IDENTIFICATION IN CLOSED LOOP
A powerful design tool
(theory, algorithms, applications)

better models, simpler controllers

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Part 2: Robust digital control – *A brief review*

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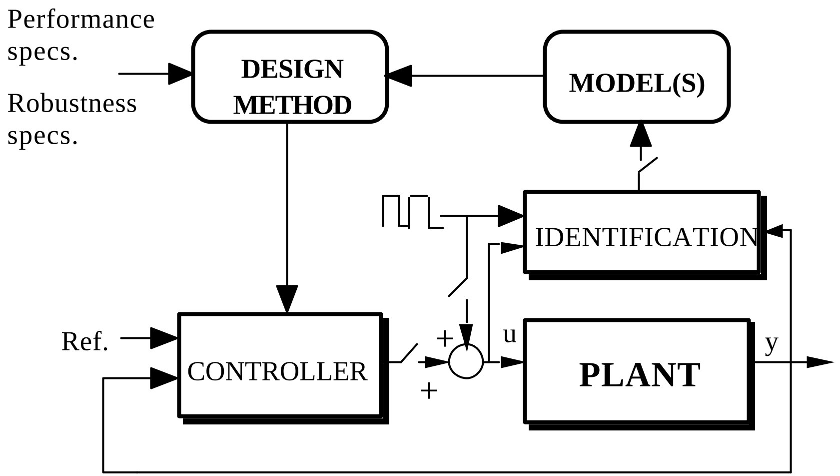
Controller Design and Validation



1) Identification of the dynamic model
2) Performance and robustness specifications
3) Compatible controller design method
4) Controller implementation
5) Real-time controller validation
   (and on site re-tuning)
6) Controller maintenance (same as 5)

Outline

Robust digital control

-The R-S-T digital controller

-Basic design

-Robustness issues

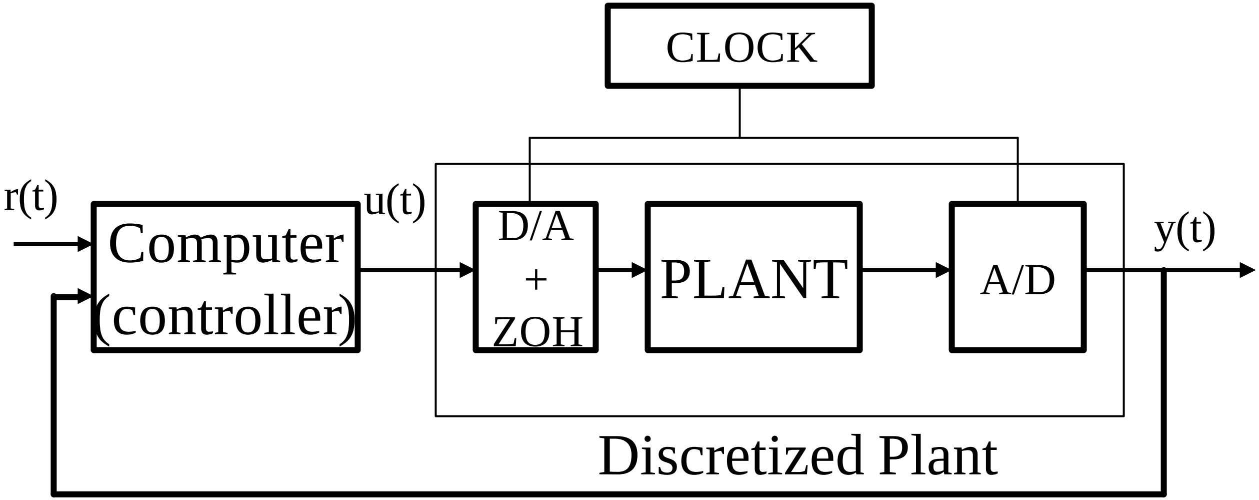
-The sensitivity functions and their properties

