Chapter III

Robust Digital Controller Design Methods

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Computer control (discrete-time controllers) Possibilities and advantages

- Large choice of strategies for controller design
- Use of more complex algorithms but with better performance than the PID
- Techniques well suited for the control of:
  - *systems with delay (dead time)*
  - *systems characterized by high order dynamic models*
  - *systems with low damped vibration modes*
- Easy combination of control design and system identification

Digital controllers – Design methods

- Digital PID controller
- Pole placement (tracking and regulation)
- Tracking and regulation with independent objectives
- Internal model control (tracking and regulation)
- Pole placement with sensitivity function shaping

Remarks:

- *All the controllers will have the RST structure (two degrees of freedom controller)*
- *The « memory » (number of parameters) depends upon the complexity of the model used for design*
- *All the design methods can be viewed as particular cases of the pole placement*
- *The design and tuning of the controllers require the knowledge of a discrete time model of the plant*

Digital PID controller

• It results from the discretization of an analog PID controller

• The computation can be rigorously applied only to:
  - plants described by a model whose order is $n \leq 2$
  - plants with a time delay smaller than T_s

• The algorithm for the parameter calculation is a particular case of the *pole placement*

Digital PID controller 1

Analog PID :

$$H_{PID}(s) = K\left[1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s}\right]$$

K – proportional gain,
T_i – integral action
T_d – derivative action
T_d/N - filtering on the derivative action

Discretization:

$$s \rightarrow (1 - q^{-1}) / T_s \ ; \quad \frac{1}{s} \rightarrow \frac{1}{1 - q^{-1}} T_s$$

Digital PID controller 1:

$$H_{PID1}(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} = K\left[1 + \frac{T_s}{T_i} \cdot \frac{1}{1 - q^{-1}} + \frac{\frac{NT_s}{T_d + NT_s}(1 - q^{-1})}{1 - \frac{T_d}{T_d + NT_s} q^{-1}}\right]$$

Digital PID controller 1

$$H_{PID1}(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + r_2 q^{-2} \qquad S(q^{-1}) = (1 - q^{-1})(1 + s'_1\, q^{-1}) = 1 + s_1 q^{-1} + s_2 q^{-2}$$

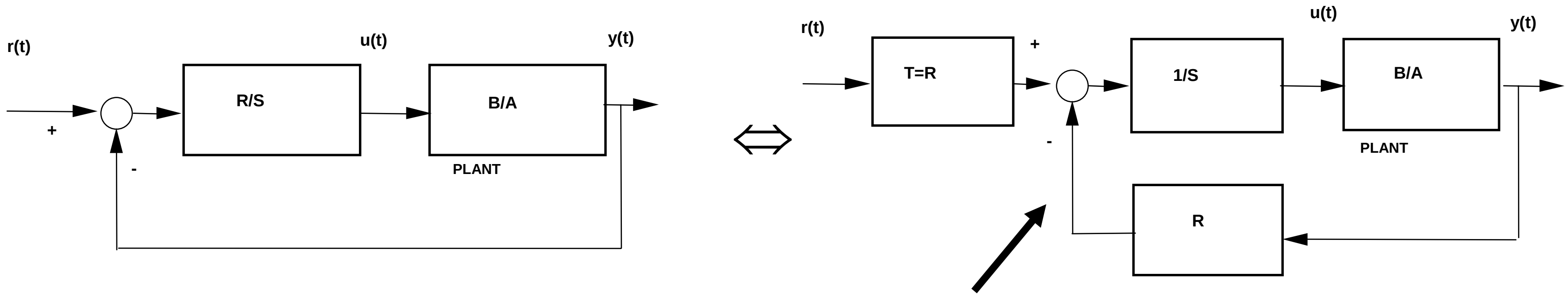
$$r_0 = K\left(1 + \frac{T_s}{T_i} - N s_1\right) \qquad r_1 = K\left[s_1\left(1 + \frac{T_s}{T_i} + 2N\right) - 1\right] \qquad r_2 = -K s_1 (1 + N)$$

$$s'_1 = -\frac{T_d}{T_d + N T_s}$$

Remark:

- The digital PID controller has 4 parameters (as the analog PID)
- Common factor in the denominator: $(1 - q^{-1})$ (integrator)
- filtering action: factor $(1 + s'_1\, q^{-1})$ in the denominator

Digital PID controller 1



Structure RST with T = R

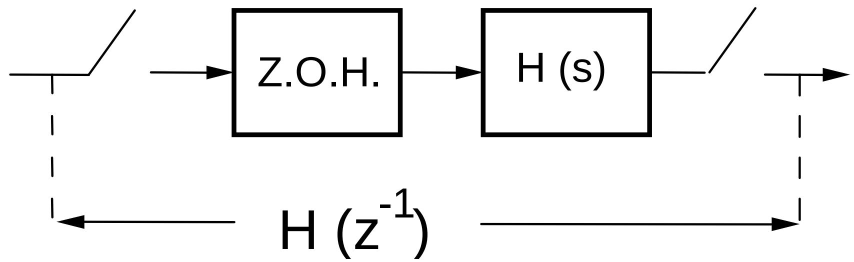
T.F. of the closed loop (r $\rightarrow$ y)

$$H_{CL}(q^{-1}) = \frac{B(q^{-1})R(q^{-1})}{A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})} = \frac{B(q^{-1})R(q^{-1})}{P(q^{-1})}$$

$P(q^{-1})$ defines the closed loop poles

The controller introduces supplementary zeros (R)

Discrete-time model of the plant



$$H(s) = \frac{Ge^{-s\tau}}{1+sT} \quad \text{or} \quad H(s) = \frac{\omega_0^2 e^{-s\tau}}{\omega_0^2 + 2\zeta\omega_0 s + s^2} \qquad (\tau < T_s)$$

$$\boxed{H(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}}}$$

The discrete-time model is obtained:

- directly by system identification (general case)
- by discretization of the continuous-time model

Parameter computation of the digital PID controller 1

Performances specifications :

$$H_{CL}(q^{-1}) = \frac{B(q^{-1})R(q^{-1})}{A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})} = \frac{B_M(q^{-1})}{P(q^{-1})} \qquad (*)$$

$B_M(q^{-1})$ cannot be imposed (as B is kept and
The controller introduces supplementary zeros)
The characteristic polynomial (*P*) of the closed loop is specified :

$$P(q^{-1}) == 1 + p'_1 q^{-1} + p'_2 q^{-2}$$

Continuous-time specification (t_M, M) $\longrightarrow$ 2nd order (ω_0, ζ) $\xrightarrow[T_s]{\text{discretization}}$ $P(q^{-1})$

$$0.25 \le \omega_0 T_s \le 1.5$$
$$0.7 \le \zeta \le 1$$

Parameter computation of the digital PID controller 1

- Known (or identified) plant model : $B(q^{-1})/A(q^{-1})$
- Desired performances (CL poles): $P(q^{-1})$

To be computed : $R(q^{-1})\,;\,S(q^{-1})$

From (*) – slide 11, one solves:

$$\boxed{P(q^{-1}) = A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})}$$

? ?

$$P(q^{-1}) = 1 + p_1' q^{-1} + p_2' q^{-2} = A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})$$

$$= (1 + a_1 q^{-1} + a_2 q^{-2})(1 - q^{-1})(1 + s_1' q^{-1})$$

$$+ (b_1 q^{-1} + b_2 q^{-2})(r_0 + r_1 q^{-1} + r_2 q^{-2})$$

$$= A'(q^{-1})S'(q^{-1}) + B(q^{-1})R(q^{-1})$$

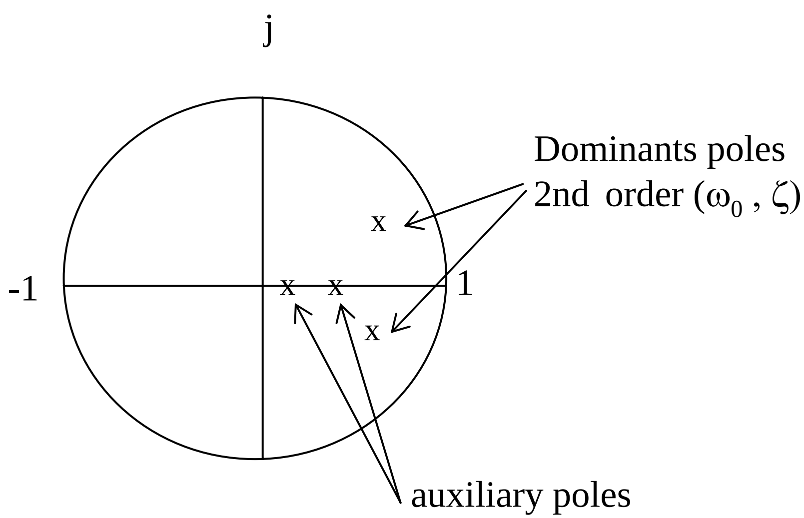
$$A'(q^{-1}) = A(q^{-1})(1 - q^{-1}) = (1 + a_1' q^{-1} + a_2' q^{-2} + a_3' q^{-3}) \qquad S'(q^{-1}) = 1 + s_1' q^{-1}$$

Tools : WinREG, *bezoutd.sci(.m)*

Choice of the polynomial *P*

- Fourth order Polynomial Equation.
- P can also be chosen as a fourth order poly. (by adding aux. poles)

$$P(q^{-1}) = (1 + p_1 q^{-1} + p_2 q^{-2} + p_3 q^{-3} + p_4 q^{-4}) =$$
$$= (1 + p_1' q^{-1} + p_2' q^{-2})(1 + \alpha_1 q^{-1})(1 + \alpha_2 q^{-2})$$



$$-0.5 \le \alpha_1, \alpha_2 \le -0.05$$

The auxiliary poles improves the closed loop robustness

Equivalent analog PID controller parameters

$$K = \frac{r_0 s'_1 - r_1 - (2 + s'_1) r_2}{(1 + s'_1)^2}$$

$$T_i = T_s \cdot \frac{K(1 + s'_1)}{r_0 + r_1 + r_2}$$

$$T_d = T_s \cdot \frac{s'_1 r_0 - s'_1 r_1 + r_2}{K(1 + s'_1)^3}$$

$$\frac{T_d}{N} = \frac{-s'_1 T_s}{1 + s'_1}$$

The continuous equivalent does not always exist!

Existence condition: $-1 \le s'_1 \le 0$ ($T_d/N > 0$)

Digital PID controller always can be implemented even if : $0 \le s'_1 \le 1$
(no equivalent achievable performance with an analog PI)

Digital PID controller 1. Examples

Plant: $$H(s) = \frac{Ge^{-s\tau}}{1+sT}$$

Discretized plant: $B(q\text{-}1) = 0.1813\ q\text{-}1\ \ +0.2122\ q\text{-}2$

$A(q\text{-}1) = 1\ \text{-}0.6065\ q\text{-}1$

$\boldsymbol{Ts = 5s,\ G = 1,\ \ T = 10s,\ \tau = 3}$

Performances → $\boldsymbol{Ts = 5s}$, $\boxed{\boldsymbol{\omega_0 = 0.05\ rad/s, \quad \zeta = 0.8}}$

***** CONTROL LAW *****

$S(q\text{-}1)\ .\ u(t) + R(q\text{-}1)\ .\ y(t) = T(q\text{-}1)\ .\ r(t)$

Controller : $R(q\text{-}1) = 0.0621 + 0.0681\ q\text{-}1$

$S(q\text{-}1) = (1 - q\text{-}1)\ .\ (1 - 0.0238\ q\text{-}1)$

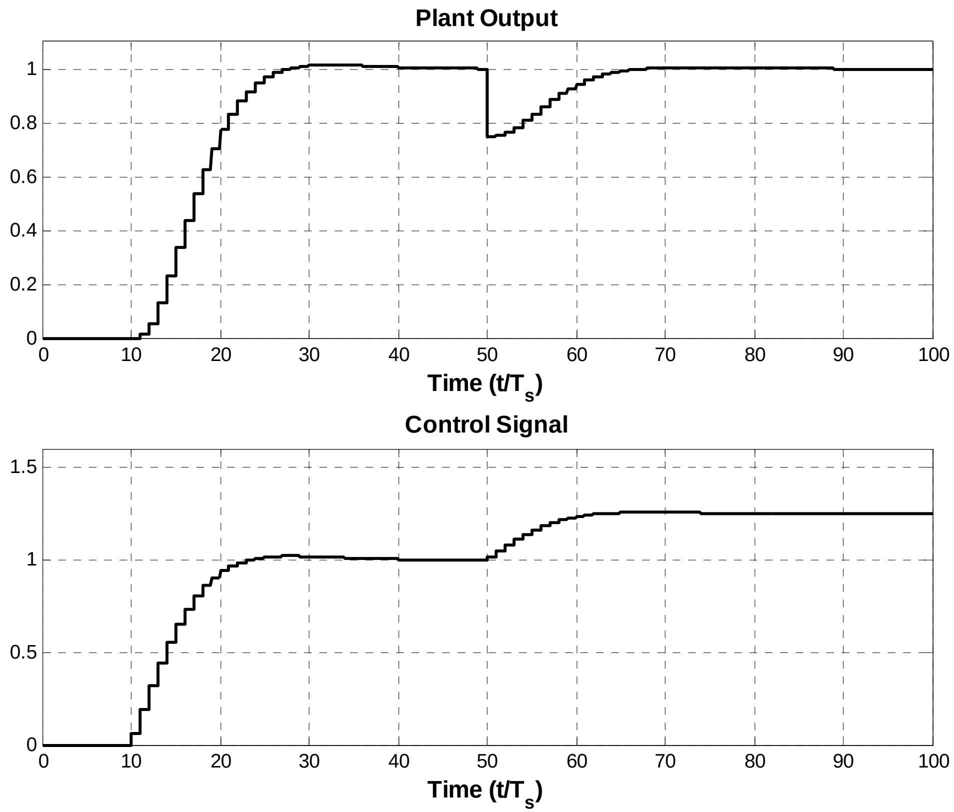
$T(q\text{-}1) = R(q\text{-}1)$

Gain Margin : 7.712 Phase Margin: 67.2 deg

Modulus Margin : 0.751 (-2.49dB) Delay Margin : 45.4 s

Analog PID : $k = -0.073$, $Ti = -2.735$, $Td = -\ 0.122$, $Td/N = 0.122$

Performances: ω_0 = 0.05 rad/s, ζ = 0.8



Closed Loop response slower than Open Loop response.
The specified ω_0 should be increased

