IDENTIFICATION IN CLOSED LOOP
A powerful design tool
(theory, algorithms, applications)

better models, simpler controllers

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Part 4: Algorithms for identification in closed loop

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Outline

- Identification in closed loop. Why ?
- An example (flexible transmission) and explanations
- Objectives of identification in closed loop
- Basic Schemes
- The CLOE Algorithms (closed loop output error)
- Properties of the algorithms
- Properties of the estimated models
- Validation of models identified in closed loop
- Iterative identification in closed loop and controller re-design
- Experimental results (flexible transmission)
- CLID – a toolbox for closed loop identification
- Use of open loop identification alg.for identification in closed loop
- Conclusions

Plant Identification in Closed Loop

Why ?

There are systems where open loop operation is not suitable (instability, drift, ..)

A controller may already exist (ex . : PID)

Re-tuning of the controller

a) to improve achieved performances
b) controller maintenance

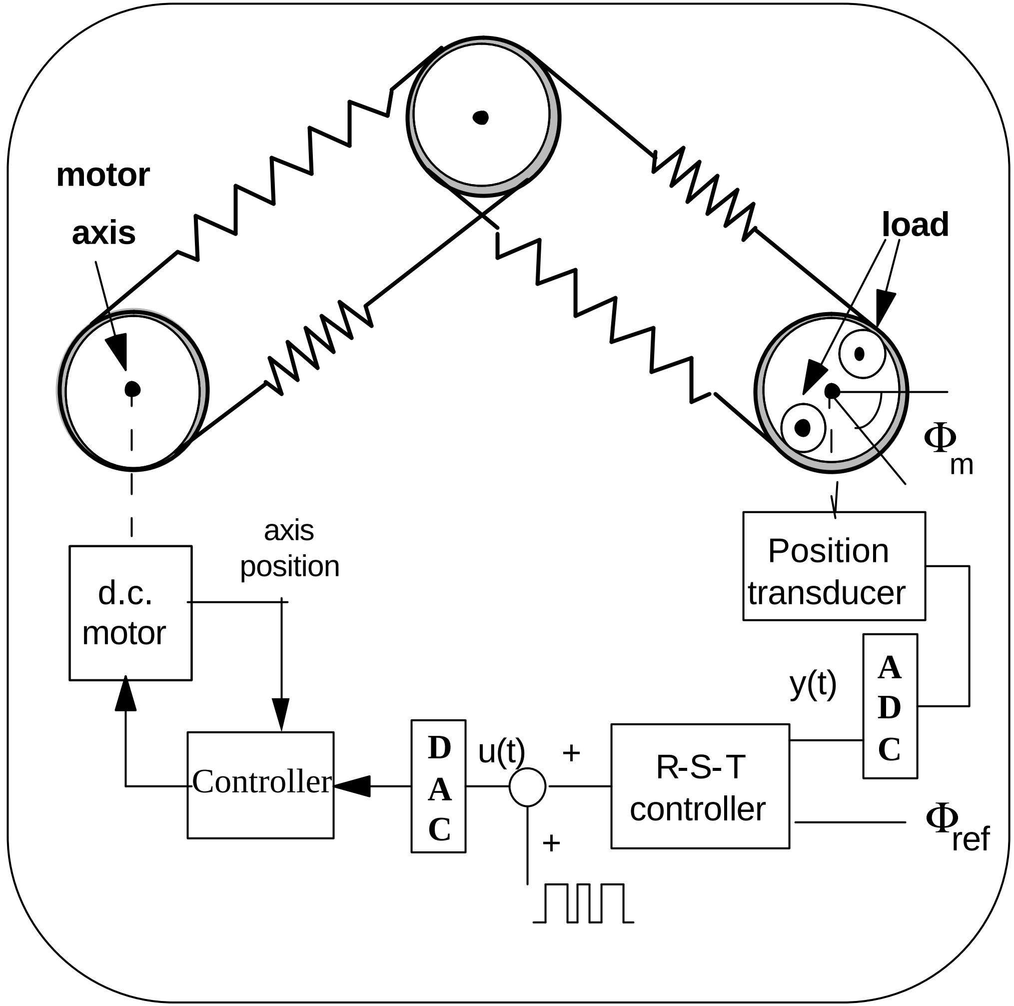
Iterative identification and controller redesign

May provide better « design » models

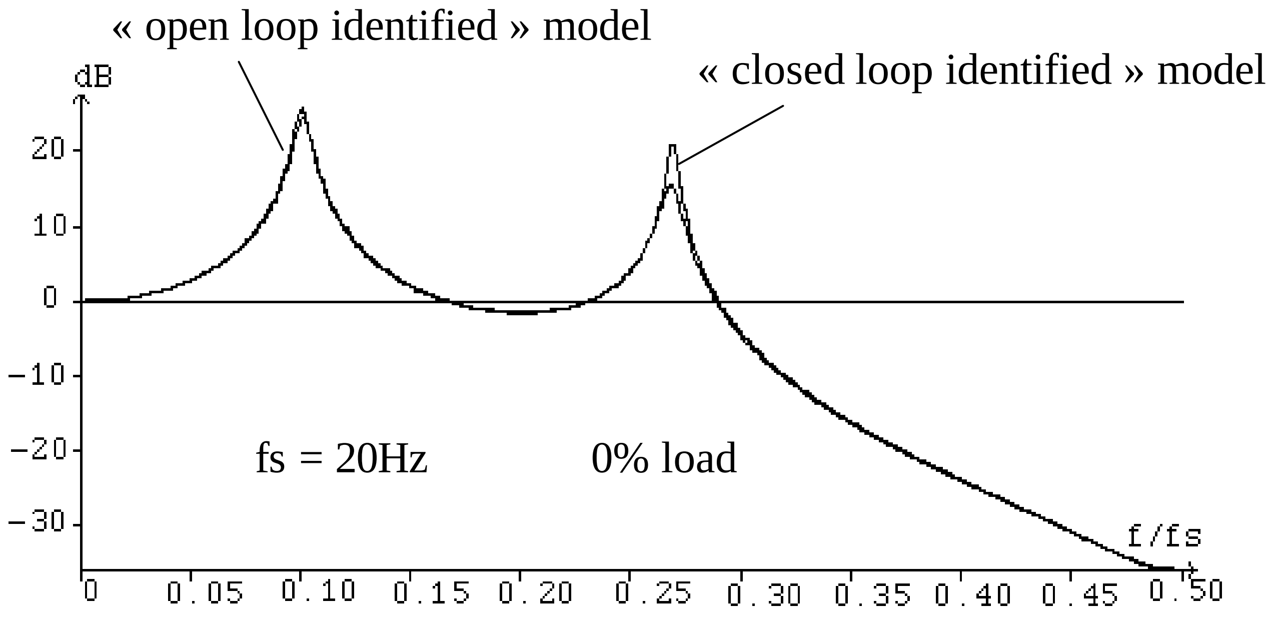
Cannot be dissociated from the controller and robustness issues

Identification in Closed Loop

The flexible transmission

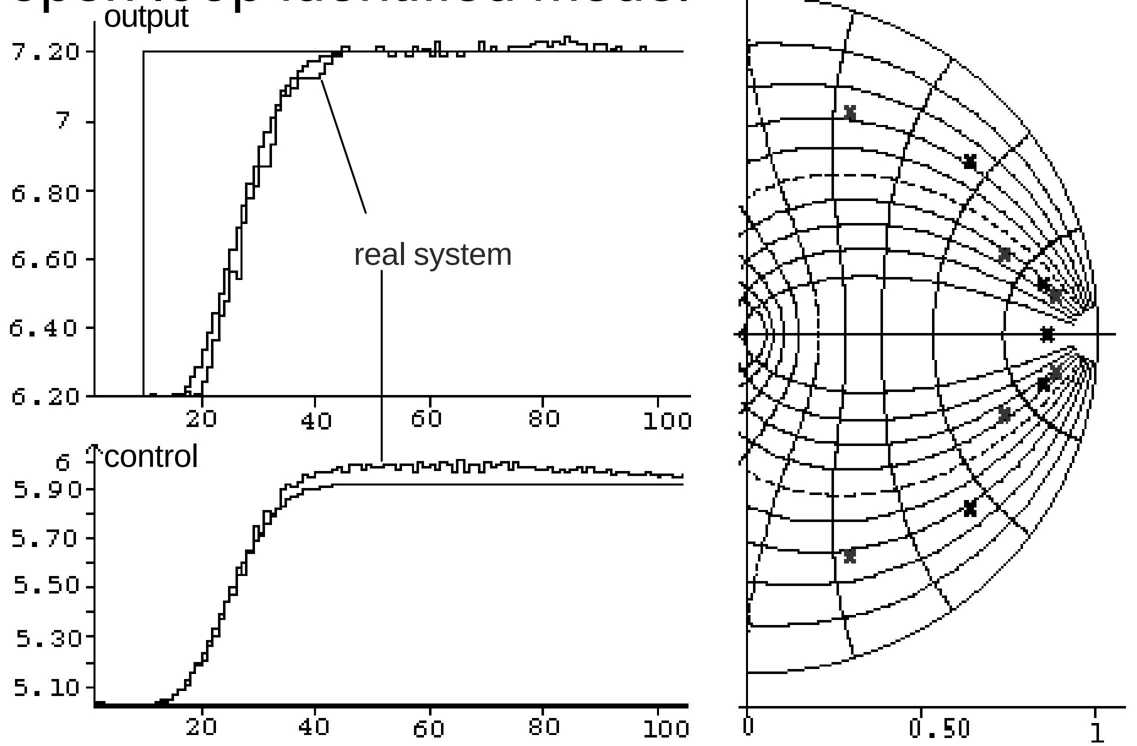


What is the *good* model (for control design) ?



Benefits of identification in closed loop (1)

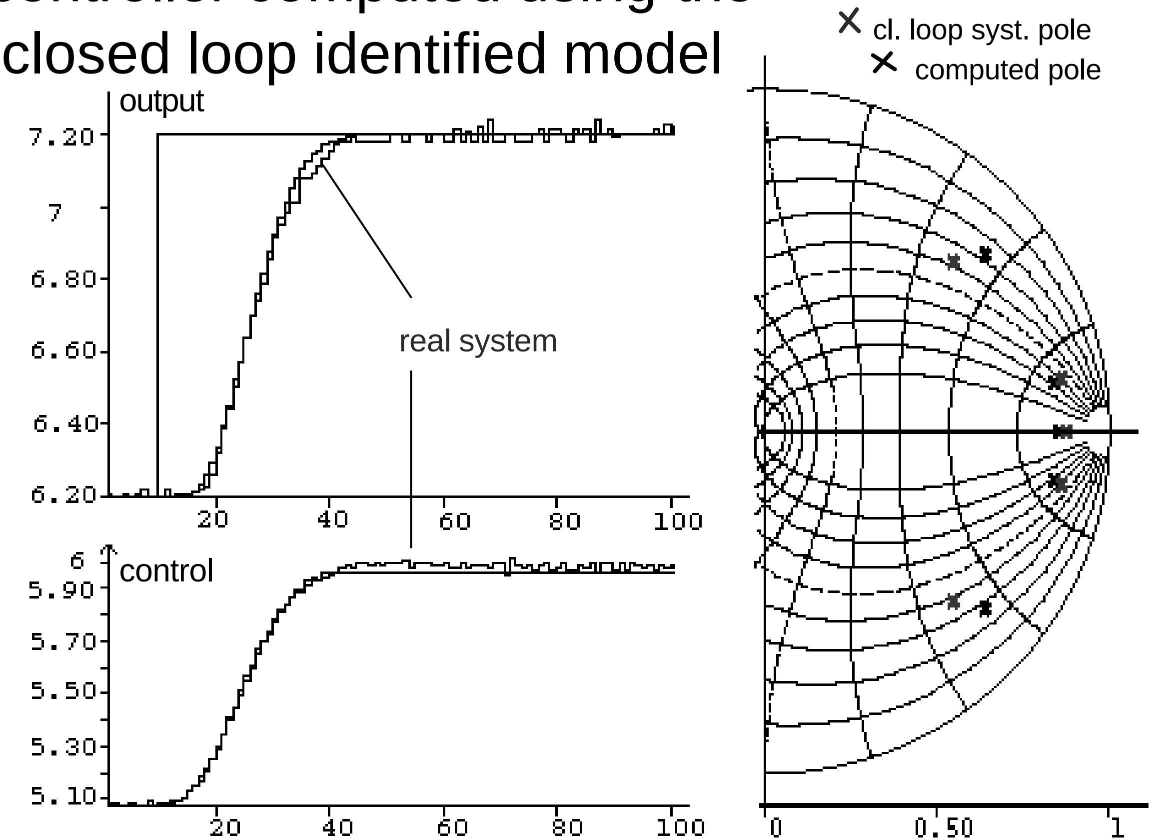
controller design using the open loop identified model



The pattern of *identified closed loop poles* is different from the pattern of *computed closed loop poles*

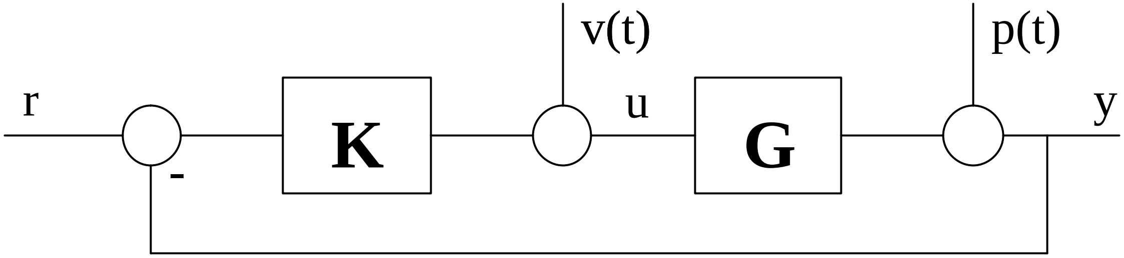
Benefits of identification in closed loop (2)

controller computed using the
closed loop identified model



The *computed* and the *identified* closed loop poles are very close

Notations



$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \qquad K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}$$

Sensitivity functions :

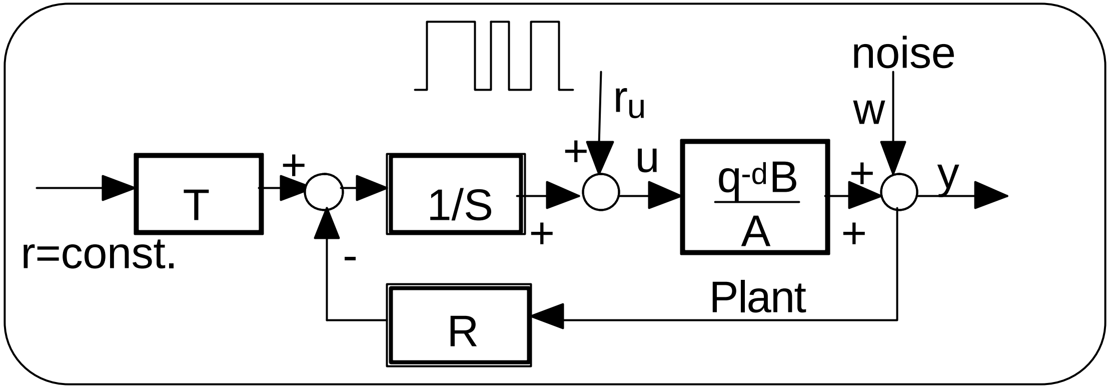
$$S_{yp}(z^{-1}) = \frac{1}{1+KG} \;;\; S_{up}(z^{-1}) = -\frac{K}{1+KG} \;;\; S_{yv}(z^{-1}) = \frac{G}{1+KG} \;;\; S_{yr}(z^{-1}) = \frac{KG}{1+KG}$$

