How to Reject Unknown Disturbances Using Adaptive Feedback Control

Application to Active Vibration Control

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Outline

- Rejection of unknown stationary disturbances
- Indirect adaptive control
- Direct adaptive control
- Rejection of unknown narrow band disturbances in active vibration control. Real-time results
- Conclusions

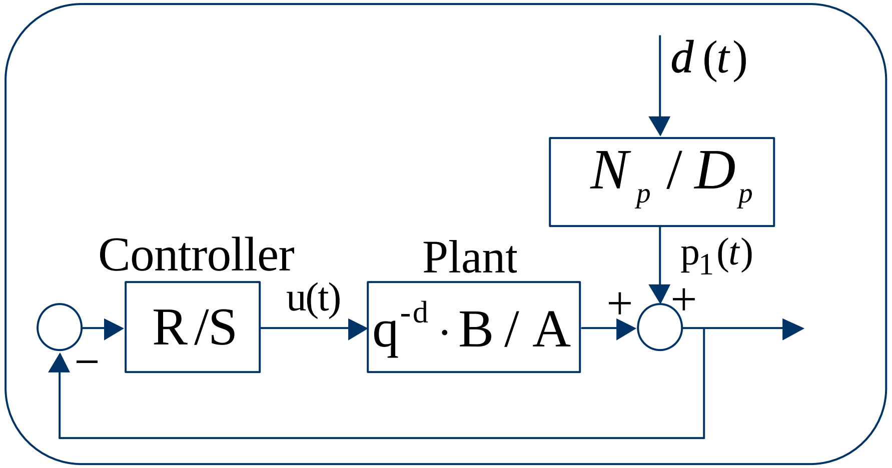
Unknown disturbance rejection

- **Problem:** Attenuation of unknown and/or variable stationary disturbances
- **Solution:** Adaptive feedback control
- **Methodology:** Based on the *Internal Model Principle*
  - Indirect adaptive control algorithm
  - Direct adaptive control algorithm
- **Objective:** Computation of a controller with an adaptive internal model of the disturbance

Rem:
Stationary disturbances models have poles on the unit circle

Hypothesis: *Plant model parameters are constant and known*

Closed loop system. Notations



$$p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot d(t): \text{deterministic disturbance}$$

$D_p \rightarrow$ poles on the unit circle; $d(t)$ = Dirac

Controller:

$$R(q^{-1}) = R'(q^{-1}) \cdot H_R(q^{-1});$$

$$S(q^{-1}) = S'(q^{-1}) \cdot H_S(q^{-1}).$$

Internal model principle: $H_S(z^{-1}) = D_p(z^{-1})$

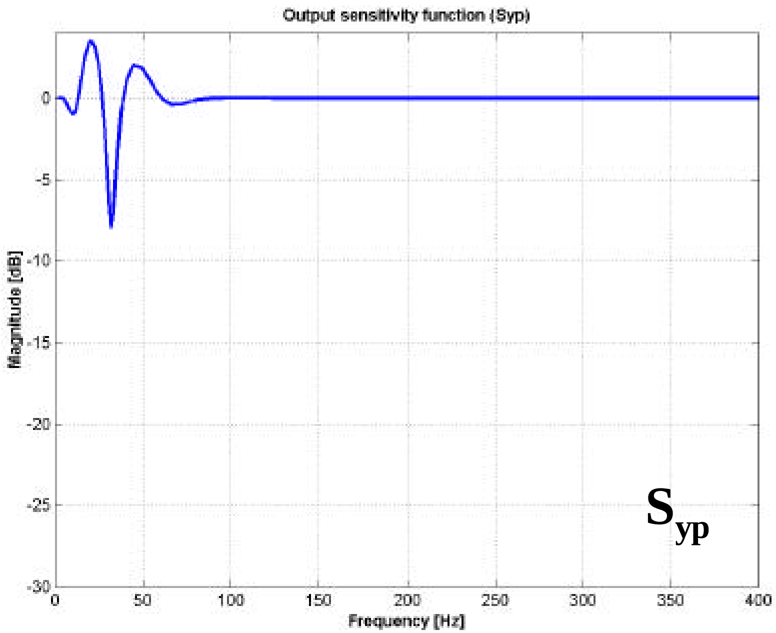
Output: $y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p_1(t) = S_{yp}(q^{-1}) \cdot p_1(t)$

