) → Robust Design

**Basic rule**: The input sensitivity function \(S_{\text{up}}\) should be small in medium and high frequencies.

*How to achieve this?*

**Pole Placement**:
- Opening the loop in high frequencies (at 0.5\(f_s\))
- Placing auxiliary closed loop poles near the high frequency poles of the plant model

**Generalized Predictive Control**:
- Appropriate weighting filter on the control term in the criterion
Adaptive Control of a Flexible Transmission

The flexible transmission

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Adaptive Control of a Flexible Transmission

Frequency characteristics for various load

Rem.: the main vibration mode varies by 100%
Robust Control Design for Adaptive Control

(Flexible Transmission)

Input sensitivity function Sup

a) Standard pole placement (1 pair dominant poles + h.f. aperiodic poles)
b) Opening the loop at 0.5f_s (H_R = 1 + q^{-1})
c) Auxiliary closed loop poles near high frequency plant poles
Part 7: Parameter Estimators for Adaptive Control

**Objective**: to reduce the effect of the disturbances upon the quality of the estimation
Classical Indirect Adaptive Control

- Uses R.L.S. type estimator (equation error)
- Sensitive to output disturbances
- Requires « adaptation freezing » in the absence of persistent excitation
- The threshold for « adaptation freezing » is problem dependent
Closed Loop Output Error Parameter Estimator for Adaptive Control

- Insensitive to output disturbances
- Remove the need for « adaptation freezing » in the absence of persistent excitation
- CLOE requires stability of the closed loop
- Well suited for « adaptive control with multiple models »
Adaptive Control – Effect of Disturbances

Classical parameter estimator (filtered RLS)

CLOE parameter estimator

Disturbances destabilize the adaptive system when using RLS parameter estimator (in the absence of a variable reference signal)
Adaptive Regulation

Part 8: Rejection of unknown narrow band disturbances. Application to active vibration control
The Active Suspension System

**Objective:**
- Reject the effect of unknown and variable narrow band disturbances
- Do not use an additional measurement

Two paths:
- Primary
- Secondary (double differentiator)

\[ T_s = 1.25 \text{ ms} \]
The Active Suspension

- Active suspension
- Residual force (acceleration) measurement
- Primary force (acceleration) (the shaker)
- Elastomer
The Active Vibration Control with Inertial Actuator

**Objective:**
- Reject the effect of unknown and variable narrow band disturbances
- Do not use an additional measurement

**Same control approach but different technology**

Two paths:
- Primary
- Secondary (double differentiator)

\[ T_s = 1.25 \text{ ms} \]
View of the active vibration control with inertial actuator

- Passive damper
- Inertial actuator (not visible)
- Primary force (acceleration) (the shaker)
- Residual force (acceleration) measurement
View of a flexible controlled structure using inertial actuators
Frequency Characteristics of the Identified Models

Primary path

Secondary path

Further details can be obtained from: http://iawww.epfl.ch/News/EJC_Benchmark/

\[ n_A = 14 ; n_B = 16 ; d = 0 \]
Direct Adaptive Control: disturbance rejection

Disturbance: Chirp

Open loop

Closed loop

Initialization of the adaptive controller
Simultaneous controller initialization and disturbance application
Initialization of the adaptive controller

Output

Input

Adaptation transient

Step frequency changes

Output

Input

Rejection of unknown finite band disturbances

• **Assumption:** Plant model almost constant and known (obtained by system identification)
• **Problem:** Attenuation of unknown and/or variable stationary disturbances without using an additional measurement
• **Solution:** Adaptive feedback control
  - Estimate the model of the disturbance (indirect adaptive control)
  - Use the internal model principle
  - Use of the Youla parameterization (direct adaptive control)

_A robust control design should be considered assuming that the model of the disturbance is known_

_A class of applications:_ suppression of unknown vibration (active vibration control)

**Attention:** The area was “dominated “ by adaptive signal processing solutions (Widrow’s adaptive noise cancellation) which require an additional transducer

**Remainder:** Models of stationary sinusoidal disturbances have poles on the unit circle
Unknown disturbance rejection – classical solution