Performances: ω_0 **= 0.15 rad/s,** ζ **= 0.8**

Discretized plant : $B(q^{-1}) = 0.1813\, q^{-1} + 0.2122\, q^{-2}$

$A(q^{-1}) = 1 - 0.6065\, q^{-2}$

$Ts = 5s$, $G = 1$, $T = 10s$, $\tau = 3$

Performances → **$Ts = 5s$, $\omega_0 = \boxed{0.15 \text{ rad/s}, \quad \zeta = 0.8}$**

***** CONTROL LAW *****

$S(q^{-1}) \,.\, u(t) + R(q^{-1}) \,.\, y(t) = T(q^{-1}) \,.\, r(t)$

Controller : $R(q^{-1}) = 1.6874 - 0.8924\, q^{-1}$

$S(q^{-1}) = (1 - q^{-1}) \,.\, (1 + 0.3122\, q^{-1})$

$T(q^{-1}) = R(q^{-1})$

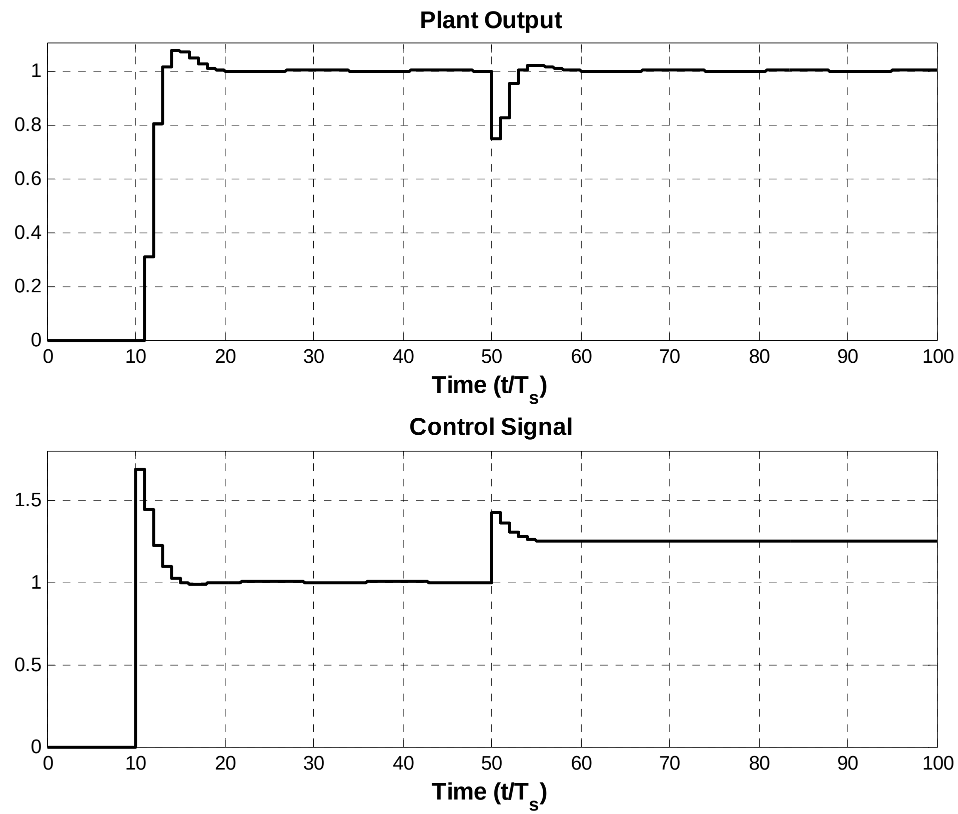
Gain Margin : 3.681 Phase Margin : 58.4 deg

Modulus Margin : 0.664 (- 3.56 dB) Delay Margin : 9.4 s

Analog PID : (no equivalent analog PID)

No equivalent analog PID as $s'_1 > 0$ (0.3122)

Performances: ω_0 = 0.15 rad/s, ζ = 0.8



- Faster response

- An overshoot appears because of the zeros introduced by R

The « good » digital PID controller (PID 2)

No supplementary zero is introduced

Desired T.F. for the closed loop: $H_{CL}(q^{-1}) = \frac{P(1)}{B(1)} \cdot \frac{B(q^{-1})}{P(q^{-1})} \Rightarrow H_{CL}(1) = 1$

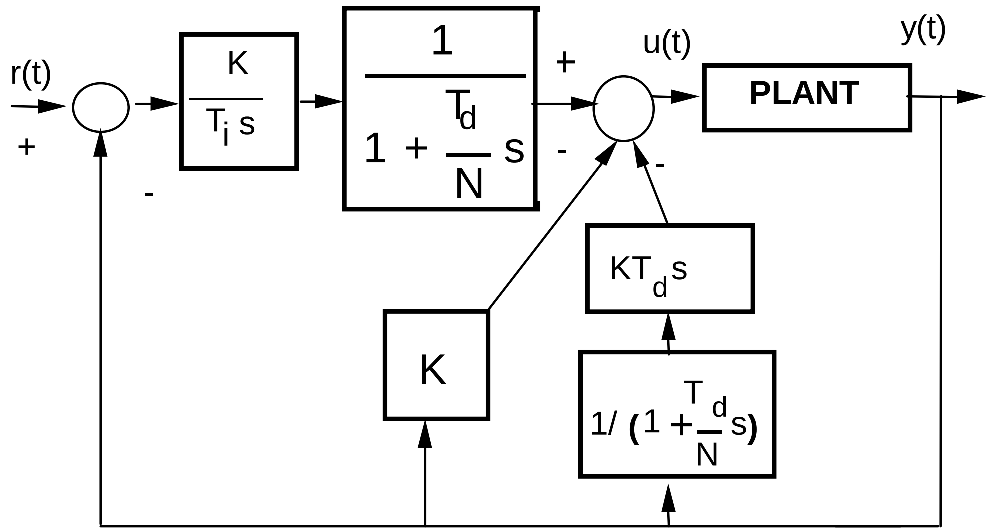
$$H_{CL}(q^{-1}) = \frac{T(q^{-1})B(q^{-1})}{A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})} = \frac{[P(1)/B(1)]B(q^{-1})}{P(q^{-1})}$$

$$\boxed{T(q^{-1}) = \frac{P(1)}{B(1)} = \frac{B(1)R(1)}{B(1)} = R(1)}$$

R and S are unchanged

Only one coefficient instead of two coeffs.

Continuous time PID corresponding to digital PID 2



The proportional and derivative actions only act on the measure

$$K = \frac{-(r_1 + 2r_2)}{1 + s_1'} \qquad T_i = T_s \cdot \frac{-(r_1 + 2r_2)}{r_0 + r_1 + r_2} \qquad T_d = T_s \cdot \frac{s_1' r_1 + (s_1' - 1) r_2}{(r_1 + 2r_2)(1 + s_1')} \qquad \frac{T_d}{N} = \frac{-s_1' T_s}{1 + s_1'}$$

Performances of the digital PID controller 2

Discretized plant : $B(q\text{-}1) = 0.1813\ q\text{-}1 + 0.2122\ q\text{-}2$

$A(q\text{-}1) = 1 - 0.6065\ q\text{-}1$

Ts = 5s, G = 1, T = 10s, $\boldsymbol{\tau = 3}$

Performances → ***Ts = 5s*** **,** $\omega_0 = \boxed{\mathbf{0.15\ rad/s, \quad \zeta = 0.8}}$

***** CONTROL LAW *****

$S(q\text{-}1)\ u(t) + R(q\text{-}1)\ y(t) = T(q\text{-}1)\ r(t)$

Controller : $R(q\text{-}1) = 1.6874 - 0.8924\ q\text{-}1$

$S(q\text{-}1) = (1 - q\text{-}1)\ (1 + 0.3122\ q\text{-}1)$

$T(q\text{-}1) = 0.795$

Gain Margin : 3.681 Phase Margin : 58.4 deg

Modulus Margin : 0.664 (- 3.56 dB) Delay Margin : 9.4 s

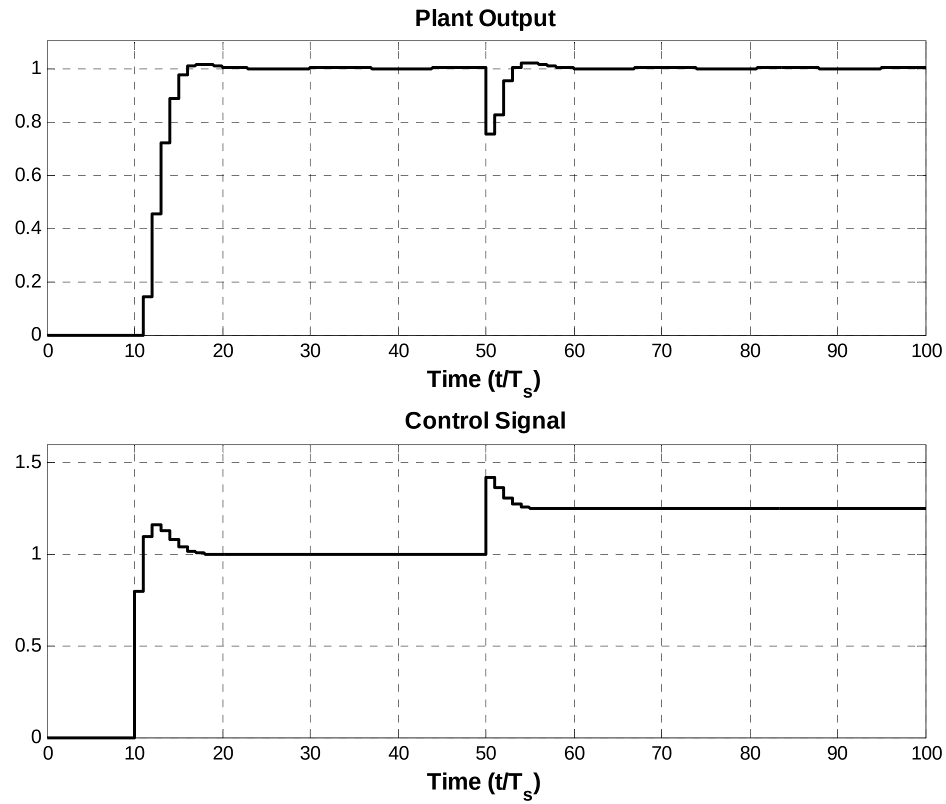
Analog PID : (no equivalent analog PID)

No equivalent analog PI as $s'_1 > 0\ (0.3122)$

To be compared with PID 1, slide 17

Performances of the digital PID controller 2

$\omega_0 = 0.15$ ***rad/s,*** $\quad \zeta = 0.8$



Reduced overshoot (corresponding to $\zeta = 0.8$).

Same response for disturbance rejection

To be compared with slide 18

Auxiliary poles effects

The auxiliary poles reduce the input sensitivity function S_{up} at high frequencies without degrading the closed loop performances

$$\Downarrow$$

Better robustness and reduction of actuator stress

Digital PID controller : conclusions

- RST Canonical structure
- Equivalent analog PID if $-1 \leq s'_1 \leq 0$
- Used with 1st or 2nd order systems with delay < T_s
- For a delay $\tau \geq 0.25T$ the analog PID leads to closed loop responses slower than open loop responses
- The digital PID controller gives better performances for systems with delay (but there is no equivalent in continuous-time)
- The digital PID controller 2 leads to a step response with a smaller overshoot than PID 1

Pole placement

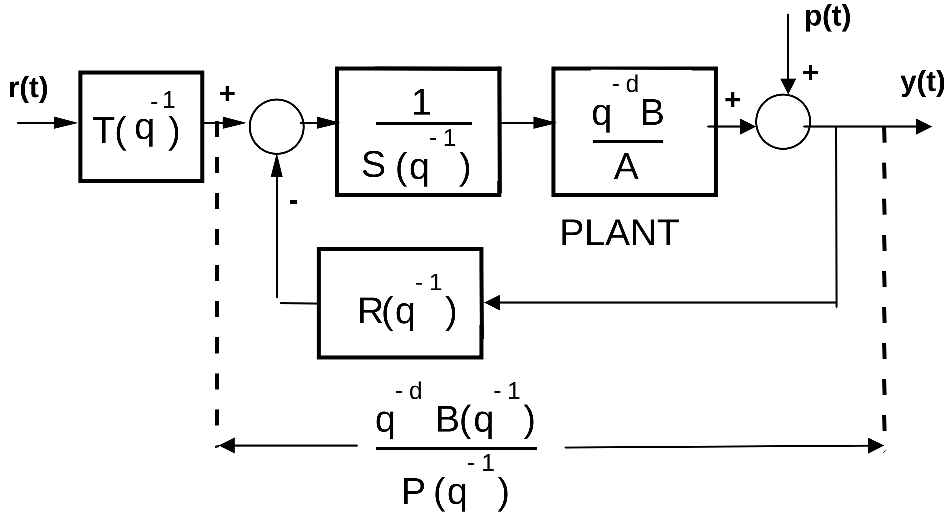
The pole placement allows to design a R-S-T controller for