CL poles: $P(q^{-1}) = A(q^{-1})S(q^{-1}) + z^{-d}B(q^{-1})R(q^{-1})$

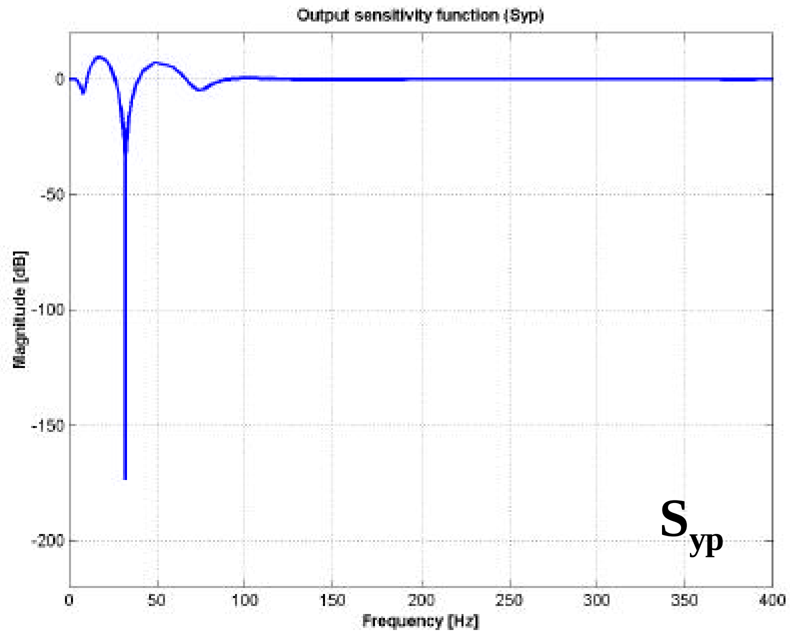
$$\rightarrow y(t) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \frac{1}{D_p(q^{-1})} \cdot d(t)$$

Internal Model Principle: $H_S(z^{-1})=D_p(z^{-1})$

Controller without internal model
$H_S=1$



Controller with internal model at 32 Hz
H_S: (f=32 Hz, ζ=0)



Indirect adaptive control

Two-step methodology:

1. Identification of the disturbance model, $D_p(q^{-1})$

2. Computation of the controller, considering $H_S(q^{-1}) = \hat{D}_p(q^{-1})$

Indirect adaptive control

Example : Sinusoidal (narrow band) disturbance

Step I : Estimation of D_P

Sinusoid $\rightarrow$ ARMA with $n_{D_p} = 2$: $D_p(q^{-1}) = 1 + d_{p_1} q^{-1} + d_{p_2} q^{-2}$;

$$N_p(q^{-1}) = 1 + n_{p_1} q^{-1}.$$

Output : $y(t+1) = -d_{p_1} y(t) - d_{p_2} y(t-1) + n_{p_1} e(t) + e(t+1)$,

where $e(t) =$ gaussian white noise.

A priori predictor : $\hat{y}^0(t+1) = \hat{q}^T(t) \cdot f(t)$; *A posteriori* predictor : $\hat{y}(t+1) = \hat{q}^T(t+1) \cdot f(t)$,

where $\hat{q}^T(t) = [\hat{d}_{p_1}(t) \quad \hat{d}_{p_2}(t) \quad \hat{n}_{p_1}(t)]$;

$$f^T(t) = [-y(t) \quad -y(t-1) \quad e(t)].$$

A priori error: $e^0(t+1) = y(t+1) - \hat{y}^0(t+1)$; *A posteriori* error: $e(t+1) = y(t+1) - \hat{y}(t+1)$

Parametric adaptation algorithm $\rightarrow \begin{cases} \hat{q}(t+1) = \hat{q}(t) + F(t+1) f(t) e^0(t+1); \\ F(t+1) = \textbf{adaptation gain.} \end{cases}$

Indirect adaptive control

Step II: Computation of the controller

Solving Bezout equation (for S' and R)

$$H_S = \hat{D}_p$$

$$A\hat{D}_p S' + q^{-d} BR = P$$

$$S = D_p S'$$

Direct adaptative control (Q-parameterization)

Nominal contr: $[R_0(q^{-1}), S_0(q^{-1})]$.

Bezout: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1})\, u(t) = -R_0\,(q^{-1})\, y(t)$

Q-parameterization :