Closed loop poles : $P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$

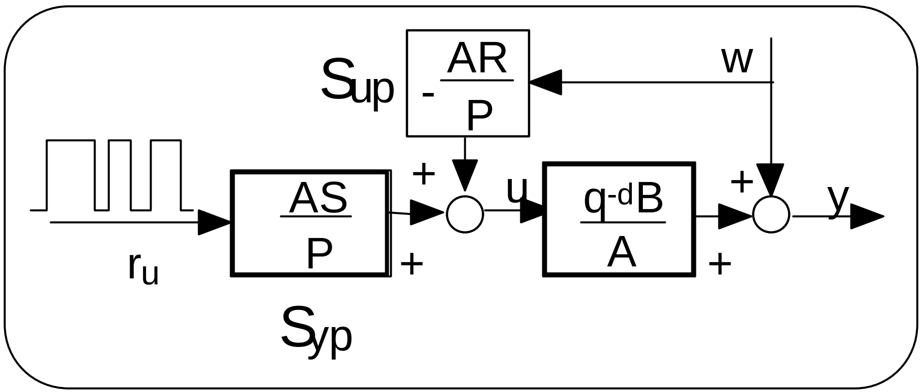
True closed loop system :(K,G), P, S_{xy}

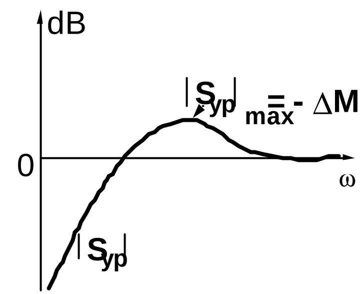
Nominal simulated(estimated) closed loop :$(K, \hat{G}), \hat{P}, \hat{S}_{xy}$

Identification in Closed Loop



Open loop interpetation





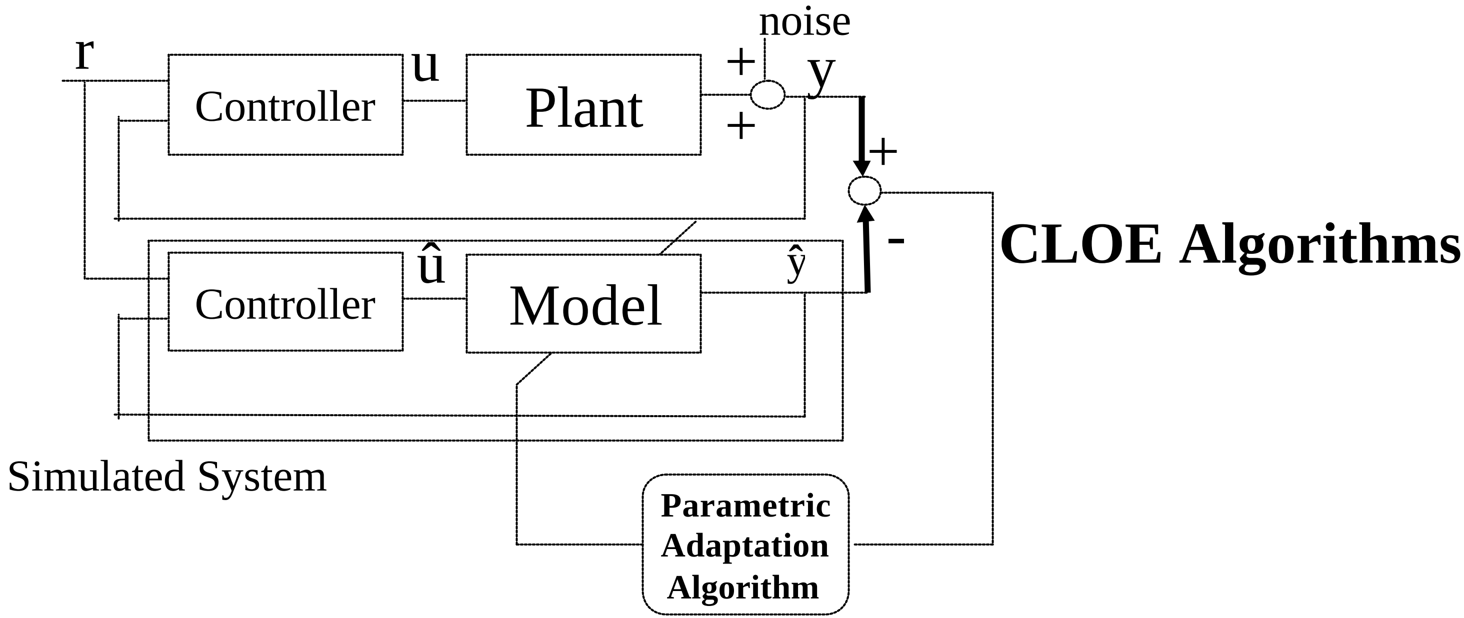
Objective : development of algorithms which:

- take advantage of the « improved » input spectrum
- are insensitive to noise in closed loop operation

Objective of the Identification in Closed Loop

(identification for control)

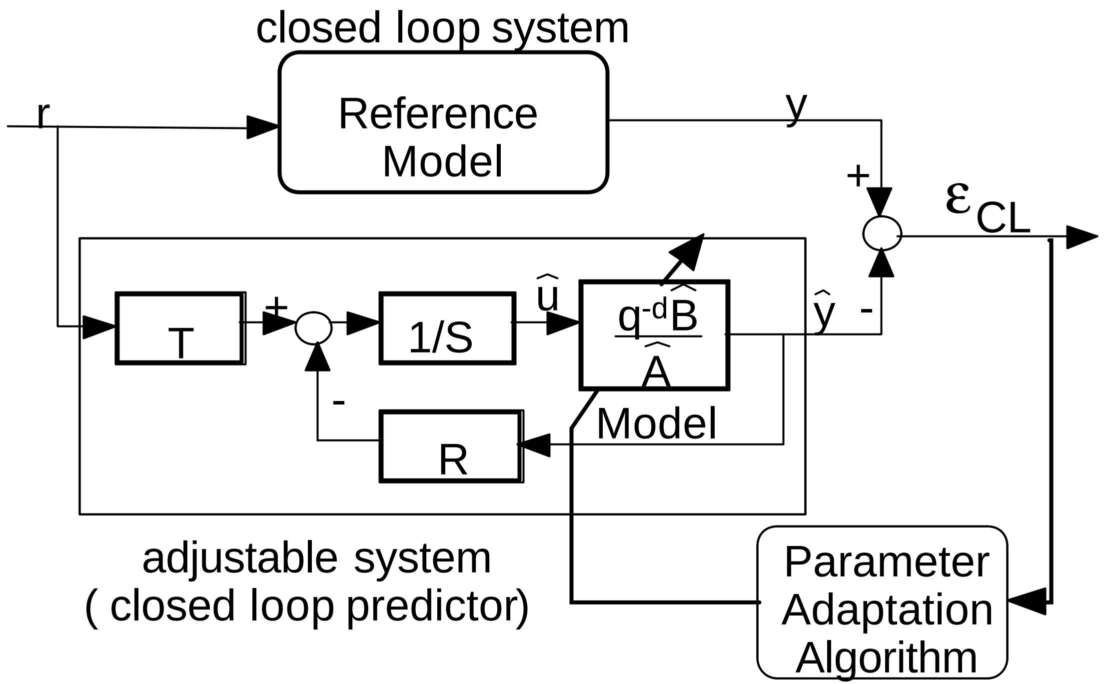
Find the « plant model » which minimizes the discrepancy between the « real » closed loop system and the « simulated » closed loop system.



Closed Loop Output Error

Identification in Closed Loop

- *M.R.A.S. point of view :*

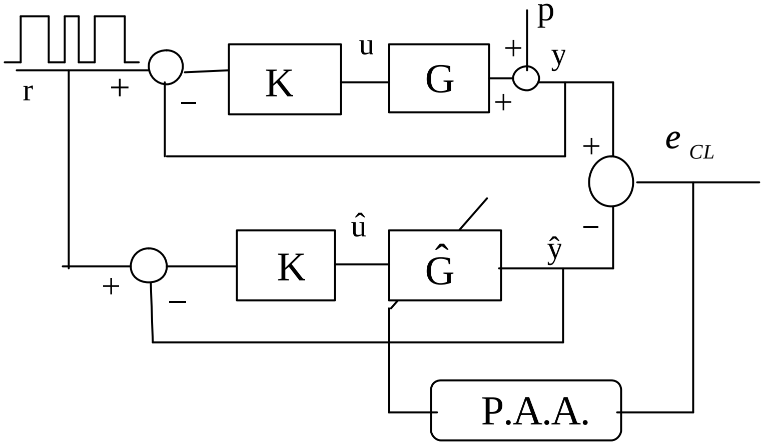


- *Identification point of view :*

A re-parametrized adjustable predictor of the closed loop

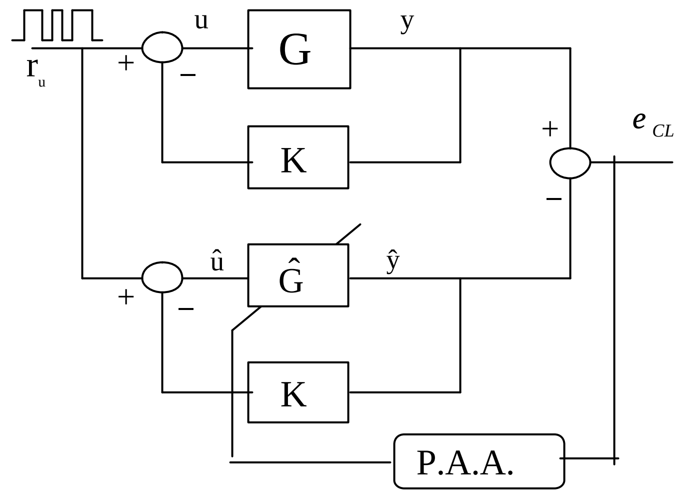
Closed Loop Output Error Identification Algorithms (CLOE)

Excitation added to reference signal



$$u = -\frac{R}{S} y + \frac{R}{S} r \qquad \hat{u} = -\frac{R}{S} \hat{y} + \frac{R}{S} r$$

Excitation added to controller output

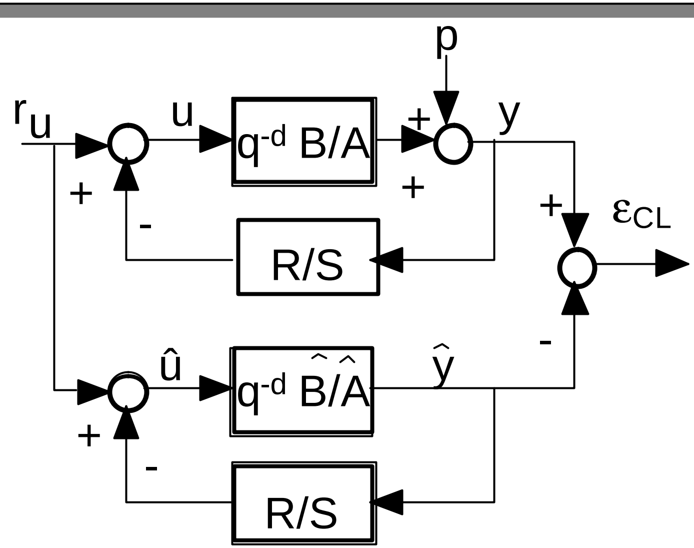


$$u = -\frac{R}{S} y + r_u \qquad \hat{u} = -\frac{R}{S} \hat{y} + r_u$$

Same algorithm but different properties of the estimated model!

Closed Loop Output Error Algorithms (CLOE)

Excitation added to the plant input



The closed loop system(for p = 0):

$$y(t+1) = -A^*(q^{-1})y(t) + B^*(q^{-1})u(t-d) = \theta^T \phi(t)$$

$$\theta^T = [a_1, \ldots a_{n_A}, b_1, \ldots b_{nB}]$$

$$\phi^T(t) = [-y(t), \ldots - y(t-n_A+1), u(t-d), \ldots u(t-d-n_B)]$$

$$u(t) = -\frac{R}{S} y(t) + r_u$$

CLOE

Adjustable predictor (closed loop)

Predicted output :

$$\hat{y}^{0}(t+1) = -\hat{A}^{*}(t,q^{-1})y(t) + \hat{B}^{*}(t,q^{-1})\hat{u}(t-d) = \hat{q}^{T}(t)f(t) \quad \textit{a priori}$$

$$\hat{y}(t+1) = \hat{q}^{T}(t+1)f(t) \quad \textit{a posteriori}$$

$$\hat{u}(t) = -\frac{R}{S}\hat{y}(t) + r_{u}$$

$$\hat{q}^{T}(t) = \left[\hat{a}_{1}(t),\ldots\hat{a}_{n_A}(t),\hat{b}_{1}(t),\ldots\hat{b}_{n_B}(t)\right]$$

$$f^{T}(t) = \left[-\hat{y}(t),\ldots-\hat{y}(t-n_{A}+1),\hat{u}(t-d),.\hat{u}(t-d-n_{B})\right]$$

Closed loop prediction (output) error

$$e^{0}_{CL}(t+1) = y(t+1) - \hat{q}^{T}(t)f(t) = y(t+1) - \hat{y}^{0}(t+1) \quad \textit{a priori}$$

$$e_{CL}(t+1) = y(t+1) - \hat{q}^{T}(t+1)f(t) = y(t+1) - \hat{y}(t+1) \quad \textit{a posteriori}$$

CLOE

The Parameter Adaptation Algorithm

$$e^{0}_{CL}(t+1) = y(t+1) - \hat{q}^{T}(t)f(t) = y(t+1) - \hat{y}^{0}(t+1)$$

$$\hat{q}(t+1) = \hat{q}(t) + F(t+1)\Phi(t)e^{0}_{CL}(t+1)$$

$$F^{-1}(t+1) = l_{1}(t)F^{-1}(t) + l_{2}(t)\Phi(t)\Phi^{T}(t)\ ;\ 0 < l_{1}(t) \leq 1\ ;\ 0 \leq l_{2}(t) < 2$$

$$\Phi(t) = f(t)$$

Updating F(t):

$$F(t+1) = \frac{1}{l_{1}(t)}\left[F(t) - \frac{F(t)\Phi(t)\Phi(t)^{T}F(t)}{\frac{l_{1}(t)}{l_{2}(t)} + \Phi(t)^{T}F(t)\Phi(t)}\right]$$

CLOE

The Parameter Adaptation Algorithm (alternative form)

$$\hat{q}(t+1) = \hat{q}(t) + F(t)\Phi(t)e_{CL}(t+1)$$

$$F^{-1}(t+1) = l_1(t)F^{-1}(t) + l_2(t)\Phi(t)\Phi^T(t)\,;\ 0 < l_1(t) \leq 1\,;\ 0 \leq l_2(t) < 2$$

$$e_{CL}(t+1) = \frac{e^0_{CL}(t+1)}{1 + \Phi^T(t)F(t)\Phi\ (t)}$$

$$e^0_{CL}(t+1) = y(t+1) - \hat{q}^T(t)f(t) = y(t+1) - \hat{y}^0(t+1)$$

Mostly used for analysis purposes

CLOE Algorithms

CLOE $$\Phi(t) = f(t)$$

F-CLOE $$\Phi(t) = \frac{S(q^{-1})}{\hat{P}(q^{-1})} f(t) \quad \hat{P} = \hat{A}(q^{-1})S(q^{-1}) + q^{-d}\hat{B}(q^{-1})R(q^{-1})$$

AF-CLOE $$\Phi(t) = \frac{S(q^{-1})}{\hat{P}(q^{-1},t)} f(t) \quad \hat{P}(q^{-1},t) = \hat{A}(q^{-1},t)S + q^{-d}\hat{B}(q^{-1},t)R$$

$$f^T(t) = \left[-\hat{y}(t),..-\hat{y}(t-n_A+1),\hat{u}(t-d),..\hat{u}(t-d-n_B)\right]$$