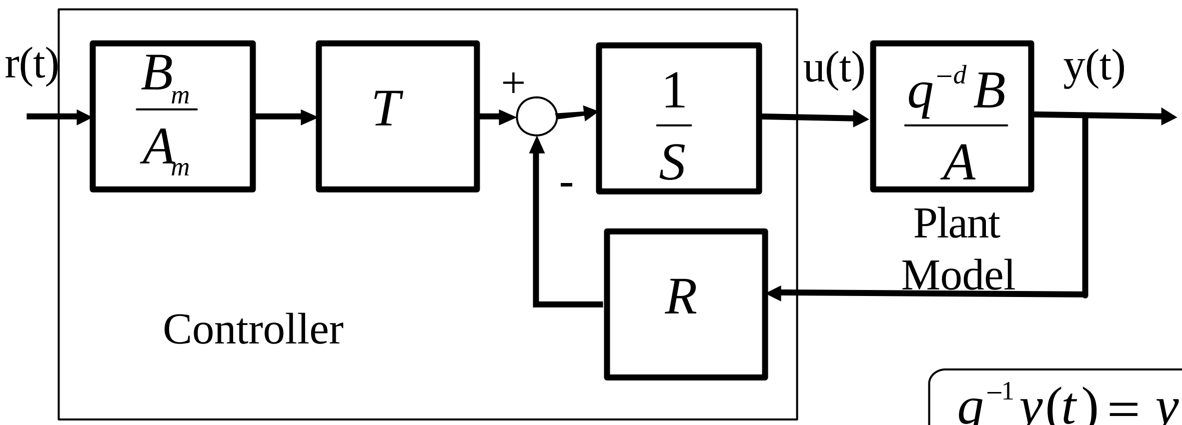
-Robustness margins

-Robust stability

-An example

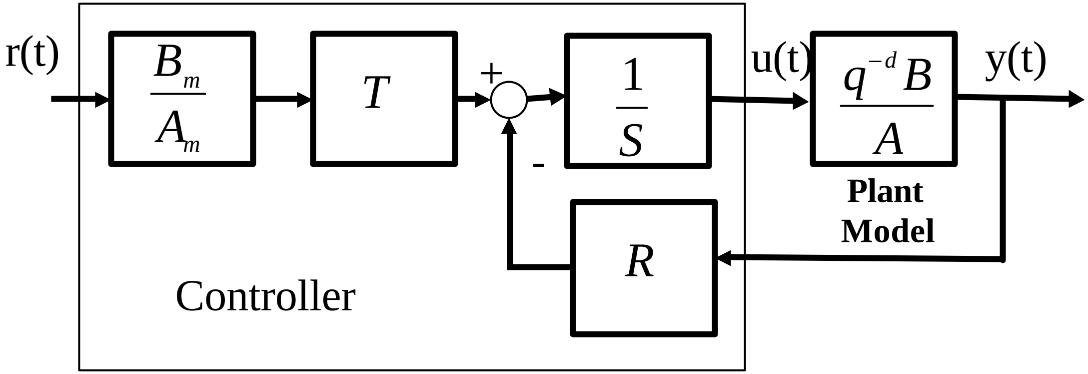
The R-S-T Digital Controller





$$q^{-1}y(t) = y(t-1)$$

The R-S-T Digital Controller



Plant Model:

$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-d-1}B^*(q^{-1})}{A(q^{-1})}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} \quad B(q^{-1}) = b_1 q^{-1} + \ldots + b_{n_B} q^{-n_B} = q^{-1}B^*(q^{-1})$$

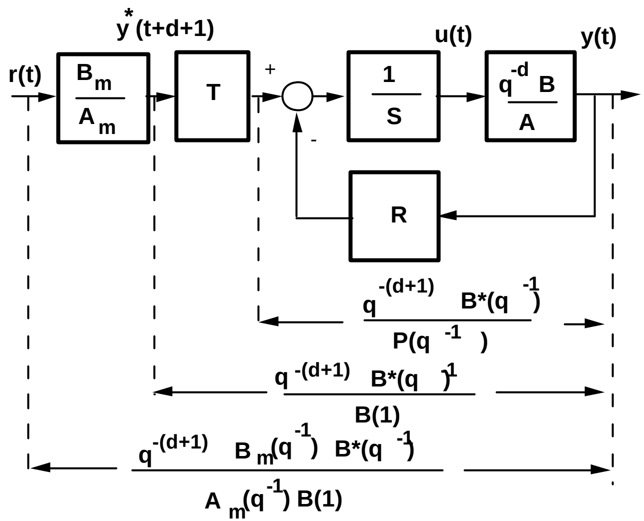
R-S-T Controller:

$$S(q^{-1})u(t) = T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)$$

Characteristic polynomial (closed loop poles):

$$P(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})$$

Pole Placement with R-S-T Controller



Controller : $R = H_R R' \,;\, S = H_S S'$ $\quad H_R, H_S :$ fixed parts

Regulation: R' and S' solutions of: $A H_S S' + q^{-d} B H_R R' = P = P_D P_F$

P_D: dominant poles; P_F: auxiliary poles

Tracking : $T = P / B(1)$

Reference trajectory: y^* — computer file, or $y^* = (B_m / A_m) r$

Connections with other Control Strategies

- Digital PID : $n_R = n_S = 2;\ H_S = 1 - q^{-1}$

-Tracking and regulation with independent objectives(MRC):

$P = B^* P_D P_F$ (Hyp.: *B* has stable damped zeros)*

- Minimum variance tracking and regulation (MVC):

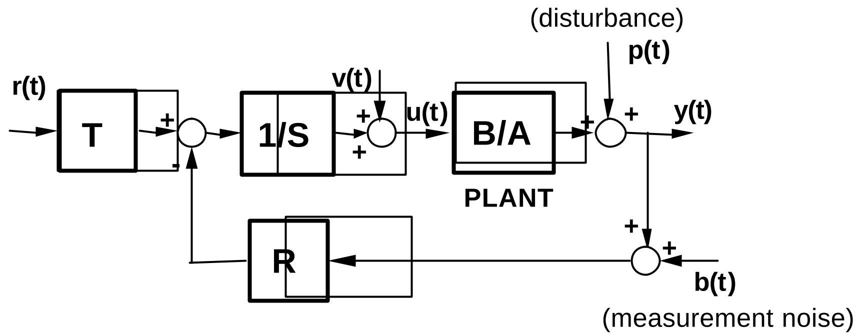
$P = B^* C$ (Hyp.: *B* has stable damped zeros)*

noise model

- Internal Model Control (IMC):

$P = A P_F$ (Hyp.: *A has stable damped poles)*

Digital control in the presence of disturbances and noise



Output sensitivity function
(p —— y)

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Input sensitivity function
(p —— u)

$$S_{up}(z^{-1}) = \frac{-A(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Noise-output sensitivity function
(b —— y)

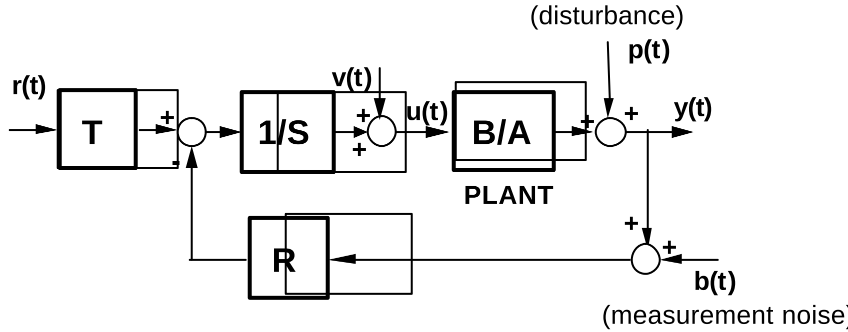
$$S_{yb}(z^{-1}) = \frac{-B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Input disturbance-output sensitivity function
(v —— y)

$$S_{yv}(z^{-1}) = \frac{B(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

All four sensitivity functions should be stable ! (see book pg.102 - 103)

