DIRECT CONTROLLER REDUCTION BY IDENTIFICATION IN CLOSED LOOP

(The Daphné Algorithms)

Application to Active Suspension Control

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CONTROLLER REDUCTION. Why ?

- Complex Models → High Order Controllers
- Robust Control Design → High Order Controllers

Example : The Flexible Transmission
(Robust control benchmark, EJC no. 2/1995 and no.2-4/1999)

Model complexity : $G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}$ $n_A = 4$; $n_B = 2$; $d = 2$

Fixed controller part : Integrator

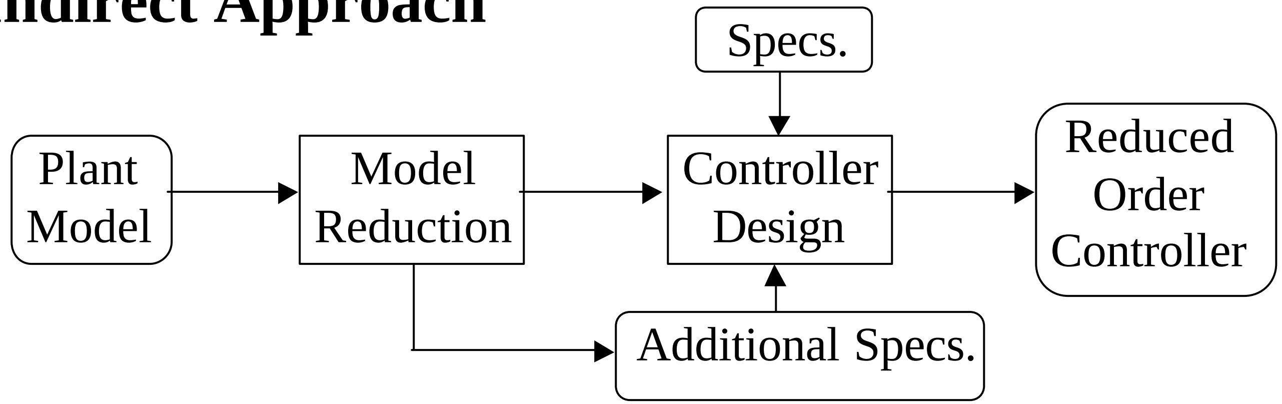
Pole placement design : $K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}$ $n_R = 4$; $n_S = 4$

Complexity of controllers achieving 100 % of specifications:

Max : $n_R = 9$; $n_S = 9$ (Nordin) **Min** : $n_R = 7$; $n_S = 7$ (Langer)

Approaches to Controller Reduction

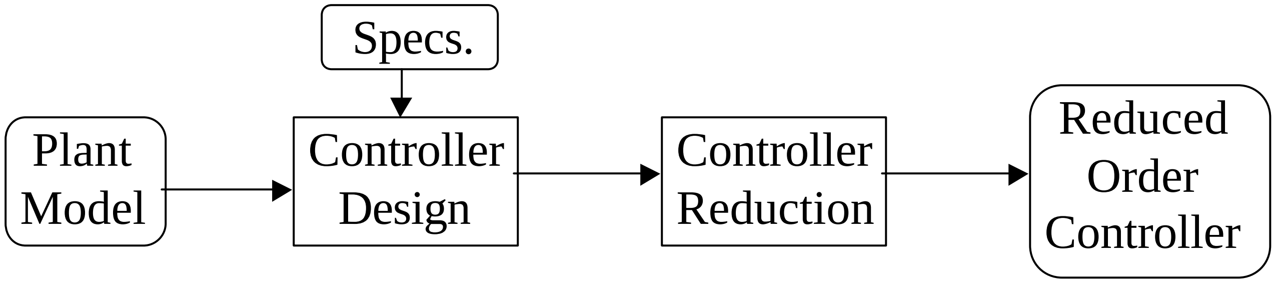
Indirect Approach



-Does not guarantee resulting controllers of desired order

- Propagation of model errors

Direct Approach



- Approximation carried in the final step
- Further controller reduction for "indirect approach"

Controller Reduction

Basic rule :

Controller reduction should be done with the aim to preserve as much as possible the closed loop properties.

Reminder :

Controller reduction without taking into account the closed loop properties can be a disaster !

Some basic references :

- Anderson &Liu : IEEE-TAC, August 1989
- Anderson : IEEE Control Magazine, August 1993

Identification in Closed Loop and Controller Reduction

- Identification in closed loop is an efficient tool for control oriented model order reduction

- Closed loop identification techniques can be used (with small changes) for direct estimation of reduced order controllers

Duality

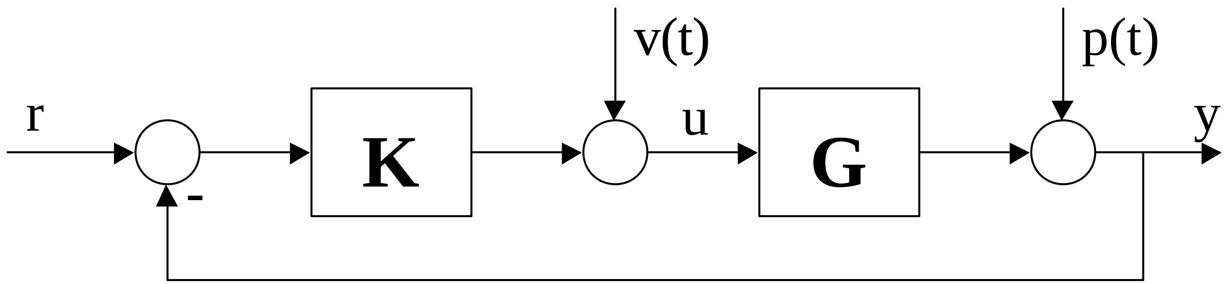
Identification of reduced order models in closed loop	⟺	Identification of reduced order controllers in closed loop

- Possibility of using “real data” for controller reduction

Outline

- Introduction
- Notations
- Specific Objectives
- Basic Schemes
- The Daphné Algorithms
- Properties of the algorithms
- Properties of the estimated reduced order controllers
- Validation of reduced order controllers
- Experimental results (Active Suspension Control)
- Practical Hints
- Conclusions

Notations



$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \qquad K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}$$

Sensitivity functions :

$$S_{yp}(z^{-1}) = \frac{1}{1+KG} \;;\; S_{up}(z^{-1}) = -\frac{K}{1+KG} \;;\; S_{yv}(z^{-1}) = \frac{G}{1+KG} \;;\; S_{yr}(z^{-1}) = \frac{KG}{1+KG}$$