Disadvantages:
- requires the use of an additional transducer
- difficult choice of the location of the transducer
- adaptation of many parameters

Not justified for the rejection of narrow band disturbances
Notations

\[ G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \]
\[ K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} = \frac{R'(q^{-1})H_R(q^{-1})}{S'(q^{-1})H_S(q^{-1})} \]

Output Sensitivity function :
\[ S_{yp}(z^{-1}) = \frac{A(z^{-1})S'(z^{-1})H_S(z^{-1})}{P(z^{-1})} \]

Closed loop poles :
\[ P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \]

The gain of \( S_{yp} \) is zero at the frequencies where \( S_{yp}(e^{j\omega}) = 0 \) (perfect attenuation of a disturbance at this frequency)
Disturbance model

Deterministic framework

\[ p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance} \]

\( D_p \rightarrow \) may have poles on the unit circle; \( \delta(t) = \text{Dirac} \)

(Sinusoid: \( D_p = 1 + \alpha q^{-1} + q^{-2}; \alpha = -2 \cos(2\pi f / f_s) \))

Stochastic framework

\[ p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot e(t) : \text{stochastic disturbance} \]

\( e(t) = \text{Gaussian white noise sequence} (0, \sigma) \)

\( D_p \rightarrow \) may have poles on the unit circle
Closed loop system. Notations

Controller $\delta(t)$

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t): \text{deterministic disturbance}$$

$D_p \rightarrow \text{poles on the unit circle}; \delta(t) = \text{Dirac}$

Controller:

$$R(q^{-1}) = R'(q^{-1}) \cdot H_R(q^{-1});$$

$$S(q^{-1}) = S'(q^{-1}) \cdot H_S(q^{-1}).$$

Internal model principle: $H_S(z^{-1}) = D_p(z^{-1})$

Output: $y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t)$

$$y(t) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \frac{1}{D_p(q^{-1})} \cdot \delta(t)$$

CL poles: $P(q^{-1}) = A(q^{-1})S(q^{-1}) + z^{-d} B(q^{-1})R(q^{-1})$
**Indirect adaptive regulation**

Two-steps methodology:

1. Estimation of the disturbance model, \( D_p(q^{-1}) \)
2. Computation of the controller, imposing \( H_s(q^{-1}) = \hat{D}_p(q^{-1}) \)

*It can be time consuming (if the plant model \( B/A \) is of large dimension)*
**Indirect adaptive control**

*Step I*: Estimation of the disturbance model  
ARMA identification (Recursive Extended Least Squares)

*Step II*: Computation of the controller  
Solving Bezout equation (for $S'$ and $R$)

\[
H_S = \hat{D}_p \\
A\hat{D}_p S' + q^{-d} BR = P \\
S = \hat{D}_p S'
\]

*Remark:*  
*It is time consuming for large dimension of the plant model*
Central contr: \([R_0(q^{-1}), S_0(q^{-1})]\).

CL poles: \(P(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^{-d}B(q^{-1})R_0(q^{-1})\).

Control: \(S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t)\)

Control: \(S(q^{-1})u(t) = -R(q^{-1})y(t)\)

\[S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t) - Q(q^{-1}) w(t),\]

where \(w(t) = A(q^{-1}) y(t) - q^{-d}B(q^{-1}) u(t)\).

CL poles: \(P(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^{-d}B(q^{-1})R_0(q^{-1})\).

Q-parametrization:
\(R(z^{-1}) = R_0(q^{-1}) + A(q^{-1})Q(q^{-1})\);
\(S(q^{-1}) = S_0(z^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\).

\[Q(q^{-1}) = q_0 + q_1 q^{-1} + \ldots q_{n_q} q^{-n_q}\]

The closed loop poles remain unchanged

The diagram shows the block diagram of the control system with the plant and model components, along with the control law and the Q-parametrization.
Yula-Kucera parametrization
An interpretation for the case $A$ asympt. stable

\[ w(t) = \frac{A N_p}{D_p} \delta(t) \]
Internal model principle and Q-parameterization

Central contr: \([R_0(q^{-1}), S_0(q^{-1})]\).
CL Poles: \(P(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^{-d}B(q^{-1})R_0(q^{-1})\).
Control: \(S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t)\)

Q-parameterization:
\(R(z^{-1}) = R_0(q^{-1}) + A(q^{-1})Q(q^{-1})\);
\(S(q^{-1}) = S_0(z^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\).