Remarks :

- F-CLOE needs an « estimated model » for filtering. This can be an «open loop model» or a model identified with CLOE or AF-CLOE
- For AF-CLOE « inital estimation » for filtering can be $\hat{A} = 1,\ \hat{B} = 0$

CLOE Properties

Case 1: The plant model is in the model set
(i.e. the estimated model has the *good order)*

- The controller is constant
- An external excitation is applied
- Measurement noise independent w.r.t. the external excitation

•*Asymptotic unbiased estimates in the presence of noise*
•subject to a (mild) sufficient passivity condition

•*CLOE:* $S / P - l / 2$

•*F-CLOE:* $\hat{P} / P - l / 2$

•*Strictly positive real tr. fct.*

$$\max_t l_2(t) \leq l < 2$$

•*AF-CLOE:* *none (local)*

A basic result – deterministic environment

Consider the PAA

$$\hat{q}(t+1) = \hat{q}(t) + F(t)\Phi(t)e_{CL}(t+1)$$

$$F^{-1}(t+1) = l_1(t)F^{-1}(t) + l_2(t)\Phi(t)\Phi^T(t) \; ; 0 < l_1(t) \le 1 ; 0 \le l_2(t) < 2$$

Assume that the *a posteriori* prediction error satisfies:

$$e_{CL}(t+1) = H(q^{-1})\left[q - \hat{q}(t+1)\right]^T \Phi(t)$$

If:

$$H(z^{-1}) - \frac{l}{2} = S.P.R.; \quad \max_t l_2(t) \le l < 2$$

Then:

$$\lim_{t\to\infty} e_{CL}(t+1) = \lim_{t\to\infty} e^0_{CL}(t+1) = 0$$

$$\|\Phi(t)\| < \infty; \forall t$$

for any initial conditions

CLOE analysis –Deterministic environment

(*a posteriori* prediction error equations)

CLOE $$e_{CL}(t+1) = \frac{S}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \phi(t)$$

F-CLOE $$e_{CL}(t+1) \approx \frac{\hat{P}}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \frac{S}{\hat{P}}\phi(t)$$

AF-CLOE $$e_{CL}(t+1) \approx \frac{\hat{P}(t)}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \frac{S}{\hat{P}(t)}\phi(t)$$

$\Phi(t)$

S.P.R. conditions for prediction error convergence

To have in addition « parameter convergence » you need a persistent excitation (i.e.: rich input like P.R.B.S.)

A basic result – stochastic environment

Assume that for any $\hat{\theta}$ along the trajectories of the algorithm:

$$e_{CL}(t+1,\hat{\theta}) = H(q^{-1})\left[\theta - \hat{\theta}\right]^T \Phi(t,\hat{\theta}) + w'(t+1)$$

and: $E\left\{\Phi(t,\hat{\theta})w'(t+1)\right\} = 0$ (regressor and noise are uncorrelated)

If: $$H(z^{-1}) - \frac{\lambda}{2} = S.P.R.;\ \max_t \lambda_2(t) \le \lambda < 2,\ \lambda_2(t) > 0$$

Decreasing adaptation gain

Then: $$Prob\left\{\lim_{t\to\infty}\hat{\theta}(t) \in D_C\right\} = 1$$

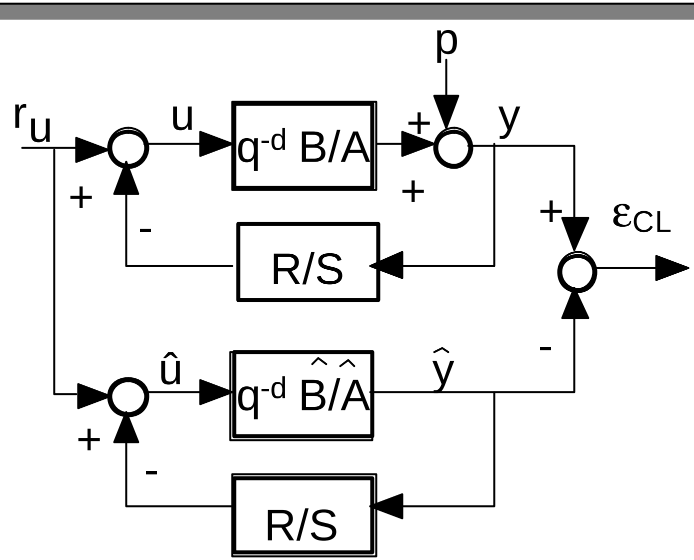
where: $$D_C = \left\{\hat{\theta} : \left[\theta - \hat{\theta}\right]^T \Phi(t,\hat{\theta}) = 0\right\}$$

Remark : With « persistent excitation » $D_C = \theta$

CLOE analysis – Stochastic environment

$w \neq 0$

Noisy case



The closed loop system:

$$y(t+1) = -A^*(q^{-1})y(t) + B^*(q^{-1})u(t-d) + Ap(t+1)$$

The a posteriori prediction error equation:

$$e_{CL}(t+1) = \frac{S}{P}\left[q - \hat{q}(t+1)\right]^T f(t) + \frac{AS}{P}p(t+1)$$

CLOE analysis – Stochastic environment

The a posteriori prediction error equation:

$$e_{CL}(t+1) = \frac{S}{P}\left[q - \hat{q}(t+1)\right]^T f(t) + \frac{AS}{P} p(t+1)$$

The « frozen » error equation:

$$e_{CL}(t+1,\hat{q}) = \frac{S}{P}\left[q - \hat{q}\right]^T f(t) + \frac{AS}{P} p(t+1)$$

H $\qquad$ w'

$$E\left\{f(t,\hat{q})w'(t+1)\right\} = 0$$

(the input and output of the estimated model do not depend on the noise for fixed estimated parameters)

Convergence condition:

$S/P - l/2 =$ S.P.R $\qquad \max_t l_2(t) \le l < 2$

CLOE Properties

Case 2: The plant model is not in the model set
(ex.: the estimated model has a *lower order)*

Basic idea for analysis of identification algorithms(Ljung) :
Convert time domain minimization criterion in frequency domain criterion using Parseval th. and Fourrier transforms

See the next slides

Analysis of identification algorithms in the frequency domain

(properties of the estimated model)

Identification criterion (time domain)

$$\hat{q}^* = \arg\min_{\hat{q}\in D} \lim_{t\to\infty}\frac{1}{N}\sum_{1}^{N} e^2(t,\hat{q}) \approx \arg\min_{\hat{q}\in D} E\left\{e^2(t,\hat{q})\right\}$$

domain of admissible parameters

Assumption: $\lim_{t\to\infty}\frac{1}{N}\sum_{1}^{N} e^2(t,\hat{q}) < \infty$

Identification criterion (frequency domain)

$$\hat{q}^* = \arg\min_{\hat{q}\in D} \frac{1}{2p}\int_{-p}^{p} f_e(e^{jw},\hat{q})\,dw$$

Spectral density of prediction error

Criterion minimized by CLOE algorithms

A recursive identification algorithm minimize a criterion of the form:

$$\min_{\hat{q}\in D} \lim_{t\to\infty} \frac{1}{N}\sum_{1}^{N} e^{2}(t,\hat{q}) \qquad (+)$$

if:

$$\hat{q}(t+1)=\hat{q}(t)+F(t)\Phi(t)e(t+1)=\hat{q}(t)+F(t)\left[-\frac{1}{2}\,grad\,e^{2}(t+1)\right]$$

For CLOE algorithms:

$$e_{CL}(t+1)=\frac{S}{P}\left[q-\hat{q}(t+1)\right]^{T} f(t)$$

$$grad\,e^{2}(t+1)=-2\frac{\partial e(t+1)}{\partial \hat{q}(t+1)}e(t+1) \qquad \frac{\partial e(t+1)}{\partial \hat{q}(t+1)}=-\frac{S}{P}f(t)\approx -\Phi_{AF-CLOE}$$

AF-CLOE minimizes a criterion of the form (+)
(CLOE and F-CLOE achieve approximatively the same optimization)

Criterion minimized by CLOE algorithms

A recursive identification algorithm minimizes a criterion of the form:

$$\min_{\hat{q}\in D} \lim_{t\to\infty} \frac{1}{N}\sum_{1}^{N} e^{2}(t,\hat{q}) \qquad (+)$$

if:

$$\hat{q}(t+1)=\hat{q}(t)+F(t)\Phi(t)e(t+1)=\hat{q}(t)+F(t)\left[-\frac{1}{2}grad\, e^{2}(t+1)\right]$$

For CLOE algorithms:

$$grad\, e^{2}(t+1)=-2\frac{\partial e(t+1)}{\partial \hat{q}(t+1)}e(t+1) \quad ; \; e(t+1)=y(t+1)-\hat{y}(t+1)$$

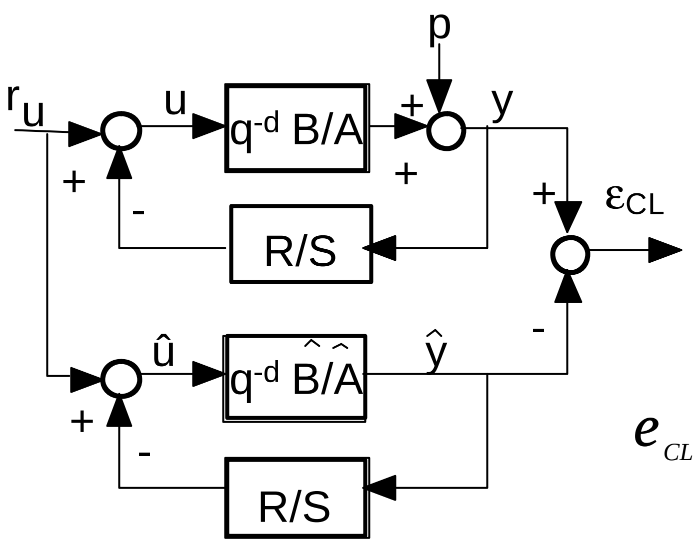
Does not depend on $\hat{q}$ (referring to $y(t+1)$)

$$\frac{\partial e(t+1)}{\partial \hat{q}(t+1)}=-\frac{\partial \hat{y}(t+1)}{\partial \hat{q}(t+1)}=-\frac{S(q^{-1})}{\hat{P}(t,q^{-1})}f(t)=-\Phi(t)_{AF-CLOE}$$

AF-CLOE minimizes a criterion of the form (+)

(CLOE and F-CLOE achieve approximatively the same optimization)

CLOE – (excitation added to plant input)



$$r_u \to y = S_{yv}$$

$$r_u \to \hat{y} = \hat{S}_{yv}$$

$$S_{yv} = \frac{G}{1+KG}$$

$$\hat{S}_{yv} = \frac{\hat{G}}{1+K\hat{G}}$$

$$e_{CL} = [S_{yv} - \hat{S}_{yv}]r_u + S_{yp}\,p$$

$f_{r_u}(w) =$ Spectral density of external excitation

$f_p(w) =$ Spectral density of the measurement noise

r_u and p are independent $\quad (f_{pr_u}(w) = 0)$

Properties of the Estimated Model (1)

Excitation added to controller output

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} [|S_{yv} - \hat{S}_{yv}|^2 f_{r_u}(\omega) + |S_{yp}|^2 f_p(\omega)] d\omega$$

$$= \arg\min_{\theta} \int_{-\pi}^{\pi} |S_{yp}|^2 [|G - \hat{G}|^2 |\hat{S}_{yp}|^2 f_{r_u}(\omega) + f_p(\omega)] d\omega$$

- $\hat{G}$ will minimize the 2 norm between the true sensitivity function and the sensitivity function of the closed loop estimator when r(t) is a white noise (PRBS)

-Plant -model error heavily weighted by the sensitivity functions

-The noise does not affect the asymptotic estimation

Properties of the Estimated Model (2)

Excitation added to reference signal

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} [|S_{yp} - \hat{S}_{yp}|^2 f_r(\omega) + |S_{yp}|^2 f_p(\omega)] d\omega$$

$$= \arg\min_{\theta} \int_{-\pi}^{\pi} |S_{yp}|^2 [|G - \hat{G}|^2 |\hat{S}_{up}|^2 f_r(\omega) + f_p(\omega)] d\omega$$

- $\hat{G}$ will minimize the 2 norm between the true sensitivity function and the sensitivity function of the closed loop estimator when r(t) is a white noise (PRBS)

-Plant -model error heavily weighted by the sensitivity functions

-The noise does not affect the asymptotic estimation

Properties of the Estimated Model (3)

Excitation added to reference signal

One has: $$\left|S_{yp} - \hat{S}_{yp}\right| = \left|S_{yr} - \hat{S}_{yr}\right|$$

Therefore one has also :

$$\hat{q}^{*} = \arg\min_{q} \int_{-p}^{p} [\left|S_{yr} - \hat{S}_{yr}\right|^{2} f_{r}(w) + \left|S_{yp}\right|^{2} f_{p}(w)]dw$$

$$= \arg\min_{q} \int_{-p}^{p} [\left|S_{yp} - \hat{S}_{yp}\right|^{2} f_{r}(w) + \left|S_{yp}\right|^{2} f_{p}(w)]dw$$

The differences with respect to the output sensitivity function and the complimentary sensitivity function are minimized

Identification in closed loop - Some remarks

- The quality of the identified model is enhanced in the critical frequency regions for control (compare with open loop id.)

CLOE $$\hat{q}^{*} = \arg\min_{q} \int_{-p}^{p} \left|S_{yp}\right|^{2} [\left|G-\hat{G}\right|^{2} \left|\hat{S}_{yp}\right|^{2} f_{r}(w) + f_{p}(w)]dw$$

OLOE $$\hat{q}^{*} = \arg\min_{q} \int_{-p}^{p} [\left|G-\hat{G}\right|^{2} f_{r}(w) + f_{p}(w)]dw$$

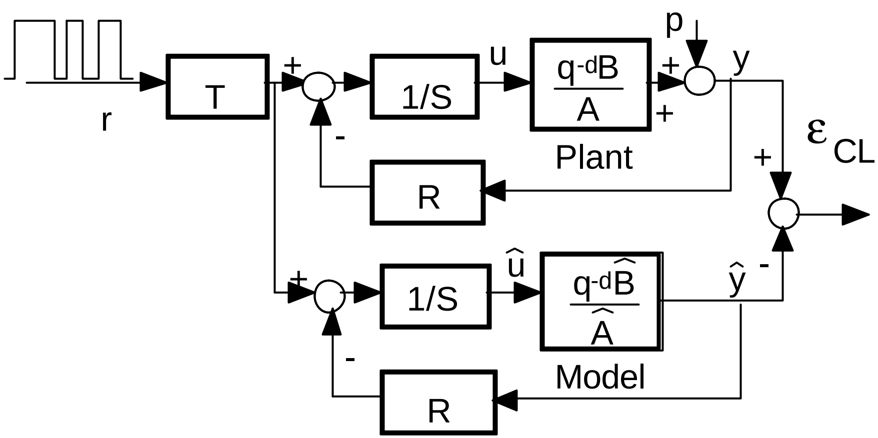
-Identification in closed loop can be used for **model reduction.**
The approximation will be good in the critical frequency regions for control.