- stable or unstable systems
- without restriction upon the degrees of A and B polynomials
- without restrictions upon the plant model zeros (stable or unstable)

It is a method that does not simplify the plant model zeros

The digital PID can be designed using pole placement

Structure



Plant: $$H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} \qquad B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n_B} q^{-n_B} = q^{-1}B^*(q^{-1})$$

Pole placement

Closed loop T.F. (r → y) *(reference tracking)*

$$H_{BF}(q^{-1}) = \frac{q^{-d}T(q^{-1})B(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{q^{-d}T(q^{-1})B(q^{-1})}{P(q^{-1})}$$

$$P(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = 1 + p_1 q^{-1} + p_2 q^{-2} + \ldots.$$

Defines the (desired)closed loop poles

Closed loop T.F. (p → y) *(disturbance rejection)*

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})}$$

Output sensitivity function

Choice of desired closed loop poles (polynomial *P*)

$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Dominant poles **Auxiliary poles**

Choice of $P_D(q^{-1})$(dominant poles)

Specification in continuous time (t_M, M) $\longrightarrow$ 2^{nd} order (ω_0, ζ) $\xrightarrow[T_e]{\text{discretization}}$ $P_D(q^{-1})$

$$0.25 \le \omega_0 T_e \le 1.5$$
$$0.7 \le \zeta \le 1$$

Auxiliary poles

- *Auxiliary poles are introduced for robustness purposes*
- *They usually are selected to be faster than the dominant poles*

Regulation(computation of *R(q⁻¹)* and *S(q⁻¹)*)

(Bezout) $A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = P(q^{-1})$ **(*)**

? **?**

A and B do not have common factors

$n_A = \deg A(q^{-1}) \quad n_B = \deg B(q^{-1})$

unique minimal solution for :

$n_P = \deg P(q^{-1}) \le n_A + n_B + d - 1$

$n_S = \deg S(q^{-1}) = n_B + d - 1 \qquad n_R = \deg R(q^{-1}) = n_A - 1$

$$S(q^{-1}) = 1 + s_1 q^{-1} + \ldots s_{n_S} q^{-n_S} = 1 + q^{-1} S^*(q^{-1})$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots r_{n_R} q^{-n_R}$$

Computation of *R(q-1)* and *S(q-1)*

Equation (*) is written as: $\boxed{Mx = p} \longrightarrow x = M^{-1}p$

$$x^T = [1, s_1, \ldots, s_{n_S}, r_0, \ldots, r_{n_R}] \qquad p^T = [1, p_1, \ldots, p_i, \ldots, p_{n_P}, 0, \ldots, 0]$$

$$\underbrace{\begin{array}{cccc|cccc}
\multicolumn{4}{c|}{\overbrace{\hspace{5cm}}^{n_B + d}} & \multicolumn{4}{c}{\overbrace{\hspace{5cm}}^{n_A}} \\
\end{array}}_{}$$

$$\underbrace{\left[\begin{array}{cccccccc}
1 & 0 & \ldots & 0 & 0 & \ldots & \ldots & 0 \\
a_1 & 1 & & . & b'_1 & & & \\
a_2 & & & 0 & b'_2 & & & b'_1 \\
 & & & 1 & . & & & b'_2 \\
 & & & a_1 & . & & & . \\
a_{n_A} & & & a_2 & b'_{n_B} & & & . \\
0 & & & . & 0 & . & . & . \\
0 & \ldots & 0 & a_{n_A} & 0 & 0 & 0 & b'_{n_B}
\end{array}\right]}_{n_A + n_B + d}\left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\\1\\1\end{array}}\right\} n_A + n_B + d$$

b'$_i = 0$ pour $i = 0, 1 \ldots d$; *b'*$_i = b_{i-d}$ pour $i > d$

Use of WinReg or *bezoutd.sci(.m)* for solving (*)

Structure of *R(q^-1^)* and *S(q^-1^)*

R and S include pre-specified fixed parts (ex: integrator)

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

H_R, H_S, - pre-specified polynomials

$$R'(q^{-1}) = r'_0 + r'_1 q^{-1} + \ldots r'_{n_{R'}} q^{-n_{R'}} \qquad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_{n_{S'}} q^{-n_{S'}}$$

- The pre specified filters H_R and H_S will allow to impose certain properties of the closed loop.
- They can influence performance and/or robustness

Fixed parts (H_R, H_S). Examples

Zero steady state error (S_{yp} should be null at certain frequencies)

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})}{P(q^{-1})}$$

Step disturbance : $H_S(q^{-1}) = 1 - q^{-1}$

Sinusoidal disturbance : $H_S = 1 + \alpha q^{-1} + q^{-2}$; $\alpha = -2\cos\omega T_s$

Signal blocking (S_{up} should be null at certain frequencies)

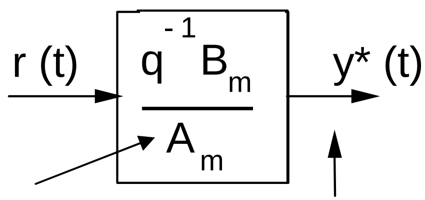
$$S_{up}(q^{-1}) = -\frac{A(q^{-1})H_R(q^{-1})R'(q^{-1})}{P(q^{-1})}$$

Sinusoidal signal: $H_R = 1 + \beta q^{-1} + q^{-2}$; $\beta = -2\cos\omega T_s$

Blocking at 0.5$f_{S:}$ $H_R = (1 + q^{-1})^n$; $n = 1,2$

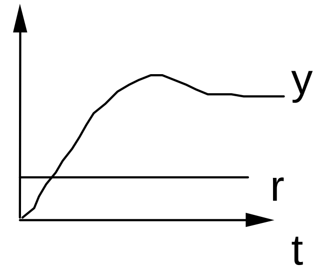
Tracking (computation of $T(q^{-1})$)

Ideal case



Tracking reference model (H_m)

desired trajectory for y (t)



$$H_m(q^{-1}) = \frac{q^{-1}B_m(q^{-1})}{A_m(q^{-1})}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots$$

$$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots$$

Specification in continuous time (t_M, M) $\longrightarrow$ 2nd order (ω_0, ζ) $\xrightarrow[T_s]{\text{discretization}}$ $H_m(q^{-1})$

$$0.25 \le \omega_0 T_s \le 1.5$$

$$0.7 \le \zeta \le 1$$

The ideal case can not be obtained (delay, plant zeros)

Objective : to approach y(t)*

$$y^*(t) = \frac{q^{-(d+1)}B_m(q^{-1})}{A_m(q^{-1})}r(t)$$

Tracking (computation of $T(q^{-1})$)

Build: $$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Choice of $T(q^{-1})$:

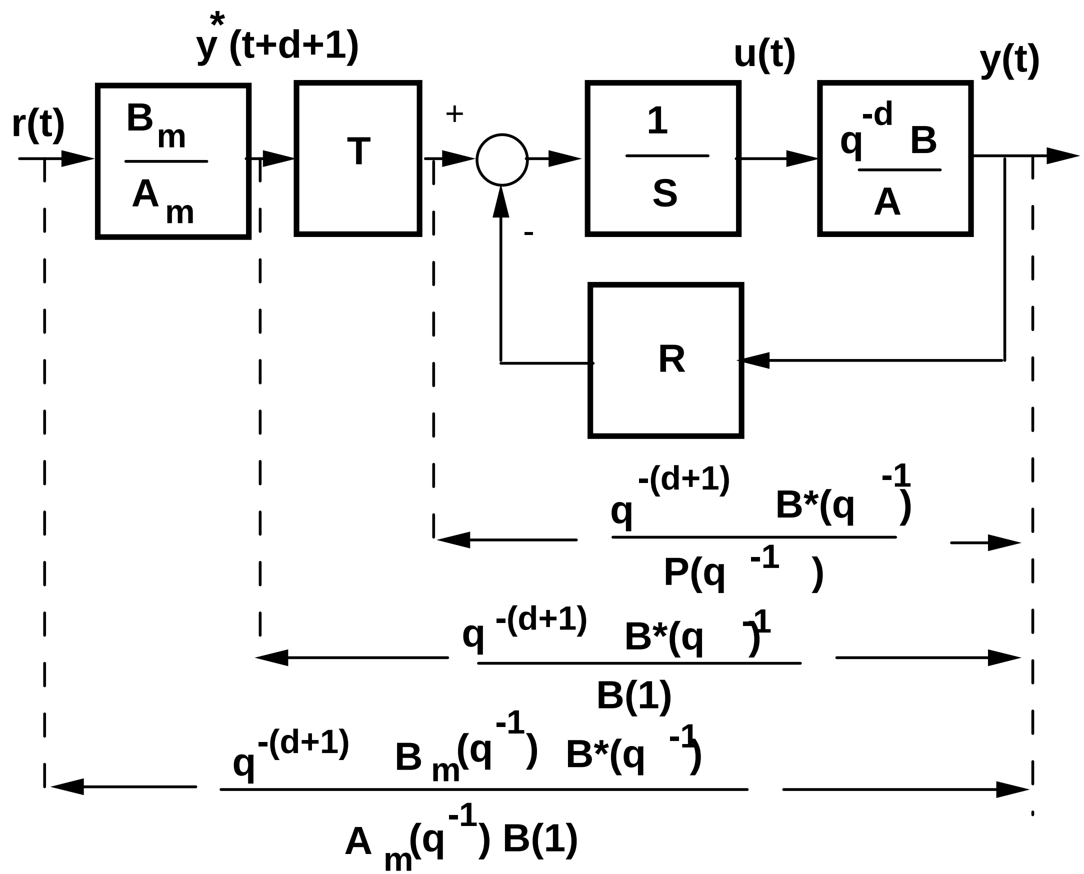
- Imposing unit static gain between y^* and y
- Compensation of regulation dynamics $P(q^{-1})$

$$T(q^{-1}) = GP(q^{-1}) \qquad G = \begin{cases} 1/B(1) & si \quad B(1) \neq 0 \\ 1 \quad si & B(1) = 0 \end{cases}$$

F.T. $r \longrightarrow y$: $$H_{BF}(q^{-1}) = \frac{q^{-(d+1)} B_m(q^{-1})}{A_m(q^{-1})} \cdot \frac{B^*(q^{-1})}{B(1)}$$

$$Particular\ case : P = A_m \qquad T(q^{-1}) = G = \begin{cases} \dfrac{P(1)}{B(1)} & si \quad B(1) \neq 0 \\ 1 \quad si & B(1) = 0 \end{cases}$$

Pole placement. Tracking and regulation



$$S(q^{-1})u(t) + R(q^{-1})y(t) = T(q^{-1})y^*(t+d+1)$$

Pole placement. Control law

$$u(t) = \frac{T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$S(q^{-1})u(t) + R(q^{-1})y(t) = GP(q^{-1})y^*(t+d+1) = T(q^{-1})y^*(t+d+1)$$

$$S(q^{-1}) = 1 + q^{-1}S^*(q^{-1})$$

$$\boxed{u(t) = P(q^{-1})Gy^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t)}$$

$$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})}r(t)$$

$$A_m(q^{-1}) = 1 + q^{-1}A_m^*(q^{-1})$$

$$\boxed{y^*(t+d+1) = -A_m^*(q^{-1})y(t+d) + B_m(q^{-1})r(t)}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots \qquad A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots$$

Pole placement. Example

Plant : $d=0$

$B(q\text{-}1) = 0.1\ q\text{-}1 + 0.2\ q\text{-}2$

$A(q\text{-}1) = 1 - 1.3\ q\text{-}1 + 0.42\ q\text{-}2$

$Bm(q\text{-}1) = 0.0927 + 0.0687\ q\text{-}1$

Tracking dynamics →

$Am\ (q\text{-}1) = 1 - 1.2451q\text{-}1 + 0.4066\ q\text{-}2$

$\boldsymbol{Ts = 1s\ ,\ \omega_0 = 0.5\ rad/s,\ \zeta = 0.9}$

Regulation dynamics → $P\ (q\text{-}1) = 1 - 1.3741\ q\text{-}1 + 0.4867\ q\text{-}2$