Closed Loop Poles remain unchanged

Internal model assignment on \(Q\) (find \(Q\) such that \(S\) contains the disturbance model):

\[ S = S_0 - q^{-d}BQ = MD_p \]

Solve:
\[ MD_p + q^{-d}BQ = S_0 \]

Will lead also to an « indirect adaptive control solution »

BUT:

\(Q\) can be used to “directly” tune the internal model without changing the closed loop poles (see next)
**Goal:** minimize $y(t)$ (according to a certain criterion).

**Hypothesis:** Identified (known) plant model $(A,B,d)$.

Consider $p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t)$: deterministic disturbance. \[ w(t) = \frac{AN_p}{D_p} \delta(t) \]

\[ y(t) = \frac{A(q^{-1})[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})]}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = \left[ \frac{S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})}{P(q^{-1})} \right] w(t) \]

\[ w(t) = \frac{A(q^{-1})N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = A(q^{-1}) \cdot y(t) - q^{-d} \cdot B(q^{-1}) \cdot w(t) \]

Define (construct): \[ \varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d}B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t). \]

Define $\varepsilon^0(t + 1)$ as the value of $y(t + 1)$ obtained with $\hat{Q}(t, q^{-1})$.

\[ \varepsilon^0(t + 1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t + 1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \hat{Q}(t, q^{-1}) \cdot w(t) \]
One can define now the \textit{a posteriori} error (using $\hat{Q}(t + 1, q^{-1})$) as:

$$
\varepsilon(t + 1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t + 1) - \frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \hat{Q}(t + 1, q^{-1}) \cdot w(t) \quad (*)
$$

\textbf{We need to express } \varepsilon(t) \text{as:}

$$
\varepsilon(t + 1) = [Q(q^{-1}) - \hat{Q}(t + 1, q^{-1})] f(t) = [\theta - \hat{\theta}(t + 1)]^T \Phi(t)
$$

Using:

$$
MD_p + q^{-d} BQ = S_0
$$

$$
\frac{S_0(q^{-1})}{P(q^{-1})} w(t + 1) = Q \frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} w(t) + \frac{M(q^{-1}) D_p ((q^{-1})}{P(q^{-1})} w(t + 1)
$$

$$(*) \text{ becomes:}
$$

\begin{align}
\varepsilon(t + 1) &= [Q(q^{-1}) - \hat{Q}(t + 1, q^{-1})] \cdot \frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t + 1) \\
\end{align}

\textbf{Details:}

$$
v(t + 1) = \frac{M(q^{-1}) D_p (q^{-1})}{P(q^{-1})} w(t + 1) = \frac{M(q^{-1}) A(q^{-1}) N_p (q^{-1})}{P(q^{-1})} \delta(t + 1)
$$

$$
\frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t) = \frac{q^{-d} B(q^{-1})}{P(q^{-1})} \cdot w(t + 1)
$$

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The Algorithm

\[ w(t+1) = A(q^{-1})y(t+1) - q^{-d}B^*(q^{-1})u(t); \quad (B(q^{-1})u(t+1) = B^*(q^{-1})u(t)) \]

**Define**:

\[ \varepsilon^0(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})}w(t+1) - \hat{Q}(t,q^{-1})\frac{q^{-d}B(q^{-1})}{P(q^{-1})}w(t+1). \]

\[ w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})}w(t+1); \quad w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}w(t); \]

\[ \hat{\theta}^T(t) = [\hat{q}_0(t), \hat{q}_1(t)]; \quad \phi^T(t) = [w_2(t), w_2(t-1)], \quad (for \quad n_{D_p} = 2 \quad since \quad n_Q = n_{D_p} - 1) \]