$\mathbf{R(z^{1})=R_0(q^{-1})+A(q^{-1})Q(q^{-1});}$

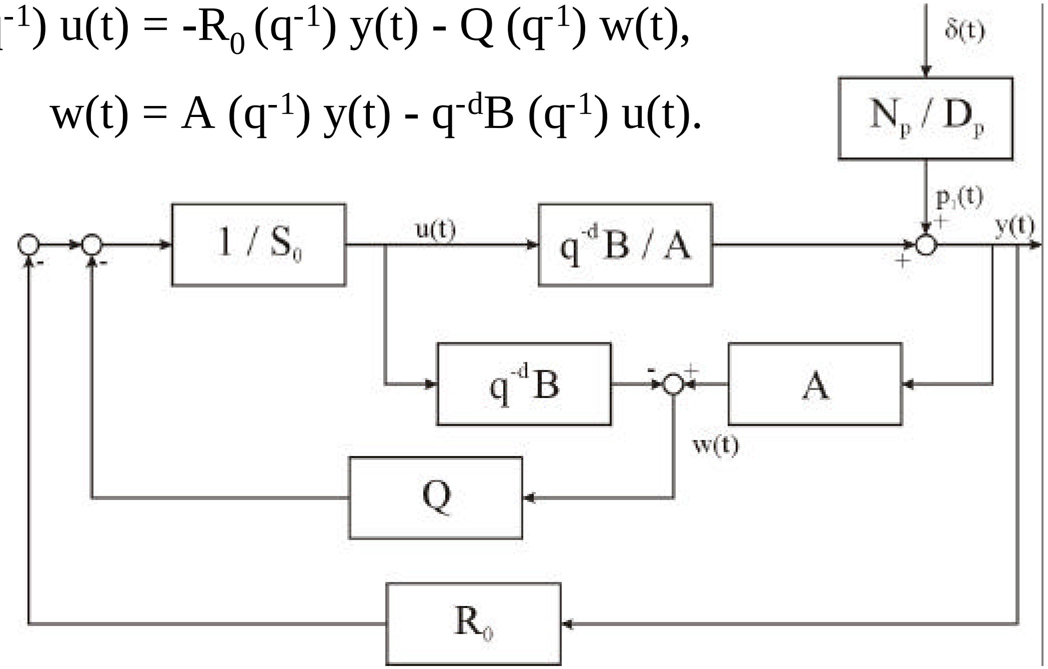
$\mathbf{S(q^{-1})=S_0(z^{-1})-q^{-d}B(q^{-1})Q(q^{-1}).}$

<u>$Q(q^{-1})$ computed such as $[R(q^{-1}),S(q^{-1})]$ contain the internal model of the disturb.</u>

$$S_0 - q^{-d} BQ = MD_p$$

Control: $S_0(q^{-1})\, u(t) = -R_0\,(q^{-1})\, y(t) - Q\,(q^{-1})\, w(t)$,

where $w(t) = A\,(q^{-1})\, y(t) - q^{-d}B\,(q^{-1})\, u(t)$.



Direct Adaptive Control (unknown D_p)

(Based on an ideea of Y. Z. Tsypkin)

Hypothesis: Identified (known) plant model (A,B,d).

Goal: minimize y(t).

Consider $p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot d(t)$: deterministic disturbance.

$$y(t) = \frac{A(q^{-1})\left[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\right]}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot d(t)$$

$$\Downarrow$$

$$e(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d}B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t).$$

Let $\hat{Q}(t,q^{-1})$ be an estimated value of $Q(q^{-1})$

We can show that

$$e(t+1) = [Q(q^{-1}) - \hat{Q}(t+1,q^{-1})] \cdot \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t+1)$$

$$(v(t+1) = \text{disturbance term} \rightarrow 0)$$

The Algorithm

A priori adaptation error: $e^0(t+1) = w_1(t+1) - \hat{q}^T(t)f(t);$

A posteriori adaptation error: $e(t+1) = w_1(t+1) - \hat{q}^T(t+1)f(t),$

where

$$\hat{q}^T(t) = [\hat{q}_0(t) \quad \hat{q}_1(t)]; \quad f^T(t) = [w_2(t) \quad w_2(t-1)], \quad (\text{for} \quad n_Q = 1)$$

and

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1);$$

$$w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t);$$

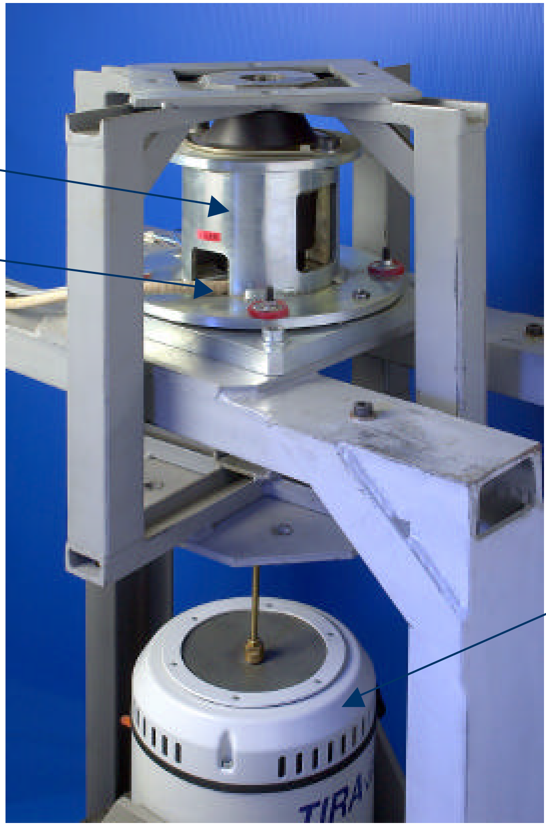
$$\mathrm{w}(\mathrm{t}+1) = \mathrm{A}(q^{-1}) \cdot y(t+1) - q^{-d}B^*(q^{-1}) \cdot u(t);$$