$\boldsymbol{Ts = 1s\ ,\ \omega_0 = 0.4\ rad/s,\ \zeta = 0.9}$

Pre-specifications : Integrator

***** CONTROL LAW *****

$S\ (q\text{-}1)\ u(t) + R\ (q\text{-}1)\ y(t) = T\ (q\text{-}1)\ y^*(t+d+1)$

$y^*(t+d+1) = [Bm(q\text{-}1)/Am(q\text{-}1)]\ r(t)$

Controller : $R(q\text{-}1) = 3 - 3.94\ q\text{-}1 + 1.3141\ q\text{-}2$

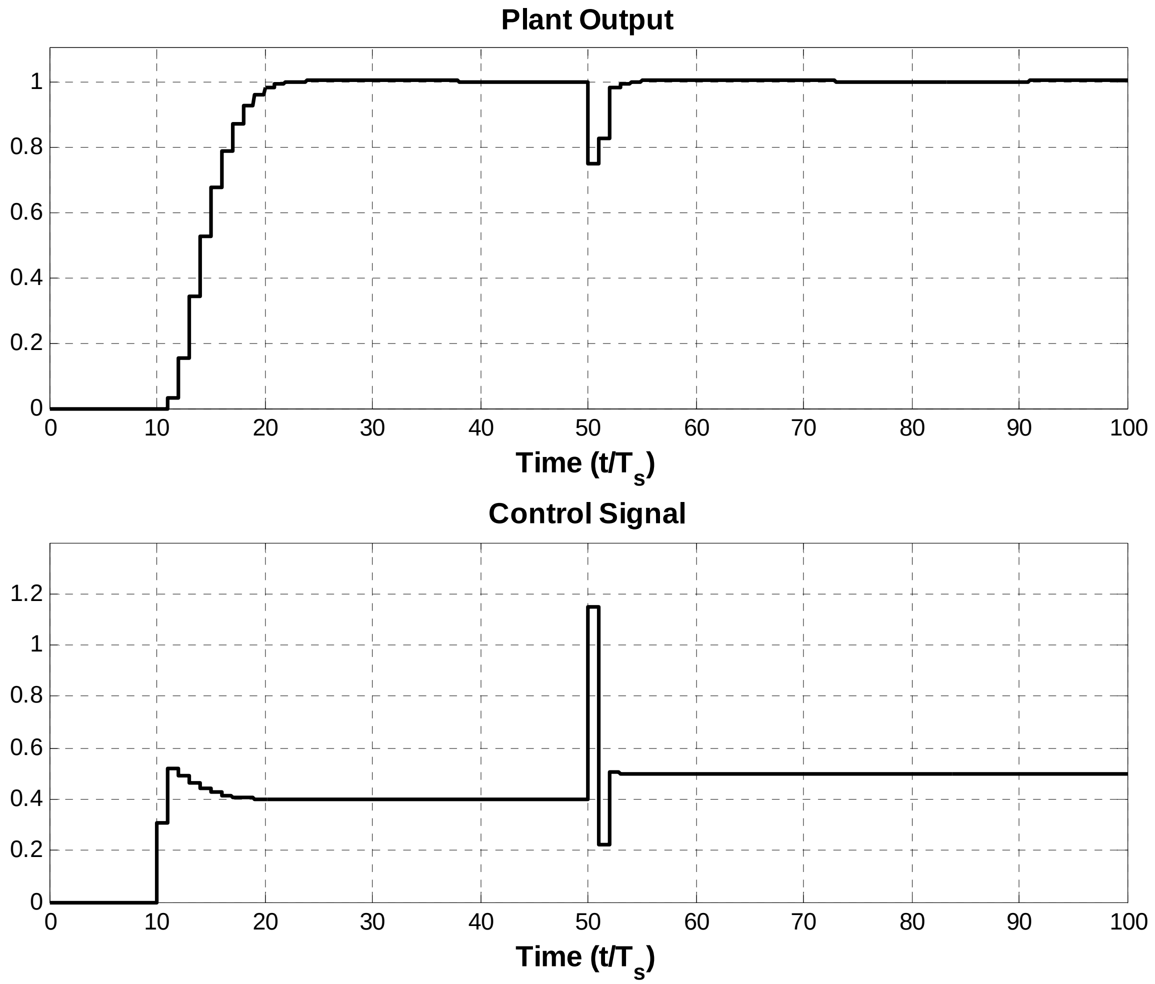
$S(q\text{-}1) = 1 - 0.3742\ q\text{-}1 - 0.6258\ q\text{-}2$

$T(q\text{-}1) = 3.333 - 4.5806\ q\text{-}1 + 1.6225\ q\text{-}2$

Gain margin : 2.703 Phase margin : 65.4 deg

Modulus margin : 0.618 (- 4.19 dB) Delay margin: 2.1. s

Pole placement. Example



Tracking and regulation with independent objectives

It is a particular case of pole placement
(the closed loop poles contain the plant zeros))

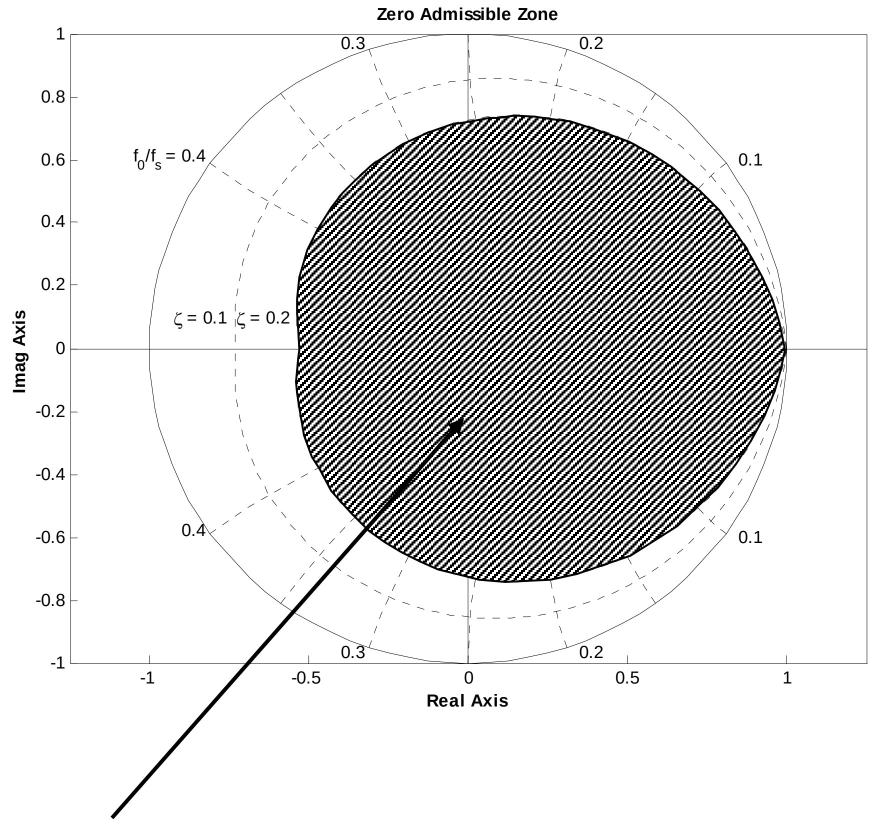
It is a method which simplifies the plant zeros
Allows exact achievement of imposed performances

Allows to design a RST controller for:
- stable or unstable systems
- without restrictions upon the degrees of the polynomials A et B
- without restriction upon the integer delay d of the plant model
- discrete-time plant models with *stable zeros!*

Does not tolerate fractional delay > 0.5 T_S *(unstable zero)*

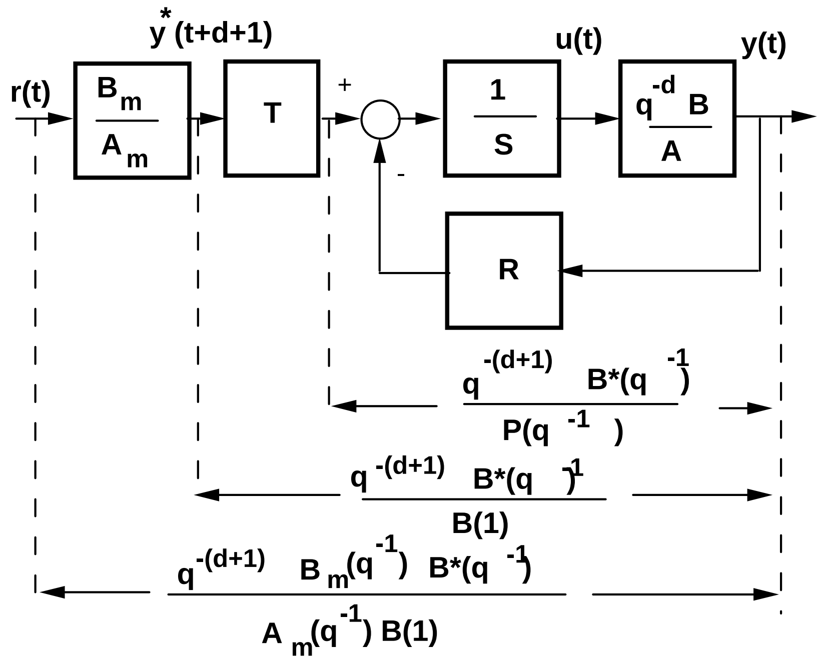
Tracking and regulation with independent objectives

The model zeros should be stable and enough damped



Admissibility domain for the zeros of the discrete time model

Tracking and regulation with independent objectives



$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Reference signal:
(tracking)

$$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Regulation (computation of *R(q^-1^)* and *S(q^-1^)*)

T.F. of the closed loop without *T:*

$$H_{CL}(q^{-1}) = \frac{q^{-d+1}B^*(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d+1}B^*(q^{-1})R(q^{-1})} = \frac{q^{-d+1}}{P(q^{-1})} = \frac{q^{-d+1}B^*(q^{-1})}{B^*(q^{-1})P(q^{-1})}$$

The following equation has to be solved :

$$A(q^{-1})S(q^{-1}) + q^{-d+1}B^*(q^{-1})R(q^{-1}) = B^*(q^{-1})P(q^{-1}) \quad (*)$$

S should be in the form: $S(q^{-1}) = s_0 + s_1 q^{-1} + \ldots + s_{n_S} q^{-n_S} = B^*(q^{-1})S'(q^{-1})$

After simplification by *B**,(*) becomes:

$$\boxed{A(q^{-1})S'(q^{-1}) + q^{-d+1}R(q^{-1}) = P(q^{-1})} \qquad (**)$$

Unique solution if: $n_P = deg\, P(q^{-1}) = n_A + d \;\; ; \;\; deg\, S'(q^{-1}) = d \;\; ; \;\; deg\, R(q^{-1}) = n_A - 1$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots r_{n_A-1} q^{-n_A-1} \qquad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_d q^{-d}$$

Regulation (computation of *R(q⁻¹)* and *S(q⁻¹)*)

(**) is written as: $Mx = p \longrightarrow x = M^{-1}p$

$$
\overbrace{}^{n_A + d + 1}
$$

$$
\left[\begin{array}{cccccccc}
1 & 0 & & & & & & 0 \\
a_1 & 1 & & & & & & . \\
a_2 & a_1 & & 0 & & & & . \\
: & : & & 1 & & & & . \\
a_d & a_{d-1} & \cdots & a_1 & 1 & & & . \\
a_{d+1} & a_d & & & a_1 & 1 & & . \\
a_{d+2} & a_{d+1} & & & a_2 & 0 & & . \\
 & & & & . & & & . \\
 & & & & . & & & 0 \\
0 & 0 & \cdots & 0 & a_{n_A} & 0 & 0 & 1
\end{array}\right]
\left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\\1\\1\\1\\1\end{array}}\right\} n_A + d + 1
$$

$$
\underbrace{}_{d + 1}\;\underbrace{}_{n_A}
$$

$x^T = [1, s'_1, \ldots, s'_d, r_0, r_1, \ldots, r_{n-1}]$ $\qquad$ $p^T = [1, p_1, p_2, \ldots, p_{n_A}, p_{n_A+1}, \ldots, p_{n_A+d}]$

Use of WinReg or *predisol.sci(.m)* for solving (**)

Insertion of pre specified parts in R and S – same as for pole placement

Tracking (computation of $T(q^{-1})$)

Closed loop T.F.: $r \longrightarrow y$

$$H_{BF}(q^{-1}) = \frac{q^{-(d+1)} B_m(q^{-1})}{A_m(q^{-1})} = \frac{B_m(q^{-1}) T(q^{-1}) q^{-(d+1)}}{A_m(q^{-1}) P(q^{-1})}$$

Desired T.F.

It results : $\boxed{T(q^{-1}) = P(q^{-1})}$

Controller equation:

$$S(q^{-1})u(t) + R(q^{-1})y(t) = P(q^{-1})y^*(t+d+1)$$

$$u(t) = \frac{P(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$u(t) = \frac{1}{b_1}\left[P(q^{-1})y^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t)\right] \qquad (s_0 = b_1)$$

Tracking and regulation with independent objectives. Examples

Plant : $d = 0$

$B(q^{-1}) = 0.2\, q^{-1} + 0.1\, q^{-2}$

$A(q^{-1}) = 1 - 1.3\, q^{-1} + 0.42\, q^{-2}$

$\rightarrow Bm\,(q^{-1}) = 0.0927 + 0.0687\, q^{-1}$

Tracking dynamics

$\rightarrow Am\,(q^{-1}) = 1 - 1.2451 q^{-1} + 0.4066\, q^{-2}$

$\boldsymbol{Ts = 1s\ ,\ \omega_0 = 0.5\ rad/s,\ \zeta = 0.9}$

Regulation dynamics $\rightarrow P\,(q^{-1}) = 1 - 1.3741\, q^{-1} + 0.4867\, q^{-2}$