Complementary sensitivity function



For $T = R$ one has:

$$S_{yr}(z^{-1}) = \frac{B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})} = -S_{yb}(z^{-1})$$

$$S_{yp}(z^{-1}) - S_{yb}(z^{-1}) = S_{yp}(z^{-1}) + S_{yr}(z^{-1}) = 1$$

Robustness of a control system

A control system is said to be *robust* for a set of given uncertainties upon the nominal plant model if it guarantees stability and performance for all plant models in this set.

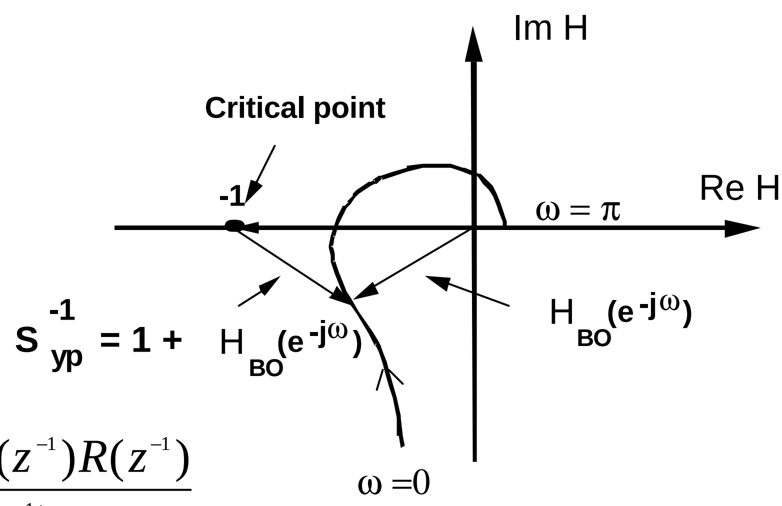
- To characterize the *robustness* of a closed loop system a frequency domain analysis is needed
- The *sensitivity functions* play a fundamental role in robustness studies
- The study of closed loop stability in the frequency domain gives valuable information for characterizing robustness

Stability of closed loop discrete time systems

The Nyquist is used like in continuous time

(can be displayed with WinReg or *Nyquist_OL.sci(.m))*

$$H_{OL}(e^{-jw}) = \frac{B(e^{-jw})R(e^{-jw})}{A(e^{-jw})S(e^{-jw})}$$



$$S_{yp}^{-1}(z^{-1}) = 1 + H_{OL}(z^{-1}) = \frac{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}$$

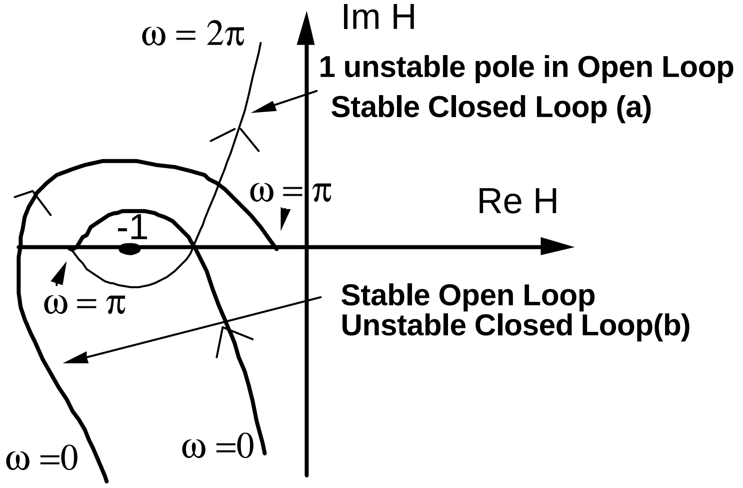
Nyquist criterion (discrete time –O.L. is stable)

The Nyquist plot of the open loop transfer fct. $H_{OL}(e^{-jw})$ traversed in the sense of growing frequencies (from 0 to $0.5f_S$) leaves the critical point[-1, j0] on the left

Stability of closed loop discrete time systems

Nyquist criterion (discrete time –O.L. is unstable)

The Nyquist plot of the open loop transfer fct. $H_{OL}(e^{-jw})$ traversed in the sense of growing frequencies (from 0 et f_S) leaves the critical point[-1, j0] on the left and the number of encirclements of the critical pointcounter clockwise should be equal to the number of unstable poles in open loop.



