Robust R-S-T Digital Control
and
Open Loop System Identification
A Brief Review

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Applications of R-S-T Controllers

Peugeot (PSA)

Double Twist Machine (Pourtier)

Sollac (Florange)
Hot Dip Galvanizing

360° Flexible Arm (LAG)
Implementation of R-S-T Digital Controllers

PLC Leroy implements R-S-T digital controllers and Data acquisition modules

ALSPA 320 implements R-S-T digital controllers and Data acquisition modules
Controller Design and Validation

1) Identification of the dynamic model
2) Performance and robustness specifications
3) Compatible controller design method
4) Controller implementation
5) Real-time controller validation
   (and on site re-tuning)
6) Controller maintenance (same as 5)
Outline

Robust digital control
- The R-S-T digital controller
- Basic design
- Robustness issues
- An example

Open loop system identification
- Data acquisition
- Model complexity
- Parameter estimation
- Validation
Robust Digital Control
The R-S-T Digital Controller

\[ r(t) \quad u(t) \quad y(t) \]

Computer (controller)

\[ D/A \quad + \quad ZOH \]

PLANT

A/D

Discretized Plant

\[ \frac{B_m}{A_m} \quad T \quad + \quad \frac{1}{S} \quad u(t) \quad q^{-d} \frac{B}{A} \]

Controller

Plant Model

\[ q^{-1}y(t) = y(t-1) \]
The R-S-T Digital Controller

Controller

Plant Model:

\[ G(q^{-1}) = \frac{q^{-d} B(q^{-1})}{A(q^{-1})} = \frac{q^{-d-1} B^*(q^{-1})}{A(q^{-1})} \]

\[ A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_n q^{-n} \]
\[ B(q^{-1}) = b_1 q^{-1} + ... + b_n q^{-n} = q^{-1} B^*(q^{-1}) \]

R-S-T Controller:

\[ S(q^{-1}) u(t) = T(q^{-1}) y^*(t + d + 1) - R(q^{-1}) y(t) \]

Characteristic polynomial (closed loop poles):

\[ P(q^{-1}) = A(q^{-1}) S(q^{-1}) + q^{-d} B(q^{-1}) R(q^{-1}) \]
Pole Placement with R-S-T Controller

Controller: \[ R = H_R R' \quad ; \quad S = H_S S' \]

\[ H_R, H_S : \text{fixed parts} \]

Regulation: \( R' \) and \( S' \) solutions of:

\[ A H_S S' + q^{-d} B H_R R' = P = P_D P_F \]

Tracking: \[ T = P / B(1) \]

Reference trajectory: \[ y^* = (B_m / A_m) r \]

Controller:

1. \[ R = H_R R' \quad ; \quad S = H_S S' \]
2. \[ H_R, H_S : \text{fixed parts} \]
3. \[ R' \] and \( S' \) solutions of:
   \[ A H_S S' + q^{-d} B H_R R' = P = P_D P_F \]
4. Tracking:
   \[ T = P / B(1) \]
5. Reference trajectory:
   \[ y^* = (B_m / A_m) r \]

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Connections with other Control Strategies

- Digital PID: \( n_R = n_S = 2; H_S = 1 - q^{-1} \)

- Tracking and regulation with independent objectives (MRC):
  \[ P = B^* P_D P_F \]  
  (Hyp.: \( B^* \) has stable damped zeros)

- Minimum variance tracking and regulation (MVC):
  \[ P = B^* C \]  
  (Hyp.: \( B^* \) has stable damped zeros)

- Internal Model Control (IMC):
  \[ P = A P_F \]  
  (Hyp.: \( A \) has stable damped poles)
The Sensitivity Functions

Output sensitivity function (p -> y)

\[ S_{yp}(q^{-1}) = \frac{AS}{AS + q^{-d}BR} = \frac{AS}{P} \]

Input sensitivity function (p -> u)

\[ S_{up}(q^{-1}) = - \frac{AR}{AS + q^{-d}BR} = - \frac{AR}{P} \]