$\boldsymbol{Ts = 1s\ ,\ \omega_0 = 0.4\ rad/s,\ \zeta = 0.9}$

Pre-specifications : Integrator

***** CONTROL LAW *****

$S\,(q^{-1})\, u(t) + R\,(q^{-1})\, y(t) = T\,(q^{-1})\, y^*(t+d+1)$

$y^*(t+d+1) = [Bm\,(q^{-1})/Am\,(q^{-1})]\, .\, r(t)$

Controller : $R(q^{-1}) = 0.9258 - 1.2332\, q^{-1} + 0.42\, q^{-2}$

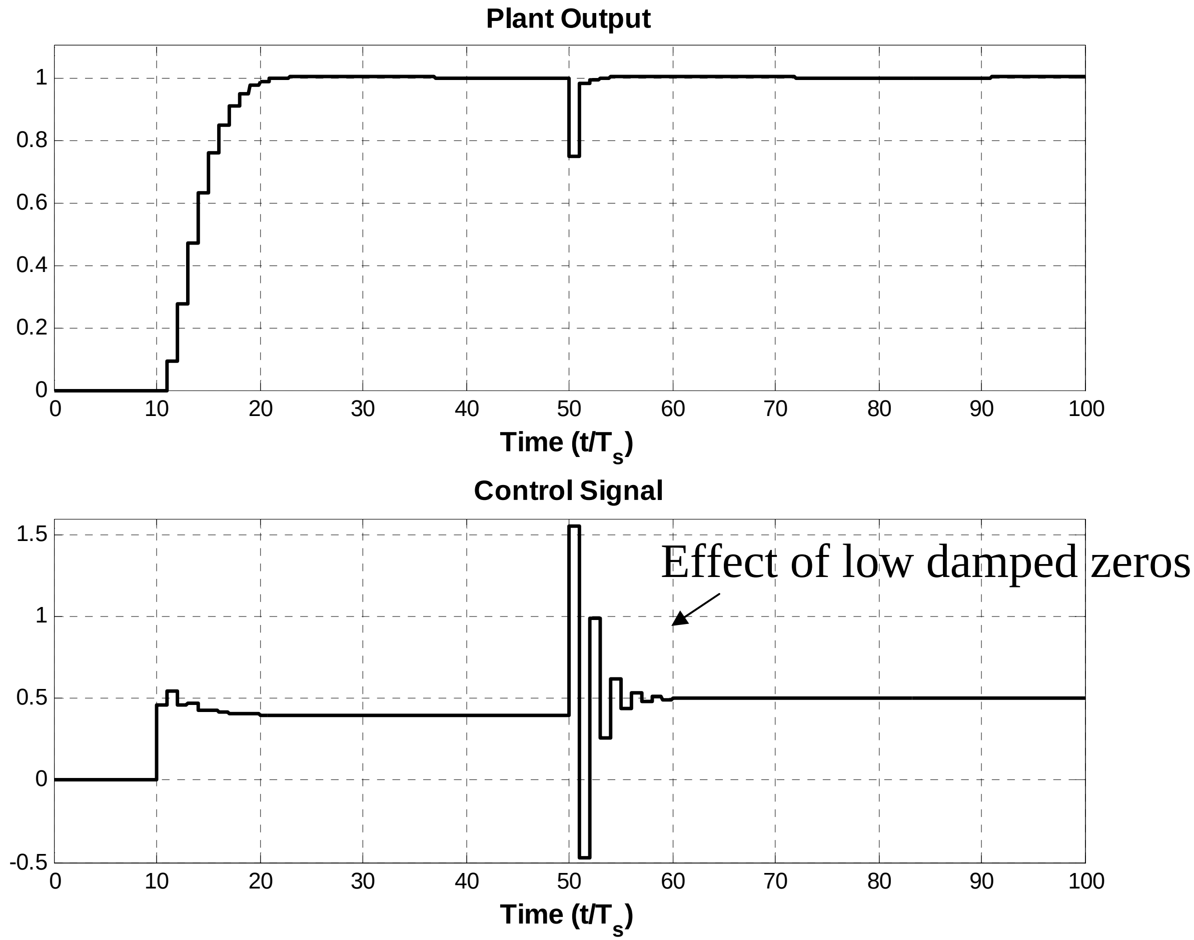
$S(q^{-1}) = 0.2 - 0.1\, q^{-1} - 0.1\, q^{-2}$

$T(q^{-1}) = P(q^{-1})$

Gain margin : 2.109 Phase margin : 65.3 deg

Modulus margin : 0.526 (- 5.58 dB) Delay margin : 1.2

Tracking and regulation with independent objectives. (d = 0)



The oscillations on the control input when there are low damped zeros can be reduced by introducing auxiliary poles

Internal model control -Tracking and regulation

It is a particular case of the pole placement

The dominant poles are those of the plant model

Does not allow to accelerate the closed loop response

Allows to design a RST controller for:
- well damped stable systems
- without restrictions upon the degrees of the polynomial A and B
- without restrictions upon the delay of the discrete time model

The plant model should be stable and well damped !

Often used for the systems featuring a large delay

Remark: The name is misleading since it has nothing in common with the "internal model principle"

Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

$$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = A(q^{-1})P_F(q^{-1}) = P(q^{-1}) \quad (*)$$

Dominant poles

$$P_F(q^{-1}) = \left(1+\alpha q^{-1}\right)^{n_{P_F}}$$

(typical choice)

R should be in the form : $R(q^{-1}) = A(q^{-1}).R'(q^{-1})$

After the cancellation of the common factor $A(q^{-1})$,(*) becomes:

$$S(q^{-1}) + q^{-d}B(q^{-1})R'(q^{-1}) = P_F(q^{-1})$$

Solution for: $S(q^{-1}) = \left(1-q^{-1}\right)S'(q^{-1})$ (typical choice)

$$R(q^{-1}) = A(q^{-1})\frac{P_F(1)}{B(1)}$$

$$S(q^{-1}) = (1-q^{-1})S'(q^{-1}) = P_F(q^{-1}) - q^{-d}B(q^{-1})\frac{P_F(1)}{B(1)}$$

Tracking (computation of $T(q^{-1})$)

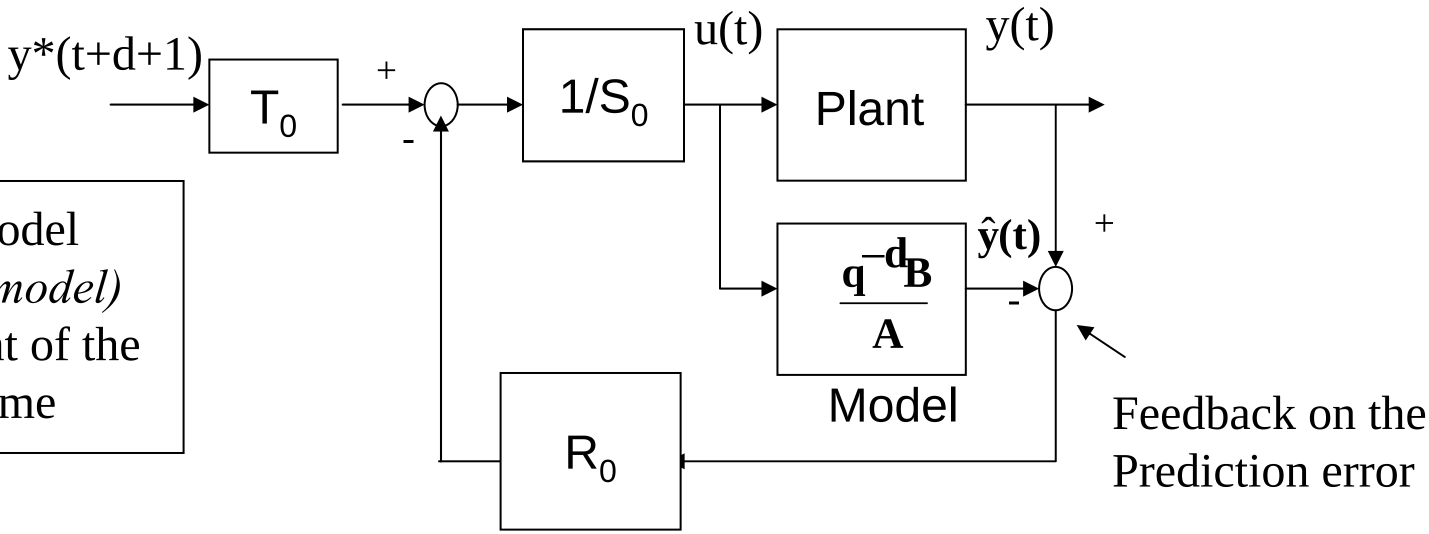
$$T(q^{-1}) = A(q^{-1})P_F(q^{-1}) / B(1)$$

Particular case : $A_m = AP_F$ (tracking dynamics = regulation dynamics)

$$T(q^{-1}) = T(1) = \frac{A(1)P_F(1)}{B(1)}$$ (cancellation of the tracking reference model)

Interpretation of the internal model control

Equivalent scheme



The plant model *(prediction model)* is an element of the control scheme

$$R_0(q^{-1}) = \frac{P_F(1)}{B(1)} A(q^{-1}) \quad \left(for\ H_R(q^{-1}) = 1\right)$$

$$S_0(q^{-1}) = P_F(q^{-1})$$

$$T_0(q^{-1}) = \frac{1}{B(1)} P(q^{-1}) = \frac{1}{B(1)} A(q^{-1}) P_F(q^{-1})$$

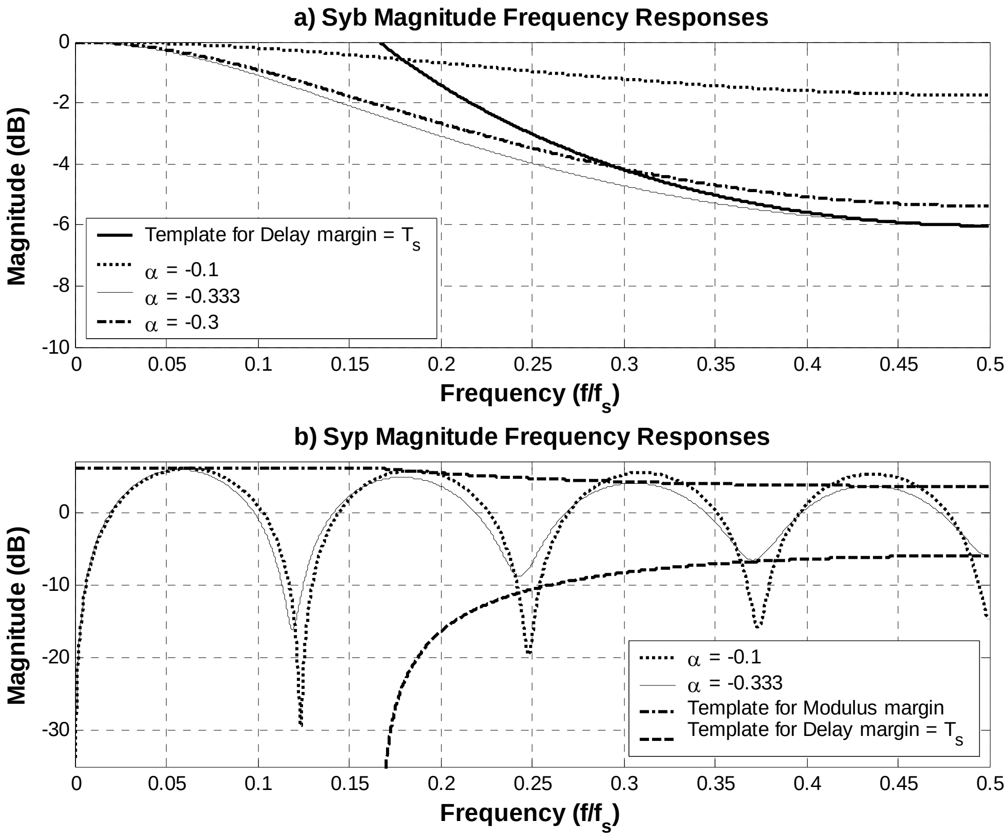
Rem.: *For all the strategies one can show the presence of the plant model in the controller*

Internal model control of a system with large delay

Plant: $d = 7;\ A = 1 - 0.2q^{-1}\ ;\ B = q^{-1}$

The « delay margin » can be satisfied by introducing auxiliary poles

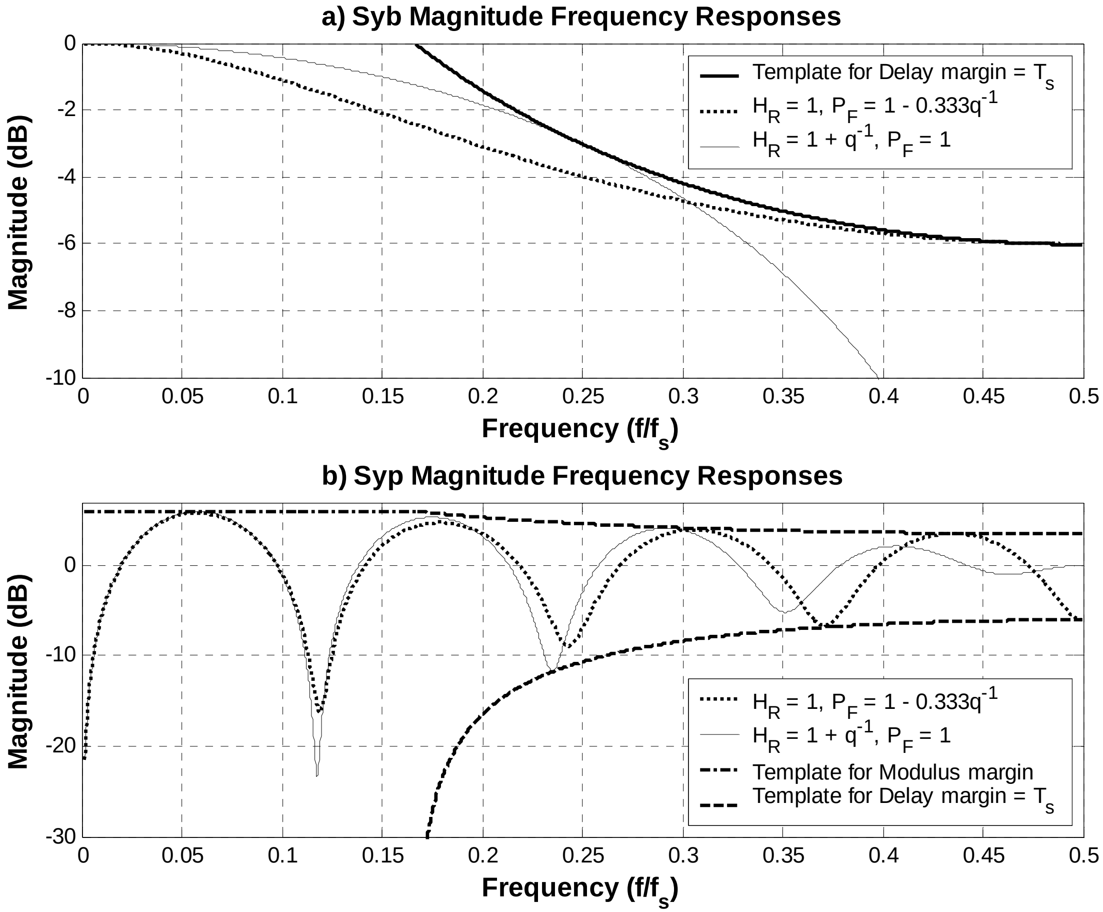
$$P_F(q^{-1}) = (1 + \alpha\, q^{-1}) \qquad -1 < \alpha < 0$$



$\alpha = -0.1; -0.3; -0.333$

Good value

Internal model control of a system with large delay



$H_R(q^{-1}) = 1 + q^{-1}$ corresponds to the opening of the loop at $0.5f_S$

See also:

I.D. Landau (1995) : Robust digital control of systems with time delay (the Smith predictor revisited) Int. J. of Control, v.62,no.2 pp 325-347

Pole placement with sensitivity functions shaping

Performance specification for pole placement :

- Desired dominant poles for the closed loop
- The reference trajectory (tracking reference model)

Questions:

- How to take into account the specifications in certain frequency regions?
- How to guarantee the *robustness* of the controllers ?
- How to take advantage from the degree of freedom for the maximum number of poles which can be assigned ?