Closed loop poles : $P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$

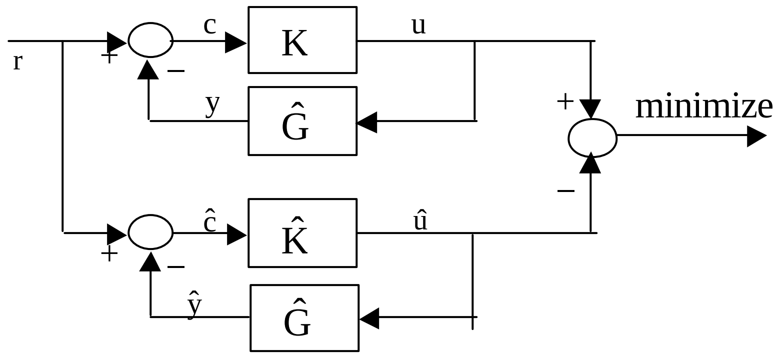
True closed loop system :(K,G), P, S_{xy}

Nominal simulated closed loop : $(K, \hat{G}), \hat{P}, \hat{S}_{xy}$

Simulated C.L. using reduced order controller : $(\hat{K}, \hat{G}), \hat{\hat{P}}, \hat{\hat{S}}_{xy}$

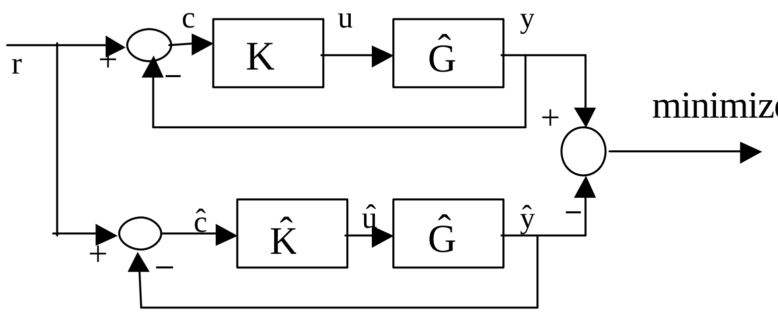
Controller Reduction - Objectives

Input matching



$$\hat{K}^* = \arg\min_{\hat{K}} \left\| \hat{S}_{up} - \hat{\hat{S}}_{up} \right\| = \arg\min_{\hat{K}} \left\| \hat{S}_{yp} (K - \hat{K}) \hat{\hat{S}}_{yp} \right\|$$

Ouput matching

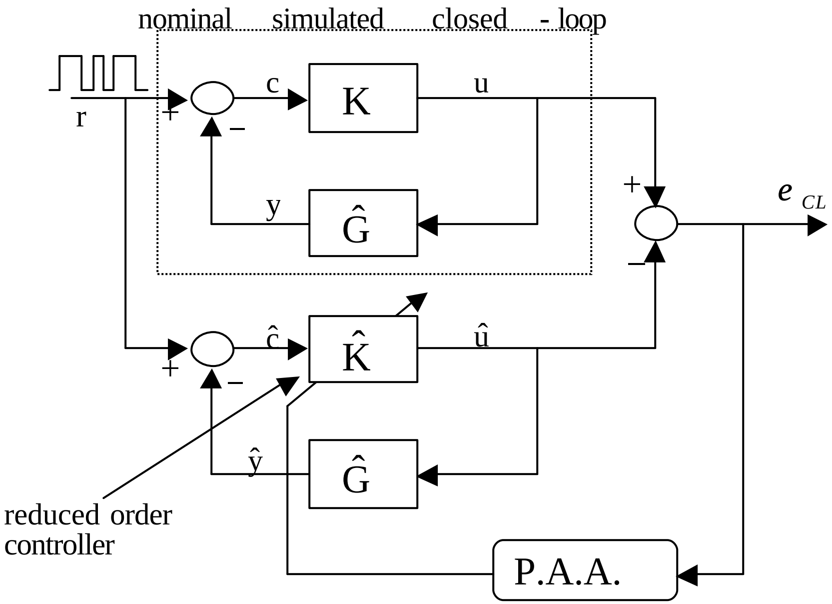


$$\hat{K}^* = \arg\min_{\hat{K}} \left\| \hat{S}_{yr} - \hat{\hat{S}}_{yr} \right\| = \arg\min_{\hat{K}} \left\| \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right\| = \arg\min_{\hat{K}} \left\| \hat{S}_{yp} (K - \hat{K}) \hat{\hat{S}}_{yv} \right\|$$

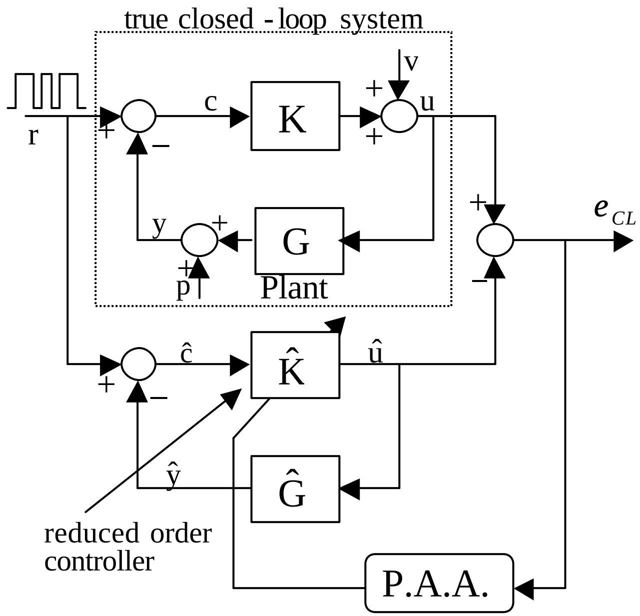
Identification of Reduced Order Controllers

Input Matching (CLIM)

Use of simulated data



Use of real data

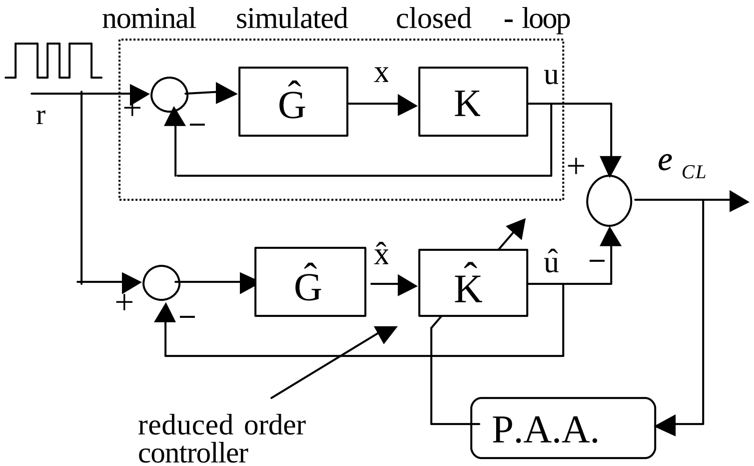


Remarks on the use of real data:

- new $\hat{G}$ identified in closed loop can be used (can be better than the design model)
- $\hat{K}$ will try to minimize the discrepancy between the two loops (will take into account $(G - \hat{G})$)

Identification of Reduced Order Controllers

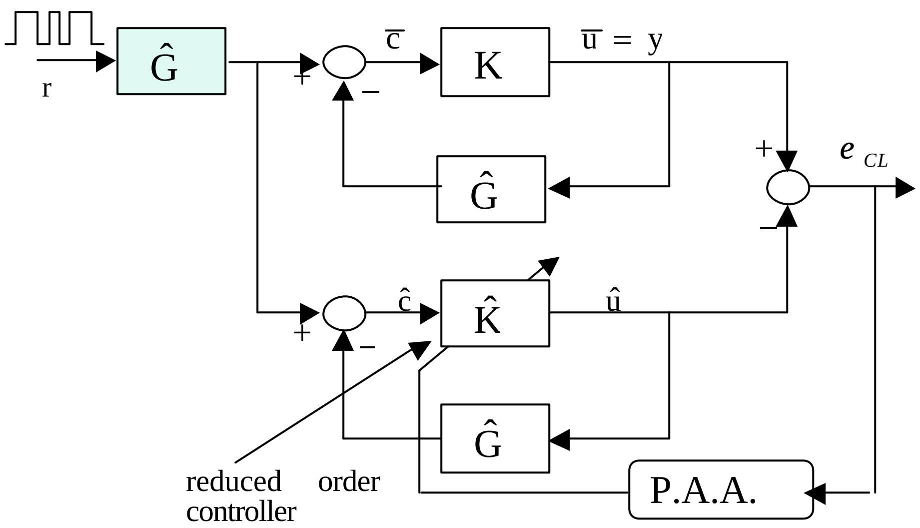
Ouput Matching (CLOM)



-The transfer fct. between r and u is S_{yp}

-The position of K and $\hat{G}$ can be interchanged

An alternative realization :

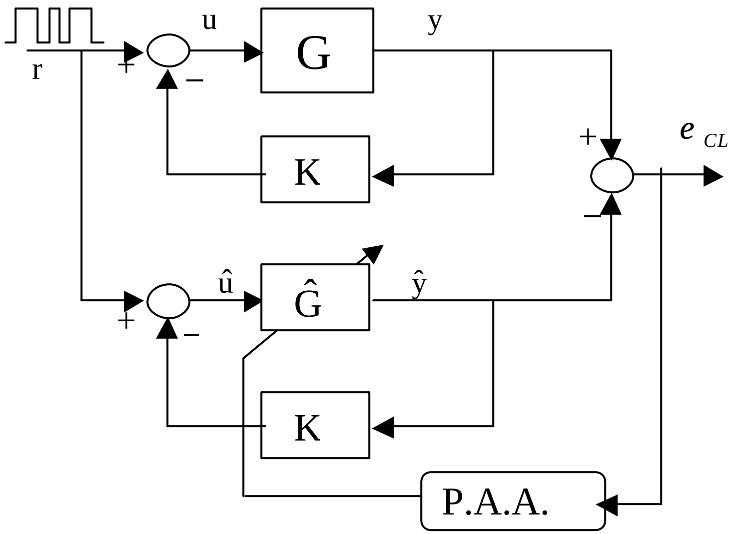


- CLIM algorithm but on filtered data

- Same asymptotic steady state properties as CLOM

Duality

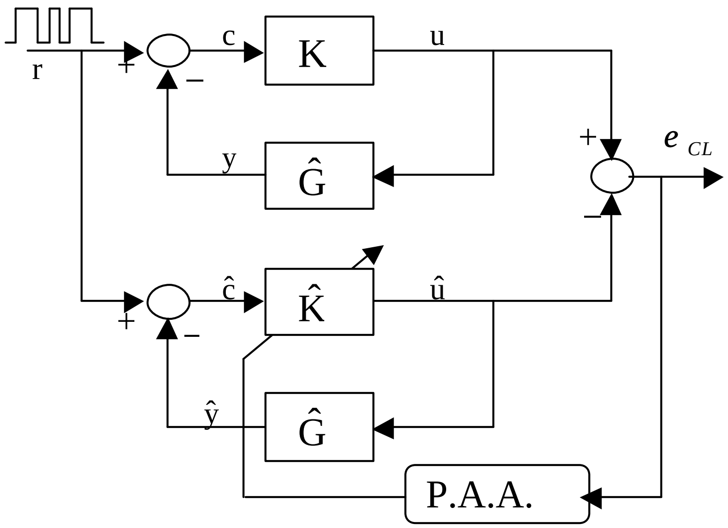
CLOE



Plant model identification in closed loop

$$G \rightarrow K$$
$$\hat{G} \rightarrow \hat{K}$$
$$K \rightarrow \hat{G}$$

Daphné (CLIM)



Reduced order controller identification in closed loop

Remark : one should take care of the structure of R and B

CLIM Algorithm

$$u(t+1) = -S^*(q^{-1})u(t) + R(q^{-1})c(t+1)\ ;\ \ (S(q^{-1}) = 1 + q^{-1}S^*(q^{-1}))$$

$$\hat{u}^0(t+1) = -\hat{S}^*(t,q^{-1})\hat{u}(t) + \hat{R}(t,q^{-1})\hat{c}(t+1) = \hat{\theta}^T(t)\phi(t)$$

$$\hat{\theta}^T(t) = \left[\hat{s}_1(t),\ldots\hat{s}_{n_{\hat{S}}}(t), \hat{r}_0(t),\ldots\hat{r}_{n_{\hat{R}}}(t)\right]$$

$$\phi^T(t) = \left[-\hat{u}(t),..-\hat{u}(t-n_{\hat{S}}+1), \hat{c}(t+1),..\hat{c}(t-n_{\hat{R}}+1)\right]$$

$$\hat{c}(t+1) = r(t+1) - \hat{y}(t+1) = r(t+1) + \hat{A}^*\hat{y}(t) - \hat{B}^*u(t-d)$$

$$\varepsilon^0_{CL}(t+1) = u(t+1) - \hat{u}^0(t+1)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1)\Phi(t)\varepsilon^0_{CL}(t+1)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^T(t)\ ;\ 0 < \lambda_1(t) \le 1\ ;\ 0 \le \lambda_2(t) < 2$$

Choice of $\Phi(t)$:

$$\text{CLIM}: \Phi(t) = \phi(t) \qquad\qquad \text{F}-\text{CLIM}: \Phi(t) = \frac{\hat{A}(q^{-1})}{\hat{P}(q^{-1})}\phi(t)$$