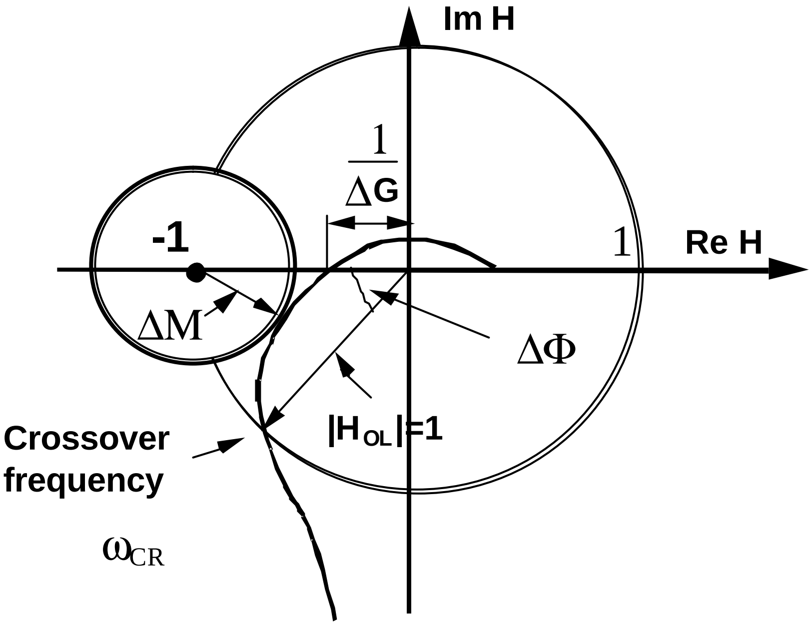
Remarks:

-The controller poles may become unstable if high performances are required without using an appropriate design method

-The Nyquist plot from $0.5f_S$ to f_S is the symmetric with respect to the real axis of the Nyquist plot from 0 to $0.5f_S$

Robustness margins

The minimal distance with respect to the critical point characterizes the robustness of the CL with respect to uncertainties on the plant model parameters(or their variations)



-Gain margin ΔG
-Phase margin $\Delta\phi$
-Delay margin $\Delta\tau$
-Modulus margin ΔM

Robustness margins

Gain margin

$$\Delta G = \frac{1}{\left|H_{OL}(jw_{180})\right|} \qquad \text{pour} \qquad \angle f(w_{180}) = -180^{\circ}$$

Phase margin

$$\Delta f = 180^{0} - \angle f(w_{cr}) \quad pour \quad \left|H_{BO}(jw_{cr})\right| = 1$$

$$\Delta f = \min_i \Delta f_i$$ If there are several intersections with the unit circle

Delay margin

$$\Delta t = \frac{\Delta f}{w_{cr}}$$

Several intersections points: $\Delta t = \min_i \frac{\Delta f_i}{w_{cr}^i}$

Modulus margin

$$\Delta M = \left|1 + H_{OL}(jw)\right|_{\min} = \left|S_{yp}^{-1}(jw)\right|_{\min} = \left(\left|S_{yp}(jw)\right|_{\max}\right)^{-1}$$

Robustness margins – typical values

Gain margin : $\Delta G \geq 2$ ($6\ dB$) [$min : 1,6$ ($4\ dB$)]

Phase margin : $30° \leq \Delta\phi \leq 60°$

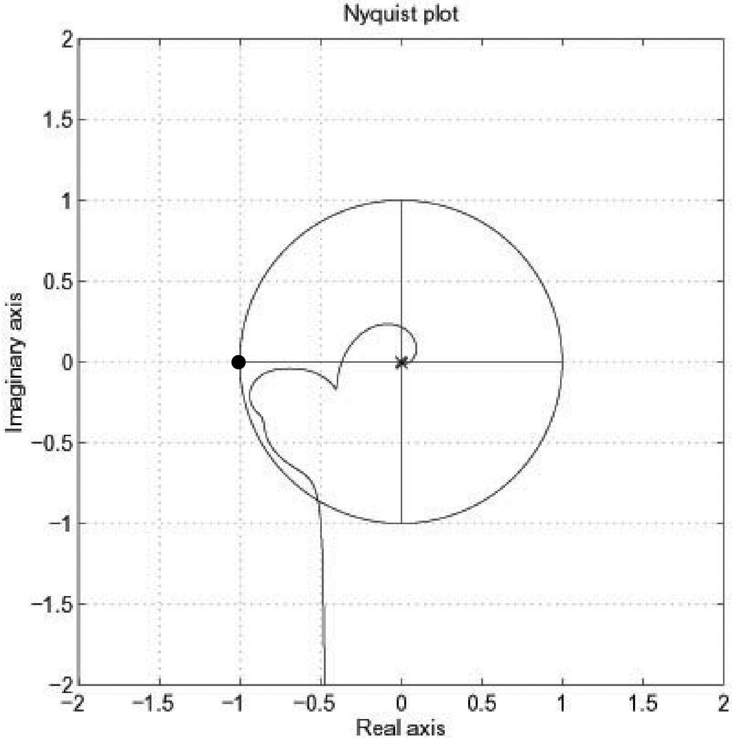
Delay margin : fraction of system delay (10%) or of time response (10%) (often $1.T_S$)

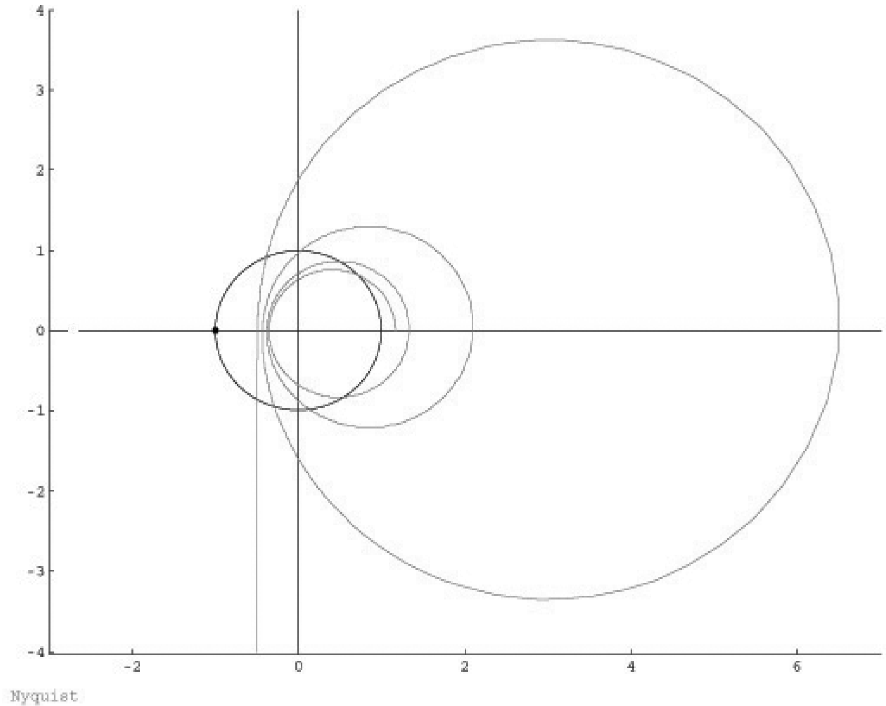
Modulus margin : $\Delta M \geq 0.5$ ($-\ 6\ dB$) [$min : 0,4$ ($-8\ dB$)]