Answer:

Shaping the sensitivity functions by:

- introducing auxiliary poles

- introducing filters in the controllers

Sensitivity functions - review

Output sensitivity function:

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})}$$

Input sensitivity function:

$$S_{up}(q^{-1}) = -\frac{A(q^{-1})R(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})}$$

Controller structure :

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

Pre specified parts (filters)

Dominant and auxiliary filters:

$$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Study of the properties of the sensitivity functions in the frequency domain: $q=z=e^{j\omega}$

Properties of the output sensitivity function

P.1- *The modulus of the output sensitivity function at a certain frequency gives the amplification or attenuation factor of the disturbance on the output*

$S_{yp}(\omega) < 1 (0\ dB)$ attenuation

$S_{yp}(\omega) > 1$ amplification

$S_{yp}(\omega) = 1$ operation in open loop

P.2

$$\Delta M = \left(\left| S_{yp}(j\omega) \right|_{\max} \right)^{-1}$$

Modulus margin

Properties of the output sensitivity function

P.3 – *The open loop (KG) being stable one has the property:*

$$\int_0^{0.5 f_S} \log\left|S_{yp}(e^{-j2\pi f/f_S})\right| df = 0$$

The sum of the areas between the curve of Syp and the axis 0dB taken with their sign is null

$$\Downarrow$$

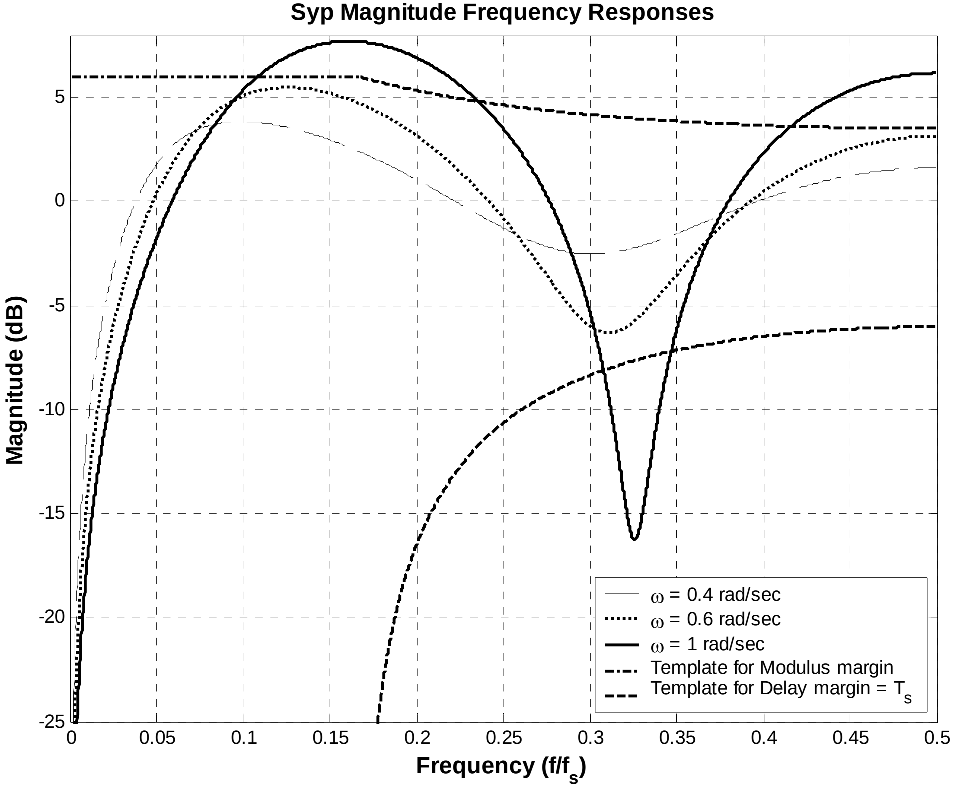
Disturbance attenuation in a frequency region implies amplification of the disturbances in other frequency regions!

Properties of the output sensitivity function

Augmenting the attenuation or widening the attenuation zone

Higher amplification of disturbances outside the attenuation zone

Reduction of the robustness (reduction of the modulus margin)

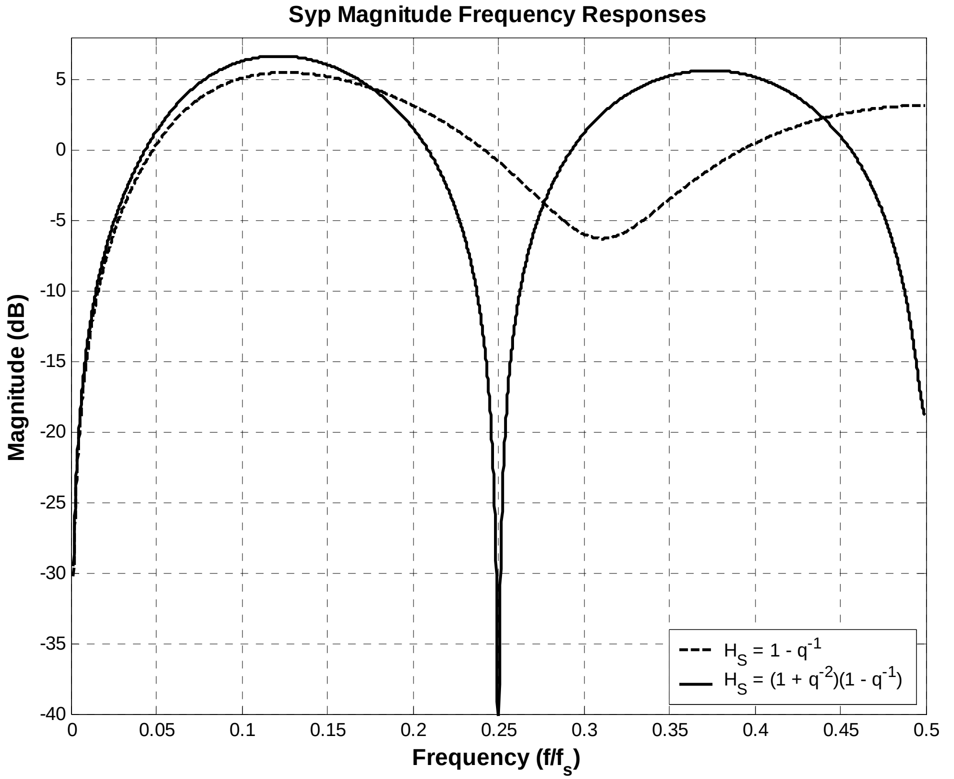


Properties of the output sensitivity function

P.4 – *Cancellation of the disturbance effect at a certain frequency:*

$$A(e^{-j\omega})\underbrace{S(e^{-j\omega})}_{\text{Zeros of } S_{yp}} = A(e^{-j\omega})H_S(e^{-j\omega})S'(e^{-j\omega}) = 0 \quad ; \quad \omega = 2\pi f / f_s$$

Allows introduction of zeros at desired frequencies

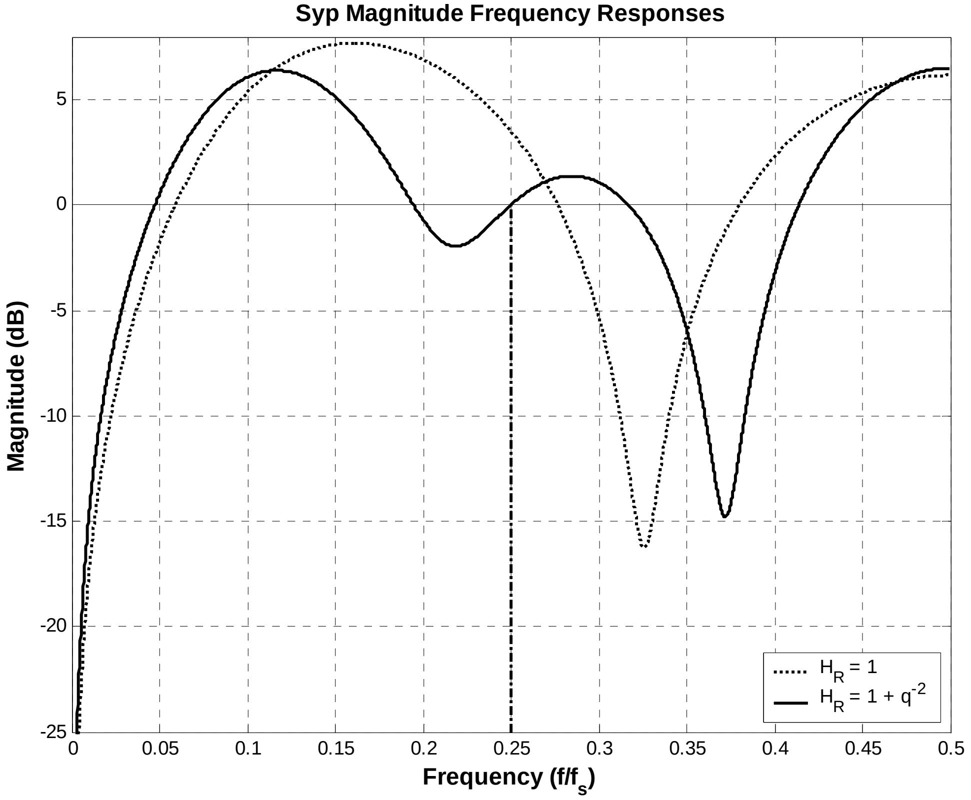


Properties of the output sensitivity function

P.5 - $\left|S_{yp}(j\omega)\right| = 1\,(0\;dB)$ *at frequencies where:*

$$B^*(e^{-j\omega})R(e^{-j\omega}) = B^*(e^{-j\omega})H_R(e^{-j\omega})R'(e^{-j\omega}) = 0\ ;\ \omega = 2\pi\ f\,/\,f_s$$

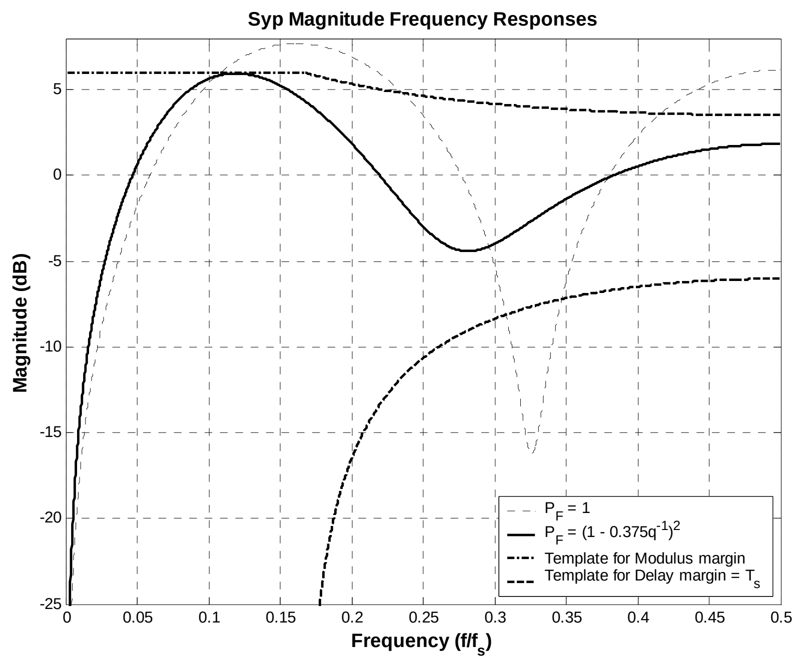
Allows introduction of zeros at desired frequencies



Properties of the output sensitivity function

P.6 – *Asymptotically stable auxiliary poles (P_F) lead (in general) to the reduction of* $\left|S_{yp}(j\omega)\right|$ *in the attenuation band of $1/P_F$*

$$P_F(q^{-1}) = (1 + p'q^{-1})^{n_{P_F}} \qquad -0.5 \leq p' \leq -0.05 \qquad n_{P_F} \leq n_P - n_{P_D}$$