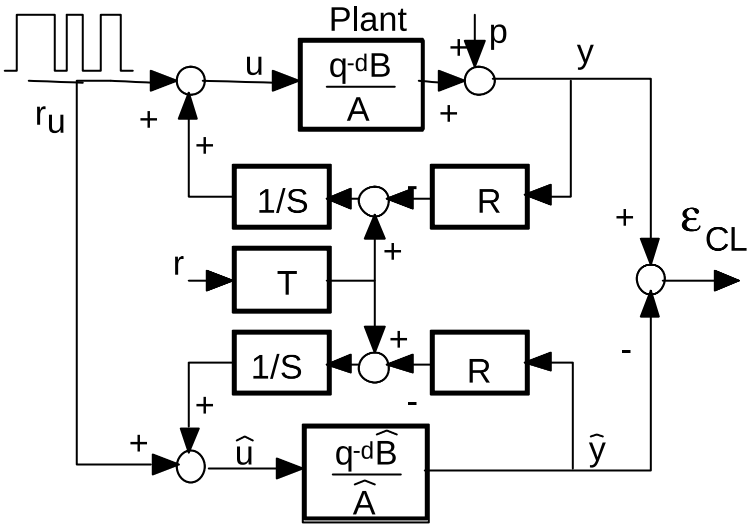
Closed Loop Output Error Identification Algorithms (CLOE)

R-S-T Controller

Excitation added to reference signal

Excitation added to controller output





$$u = -\frac{R}{S}y + \frac{T}{S}r \qquad \hat{u} = -\frac{R}{S}\hat{y} + \frac{T}{S}r$$

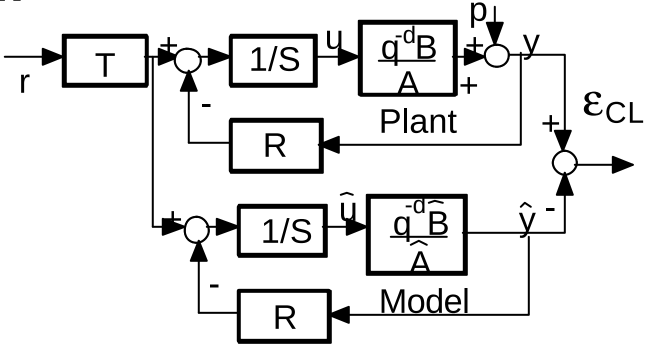
$$u = -\frac{R}{S}y + r_u \qquad \hat{u} = -\frac{R}{S}\hat{y} + r_u$$

Use of the prefilter T (R-S-T controller)

Difference between closed loop transfer functions (excitation through *T*)

$$\frac{BT}{AS+BR}-\frac{\hat{B}T}{\hat{A}S+\hat{B}R}=\frac{T}{R}\left[\frac{BR}{AS+BR}-\frac{\hat{B}R}{\hat{A}S+\hat{B}R}\right]=\frac{T}{R}\left[S_{yr}-\hat{S}_{yr}\right]$$

$$=\frac{T}{S}\left[\frac{BS}{AS+BR}-\frac{\hat{B}S}{\hat{A}S+\hat{B}R}\right]=\frac{T}{S}\left[S_{yv}-\hat{S}_{yv}\right]$$



Properties of the estimated model:

$$\hat{q}^{*}=\arg\min_{q}\int_{-p}^{p}\left[\left|S_{yr}-\hat{S}_{yr}\right|^{2}\left|\frac{T}{R}\right|^{2}f_{r}(w)+\left|S_{yp}\right|^{2}f_{p}(w)\right]dw$$

$$=\arg\min_{q}\int_{-p}^{p}\left[\left|S_{yv}-\hat{S}_{yv}\right|^{2}\left|\frac{T}{S}\right|^{2}f_{r}(w)+\left|S_{yp}\right|^{2}f_{p}(w)\right]dw$$

$T=S$ *Excitation added to the plant input*

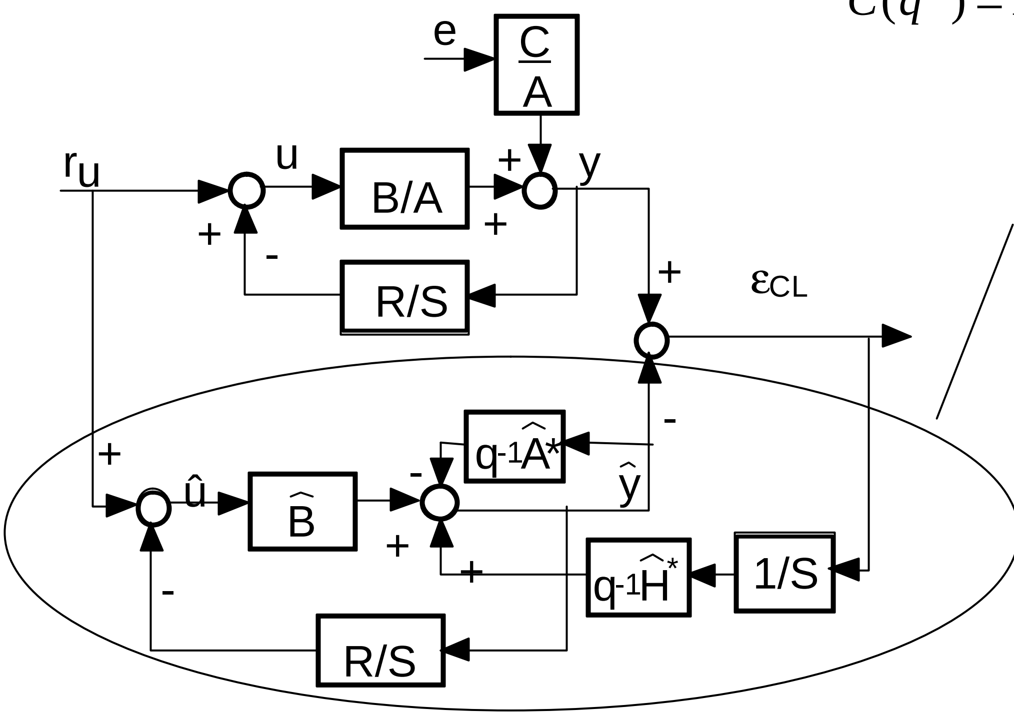
$T=R$ *Excitation added to the controller input (measure)*

Identification in Closed Loop of ARMAX Models

X-CLOE *Extended Closed Loop Output Error*

$$y(t+1) = -A^*(q^{-1})y(t) + B^*(q^{-1})u(t-d) + C^*e(t) + e(t+1)$$

$$C(q^{-1}) = 1 + q^{-1}C^*(q^{-1})$$



Optimal predictor for the closed loop when :

$$\hat{A} = A;\ \hat{B} = B;\ \hat{C} = C$$

$$H^* = C^*S - A^*S - B^*R;\ H = 1 + q^{-1}H^*;\ nH \cong nP$$

X-CLOE – the algorithm

Predicted output :

$$\hat{y}^{\circ}(t+1) = \hat{\theta}_e^T(t)\phi_e(t) \qquad \textit{a priori}$$

$$\hat{u}(t) = -\frac{R}{S}\hat{y}(t) + r_u$$

$$\hat{\theta}_e^T(t) = \left[\hat{a}_1(t),\ldots\hat{a}_{n_A}(t),\hat{b}_1(t),\ldots\hat{b}_{n_B}(t),\hat{h}_1(t),\ldots\hat{h}_{n_H}(t)\right]$$

$$\varepsilon_{CLf}(t) = \frac{1}{S}\varepsilon_{CL}(t)$$

$$\phi^T(t) = \left[-\hat{y}(t),.-\hat{y}(t-n_A+1),\hat{u}(t-d),.\hat{u}(t-d-n_B),\varepsilon_{CLf}(t),.\varepsilon_{CLf}(t-n_H+1)\right]$$

Closed loop prediction (output) error

$$\varepsilon_{CL}^0(t+1) = y(t+1) - \hat{\theta}_e^T(t)\phi_e(t) = y(t+1) - \hat{y}^0(t+1) \qquad \textit{a priori}$$

X-CLOE – the algorithm

The Parameter Adaptation Algorithm

$$e^0_{CL}(t+1) = y(t+1) - \hat{q}^T_e(t) f_e(t) = y(t+1) - \hat{y}^0(t+1)$$

$$\hat{q}(t+1) = \hat{q}(t) + F(t+1)\Phi(t) e^0_{CL}(t+1)$$

$$F^{-1}(t+1) = l_1(t) F^{-1}(t) + l_2(t)\Phi(t)\Phi^T(t)\ ;\ 0 < l_1(t) \leq 1\ ;\ 0 \leq l_2(t) < 2$$

$$\Phi(t) = f_e(t)$$

X-CLOE Properties

Case 1: The plant model is in the model set

Deterministic case:

• Global convergence does not require any S.P.R. condition *(works allways)*

Stochastic case (noise)

• Asymptotic unbiased estimates

• Convergence condition: $1/C - \lambda/2 =$ S.P.R *(like in open loop for ELS and OEEPM)*

Case 2: The plant model is not in the model set

• Slightly less good « approximation » properties than CLOE

• Provides better results than « open loop » identification alg.

Validation of Models Identified in Closed Loop

Controller dependent validation !

1) Statistical Model Validation

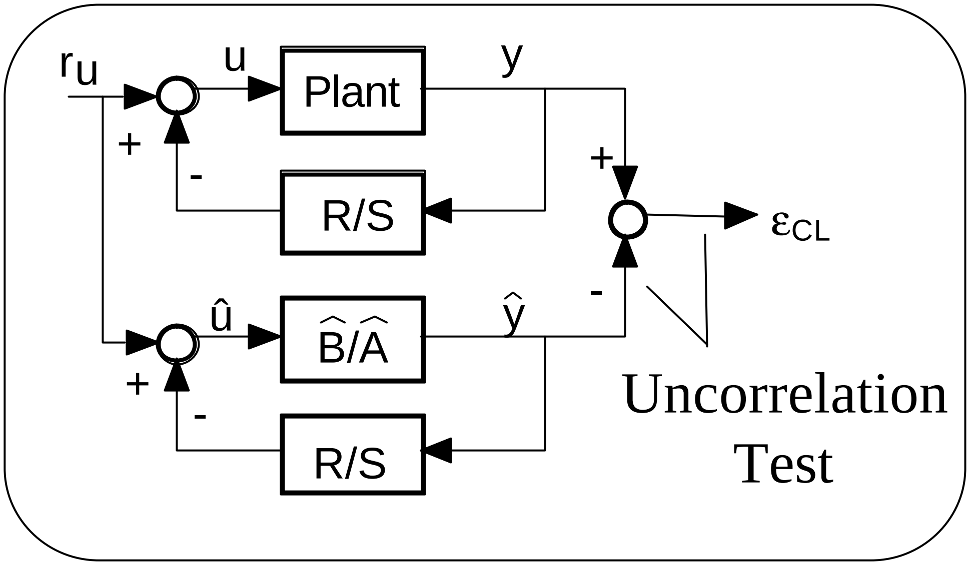
2) Pole Closeness Validation

3) Sensitivity Functions ClosenessValidation

4) Time Domain Validation

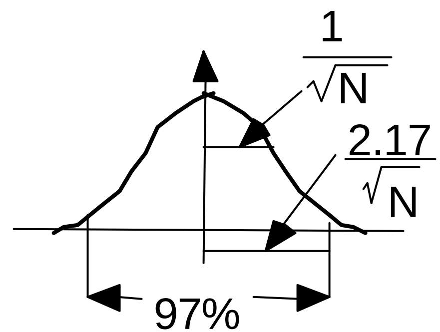
Identification in Closed Loop

Statisitical Model Validation



$$|RN(i)| \le \frac{2.17}{\sqrt{N}};\ i \ge 1$$

normalized crosscorelations — number of data



Controller dependent validation !

$$N = 256 \rightarrow |RN(i)| \le 0.136$$

pratical value : $|RN(i)| \le 0.15$

« Uncorrelation » Test

$\{e_{CL}(t)\}$: centered sequence of residual closed loop prediction errors

One computes:

$$R(i) = \frac{1}{N}\sum_{t=1}^{N} e_{CL}(t)\hat{y}(t-i) \quad ; \quad i = 0,1,2,\ldots,i_{\max} \qquad ; \qquad i_{\max} = \max(n_A, n_B + d)$$

$$RN(i) = \frac{R(i)}{\left[\left(\frac{1}{N}\sum_{t=1}^{N}\hat{y}^2(t)\right)\left(\frac{1}{N}\sum_{t=1}^{N}e_{CL}^{\;2}(t)\right)\right]^{1/2}} \quad ; \quad i = 0,1,2,\ldots,i_{\max}$$

Remark: $RN(0) \neq 1$

Theoretical values: *RN(i) =0; i = 1, 2…*i_{max}

Real situation:

- Finite number of data
- Residual structural errors (orders, nonlinearities, noise)
- Objective: to obtain « good » simple models

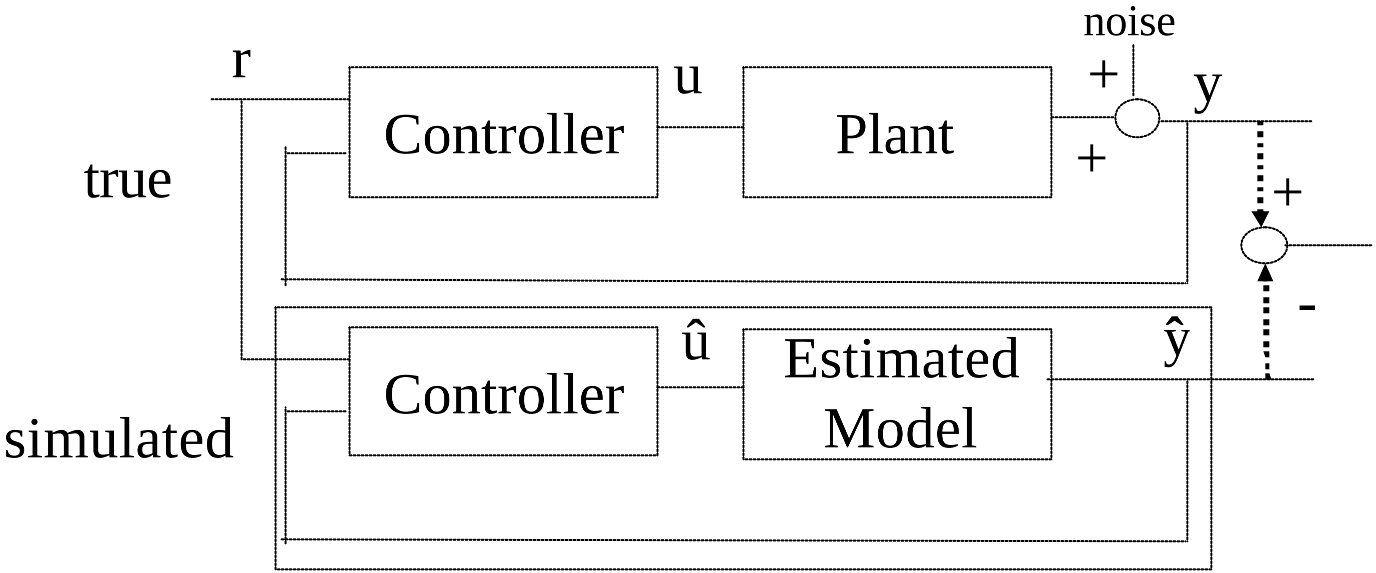
Validation criterion (N = number of data):

$$|RN(i)| \leq \frac{2.17}{\sqrt{N}} \quad ; \quad i \geq 1$$

or: $|RN(i)| \leq 0.15 \; ; \; i = 1,\ldots,i_{\max}$

Identification in Closed Loop

Pole Closeness Validation

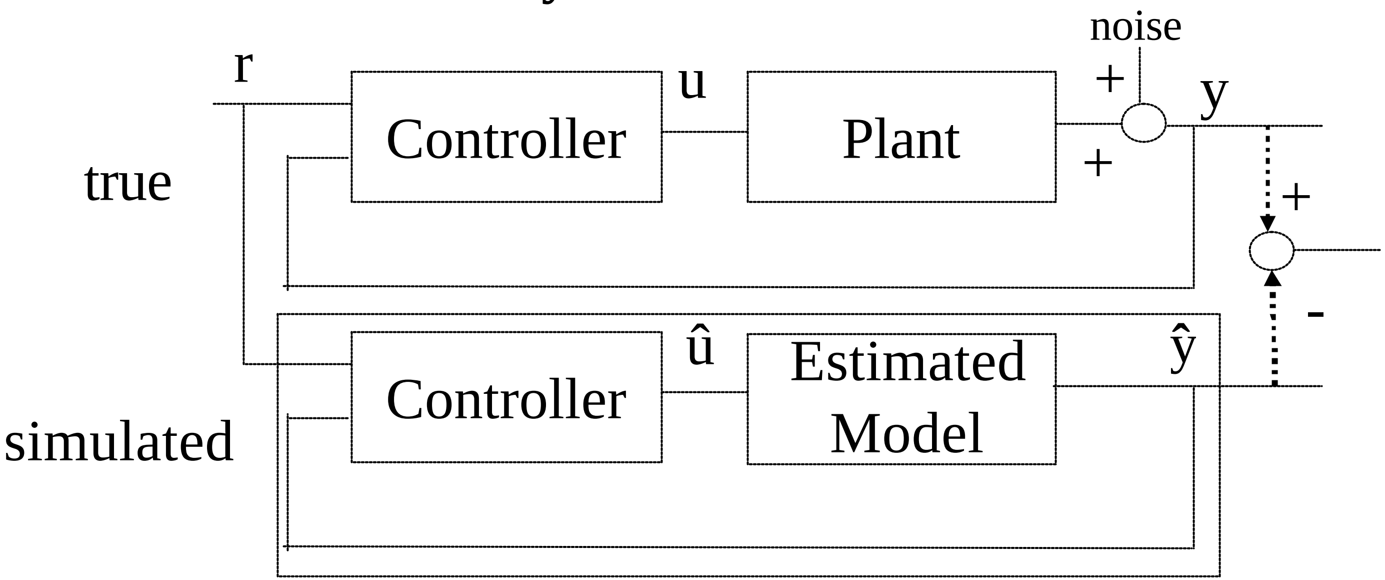


If the estimated model is good, the poles of the « true » loop and of the « simulated » loop should be *close*