A modulus margin $\Delta M \geq 0.5$ implies $\Delta G \geq 2$ *et* $\Delta\phi > 29°$
Attention ! The converse is not generally true

The *modulus margin* defines also the tolerance with respect to nonlinearities

Robustness margins





Good gain and phase margin
Bad modulus margin

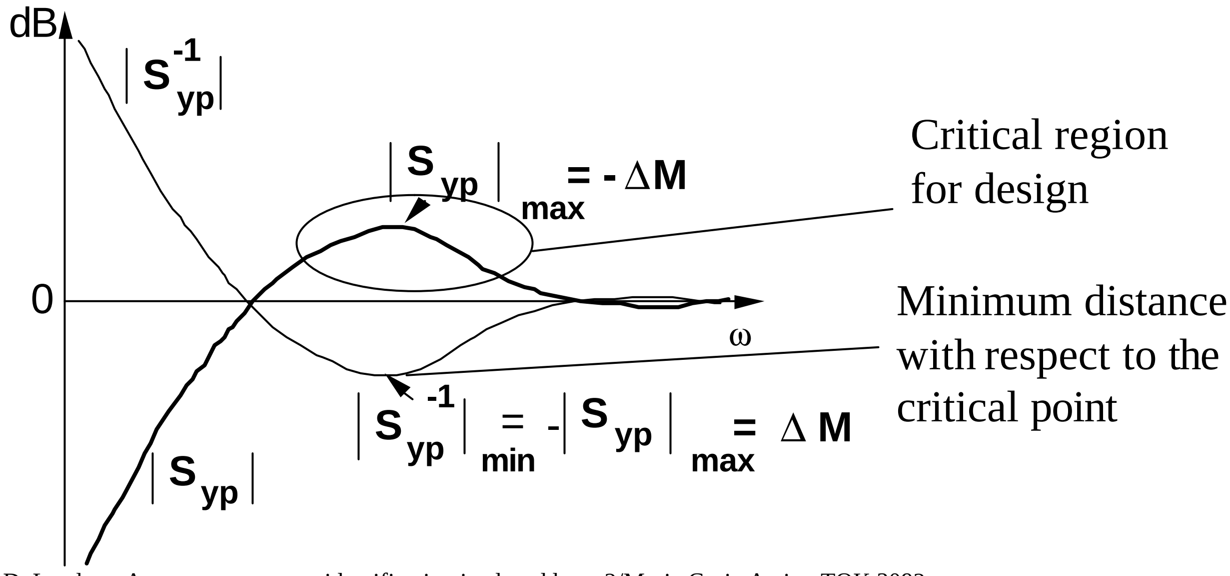
Good gain and phase margin
Bad delay margin

Modulus margin and sensitivity function

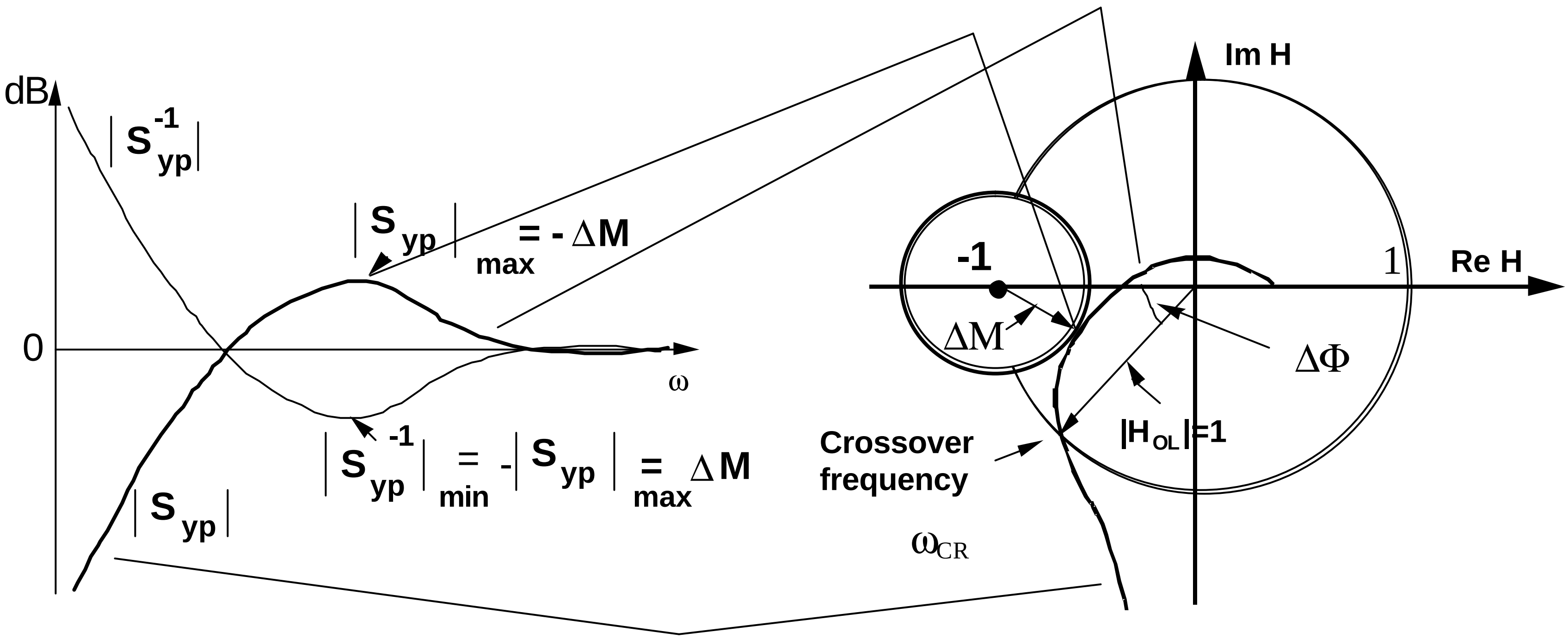
$$\Delta M = \left|1 + H_{OL}(z^{-1})\right|_{\min} = \left|S_{yp}^{-1}(z^{-1})\right|_{\min} = \left(\left|S_{yp}(z^{-1})\right|_{\max}\right)^{-1} =$$