Noise sensitivity function (b -> y)

\[ S_{yb}(q^{-1}) = - \frac{q^{-d}BR}{AS + q^{-d}BR} = - \frac{q^{-d}BR}{P} \]

\[ S_{yp} - S_{yb} = 1 \]
Robustness Margins

Typical values:

\[ \Delta M \geq 0.5 \, \text{(-6dB)} , \quad \Delta \tau > T_s \]

\[ \Delta M \geq 0.5 \Rightarrow \Delta G \geq 2 ; \, \Delta \Phi > 29^\circ \]

The inverse is not necessarily true!

Modulus Margin:

\[ \Delta M = \left| 1 + H_{OL}(z^{-1}) \right|_{\text{min}} = \left| S_{yp}(z^{-1}) \right|_{\text{max}}^{-1} = \left\| S_{yp} \right\|_{\infty}^{-1} \]

Delay Margin:

\[ \Delta \tau = \min_i \frac{\Delta \Phi_i}{\omega_{CR}} \]
Robust Stability

Family of plant models:

\[ G' \in F(G, \delta, W_{xy}) \]

\[ G - nominal\ model; \quad \left\| \delta (z^{-1}) \right\|_\infty \leq 1 \]

\[ W_{xy} (z^{-1}) - size\ of\ uncertainty \]

Robust stability condition:

- a related sensitivity function

\[ \left\| S_{xy} W_{xy} \right\|_\infty < 1 \]

defines the size of the tolerated uncertainty

- a type of uncertainty

\[ \left| S_{xy} \right| < \left| W_{xy} \right|^{-1} \]

defines an upper template for the modulus of the sensitivity function

There also lower templates (because of the relationship between various sensitivity fct.)
\( |S_{yp}|_{\text{max}} = -\Delta M \)

Output Sensitivity Function

Input Sensitivity Function

\( |S_{up}|_{\text{max}}^{1} \)

size of the tolerated additive uncertainty \( W_{a} \)

( \( G' = G + \delta W_{a} \) )

nominal perform.
Robust Controller Design

Pole placement with sensitivity functions shaping

Nominal performance: $P_D$ and part of $H_R$ and $H_S$

$P = P_D P_F$
$R = R'(H_R)$
$S = S'(H_S)$

Allow to shape the sensitivity functions

Several approaches to design:

- Iterative
  Choosing $P_F$ and using band stop filters $H_{Ri} / P_{Fi}$, $H_{Sj} / P_{Fj}$

- Convex optimization
  (see Langer, Landau, Automatica, June99, Optreg (Adaptech))
360° Flexible Arm

- Mirror
- Detector
- Rigid frames
- Aluminium
- Light source
- Tach
- POT.
- Encoder
- Local position servo
- Computer

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360° Flexible Arm

Frequency characteristics

Poles-Zeros

(Identified Model)
Shaping the Sensitivity Functions

Output Sensitivity Function - $S_{yp}$

Input Sensitivity Function - $S_{up}$

A- without auxiliary poles
B- with auxiliary poles
C- with stop band filter $H_{S1} / P_{F1}$
D- with stop band filter $H_{S2} / P_{F2}$
Open Loop System Identification
I/O Data Acquisition under an Experimental Protocol

Model Complexity Estimation (or Selection)

Choice of the Noise Model

Parameter Estimation

Model Validation

Yes

No

Control Design

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Signal: a P.R.B.S sequence
Magnitude: few % of the input operating point
Clock frequency: $f_{\text{clock}} = \left(\frac{1}{p}\right)f_s$; $p = 1, 2, 3$ ($f_s = \text{sampling frequency}$)
Length: $(2^{N-1} - 1)pT_s$; $N = \text{number of cells}$, $T_s = 1/f_s$
Largest pulse: $NpT_s$