- *The poles of the « simulated » system can be computed*
- *The poles of the « true » system should be estimated*

Identification in Closed Loop

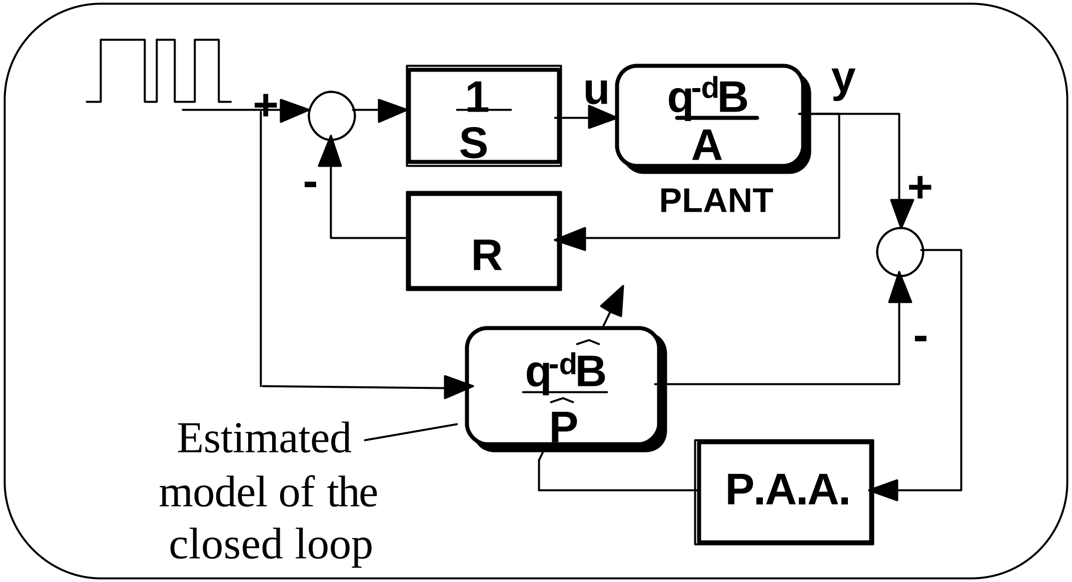
Sensitivity Function Closeness Validation



If the estimated model is good, the sensitivity functions of the « true » loop and of the « simulated » loop should be *close*

- *The sensitivity fct. of the « simulated » system can be computed*
- *The sensitivity fct. of the « true » system should be estimated*

Closed loop poles/Sensitivity functions Estimation



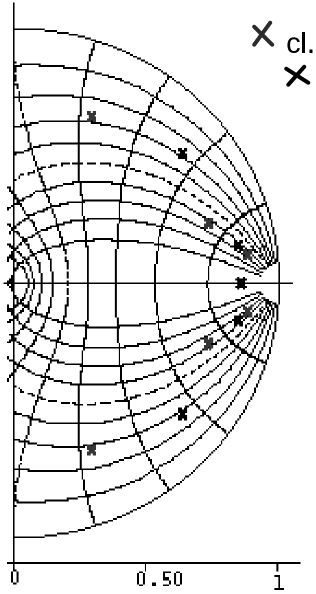
Rem:

- use of open loop identification algorithms
- same signals as those used for the identification of the *plant model* in closed loop operation

How to asses the poles/sensitivity fct. closeness ?

Poles:

- Patterns of the poles map
- Closeness of $1/\hat{P}$ and $1/\hat{\hat{P}}$



Sensitivity functions:

- Closeness of $\hat{S}_{xy}$ and $\hat{\hat{S}}_{xy}$

The closeness of two transfer functions can be measured by the « Vinnicombe distance » (ν gap) (min = 0, max = 1)
(*will be discussed later)*

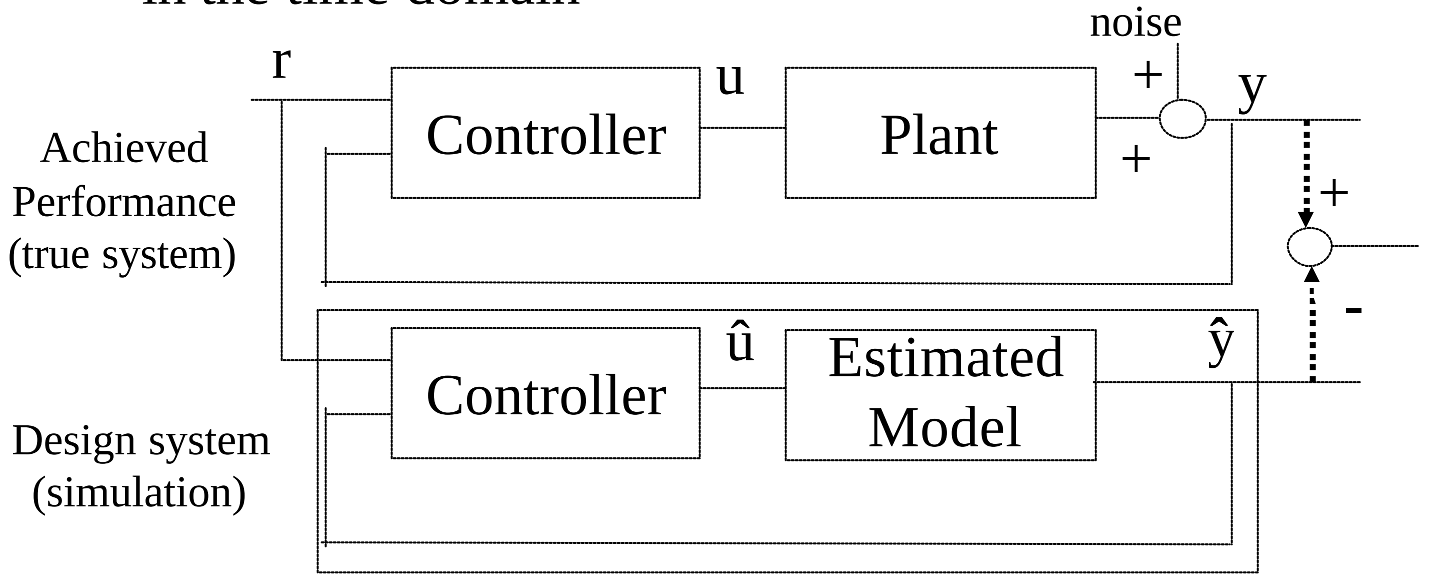
$\hat{X}$: Closed loop transfer function computed with the estimated plant model

$\hat{\hat{X}}$: Estimated closed loop transfer function

Identification in Closed Loop

Time Domain Validation

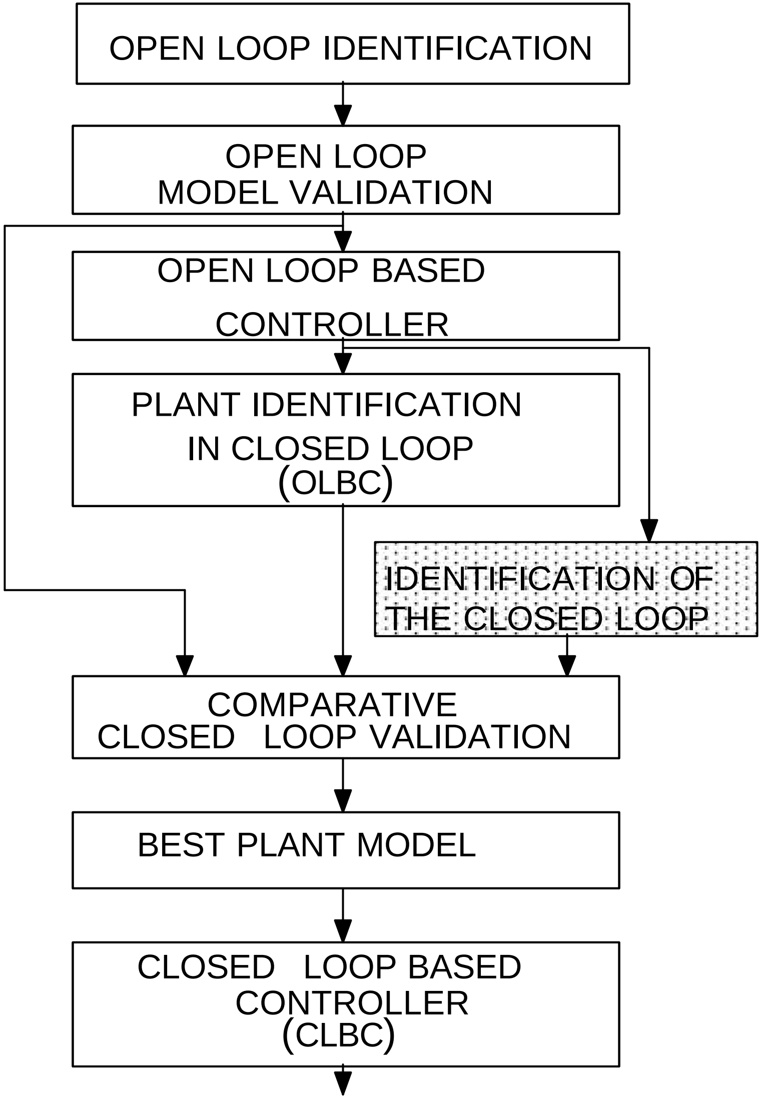
Comparison of « achieved » and « simulated » performance in the time domain



Rem.:

- not enough accuracy in many cases
- difficult interpretation of the results in some cases

Methodology of Plant Model Identification in Closed Loop



Closed loop identification schemes

Two possibilities for error generation:
-output error (CLOE)
-input error (CLIE)

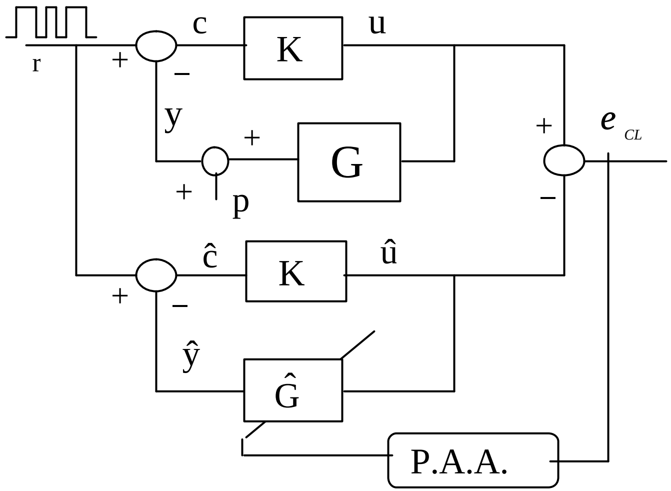
Two possibilities for applying the external excitation:
-added to the controller input (reference)
-added to the plant input

What is in fact important ?
The nominal sensitivity function we would like to approximate by the closed loop predictor (the identification criterion)

Remark:
Once a scheme is selected, to process the data one can use all the versions of the algorithms (choice of the regressor vector)

Closed loop input error (CLIE)

Excitation added to controller intput (reference)



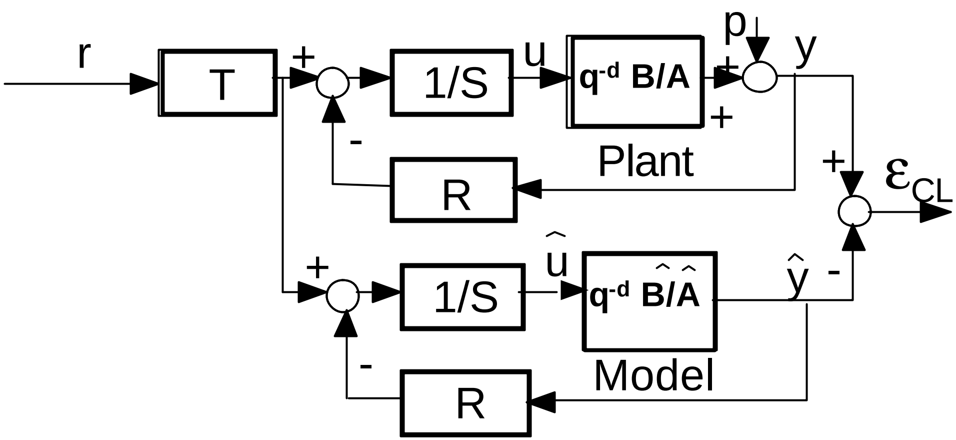
For details, see:

Landau I.D., Karimi A., (2002) : « A unified approach to closed-loop plant identification and direct controller reduction », *European Journal of Control*, vol.8, no.6

Selection of closed loop identification schemes

Identification criterion	Closed loop identification scheme
$\min \left\| S_{yp} - \hat{S}_{yp} \right\|$ or $\min \left\| S_{yr} - \hat{S}_{yr} \right\|$	CLOE with external excitation added to the controller input equivalent to CLIE with external excitation added to the plant input
$\min \left\| S_{up} - \hat{S}_{up} \right\|$	CLIE with external excitation added to the controller input
$\min \left\| S_{yv} - \hat{S}_{yv} \right\|$	CLOE with external excitation added to the controller input

Iterative Identification in Closed Loop and Controller Re-Design



Step 1 : Identification in Closed Loop

-Keep controller constant

-Identify a new model such that ε_{CL} ↘

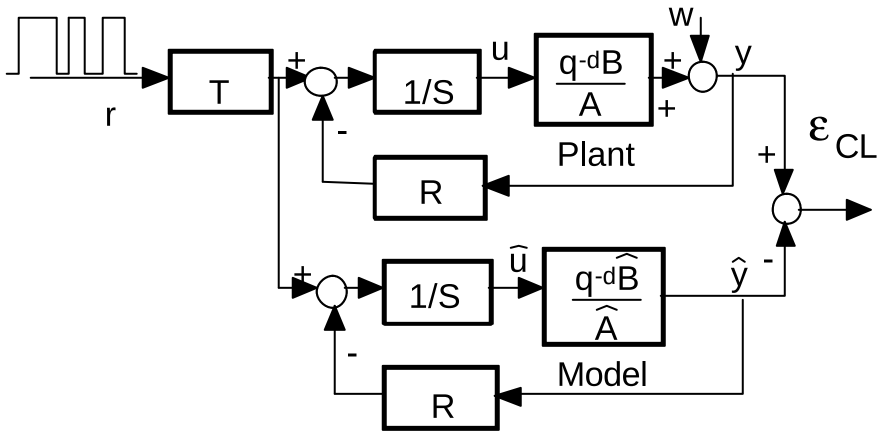
Step 2 : Controller Re – Design

- Compute a new controller such that ε_{CL} ↘

Repeat 1, 2, 1, 2, 1, 2,…

An interesting connection CL/OL

Open loop identification algorithms are particular cases of closed loop identification algorithms