CLOM Algorithm

Idem CLIM but r is added to the plant input

Forcing fixed parts in the reduced order controller:

$\hat{K} = K_F \hat{K}', \quad K_F$ is known

Same algorithm but $\hat{c}$ is replaced by $K_F \hat{c}$

Stability Analysis

A) $n_{\hat{R}} = n_R \; ; \; n_{\hat{S}} = n_S$

$$\lim_{t\to\infty} \varepsilon_{CL}(t+1) = \lim_{t\to\infty} \varepsilon^0_{CL}(t+1) = 0$$

if:

$$(*) \quad H'(z^{-1}) = H(z^{-1}) - \frac{\lambda}{2}; \; \max_t \lambda_2(t) \le \lambda < 2$$

is a *strictly positive real transfer function* where:

$$H = \begin{cases} \hat{A}/\hat{P} & \text{for CLIM} \\ 1 & \text{for F-CLIM} \end{cases}$$

B) $n_{\hat{R}} < n_R \; ; \; n_{\hat{S}} < n_S$

Hypotheses:

A stabilizing controller with orders $n_{\hat{R}}$ and $n_{\hat{S}}$ exists

$$u(t+1) = -\hat{S}^*(q^{-1})u(t) + \hat{R}(q^{-1})c(t+1) + \eta(t+1)$$

$r(t), \eta(t) =$ norm bounded

All signals are norm bounded under the passivity condition (*)

Asymptotic Properties of the Estimated Controller

$\hat{\theta}^*$ - vector of the estimated controller parameters

Closed Loop Input Matching

Simulated Data

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \hat{S}_{up} - \hat{\hat{S}}_{up} \right|^2 \phi_r(\omega) d\omega = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \hat{S}_{yp} \right|^2 \left| K - \hat{K} \right|^2 \left| \hat{\hat{S}}_{yp} \right|^2 \phi_r(\omega) d\omega$$

- $\left\| \hat{S}_{up} - \hat{\hat{S}}_{up} \right\|_2$ is minimized if r(t) is white noise
- The frequency distribution of $\left| K - \hat{K} \right|^2$ is weighted by the output sensitivity functions for the nominal and for the reduced order controller
- The frequency distribution of $\left| K - \hat{K} \right|^2$ can be tuned by the choice of r(t)

Asymptotic Properties of the Estimated Controller

Closed Loop Input Matching

Use of Real Data

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left\{ \left| S_{up} - \hat{\hat{S}}_{up} \right|^2 \phi_r(\omega) + \left| S_{yp} \right|^2 \phi_{v'}(\omega) \right\} d\omega$$

$v'(t) = v(t) - Kp(t)$: equivalent input noise

- The noise does not affect estimation of controller parameters

- When using real data, the closed loop system with reduced order controller approximates the real closed loop system (instead of the *nominal simulated system*)

Asymptotic Properties of the Estimated Controller

Closed Loop Output Matching

Simulated Data

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right|^2 \phi_r(\omega) d\omega = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \hat{S}_{yp} \right|^2 \left| K - \hat{K} \right|^2 \left| \hat{\hat{S}}_{yv} \right|^2 \phi_r(\omega) d\omega$$

- $\left\| \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right\|_2$ is minimized if r(t) is white noise
- The frequency distribution of $\left| K - \hat{K} \right|^2$ is weighted by $\hat{S}_{yp}$ and $\hat{\hat{S}}_{yv}$
- The frequency distribution of $\left| K - \hat{K} \right|^2$ can be tuned by the choice of r(t)

Asymptotic Properties of the Estimated Controller

Closed Loop Output Matching

Use of Real Data

$$\hat{q}^* = \arg\min_{q} \int_{-p}^{p} \left\{ \left| \hat{\hat{S}}_{up}(G-\hat{G})S_{yp} - \hat{\hat{S}}_{yp}(K-\hat{K})S_{yv} \right|^2 f_r(w) + \left| S_{yp} \right|^2 f_{v'}(w) \right\} dw = \arg\min_{q} \int_{-p}^{p} \left\{ \left| S_{yp} - \hat{\hat{S}}_{yp} \right|^2 f_r(w) + \left| S_{yp} \right|^2 f_{v'}(w) \right\} dw$$

- The noise does not affect estimation of controller parameters
- Minimization of $\left| K - \hat{K} \right|^2$ in the frequency regions where the $\left| S_{yp} \right|$ and $\left| \hat{\hat{S}}_{yv} \right|$ are high
- Minimization of the gain of $\hat{\hat{S}}_{up}$ at the frequencies where important additive modeling errors exist and the gain of the estimated model is low

Validation of Estimated Reduced Order Controllers

Simulated Data

-The reduced order controller should **stabilize** the nominal model

- The (reduced) sensitivity functions should be **close** to the nominal ones in the critical regions for performance and robustness

- The generalized stability margin for the reduced order system should be **close** to the nominal one

Validation tools :

– ν-gap between nominal and "reduced order" sensitivity fct.

$$\delta_\nu\left(S,\hat{S}\right)=\left\|(1+S'S)^{-1/2}(S-\hat{S})(1+\hat{S}'\hat{S})^{-1/2}\right\|_\infty<1$$

(+ winding number condition. S' denotes complex conjugate of S)

- Visual comparison

One assumes: $\hat{G}=G$ *! (as everybody in reduction business)*

Vinnicombe Stability Test

Initial robust design: $d_n(G_1, G_2) \leq b(K_{nom}, G_1)$

$$b(K,G) = \begin{cases} \|T(K,G)\|_\infty^{-1} & \text{if } (K,G) \text{ is stable} \\ 0 & \text{otherwise} \end{cases}$$

$$T(K,G) = \begin{bmatrix} S_{yr} & S_{yv} \\ -S_{up} & S_{yp} \end{bmatrix}$$

We would like to have: $d_n(G_1, G_2) \leq b(\hat{K}, G_1)$

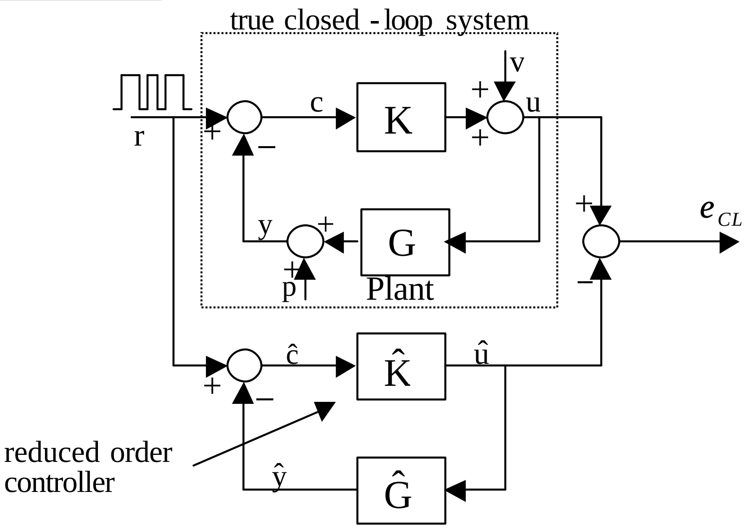
Validation test: $\left| b(K, G_1) - b(\hat{K}, G_1) \right| < e \; ; \; e > 0$