$$\left(\left|\frac{A(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}\right|_{\max}\right)^{-1} \quad pour \quad z^{-1} = e^{-j2pf}$$

$$\left|S_{yp}(e^{-jw})\right|_{\max} dB = \Delta M^{-1} dB = -\Delta M \ dB$$



Correspondance Output Sensitivity → Nyquist Plot



Properties of the output sensitivity function

– *The open loop being stable, one has the property:*

$$\int_0^{0.5 f_S} \log\left|S_{yp}(e^{-j2pf/f_S})\right| df = 0$$

The sum of the areas between the curve of Syp and the axis 0dB taken with their sign is null

↓

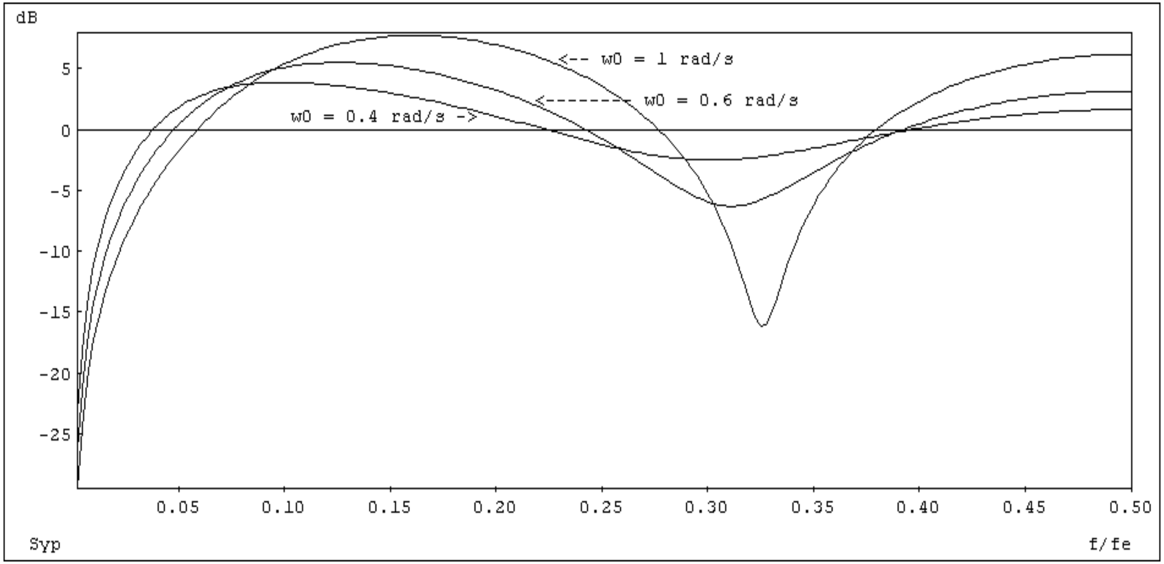
Disturbance attenuation in a frequency region implies amplification of the disturbances in other frequency regions!

Properties of the output sensitivity function

Augmenting the attenuation or widening the attenuation zone

Higher amplification of disturbances ouside the attenuation zone

Reduction of the robustness (reduction of the modulus margin)



Robust stability

To assure stability in the presence of uncertainties (or variations) on the dynamic chatacteristics of the plant model

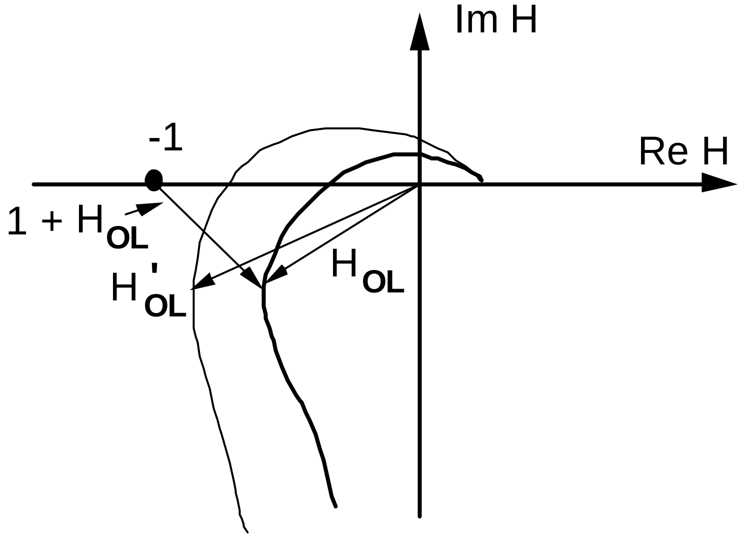
H_{OL} – nominal F.T.; H'_{OL} –Different from H_{OL} (perturbed)