$$B(q^{-1}) \cdot u(t+1) = B^*(q^{-1}) \cdot u(t).$$

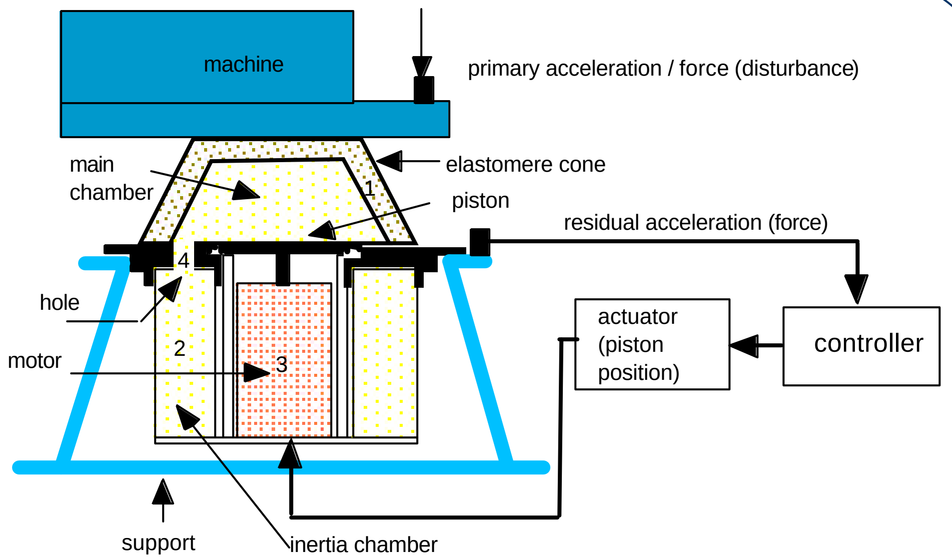
Parameter adaptation algorithm:

$$\begin{cases} \hat{q}(t+1) = \hat{q}(t) + F(t+1)f(t)e^0(t+1); \\ F^{-1}(t+1) = l_1(t)F^{-1}(t) + l_2(t)f(t)f^T(t). \end{cases}$$

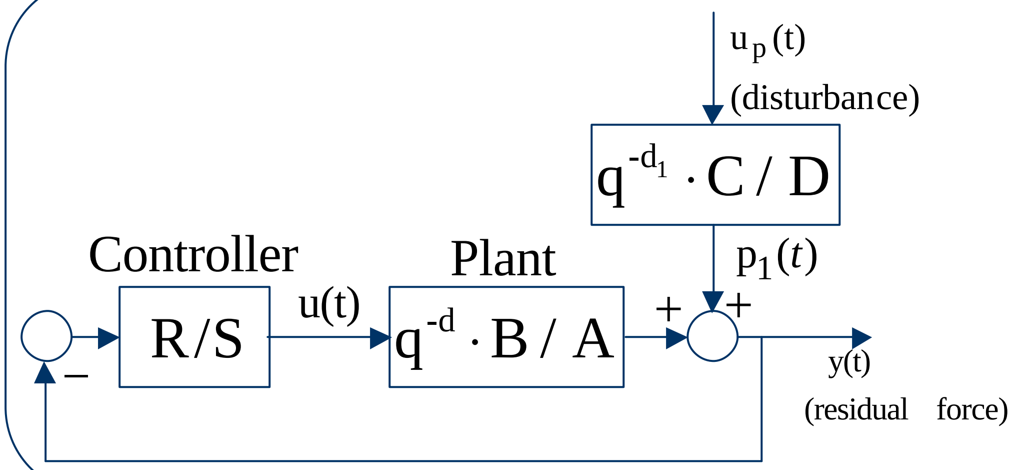
The Active Suspension



The Active Suspension System



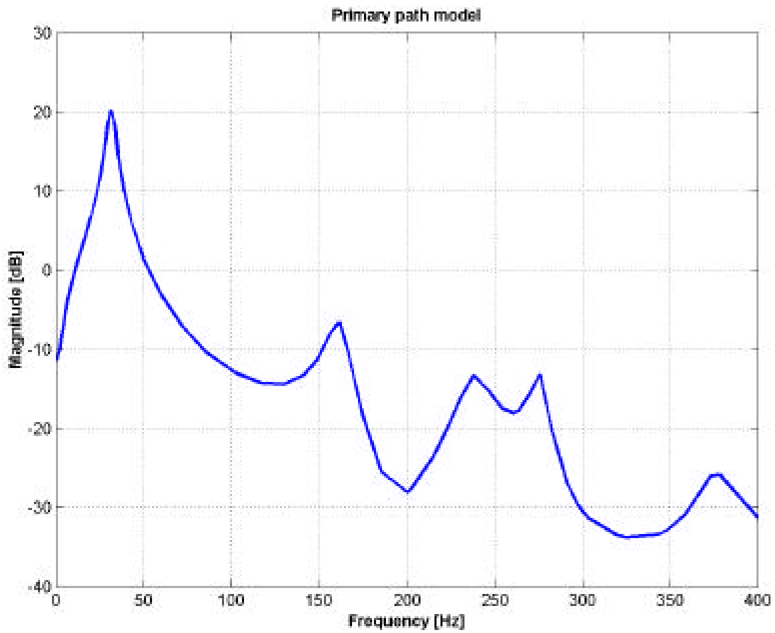
Two paths :
- Primary
- Secondary (double differentiator)