Validation of Estimated Reduced Order Controllers

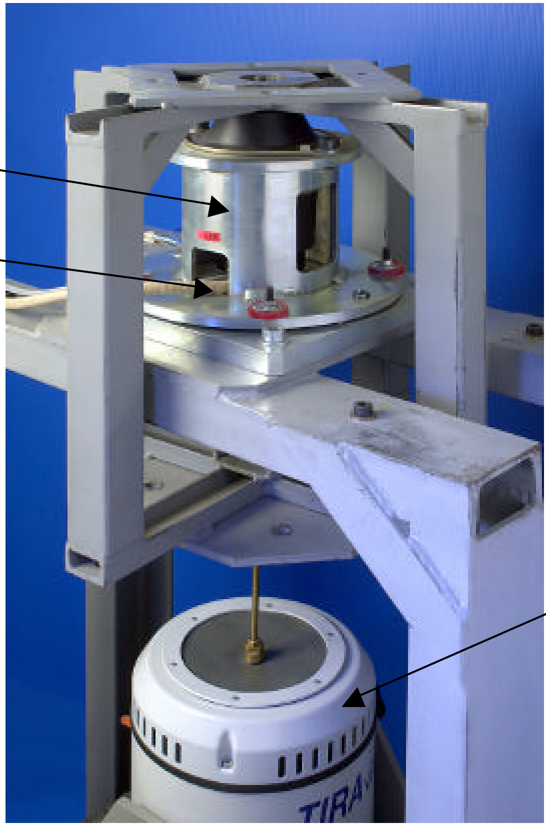
Use of Real Data

- Statistical tests (like in closed loop identification)
  - *variance of residual closed loop error*
  - *cross-correlations* $(\varepsilon_{CL} / \hat{u})$
- Vinnicombe gap between :

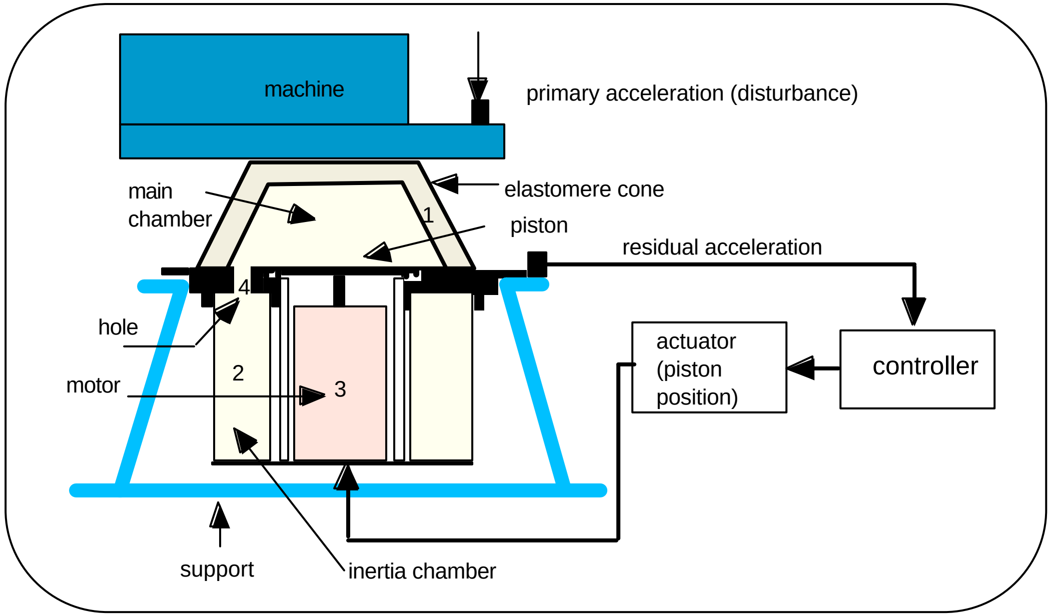
Identified T.F. of the true nominal closed loop **and** *Computed T.F. of the simulated closed loop with reduced order controller*