In many applications, introduction of high frequency auxiliary poles is enough for assuring the required robustness margins

Properties of the output sensitivity function

P.7 – *Simultaneous introduction of a fixed part* H_{Si} *and of a pair of auxiliary poles* P_{Fi} *having the form:*

$$\frac{H_{S_i}(q^{-1})}{P_{F_i}(q^{-1})} = \frac{1+\beta_1 q^{-1}+\beta_2 q^{-2}}{1+\alpha_1 q^{-1}+\alpha_2 q^{-2}}$$

resulting from the dicretization of :

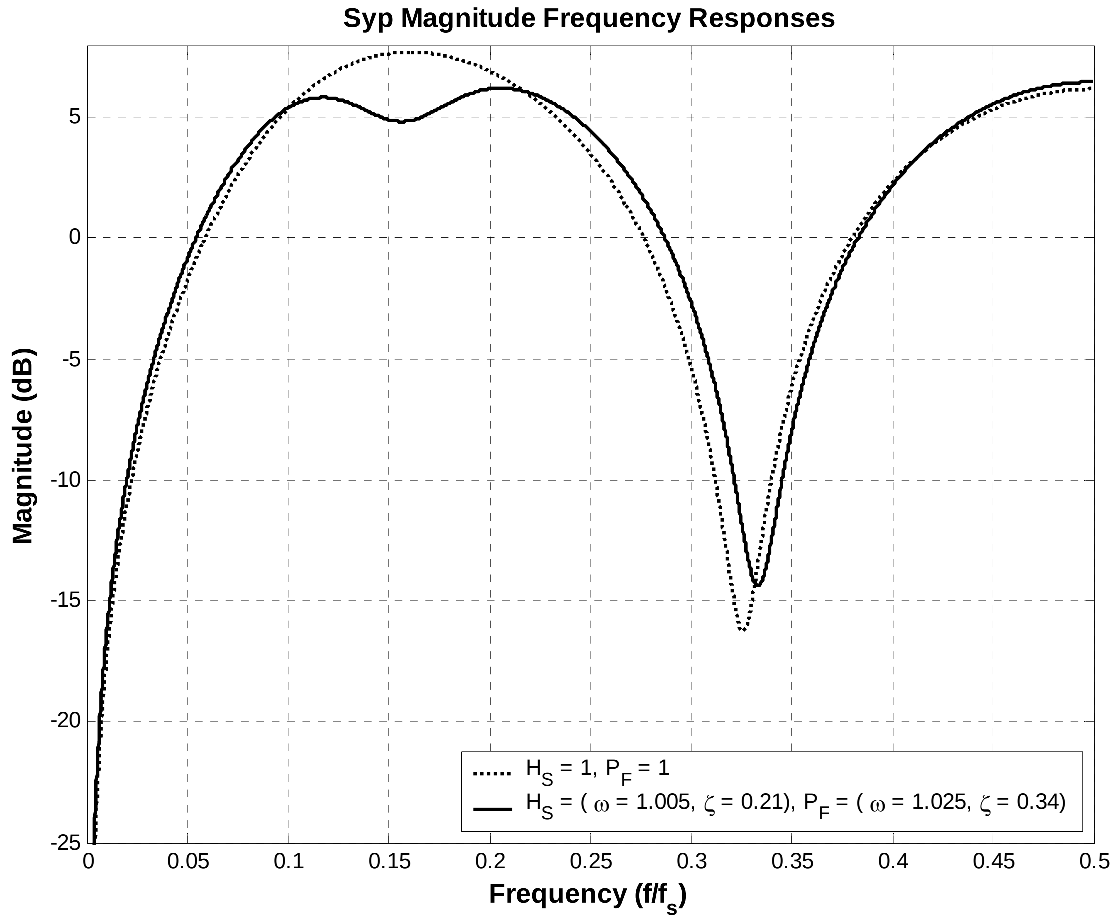
$$F(s) = \frac{s^2+2\zeta_{num}\omega_0 s+\omega_0^2}{s^2+2\zeta_{den}\omega_0 s+\omega_0^2}$$ *with:* $$s = \frac{2}{T_e}\frac{1-z^{-1}}{1+z^{-1}}$$

introduces an attenuation at the normalized discretized frequency:

$$\omega_{disc} = 2\arctan\left(\frac{\omega_0 T_e}{2}\right)$$ *with the attenuation:* $$M_t = 20\log\left(\frac{\zeta_{num}}{\zeta_{den}}\right) \quad (\zeta_{num} < \zeta_{den})$$

and with negligible effect at $f << f_{disc}$ *and at* $f >> f_{disc}$

Properties of the output sensitivity function



Effective computation with the function: *filter22.sci (.m)*

Properties of the input sensitivity function

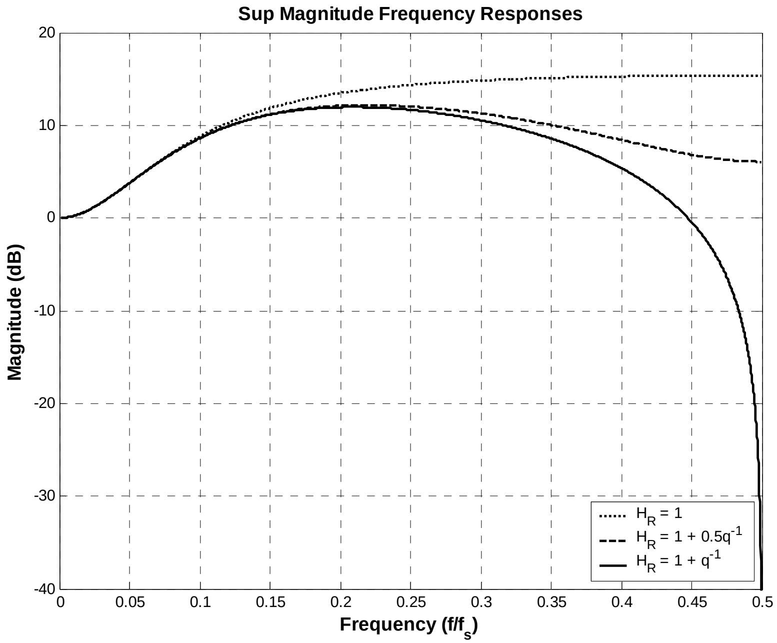
P.1 – *Cancellation of the disturbance effect on the input at a certain frequency (S_{up} = 0):*

$$A(e^{-j\omega})H_R(e^{-j\omega})R'(e^{-j\omega}) = 0 \quad ; \quad \omega = 2\pi\, f / f_s$$

Allows introduction of zeros at desired frequencies

$$H_R(q^{-1}) = 1 + \beta q^{-1} \qquad 0 < \beta \le 1$$

(active at $0.5f_S$)



Rem: The system operate in open loop at this frequency

Properties of the input sensitivity function

P.2 – *At frequencies where:*

$$A(e^{-j\omega})H_S(e^{-j\omega})S'(e^{-j\omega}) = 0 \quad ; \quad \omega = 2\pi f / f_s$$

One has:

$$|S_{yp}(j\omega)| = 0 \qquad \qquad |S_{up}(e^{-j\omega})| = \left|\frac{A(e^{-j\omega})}{B(e^{-j\omega})}\right|$$

Inverse of the system gain

Consequence : strong attenuation of the disturbances should be done only in the frequency regions where the system gain is enough large (in order to preserve robustness and avoid too much stress on the actuator)

Remember: $|S_{up}(j\omega)|^{-1}$ gives the tolerance with respect to additive uncertainties on the model (high $|S_{up}(j\omega)|$ = weak robustness)

Properties of the input sensitivity function

P.3 – *Simultaneous introduction of a fixed part* H_{Ri} *and of a pair of auxiliary poles* P_{Fi} *having the form:*

$$\frac{H_{R_i}(q^{-1})}{P_{F_i}(q^{-1})} = \frac{1+\beta_1 q^{-1} + \beta_2 q^{-2}}{1+\alpha_1 q^{-1} + \alpha_2 q^{-2}}$$

resulting from the dicretization of :

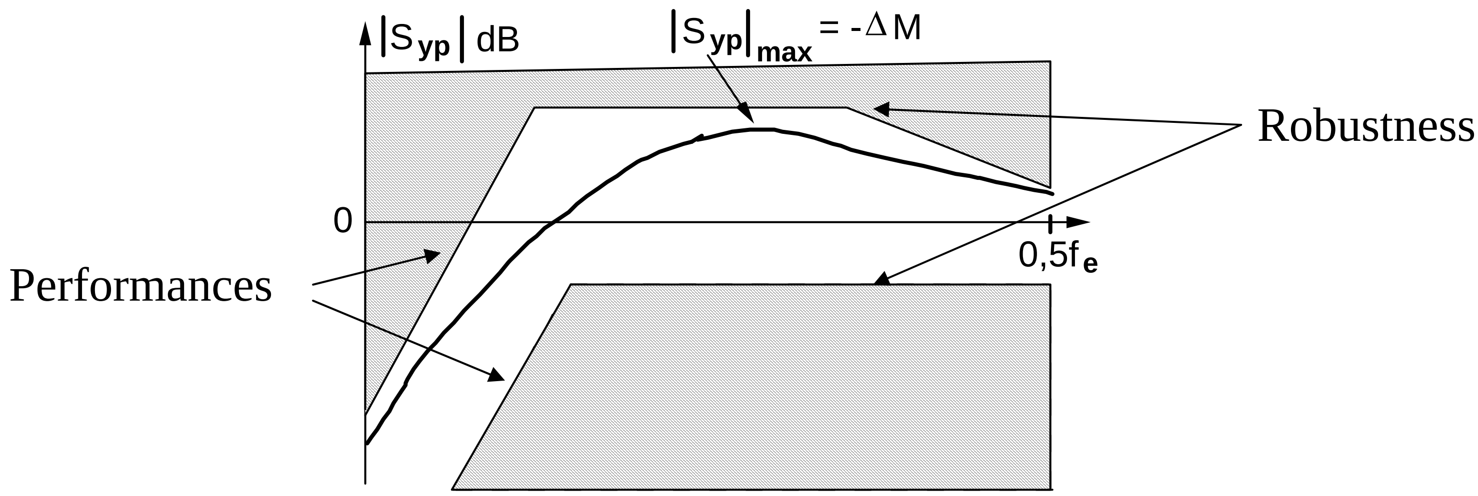
$$F(s) = \frac{s^2 + 2\zeta_{num}\omega_0 s + \omega_0^2}{s^2 + 2\zeta_{den}\omega_0 s + \omega_0^2} \quad \text{with:} \quad s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}$$

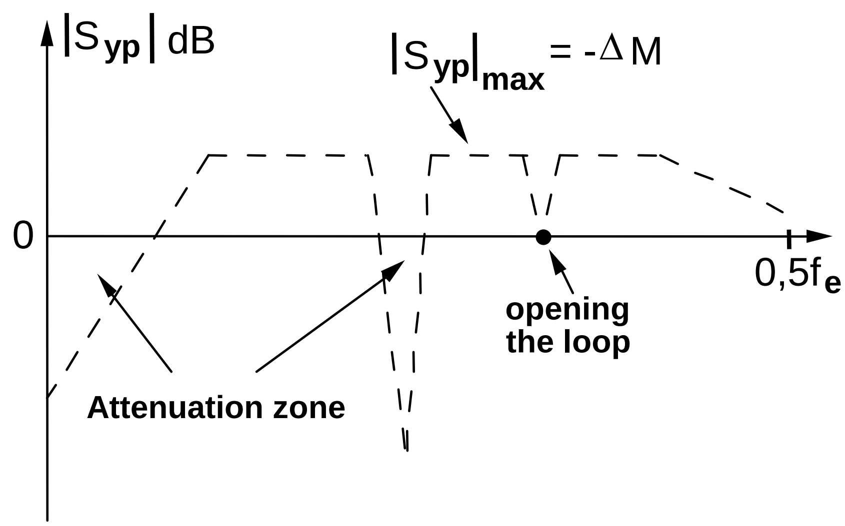
introduces an attenuation at the normalized discretized frequency:

$$\omega_{disc} = 2\arctan\left(\frac{\omega_0 T_e}{2}\right) \quad \text{with the attenuation:} \; M_t = 20\log\left(\frac{\zeta_{num}}{\zeta_{den}}\right) \quad (\zeta_{num} < \zeta_{den})$$