Use R=0, S=T=1 and you get the open loop identification algorithms

CLOE → (OL)OE

X-CLOE → (OL)OEEPM

Experimental Results

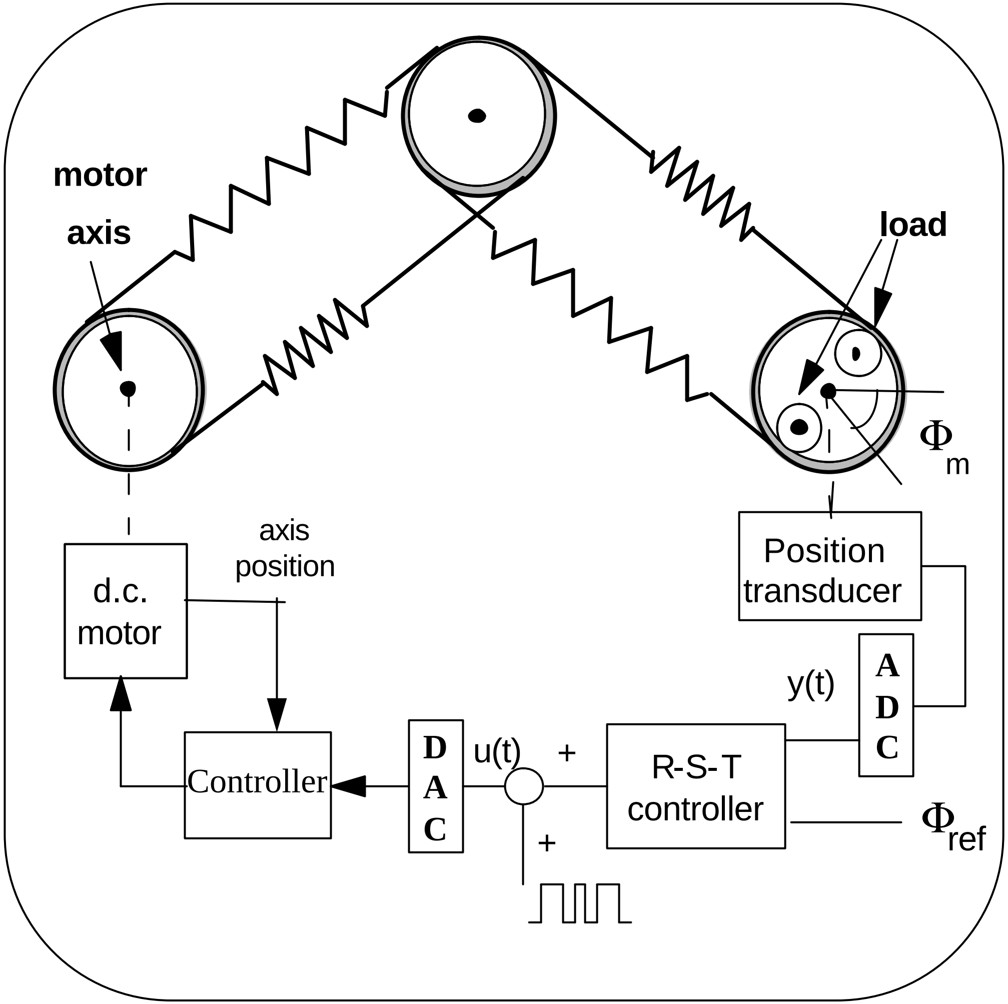
Identification in closed loop and controller re-design

for

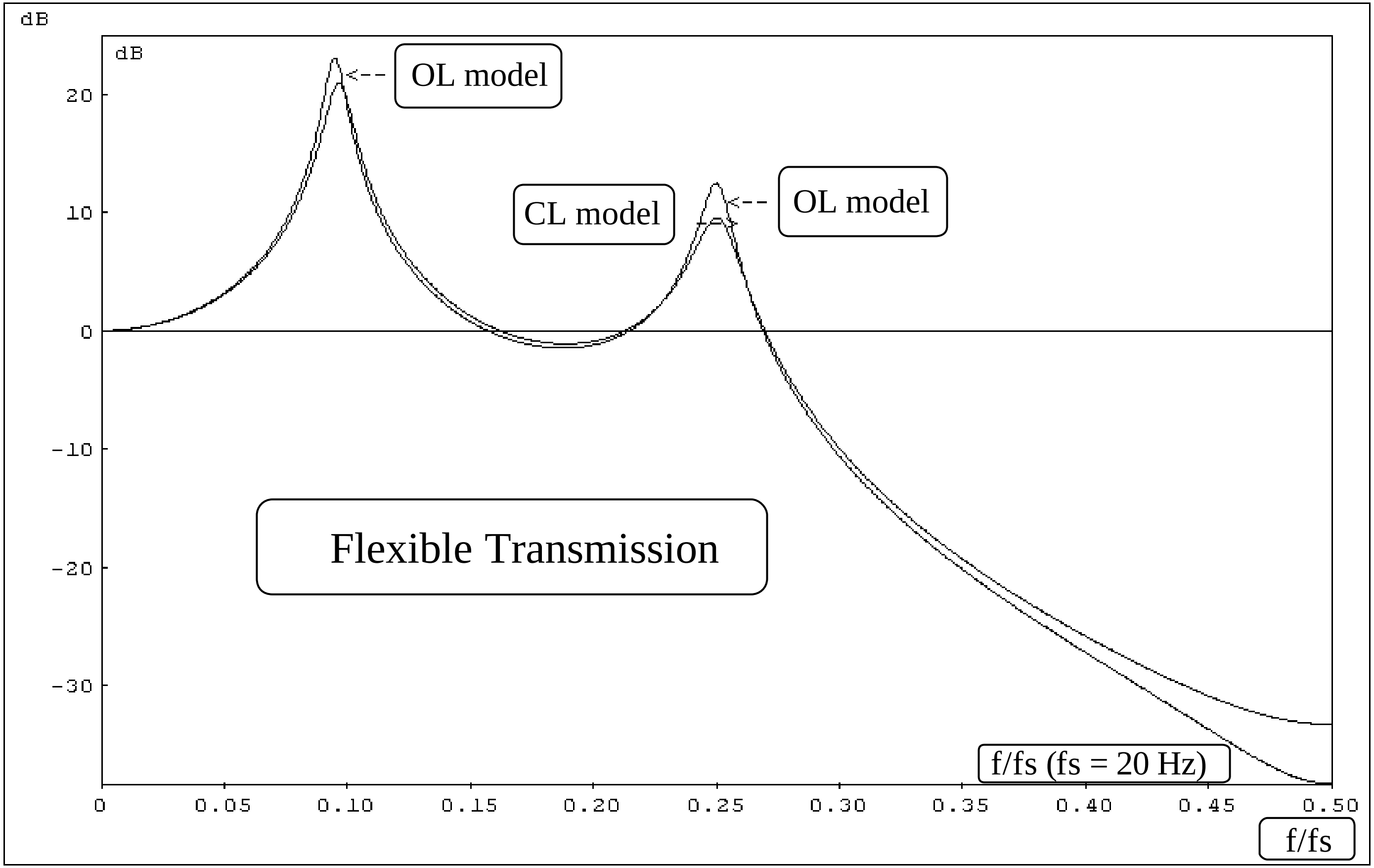
a flexible transmission

Identification in Closed Loop

The flexible transmission



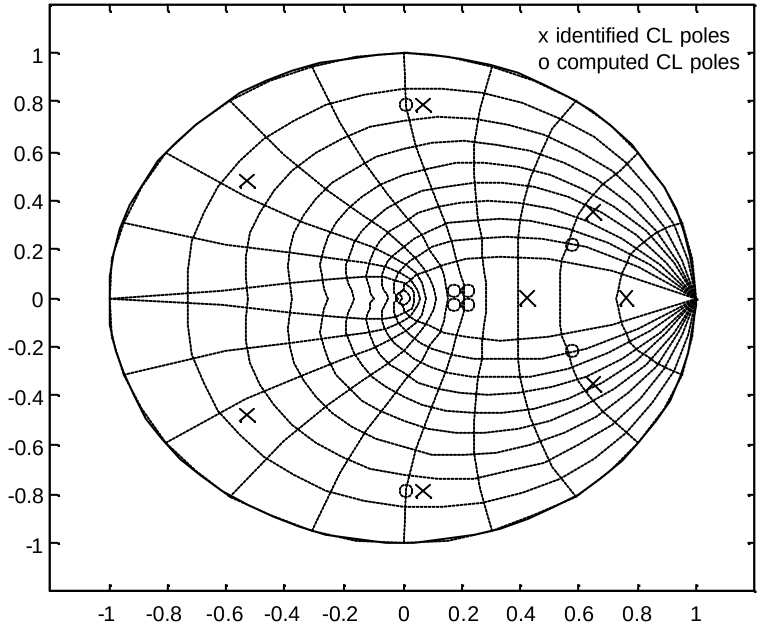
Flexible Tranmission
Frequency Characteristics of the Identified Models

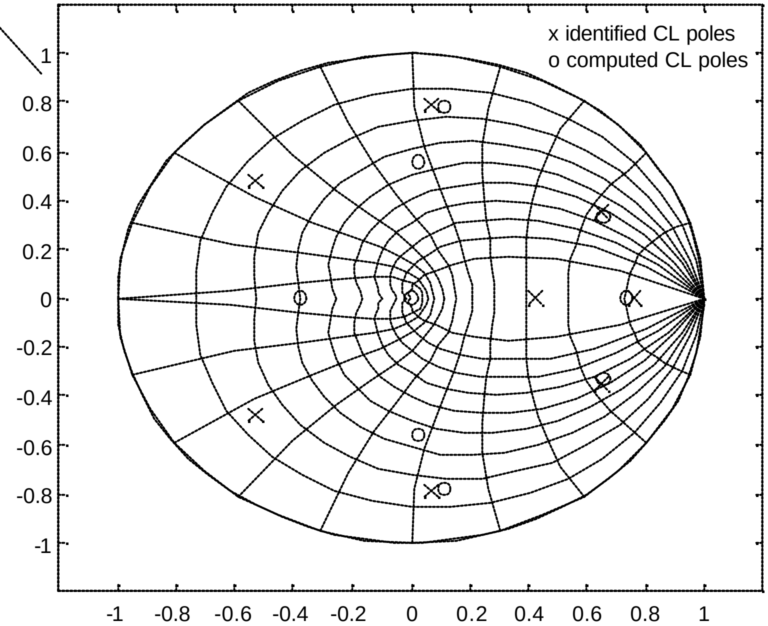


Model Validation in Closed Loop

Poles Closeness Validation

Controller computed using open loop identified controller (OLBC)





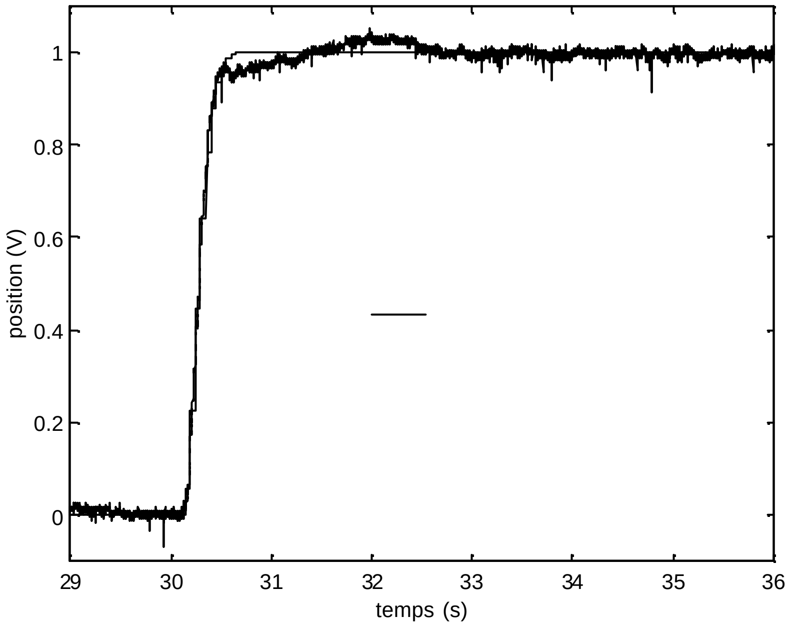
Model identified in open loop   Model identified in closed loop

The model identified in closed loop provides « computed » poles closer to the « real » poles than the model identified open loop

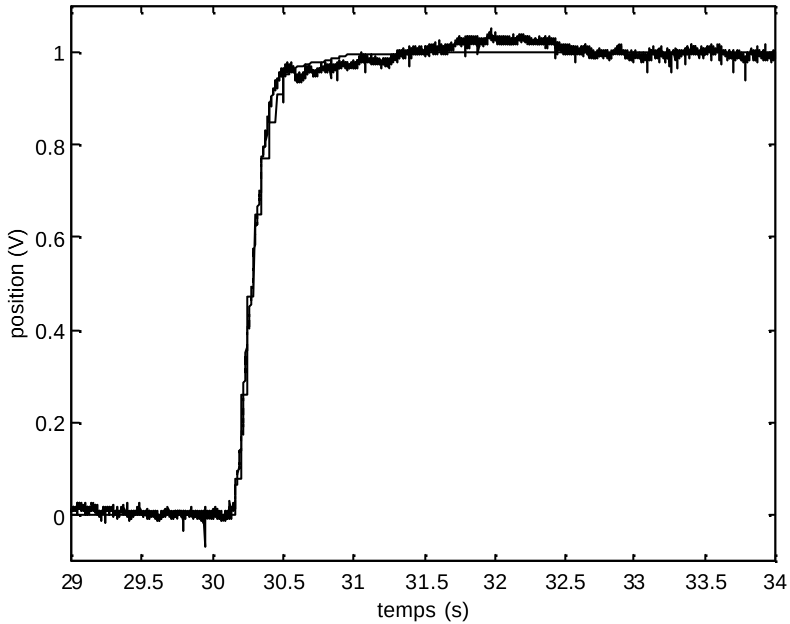
Model Validation in Closed Loop

Time Domain Validation

O.L.B.C.