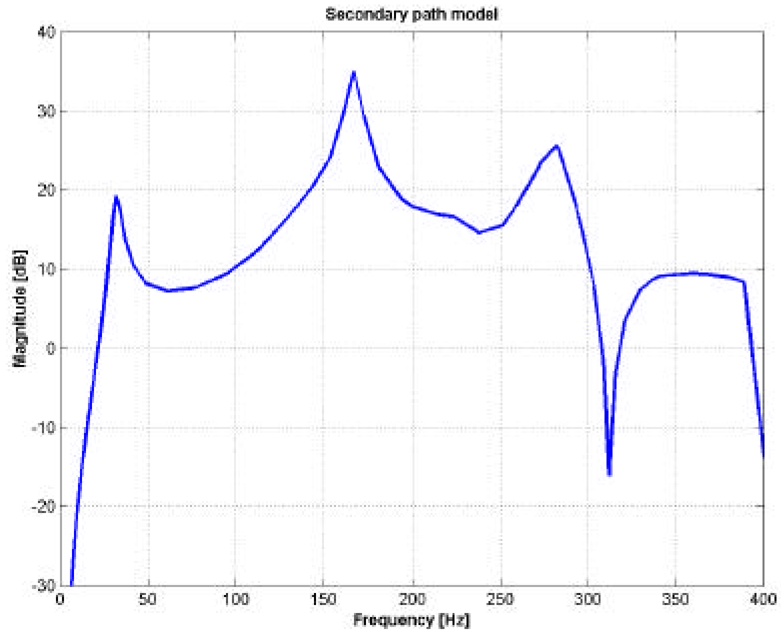
$$T_s = 0.00125\,s$$

Frequency Characteristics of the Identified Models

Primary path



Secondary path



$n_A = 14 \, ; n_B = 16 \, ; d = 0$

V. Real-time results

Narrow band disturbances = variable frequency sinusoid $\Rightarrow n_Q = 1$

Frequency range: 25 ÷ 47 Hz

Evaluation of the two algorithms in **real-time**

Nominal controller $[R_0(q^{-1}),S_0(q^{-1})]$: $n_{R0}=14$, $n_{S0}=16$

Implementation protocol 1: Self-tuning

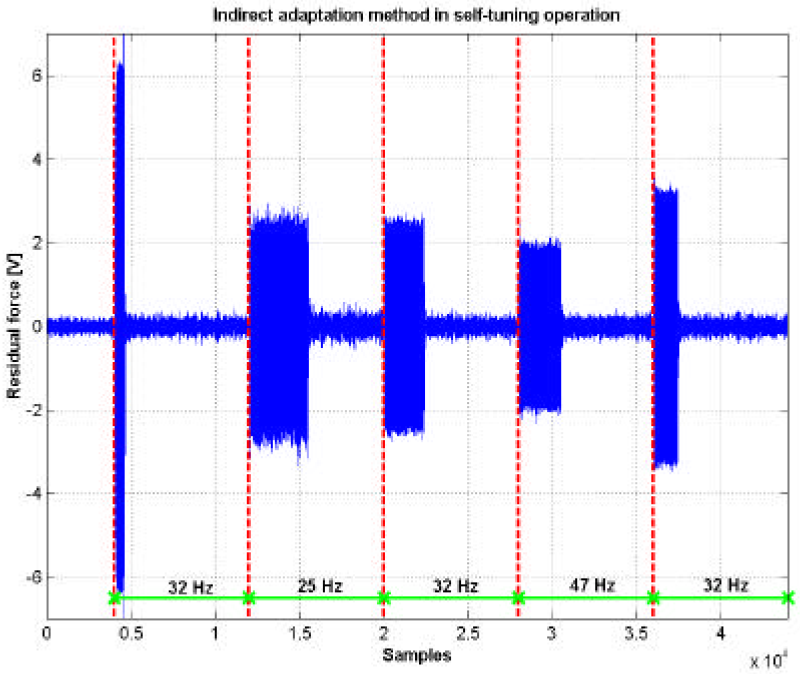
- The algorithm stops when it converges and the controller is applied.
- It restarts when the variance of the residual force is bigger than a given threshold.
- As long as the variance is not bigger than the threshold, the controller is constant.