**A priori** adaptation error:

\[ \varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t) \]

**A posteriori** adaptation error:

\[ \varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t) \]

**Parameter adaptation algorithm**:

\[
\begin{align*}
\hat{\theta}(t+1) &= \hat{\theta}(t) + F(t+1)\phi(t)\varepsilon^0(t+1); \\
F^{-1}(t+1) &= \lambda_1(t)F^{-1}(t) + \lambda_2(t)\phi(t)\phi^T(t).
\end{align*}
\]

Various choices possible for \( \lambda_1 \) and \( \lambda_2 \) which define the adaptation gain time profile

*(For a stability proof see Automatica, 2005, no.4 pp. 563-574)*
Direct adaptive rejection of unknown disturbances

- The order of the $Q$ polynomial depends upon the order of the disturbance model denominator ($D_p$) and not upon the complexity of the plant model
- Less parameters to estimate than for the identification of the disturbance model
- Operation in “self–tuning” mode (constant unknown disturbance) or “adaptive” mode (time varying unknown disturbance)
Further experimental results on the active suspension
*Comparison between direct/indirect adaptive control*
Real-time results – Active Suspension (continuation)

Narrow band disturbances = variable frequency sinusoid ⇒ \( n_0 = 1 \)
Frequency range: 25 ÷ 47 Hz

Evaluation of the two algorithms in real-time

Nominal controller \([R_0(q^{-1}), S_0(q^{-1})]\): \( n_{R0} = 14, n_{S0} = 16 \)

Implementation protocol 1: Self-tuning

- The algorithm stops when it converges and the controller is applied.
- It restarts when the variance of the residual force is bigger than a given threshold.
- As long as the variance is not bigger than the threshold, the controller is constant.

Implementation protocol 2: Adaptive

- The adaptation algorithm is continuously operating
- The controller is updated at each sample
Frequency domain results – direct adaptive method

Spectral densities of the residual force. Direct method in self-tuning operation

- Open loop (25 Hz)
- Open loop (32 Hz)
- Open loop (47 Hz)
- Closed loop (25 Hz)
- Closed loop (32 Hz)
- Closed loop (47 Hz)

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Frequency domain results – indirect adaptive method

Spectral densities of the residual force. Indirect method in self-tuning operation
Direct Adaptive Control

Self-tuning Mode

Adaptive Mode

• Direct adaptive control in adaptive mode operation gives better results than direct adaptive control in self-tuning mode
• Direct adaptive control leads to a much simpler implementation and better performance than indirect adaptive control
Active vibration control using an inertial actuator

Real-time results

Rejection of two simultaneous sinusoidal disturbances
Active vibration control using an inertial actuator

Frequency Characteristics of the Identified Models

Complexity of secondary path: \( n_A = 10; n_B = 12; d = 0 \)
Frequency domain results – direct adaptive method

Rejection of two simultaneous sinusoidal disturbances

Power Spectral Density Estimate

- Rejection of two simultaneous sinusoidal disturbances

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Time Domain Results – Direct adaptive control

Adaptive Operation

Simultaneous rejection of two time varying sinusoidal disturbances
Time Domain Results – Direct adaptive control

Evolution of the $Q$ parameters

Simultaneous rejection of two time varying sinusoidal disturbances
Time Domain Results – Direct adaptive control

*Evolution of the control input*

Simultaneous rejection of two time varying sinusoidal disturbances
Comparison direct/indirect adaptive regulation

Time domain results – Adaptive regime
Active vibration control using an inertial actuator

Indirect adaptive method

Direct adaptive method

Direct adaptive control leads to a much simpler implementation and better performance than Indirect adaptive control
Conclusions

- Using internal model principle, adaptive feedback control solutions can be provided for the rejection of unknown disturbances
- Both direct and indirect solutions can be provided
- Two modes of operation can be used: self-tuning and adaptive
- Direct adaptive control is the simplest to implement
- Direct adaptive control offers better performance
- The methodology has been extensively tested on:
  • active suspension system
  • active vibration control with an inertial actuator