Model identified in open loop

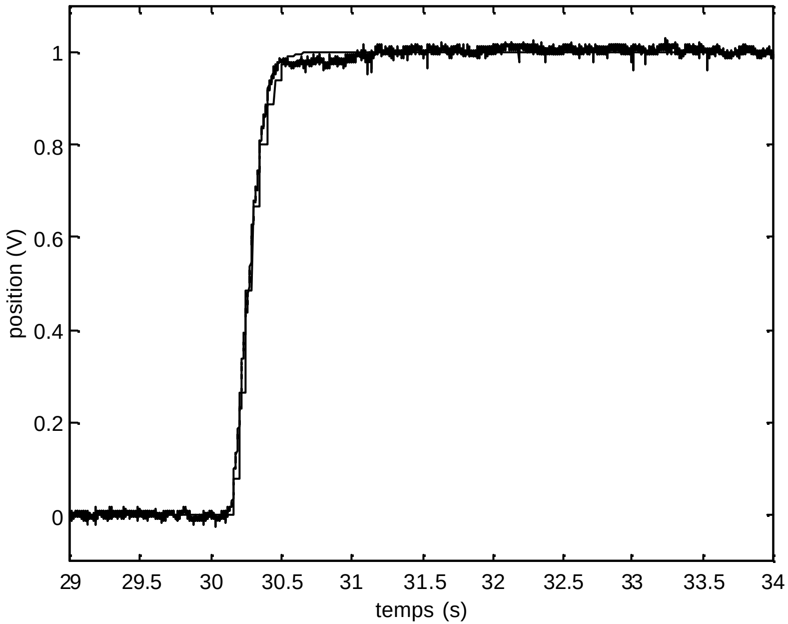


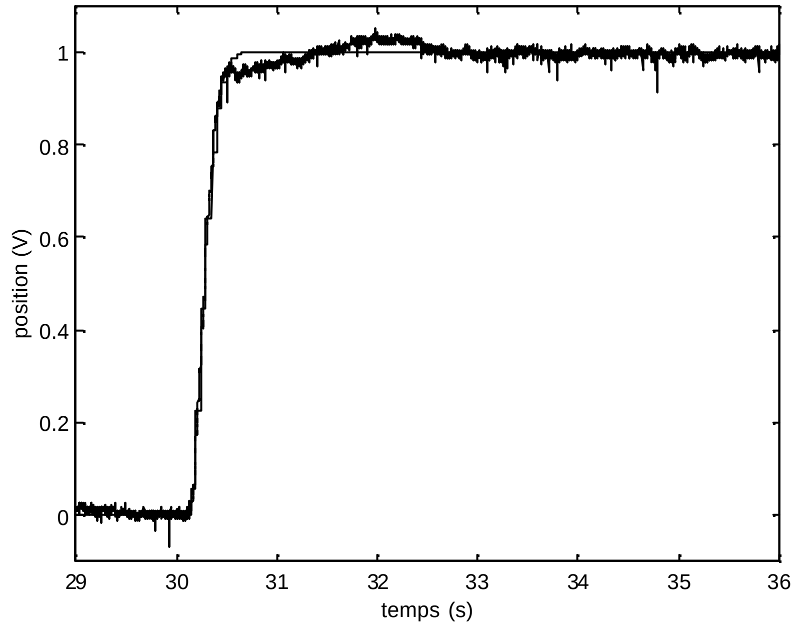
Model identified in closed loop

The simulation using the model identified in C.L. is closer to the real response than the simulation using the O.L.identified model

Controller Re-design Based on the Model Identified in Closed Loop

(on-site controller re-tuning)





Re-designed controller (CLBC) Initial controller (OLBC)

The CLBC controller provides performance which is closer to the designed performance than that provided by the OLBC controller

CLID^{TM}
(Matlab) Toolbox for Closed Loop Identification

To be downloaded from the web site:
http//:landau-bookic.lag.ensieg.inpg.fr

- files(.p and.m)
- examples (type :democlid)
- help.htm files (condensed manual)

CLID Toolbox

>> help clid

CLOSED LOOP IDENTIFICATION MODULE
by :
ADAPTECH
4 rue du Tour de l'Eau, 38400 Saint Martin d'Heres, France
info@adaptech.com
Copyright by Adaptech, 1997-1999

List of functions

cloe - Closed Loop Output Error Identification

fcloe - Filtered Closed Loop Output Error Identification

afcloe - Adaptive Filtered Closed Loop Output Error Identification

xcloe - Extended Closed Output Error Identification

clvalid - Validation of Models Identified in Closed Loop using also Vinnicombe gap

clie - Closed Loop Input Error Identification

```
>> help cloe

 CLOE       is used to identify a discrete time model of a plant operating in
                        closed-loop with an RST controller based on the CLOE method.

                        [B,A]=cloe(y,r,na,nb,d,R,S,T,Fin,lam1,lam0)

                        y and r are the column vectors containing respectively the output and the
                        excitation signal.

                        na, nb are the order of the polynomials A,B and d is the pure time delay

                        R, S and T are the column vectors containing the parameters of a two
                        degree of freedom controller. S*u(t)=-R*y(t)+T*r(t)
                        Remark: when the excitation signal is added to the measured output
                        (i.e. the controller is in feedforward with unit feedback) we have T=R and
                        when the excitation signal is added to the control input (i.e. the controller
                        is in feedback) we have T=S.

                        Fin is the initial gain F0=Fin*(na+nb)*eye(na+nb) (Fin=1000 by default)

                        lam1 and lam0 make different adaptation algoritms as follows:

                        lam1=1;lam0=1              :decreasing gain (default algorithm)
                        0.95<lam1<1;lam0=1 :decreasing gain with fixed forgetting factor
                        0.95<lam1,lam0<1           :decreasing gain with variable forgetting factor

                        See also FCLOE, AFCLOE, XCLOE and CLVALID.
                        Copyright by Adaptech, 1997-1999
```

CLOE – closed output error identification function

>> help cloe

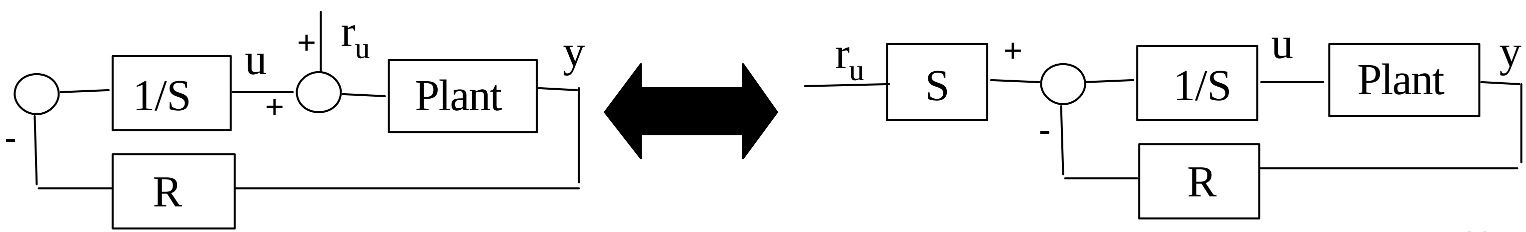
```
[B,A]=cloe(y,r,na,nb,d,R,S,T,Fin,lam1,lam0)
```

- [B,A]: Model Polynomials (identified)
- y: Measured output
- r: Excitation
- na, nb, d: Model orders
- R, S, T: Controller Polynomials
- Fin: Initial adaption gain (Fin=1000 by default)
- lam1: λ_1
- lam0: λ_0

lam1=1;lam0=1 : decreasing gain (default algorithm)
0.95<lam1<1; lam0=1 : decreasing gain with fixed forgetting factor
0.95<lam1,lam0<1 : decreasing gain with variable forgetting factor

- Excitation superposed to the reference: ***Need to specify R, S and T=R***

- Excitation superposed to the controller output (i.e. plant input): ***Need to take T=S***



CLVALID – closed loop model validation function

>> help clvalid

[lossf,gap,Pcal,Piden,yhat]=clvalid(B,A,R,S,T,y,r,pcl)

B,A: Model Polynomials (identified)

S,T: Controller Polynomials

lossf : $\frac{1}{N}\sum_{1}^{N}\left[y(t)-\hat{y}(t)\right]^2$

gap : Vinnicombe gap metric between *identified* and *computed* closed loop transfer function (BT/P)

Pcal : Computed closed loop poles by given model and controller

Piden : Identified closed loop poles from [y r] data

yhat : Closed loop estimated output

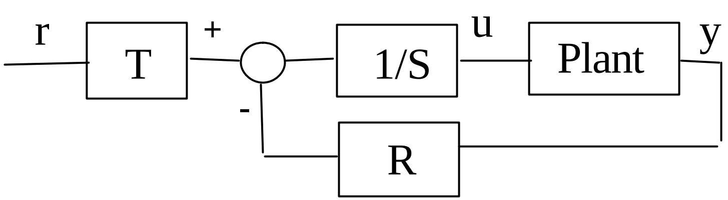
pcl = 1 : performs pole closeness validation (poles map display)
pcl =0; default

DEMOCLID –demo function

>> democlid

Load input (PRBS), output data (simubf file), controller
and values for Fin (=1000), lam1(= 1), lam0(= 0)

Data are generated in closed loop with an RST controller
The external excitation is superposed to the reference



Plant model
for data generation

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})e(t)$$

$$A(q^{-1}) = 1 - 1.5q^{-1} + 0.7q^{-2} \; ; \quad B(q^{-1}) = q^{-1} + 0.5q^{-2} \; ; \quad d = 0$$

$$C(q^{-1}) = 1 + 1.6q^{-1} + 0.9q^{-2}$$

Controller
for data generation

$$R(q^{-1}) = 0.8659 - 1.2763q^{-1} + 0.5204\,q^{-2}$$

$$S(q^{-1}) = 1 - 0.6283\,q^{-1} - 0.3717\,q^{-2}$$

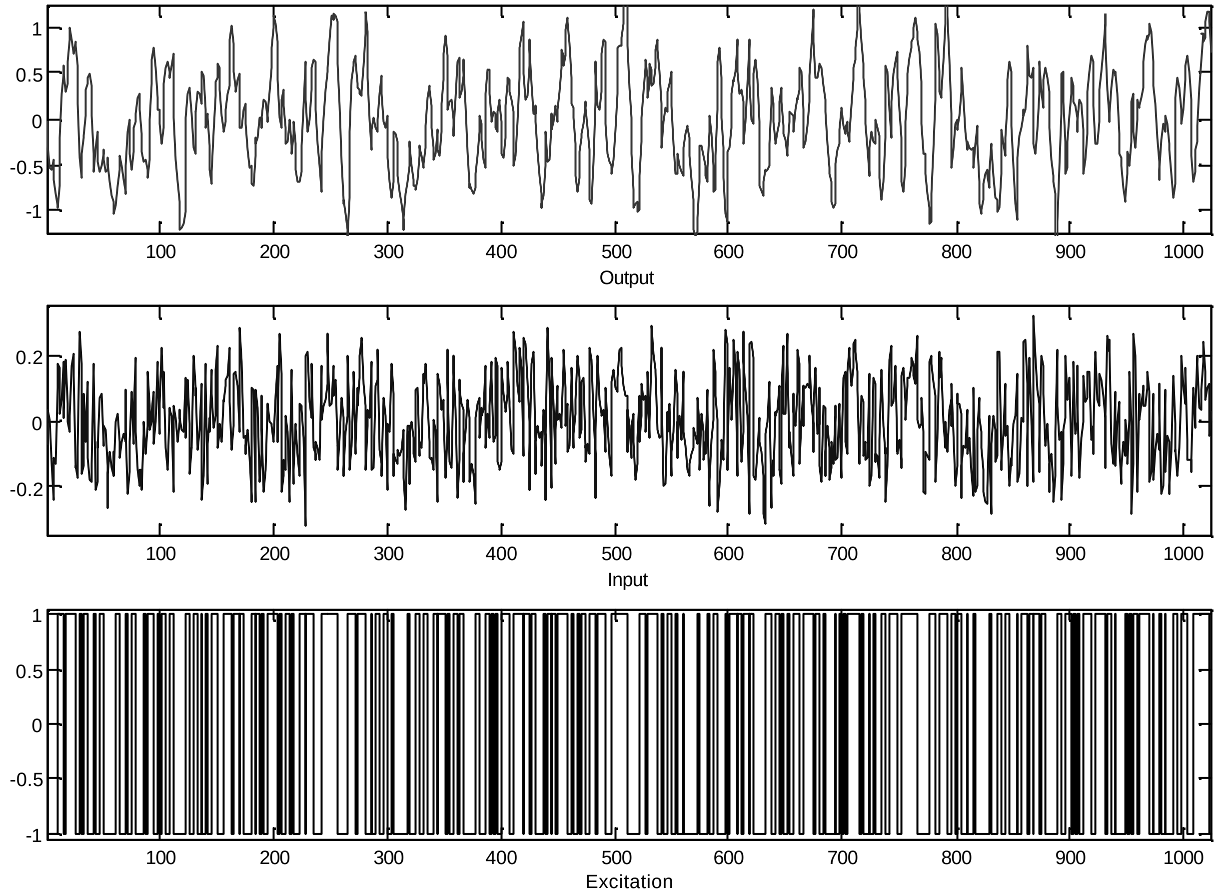
$$T(q^{-1}) = 0.11$$

File SIMUBF

Output (y)

Plant input (u)

External excitation (r)



Excitation superposed to the reference

Identification results

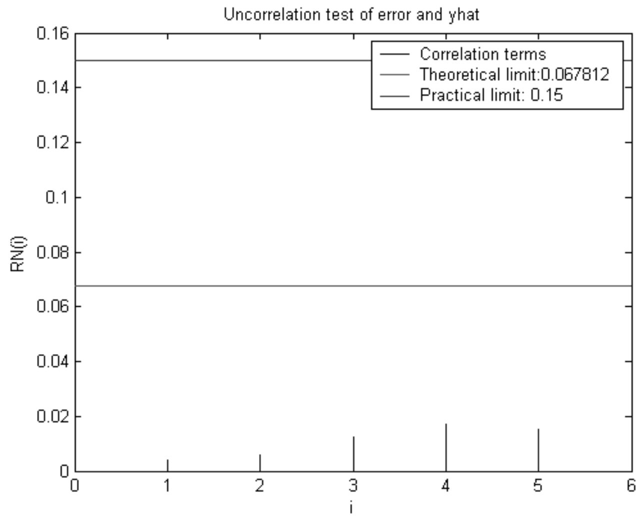
CLOE

```
>> [B,A]=cloe(y,r,na,nb,d,R,S,T,Fin,lam1,lam0)

B =

     0    0.9527    0.4900


A =

  1.0000   -1.4808    0.6716
```

AF-CLOE

```
>> [B,A]=afcloe(y,r,na,nb,d,R,S,T,Fin,lam1,lam0)

B =

     0    0.9684    0.4722


A =

  1.0000   -1.4844    0.6821
```

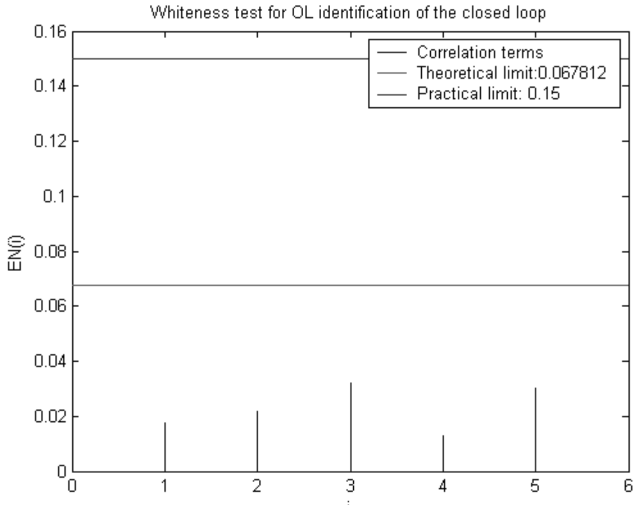
Statistical Validation of the model identified in closed loop



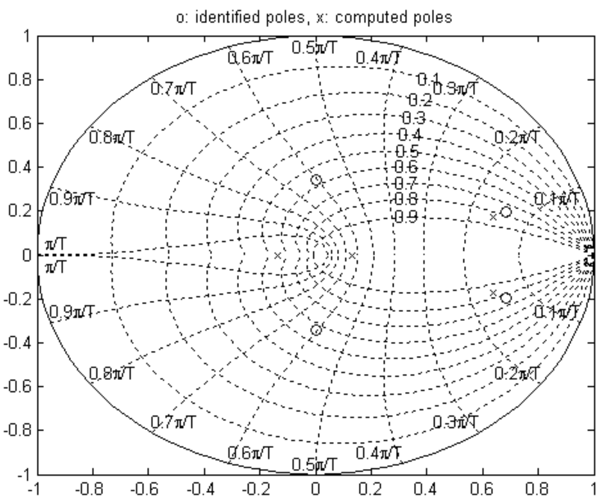
Model of the plant identified in closed loop with AF-CLOE