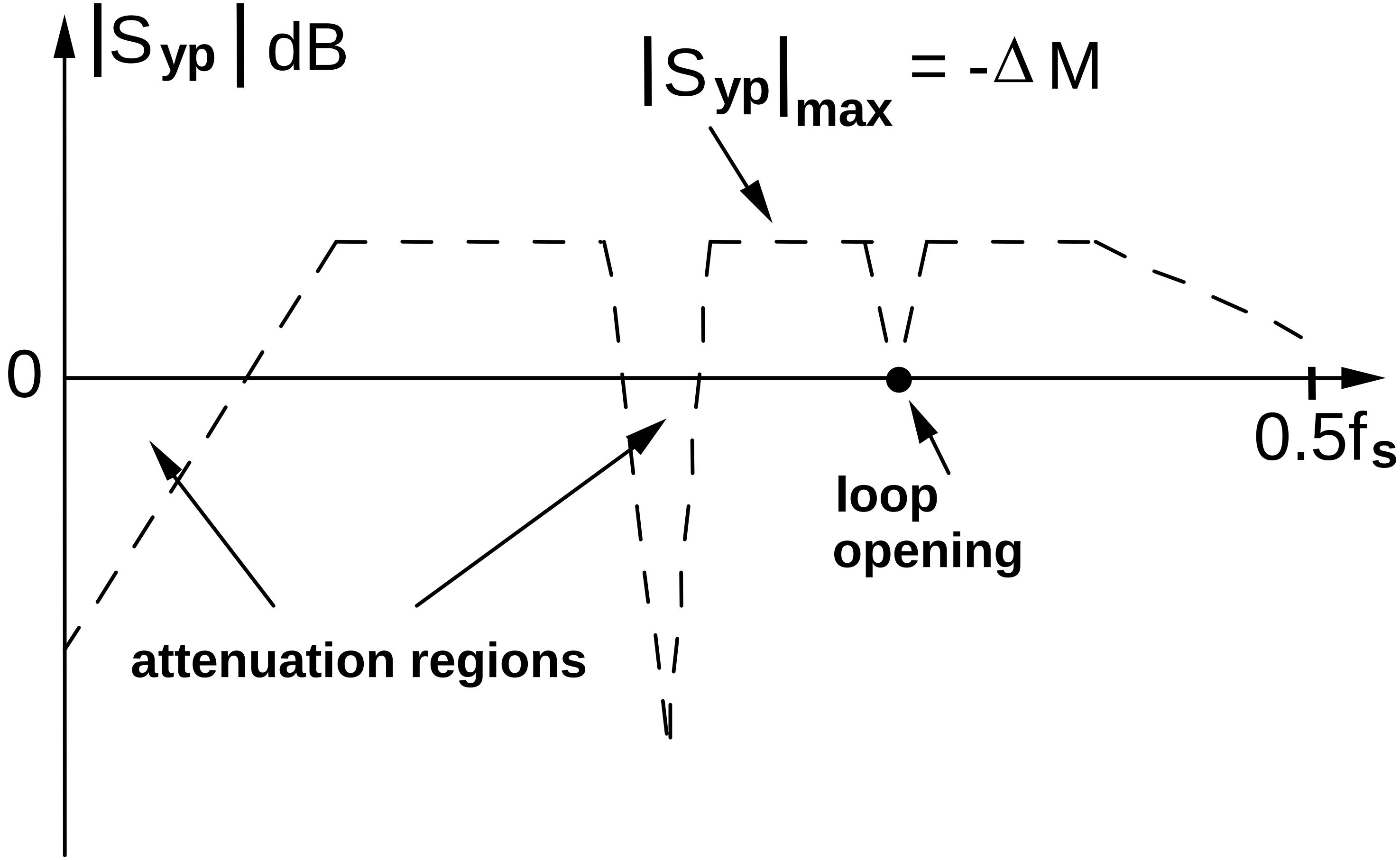
and with negligible effect at $f << f_{disc}$ *and at* $f >> f_{disc}$

Templates for the output sensitivity functions S_{yp}





Templates for the input sensitivity function S_{up}



Shaping the sensitivity functions

1. Choice of the dominants et auxiliary poles of the closed loop
2. Choice of the fixed part of the controller (H_S and H_R)
3. Simultaneous choice of the fixed parts and the auxiliary poles

Procedure:

Basic shaping : use 1 and 2
Fine shaping: use 3

Tools for sensitivity shaping: WinReg (Adaptech) and *ppmaster.m*

There exist also tools for automatic sensitivity function shaping based on convex optimization (Optreg from Adaptech)

Shaping the sensitivity functions - Example I

Plant: $A = 1 - 0.7q^{-1}$; $B = 0.3q^{-1}$; $d = 2$; $T_e = 1s$

Specifications:

- Integrator
- Dominant poles: discretization of a cont. time 2nd order system : $\omega_0 = 1$ rad/s, $\zeta = 0.9$

Controller A :

Attenuation band: *0* up to *0.058 Hz* but $\Delta M < -6\ dB$ *and* $\Delta\tau < T_s$

Objective: same attenuation band but with $\Delta M > -6\ dB$ *and* $\Delta\tau > T_s$

- insertion of auxiliary poles: $P_F = \left(1 - 0.4q^{-1}\right)^2$

Controller B : good margins but reduction of the attenuation band

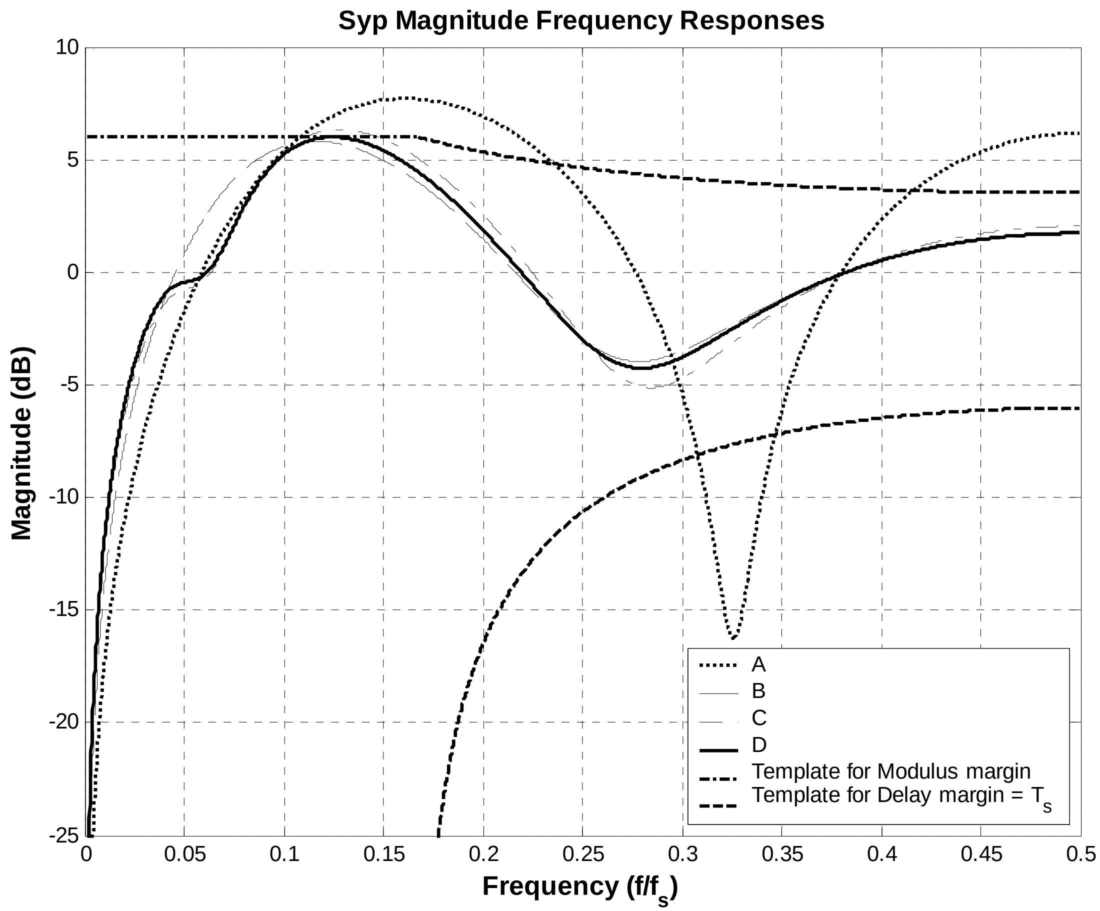
-insertion of pole-aero filter H_S/P_F centered at $\omega_0 = 0.4$ rad/s (0.064 Hz)

Controller C : good attenuation band but $S_{yp} > 6\ dB$

- larger (slower) auxiliary poles (0.4 ➞ 0.44)

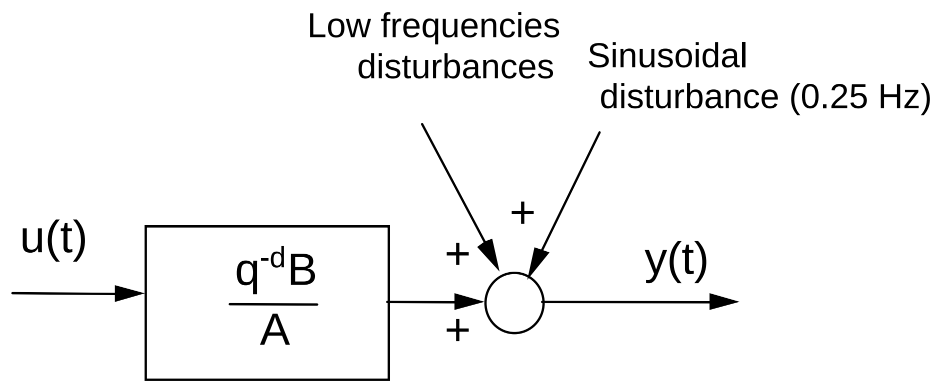
Controller D : Correct

Shaping the sensitivity functions - Example I



Shaping the sensitivity functions - Example II

Plant (integrator): $A = 1 - q^{-1}$; $B = 0.5q^{-1}$; $d = 2$; $T_s = 1s$



Specifications:

1. No attenuation of the sinusoidal disturbance at (0.25 Hz)
2. Attenuation band at low frequencies : 0 à 0.03 Hz
3. Disturbances amplification at 0.07 Hz: < 3dB
4. Modulus margin > -6 dB and Delay margin > T
5. No integrator in the controller

Shaping the sensitivity functions - Example II

- Fixed parts design : $H_R = 1 + q^{-2}$; $H_S = 1$

Opening the loop at 0.25 Hz

-Dominant poles: discretization of a cont. time 2nd order system:

$\omega_0 = 0.628$ rad/s, $\zeta = 0.9$

Controller A : the specs. at 0.07 Hz are not fulfilled

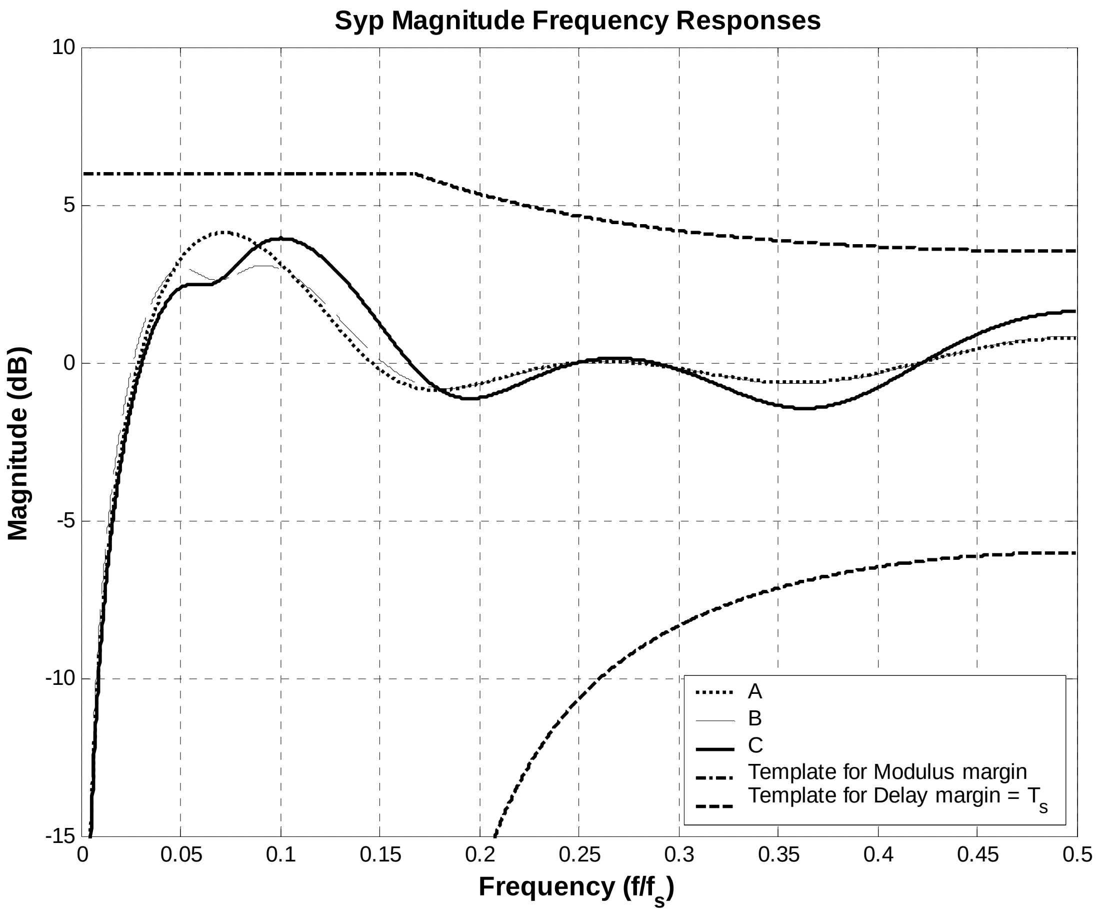
- insertion of a pole-zero filter H_S/P_F centered at $\omega_0 = 0.44$ rad/s

Controller B : Attenuation band smaller than that specified

- dominant poles acceleration: $\omega_0 = 0.9$ rad/s

Controller C : Correct

Shaping the sensitivity functions - Example II



Some concluding remarks

- All the digital controllers has a three branches structure(R-S-T).
- They have two degrees of freedom (tracking and regulation)
- Controller design is done in two steps:
  1) *R et S* (regulation)   2) *T* (tracking)
- Controller complexity depends upon the plant model complexity
- *Pole placement* is the basic control strategy
- *Tracking and regulation with independent objectives* is applicable to discrete time models with stable zeros
- *Internal model control* is applicable only to stable and well damped plants
- Design of digital PID is a particular case of pole placement. Can be used for the control of simple plants (order max. = 2)
- All the digital controllers presented implement a *predictive control* and they contain implicitly a *predicition model of the plant*