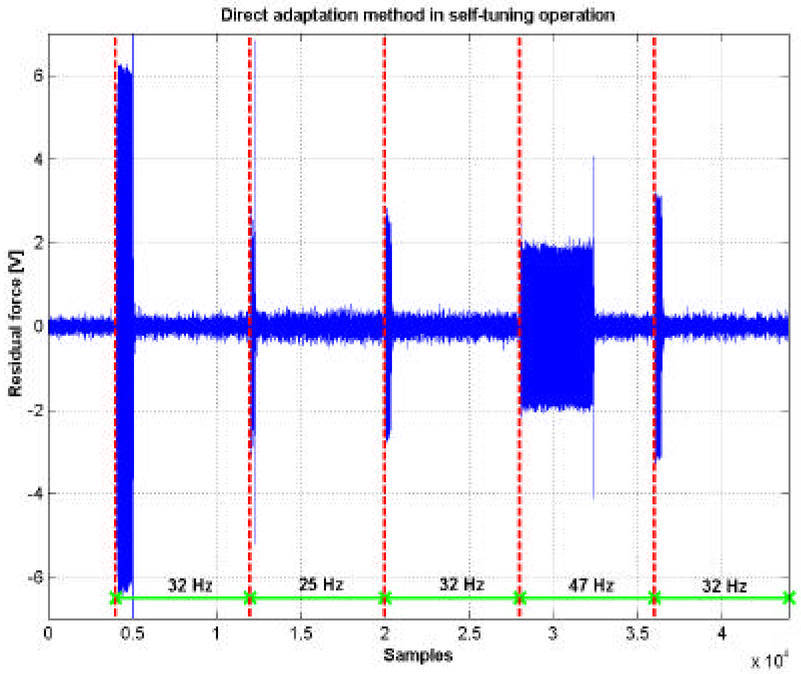
Implementation protocol 2: Adaptive

- The adaptation algorithm is continuously operating
- The controller is updated at each sample

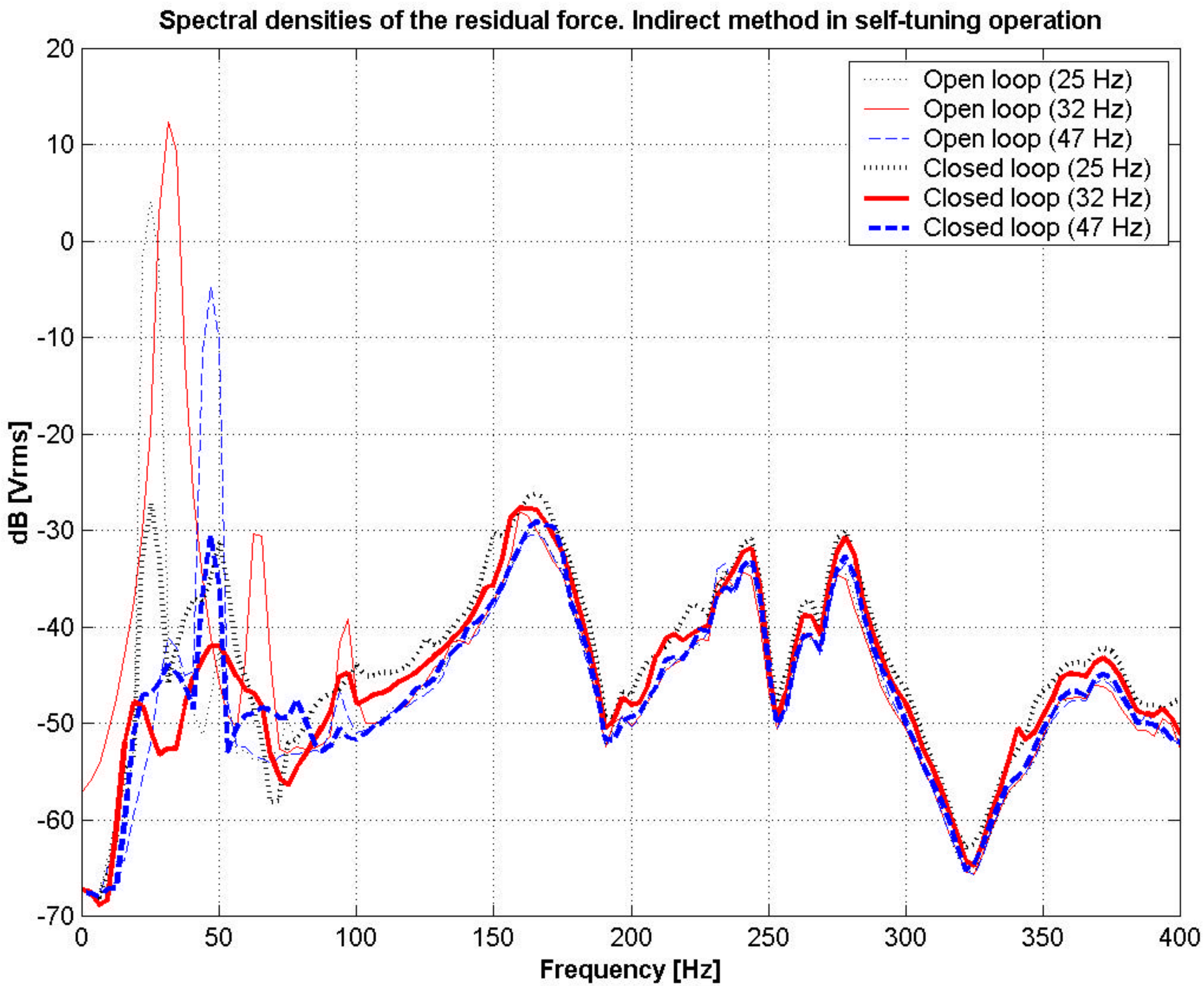
Time Domain Results
Self-tuning Operation