Is OK. $$\left|RN(i)\right| \leq \frac{2.17}{\sqrt{N}} = \frac{2.17}{\sqrt{1024}} = 0.0678; \quad i \geq 1$$

Poles closeness validation and n-gap validation



Stochactic validation of the identified model of the closed loop
Is OK.
Can be used for poles closeness validation and ν-gap validation



Poles map
o identifed poles of the true CL system
x computed poles of the simulated CL system
Is OK

ν–gap = 0.0105 (min = 0, max =1)

File SIMUBF – comparison of identified models

Closed Loop Statisitical Validation

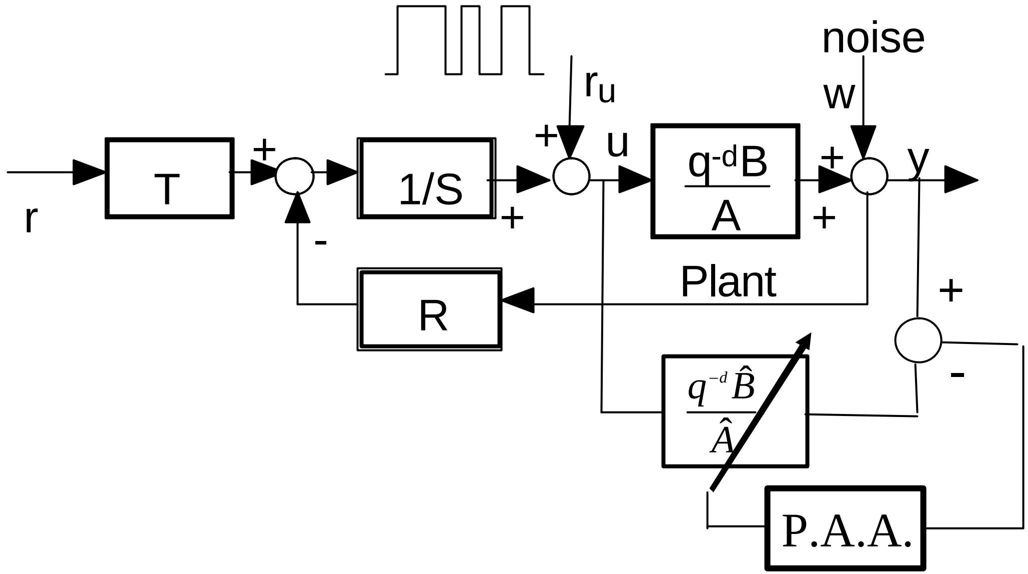
Méthod	a_1	a_2	b_1	b_2	CL Error Varaince R(0)	Normalized Intercorrelations (validation bound 0.068) |RN(max)|.
Nominal Model	-1.5	0.7	1	0.5		
AF-CLOE	-1.4689	0.6699	0.991	0.5276	0.03176	0.0092
CLOE	-1.476	0.6674	0.9592	0.4862	0.03181	0.0284
F-CLOE	-1.4692	0.6704	0.9591	0.5152	0.03175	0.0085
X-CLOE	-1.49	0.6822	0.9668	0.3775	0.0312	0.0237
OL type identification (RLS.)	-1.3991	0.6034	0.975	0.508	0.0323	0.0843

Best results

Open loop type identification
(between u and y ignoring the controller)

Can we use
“open loop identification algorithms “
for identification in closed loop ?

Identification in closed loop – direct approach



-To be used when no any a priori information upon controller and plant model are available

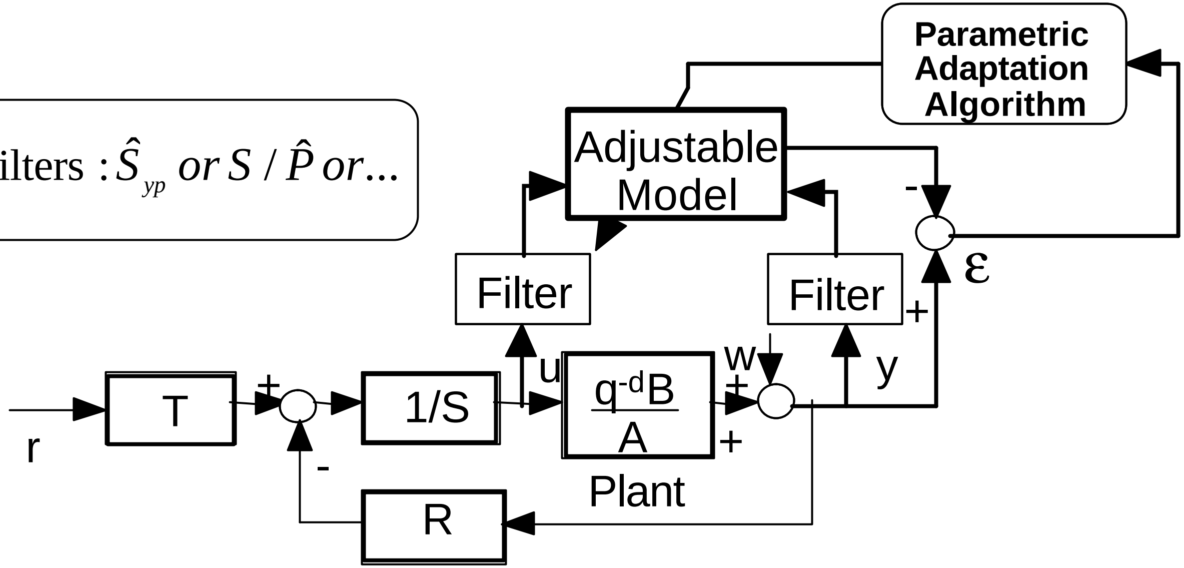
-For theoretical reasons one prefers OL alg. for ARMAX (pred.error)

-Allows to get a first model to be used for controller re-design

- Not good results in general.

Identification in Closed Loop
Filtered Open Loop (FOL)Identification Algorithms

I/O Data Filters : $\hat{S}_{yp}$ $or\ S\ /\ \hat{P}\ or...$



- -Biased estimates (except very particular situations)
- -Require (theoretically) time varying filters
- -FOL alg. can be seen as approximations of CLOE alg.
- -Require an a priori information upon plant and controller
- -Are used in standard indirect adaptive control

For details see : Landau et al « Adaptive Control », Springer, 1997

Identification in Closed Loop
Filtered Open Loop (FOL)Identification Algorithms

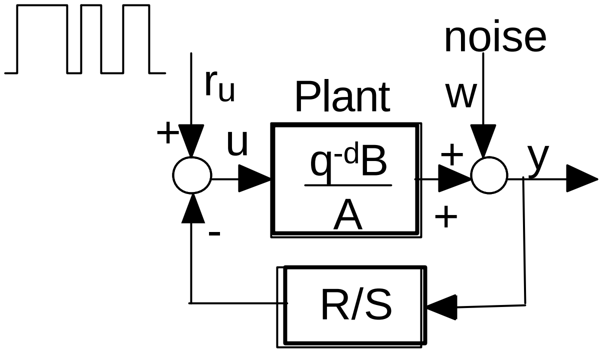
-The basic idea is that one should process data in the frequency range critical for control (near the Nyquist point)

-For OL output error the data should be filtered through $\hat{S}_{YP}$

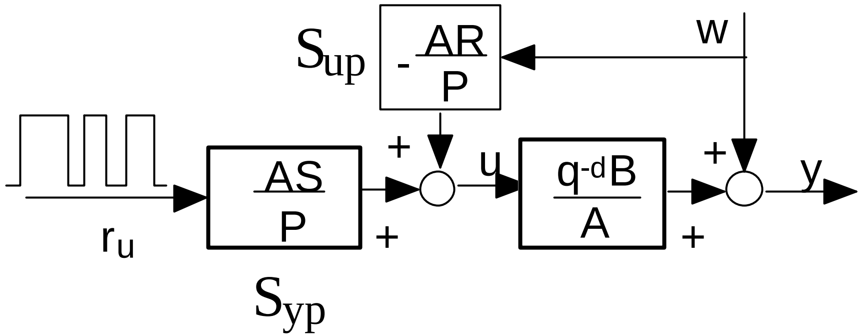
-For OL RLS and alg. based on the whitening of the prediction error, the data should be filtered by $S / \hat{P}$

- One needs a priori information upon the plant and controller or upon the closed loop (the sensitivity function)

Identification in Closed Loop without Controller knowledge
The instrumental variable approach (Hof/Shrama)



Experimental setting



Equivalent *open loop* representation

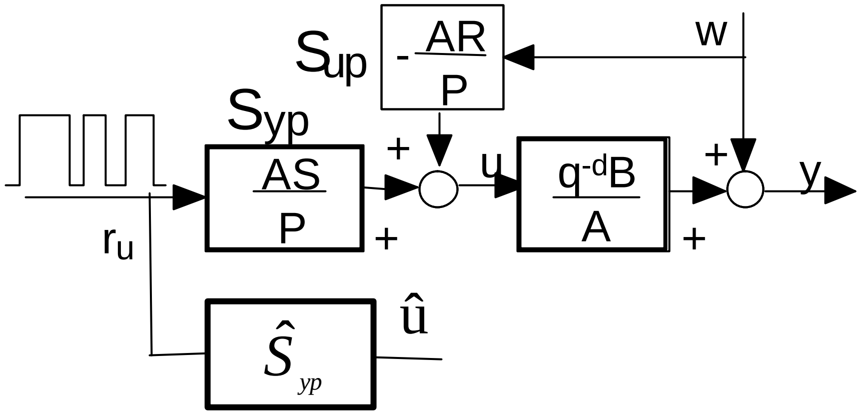
1) Identify the transfer between r_u and *u:* $S_{yp} = AS/P$

2) Create an *instrumental variable û:* $\hat{u} = \hat{S}_{yp} r_u$

3) Identify the plant model between {û} and {y} using OLOE alg.

- *Does not require the knowledge of the controller*
- *Theoretically one can get unbiased estimates*
- *To enhance « control » aspects filters should be used*

Identification in Closed Loop
The instrumental variable approach



Identification of the sensitivity fct. S_{yp} between r_u and u:

1rst approach:

- select an appropriate order (approx : nA + nB +d-1)
- the numerator of S_{yp} starts with q^0

2nd approach:

- use a FIR filter of large dimension

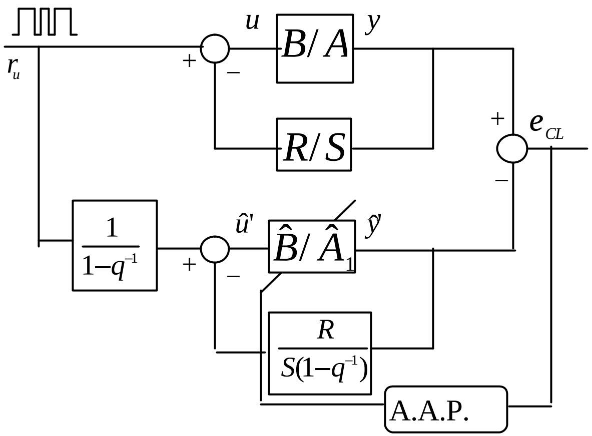
û is not contaminated by the measurement noise

Concluding Remarks

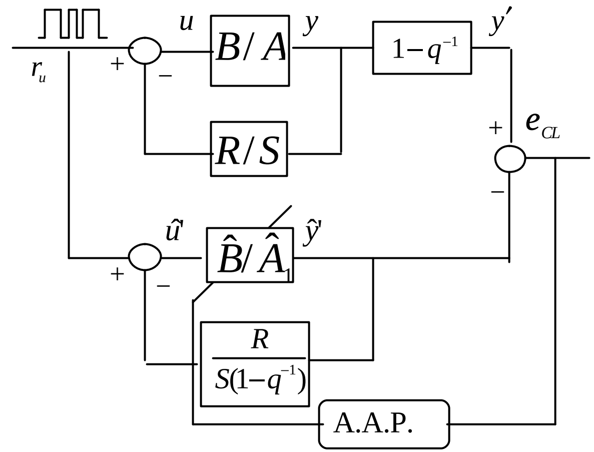
- Methods are available for efficient identification in closed loop
- CLOE algorithms provide unbiased parameter estimates
- CLOE provides “control oriented “reduced order” models (precision enhanced in the critical frequency regions for control)
- The knowledge of the controller is necessary (for CLOE and FOL)
- **In many cases the models identified in closed loop allow to improve the closed loop performance**
- **For controller re-tuning, opening the loop is no more necessary**
- Identification in closed loop can be used for “model reduction”
- By duality arguments one can use the algorithms for controller reduction
- **Successful use in practice**
- A MATLAB Toolbox is available (CLID- see website))
- A stand alone software is available (WinPIM/Adaptech)

Appendix

How to identify in closed loop systems with integrators ?



Replace the input of the closed loop predicor by its integral



Replace the measured output by its variations

Attention :

- the controller in the predictor has to be modified
- one identifies the plant model without integrator

“Personal” references directly related to the course

Books:

I.D. Landau, R. Lozano, M.M'Saad "Adaptive Systems", Springer Verlag, London 1997

I.D.Landau “Commande de Systèmes - conception, identification et mise en oeuvre” Hermès, Paris, Juin 2002 (Chapter 9 – translation available)

I.D.Landau, A. Besançon (Editors) "Identification des Systèmes", Traité des Nouvelles Technologies, Hermès, Paris,2001

Web site:

http://landau-bookic.lag.ensieg.inpg.fr

« Slides » for chapters and tutorial can be downloaded

Free routines (matlab, scilab) can be downloaded (including CLID)

« Personal » References

Papers:

Landau I.D., Karimi A., (1997) : « Recursive algorithms for identification in closed -loop – a unified approach and evaluation », *Automatica*, vol. 33, no. 8, pp. 1499-1523.

Landau I.D., Karimi A., (1997) : « An output error recursive algorithm for unbiased estimation in closed loop » *Automatica*, vol. 33, no. 5, pp. 933-938.

Karimi A., Landau I.D.(1998): "Comparison of the closed-loop identification methods in terms of the bias distribution" *Systems and Control Letters*, 34, 159-167, 1998

Landau I.D., Karimi A. (1999): « A recursive algorithm for ARMAX model identification in closed loop », IEEE Trans. On AC. Vol 44, no 4, pp 840-843

Landau I.D., (2001) : « Identification in closed loop : a powerful design tool (better models, simpler controllers) », *Control Engineering Practice*, vol. 9, no.1, pp.51- 65.

J.Langer,I.D.Landau (1996): "Improvement of robust digital control by identification in the closed loop. Application to a 360° flexible arm" *CEP*, vol 4,no 12, pp. 1637-1646,

I.D.Landau, A. Karimi (1999): "A recursive algorithm for ARMAX model identification in closed loop" IEEE Trans. on Automatic Control 44, 840-843,

I.D. Landau, A.Karimi, (2002) :« A unified approach to closed-loop plant identification and direct controller reduction », *European J. of Control*, vol.8, no.6

Important References

L. Ljung(2002), *System Identification*, Prentice Hall, N.J.2nd ed. 2002

M. Gevers(1993) "Towards a joint design of identification and control" (ECC 93), in (H. Trentelman,J. Willems eds) "*Essays on Control: perspectives in the theory and its application*", Birkhauser, Boston, pp 111-152

M.Gevers (2004) "Identification for control. Achievements and open problems". Proc. 7th IFAC Symp. on Dynamics and control of process systems(DYCOPS 2004), Cambridge, Mass. USA, July *(contains an extensive list of references for the interaction between identification and control)*

P.M. Van den Hof, P. Shrama(1995) "Identification and control – closed-loop issues", *Automatica* 31(12), 1751-1770