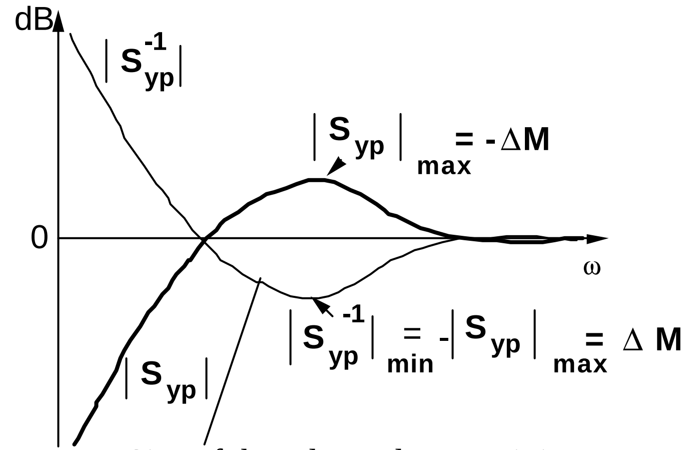
Robust stability condition (sufficient cond.):

$$\left|H'_{OL}(z^{-1}) - H_{OL}(z^{-1})\right| < \left|1 + H_{OL}(z^{-1})\right| = \left|S_{yp}^{-1}(z^{-1})\right| =$$

$$\left|\frac{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}\right| = \left|\frac{P(z^{-1})}{A(z^{-1})S(z^{-1})}\right| \quad ; \quad z^{-1} = e^{-jw} \qquad (*)$$

$$\left|S_{yp}(z^{-1})\right| < \left|H'_{OL}(z^{-1}) - H_{OL}(z^{-1})\right|^{-1}$$





Size of the tolerated uncertainity on H_{OL} at each frequency (radius)

Tolerance to plant additive uncertainty

From previous slide :

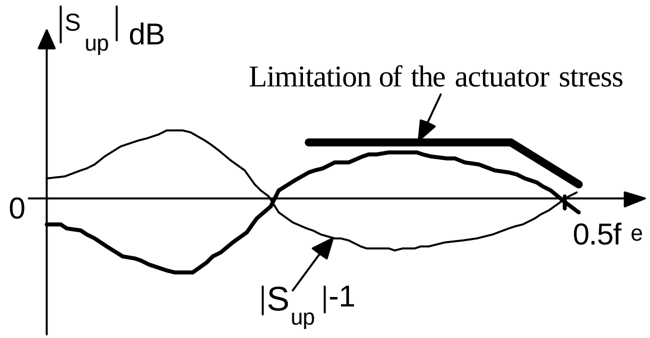
$$\left|\frac{B'(z^{-1})R(z^{-1})}{A'(z^{-1})S(z^{-1})}-\frac{B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}\right|=\left|\frac{R(z^{-1})}{S(z^{-1})}\right|\cdot\left|\frac{B'(z^{-1})}{A'(z^{-1})}-\frac{B(z^{-1})}{A(z^{-1})}\right|<\left|\frac{A(z^{-1})S(z^{-1})+B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1})}\right| \quad (*)$$

H'_{OL} H_{OL} G' G

$$\left|\frac{B'(z^{-1})}{A'(z^{-1})}-\frac{B(z^{-1})}{A(z^{-1})}\right|<\left|\frac{A(z^{-1})S(z^{-1})+B(z^{-1})R(z^{-1})}{A(z^{-1})R(z^{-1})}\right|=\left|\frac{P(z^{-1})}{A(z^{-1})R(z^{-1})}\right|=\left|S_{up}^{-1}(z^{-1})\right| \quad (**)$$

G' G

$$\left|S_{up}(z^{-1})\right|<\left|G'(z^{-1})-G(z^{-1})\right|^{-1}$$



(Size of tolerated uncertainties)

Tolerance to plant normalized uncertainty (multiplicative uncertainty)

From (**), previous slide:

$$\frac{\left|\frac{B'(z^{-1})}{A'(z^{-1})}-\frac{B(z^{-1})}{A(z^{-1})}\right|}{\left|\frac{B(z^{-1})}{A(z^{-1})}\right|}<\left|\frac{A(z^{-1})S(z^{-1})+B(z^{-1})R(z^{-1})}{B(z^{-1})R(z^{-1})}\right|=\left|\frac{P(z^{-1})}{B(z^{-1})R(z^{-1})}\right|=\left|S_{yb}^{-1}(z^{-1})\right|$$

The inverse of the modulus of the "complementary sensitivity function" gives at each frequency the tolerance with respect to "normalized (multiplicative) uncertainty"

Relation between additive and multiplicative uncertainty:

$$G' = G + (G' - G) = G\left(1 + \frac{G' - G}{G}\right)$$

Important message

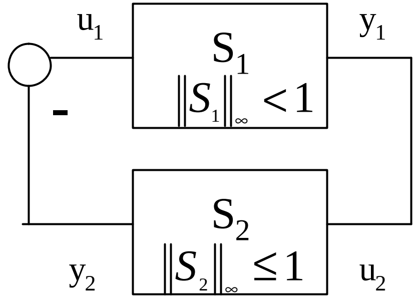
Large values of the modulus of the sensitivity functions in a certain frequency region