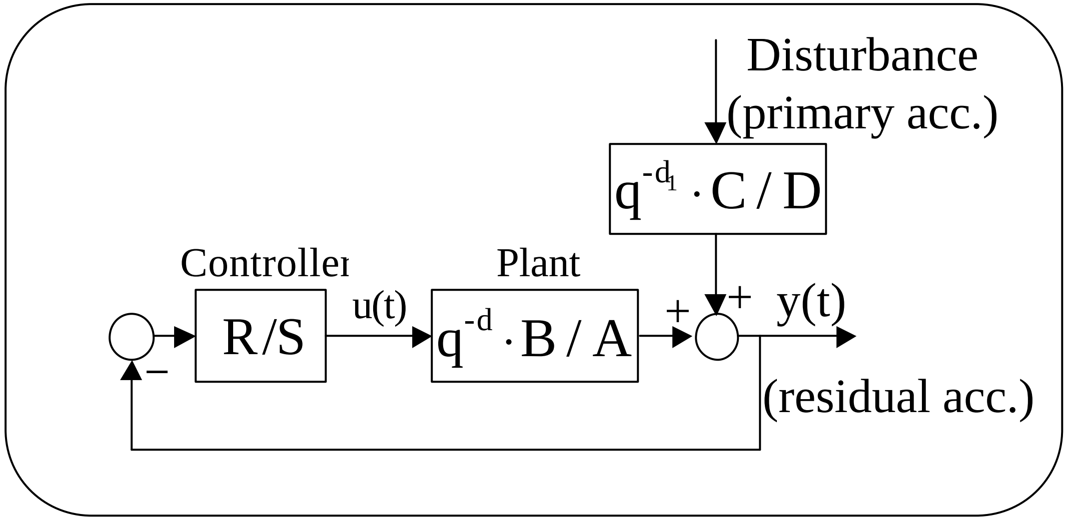


The Active Suspension



Experimental Results - Control of an Active Suspension





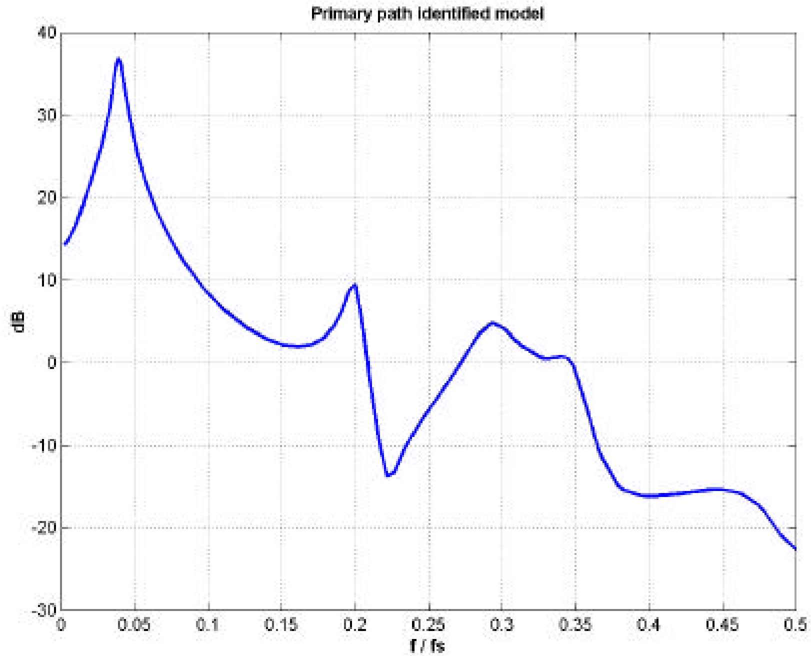
- controller: PC
- sampling freq.: 800 Hz

Interesting frequency range for vibration attenuation: 0 - 200 Hz

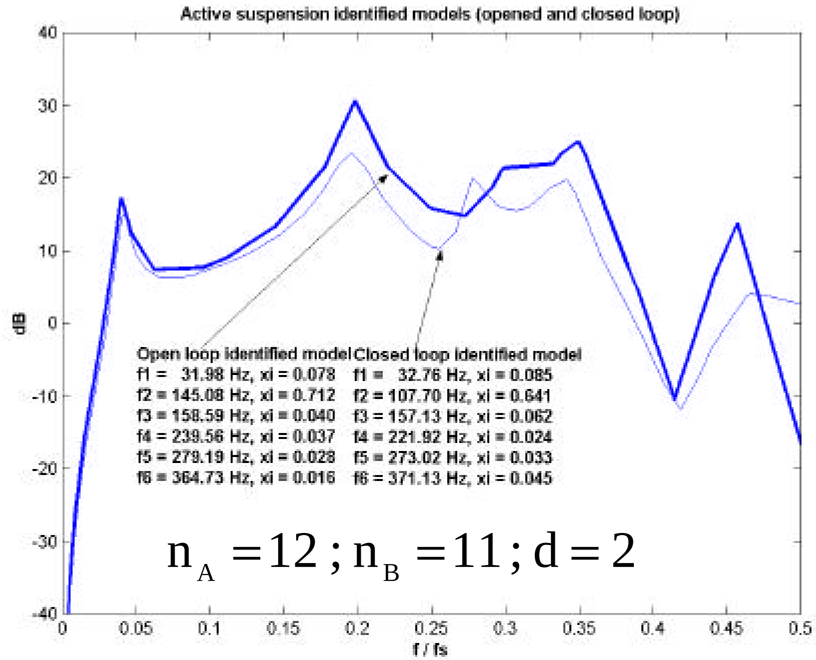
Active Suspension

Frequency Characteristics of the Identified Model

Primary path



Secondary path



$n_A = 12 ; n_B = 11 ; d = 2$

Control objectives :

- Minimize residual acc. around first vibration mode
- Distribute amplification of disturb. over high frequency region

- Open loop identified model (design model)
- Closed loop identified model used for controller reduction (better C.L. validation)

The Nominal Controller

Important attenuation of S_{yp} at the frequency of the first vibration mode (32 Hz)

Design method: Pole placement with sensitivity shaping using convex optimization

Dominant poles : first vibration mode with ξ=0.8 (instead of 0.078)