Indirect adaptive method **Direct adaptive method**

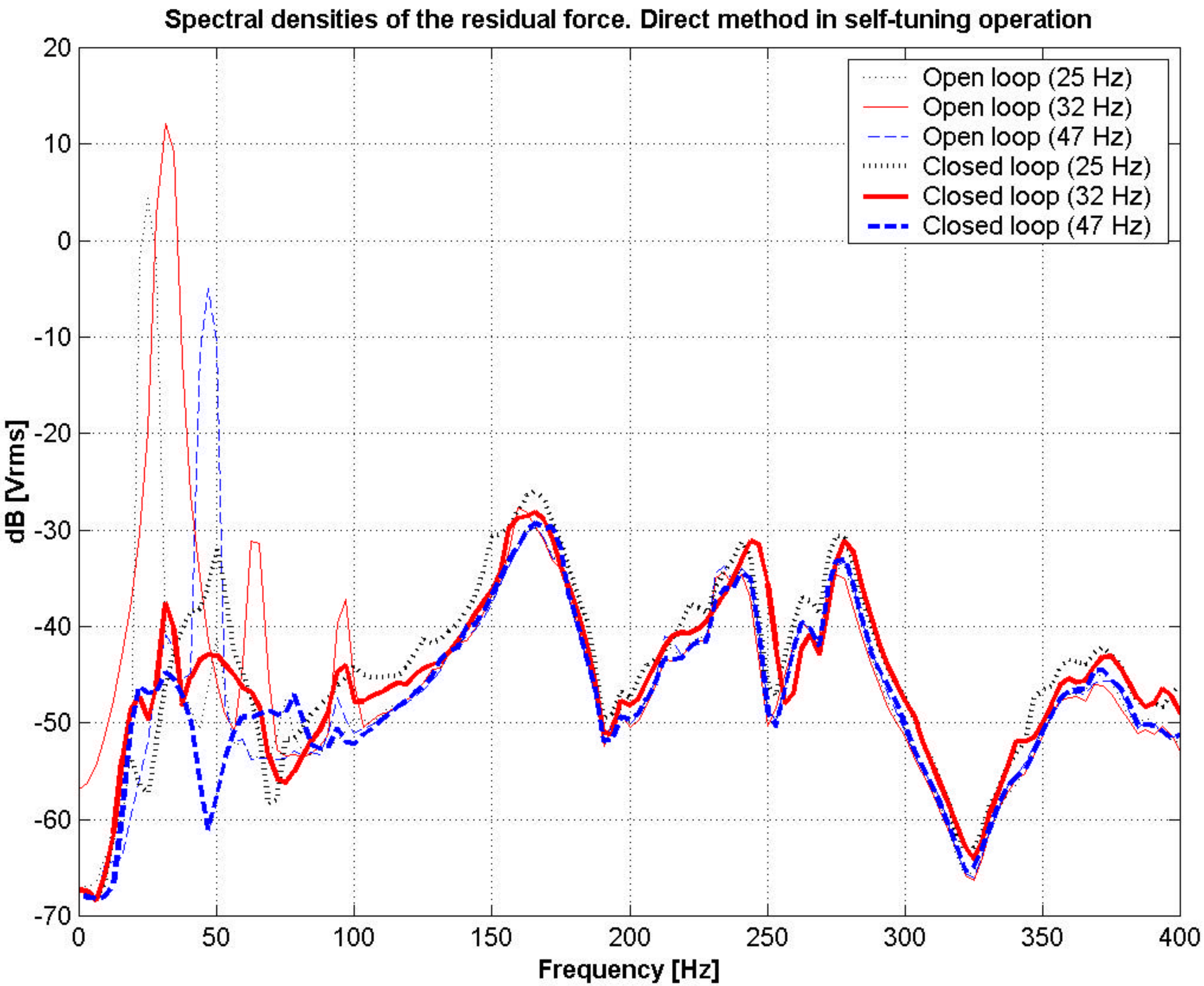




Frequency domain results – indirect adaptive method



Frequency domain results – direct adaptive method

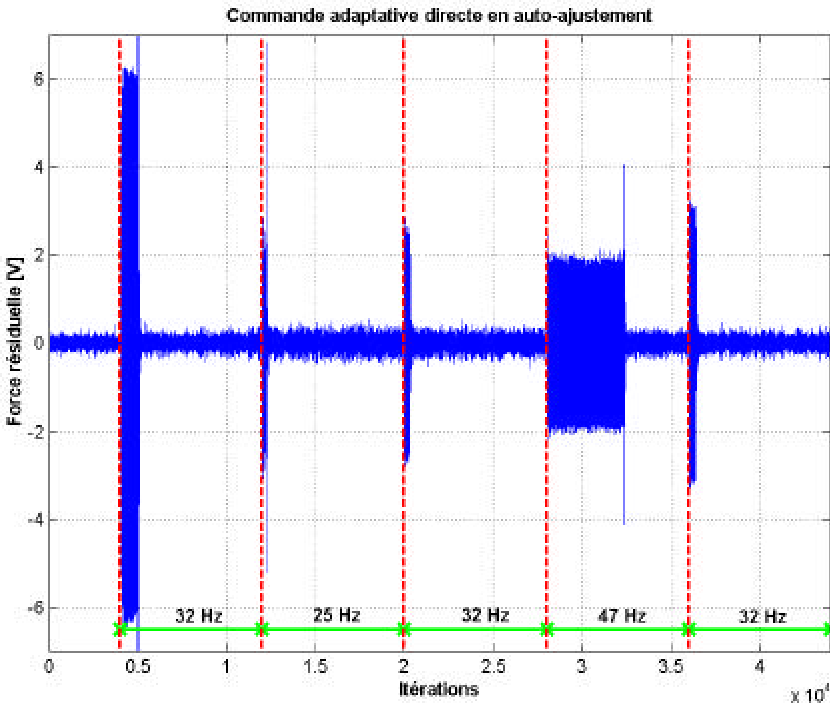


Real-time performances – indirect and direct methods

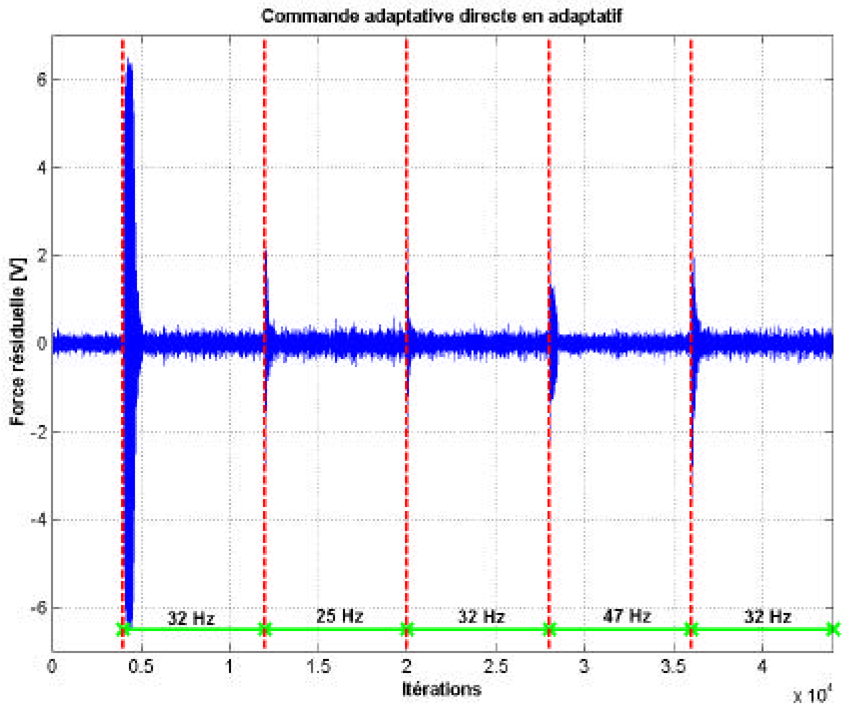
Method	**Indirect**			**Direct**		
Frequency [Hz]	25	32	47	25	32	47
Attenuation [dB]	31.32	64.23	25.72	61.65	49.64	55.79

Direct Adaptive Control

Self-tuning Mode



Adaptative Mode



- *Direct adaptive control in adaptive mode operation gives better results than direct adaptive control in self-tuning mode*
- *Direct adaptive control leads to a much simpler implementation than indirect adaptive control*

Conclusions

-Using internal model principle, adaptive control solutions can be provided for the tejection of unknown disturbances

-Both direct and indirect solutions can be provided

-Two modes of operation can be used : self-tuning and adaptive

-Direct adaptive control is the simplest to implement

-Direct adaptive control offers better performance

-The methodology has been extensively tested on an active suspension system