**Length:** < allowed duration of the experiment
**Largest pulse:** $\geq t_R$ (rise time)
An I/O File

![Diagram of a control system with components such as a power amplifier, filter, d.c. motor, tacho generator, and an input-output file. The diagram shows the flow of signals represented as functions u(t) and y(t).]
Complexity Estimation from I/O Data

Objective:
To get a good estimation of the model complexity \((n_A, n_B, d)\) directly from noisy data

\[ n = \max(n_A, n_B + d) \]

\[ \hat{n}_{opt} = \min_{\hat{n}} CJ = \min_{\hat{n}} [J(\hat{n}) + S(\hat{n}, N)] \]

To get a good order estimation, \(J\) should tend to the value for noisy free data when \(N \rightarrow \infty\) (use of instrumental variables)
Parameter Estimation

Discretized plant

DAC + ZOH \rightarrow \text{Plant} \rightarrow \text{ADC} \rightarrow y(t)

\epsilon(t) \rightarrow y(t) \rightarrow \hat{y}(t)

Estimated model parameters $\hat{\theta}(t)$

Parameter adaptation algorithm
\( S1: A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + e(t) \)

\( S2: A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + A(q^{-1})w(t) \)

\( S3: A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})e(t) \)

\( S4: A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + \left[1/C(q^{-1})\right]e(t) \)

- **Extended Least Squares**
  - O.E. with Extended Prediction Model
  - (Recursive) Maximum Likelihood

- **Generalized Least Squares**
  - Instrumental Variable...

- **Recursive Least Squares**

**Plant + Noise » Models**
Parameter Estimation Methods

Plant Model

\[ y(t + 1) = -A^*(q^{-1})y(t) + B^*(q^{-1})u(t - d) = \theta^T \psi(t) \]

\( \theta \) – parameter vector; \( \psi \) – measurement vector

Estimated model

\[ \hat{y}^0(t + 1) = \hat{\theta}^T(t)\phi(t) \]

\( \hat{\theta} \) – estimated parameter vector; \( \phi \) – observation vector

Prediction error

\[ \varepsilon^0(t + 1) = y(t + 1) - \hat{\theta}^T(t)\Phi(t) = y(t + 1) - \hat{y}^0(t + 1) \]

Parameter adaptation algorithm

\[ \hat{\theta}(t + 1) = \hat{\theta}(t) + F(t + 1)\Phi(t)\varepsilon^0(t + 1) \]

\[ F^{-1}(t + 1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^T(t) \]

\( 0 < \lambda_1(t) \leq 1; 0 \leq \lambda_2(t) < 2 \)

\[ \Phi(t) = f[\phi(t)] \]
Parameter Estimation Methods

I- Based on the asymptotic whitening of the prediction error
(Recursive Least Squares, Extended Least Squares, Recursive Max. Likelihood, O.E. with Extended Predictiton Model )

II- Based on the asymptotic decorrelation between the prediction error and the observation vector
(Output Error, Instrumental Variable)
Validation of Identified Models

Statistical Validation

ARMAX (ARARX) predictor

Output Error Predictor

Plant

Model

$y + \varepsilon$ Whiteness Test

$u$ $q^{-1}$

$y - \varepsilon$

$|RN(i)| \leq \frac{2.17}{\sqrt{N}} ; i \geq 1$

normalized crosscorrelation number of data

$1 \div \sqrt{N}$

$2.17 \div \sqrt{N}$

97%

$N = 256 \rightarrow |RN(i)| \leq 0.136$

practical value: $|RN(i)| \leq 0.15$

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Software Tools for Implementing the Methodology

System Identification
- Winpim (Adaptech)
  identification in open loop and closed loop operation
- CLID (Adaptech)
  identification in closed loop (Matlab Toolbox)

Controller Design
- Winreg (Adaptech)
  design and optimisation of R-S-T digital controllers
- Optreg (Adaptech)
  automated design of robust digital controllers (under Matlab)

Real-time implementation
- Wintrac (Adaptech): cascade digital control
« Personal » References