Opening of the loop at 0.5f_s : $H_R = 1 + q^{-1}$; $(R = H_R R')$

Nominal controller complexity : $n_R = 27$; $n_S = 28$

Pole placement complexity : $n_R = 12$; $n_S = 13$

Direct Controller Reduction

CLIM algorithm/ simulated data

r(t) = PRBS, L= 4096 , clock = 0.5f_S , N=10

P.A.A.: *variable forgetting factor*

$H_R = 1 + q^{-1}$; $(\hat{K} = H_R \hat{K}')$

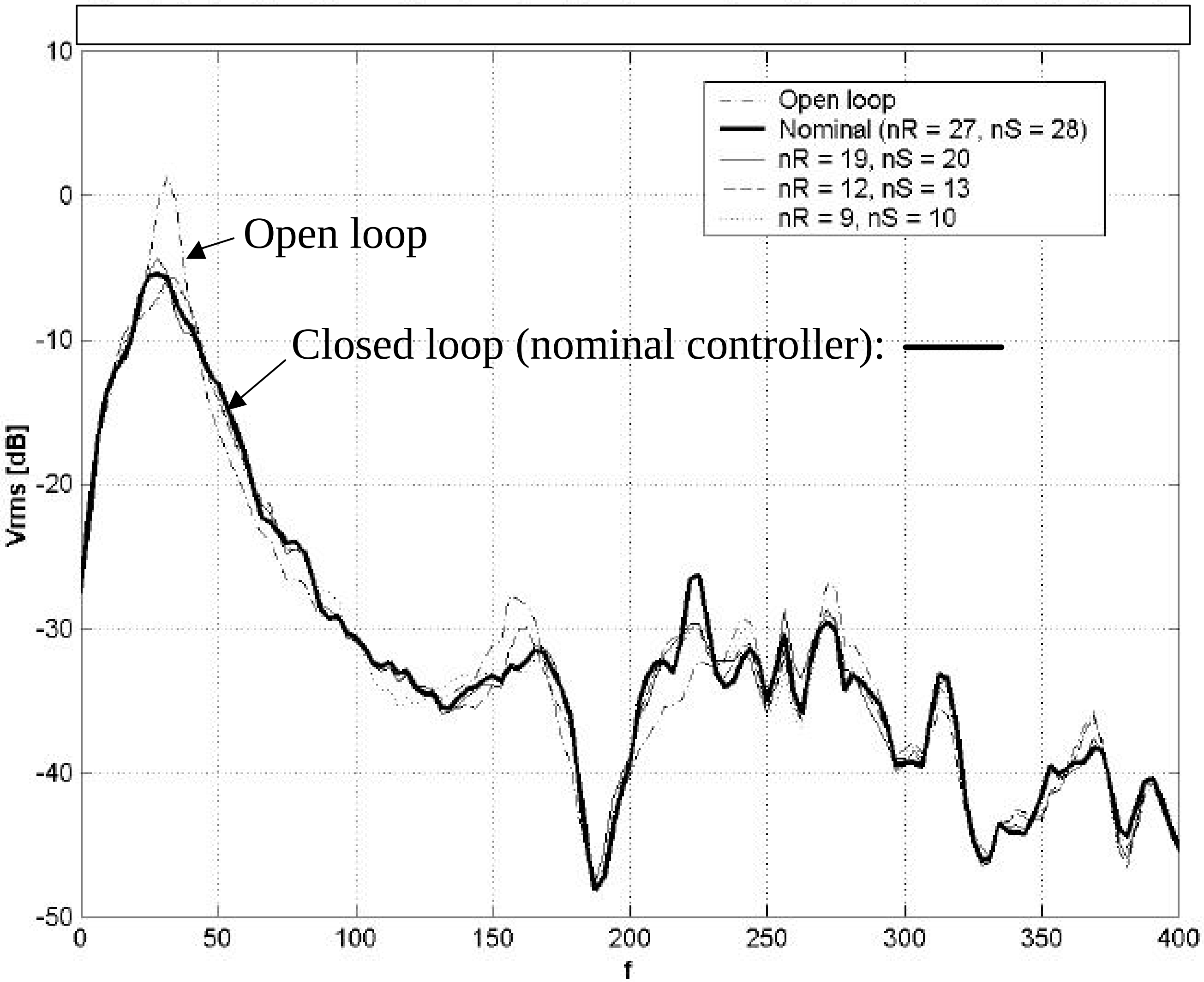
	Controller	K_n $n_R = 27$ $n_S = 28$	K_1 $n_R = 19$ $n_S = 20$	K_2 $n_R = 12$ $n_S = 13$	K_3 $n_R = 9$ $n_S = 10$
	$d_n(K_n, K_i)$	0	0.1810	0.5049	0.5180
	$d_n(S^n_{up}, S^i_{up})$	0	0.1487	0.4388	0.4503
	$d_n(S^n_{yp}, S^i_{yp})$	0	0.0928	0.1206	0.1233
	$b(k)$	0.0800	0.0786	0.0685	0.0810
real time experiments	$d_n(CL(K_n), CL(K_i))$	0.1296	0.2461	0.5435	0.5522
real time experiments	*C.L. error variance*	0.0023	0.0083	0.0399	0.0398

Performances of the reduced order controllers are very close to those of the nominal controller (see next slide)

Direct Controller Reduction

CLIM algorithm/ simulated data

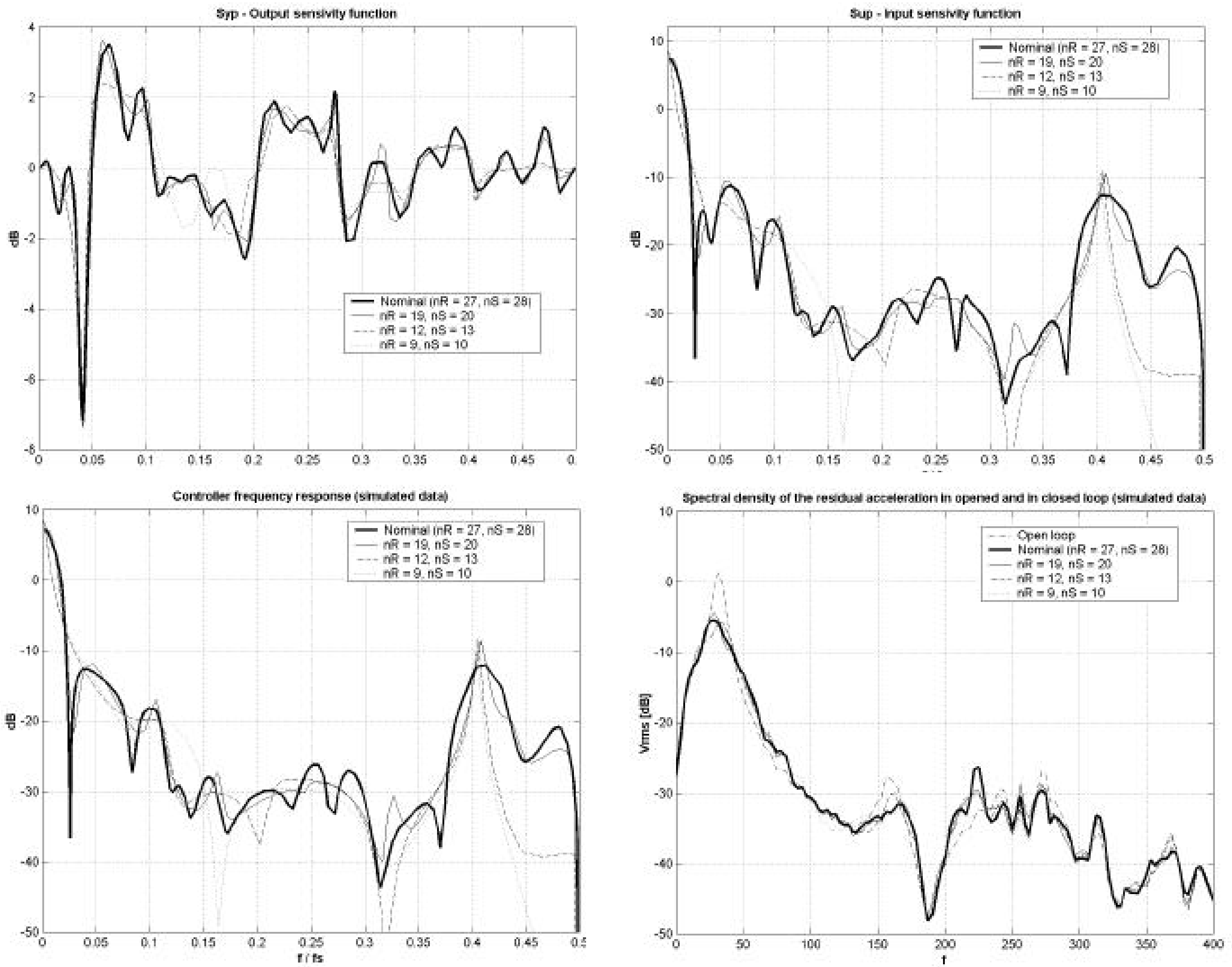
Spectral density of the residual acceleration (performance)



Direct Controller Reduction

CLIM algorithm/ simulated data

S_{yp} S_{up}



K

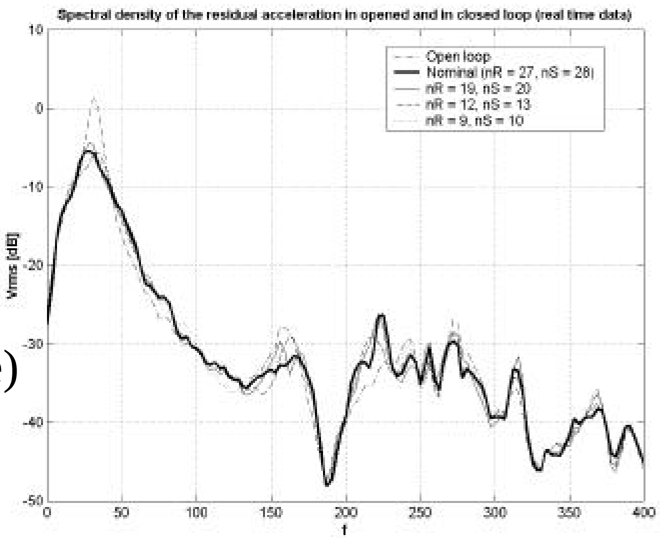
Spectral density of residual acceleration (performance)

Direct Controller Reduction

CLIM algorithm/ use of real data

Controller	K_n $n_R = 27$ $n_S = 28$	K_1 $n_R = 19$ $n_S = 20$	K_2 $n_R = 12$ $n_S = 13$	K_3 $n_R = 9$ $n_S = 10$
$d_n(K_n, K_i)$	0	0.1500	0.4870	0.5216
$d_n(S_{up}^n, S_{up}^i)$	0	0.1285	0.4197	0.4560
$d_n(S_{yp}^n, S_{yp}^i)$	0	0.1719	0.1639	0.1150
$b(k)$	0.0800	0.0722	0.0605	0.0823
real time experiments { $d_n(CL(K_n), CL(K_i))$	0.1296	0.1959	0.5230	0.5602
real time experiments { *C.L. error variance*	0.0023	0.0072	0.0359	0.0422

Spectral density of residual acceleration (performance)



Results are very close to those obtained with simulated data

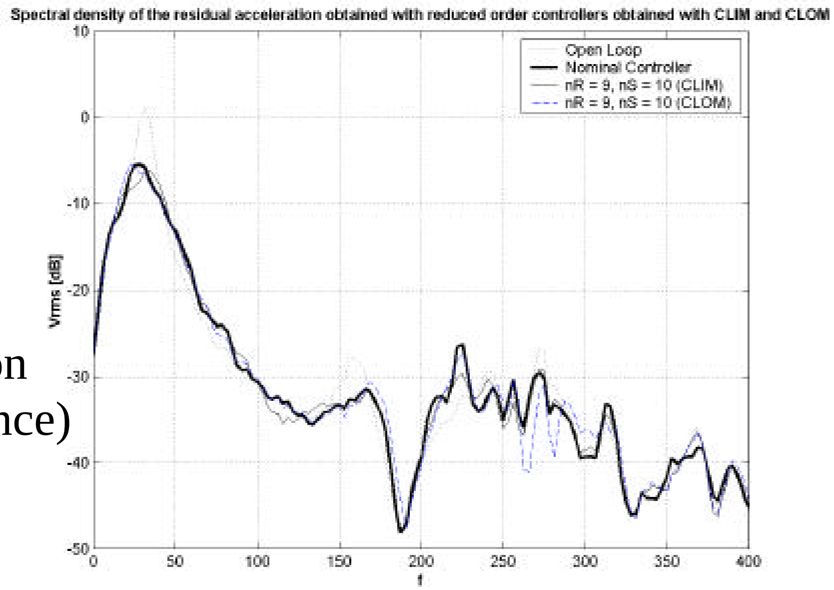
Explanation :
Quality of the model used for controller reduction

Direct Controller Reduction

CLOM algorithm/ simulated data

Controller	K_n $n_R = 27$ $n_S = 28$	K_2 $n_R = 12$ $n_S = 13$	K_3 $n_R = 9$ $n_S = 10$
$d_n(K_n, K_i)$	0	0.7287	0.7743
$d_n(S^n_{up}, S^i_{up})$	0	0.7144	0.7709
$d_n(S^n_{yp}, S^i_{yp})$	0	0.0975	0.1007
$b(k)$	0.0800	0.0786	0.0796

Spectral density of residual acceleration (performance)



– Smaller $\delta_\nu(S^n_{yp}, S^i_{yp})$ with CLOM

– Smaller $\delta_\nu(S^n_{up}, S^i_{up})$ with CLIM

(coherent with the theory)

CLIM and CLOM provide reduced order controllers with good performances

Practical Hints

A) No access to real-time data

Classical situation for controller reduction techniques

Given : nominal plant model, nominal controller

B) Access to the real system

- Improve the quality of the model by identification in closed loop
- Use also real data for direct controller reduction
- Do real time validation of the reduced order controllers

COHERENCE

What closed loop plant model identification scheme should be used when a criterion for controller reduction is given ?

Answer: *Same criterion for identification in closed loop and controller reduction*

- ***Tracking and output disturbance rejection***

CLOE with excitation added to controller input → Model identification or Model reduction

CLIM with excitation added to plant input → Controller reduction

In both schemes: $\left\| \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right\|_2$ **is minimized**

Concluding Remarks

- The Daphné algorithms (CLIM,CLOM) allow to directly estimate reduced order controllers
- The algorithms achieve a two norm minimization between nominal and reduced order sensitivity functions
- They have the unique feature of using also real data (this allows to take in account to a certain extent the modeling error)
- Direct estimation of reduced order controllers can be interpreted as the *dual* of reduced order plant model identification in closed loop
- **Successful use in practice**
- A MATLAB Toolbox is available (REDUC)
- There is an interaction between closed loop identification and direct controller reduction (*coherence*)

Future work :
- multivariable case

References

Anderson B.D.O., Liu Y., (1989) : « Controller reduction : concepts and approaches », *IEEE Trans. on Automatic Control*, vol. 34, no. 8, pp. 802-812.

Anderson B.D.O., (1993) : « Controller reduction : moving from theory to practice », *IEEE Control Magazine*, vol. 13, pp. 16-25.

Landau I.D., Karimi A., Constantinescu A., (2001) : « Direct controller reduction by identification in closed loop », *Automatica*, vol. 37, no. 11, pp. 1689-1702.

Landau I.D., Karimi A., (2001a) : « A unified approach to closed-loop plant identification and direct controller reduction », *Proceedings European Control Conference 2001 (ECC01)*, Porto, Portugal, Septembre.

Landau I.D., (2002) *Identification et Commande des Systèmes*, 3rd edition, Hermes, Paris (June)

Vinnicombe G., (1993) : « Frequency domain uncertainty and the graph topology », *IEEE Trans. on Automatic Control*, vol. 38, no. 9, pp. 1371-1383.