⇒ Low tolerance to model uncertainty

⇓

Critical regions for control design
Need for a good model in these regions

Small gain theorem



S_1: linear time invariant (state *x*)

$$\|S_1\|_\infty < 1$$

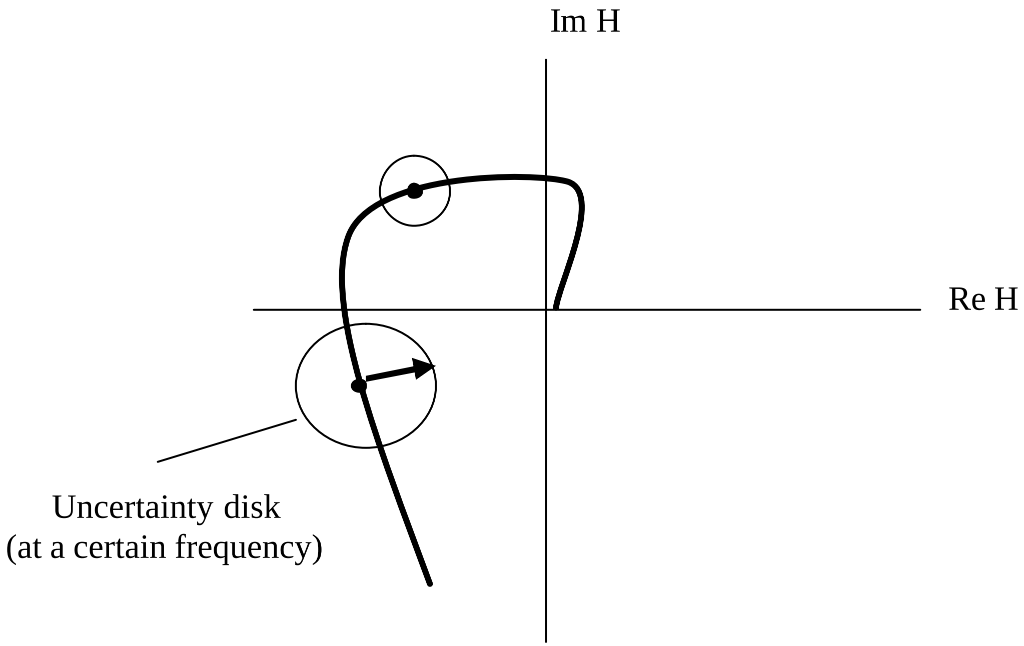
S_2: $\|S_2\|_\infty \le 1$

Then:

$$\lim_{t\to\infty} x(t) = 0\ ;\ \lim_{t\to\infty} u_1(t) = 0\ ;\ \lim_{t\to\infty} y_1(t) = 0$$

It will be used to characterize "robust stability"

Description of uncertainties in the frequency domain



1) It needs a description by a transfer function which may have any phase but a modulus < 1
2) The size of the radius will vary with the frequency and is characterized by a transfer function

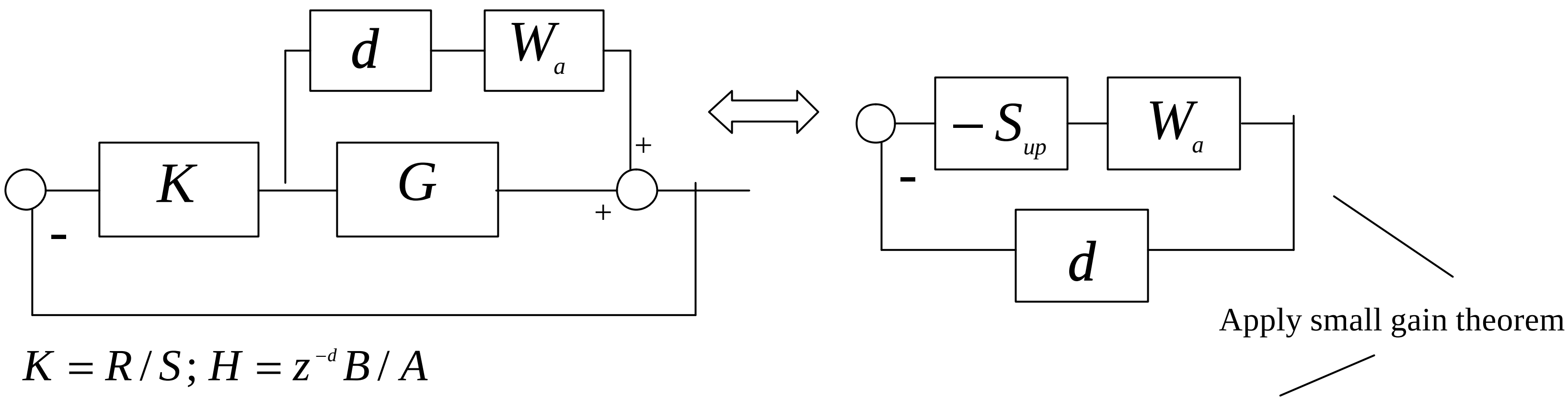
Additive uncertainty

$$G'(z^{-1}) = G(z^{-1}) + d(z^{-1})W_a(z^{-1})$$

$d(z^{-1})$ any stable transfer function with $\left\| d(z^{-1}) \right\|_\infty \leq 1$

$W_a(z^{-1})$ a stable transfer function

$$\left| G'(z^{-1}) - G(z^{-1}) \right|_{\max} = \left\| G'(z^{-1}) - G(z^{-1}) \right\|_\infty = \left\| W_a(z^{-1}) \right\|_\infty$$



$K = R/S;\ H = z^{-d}B/A$

Robust stability condition: $\left\| S_{up}(z^{-1})W_a(z^{-1}) \right\|_\infty < 1$

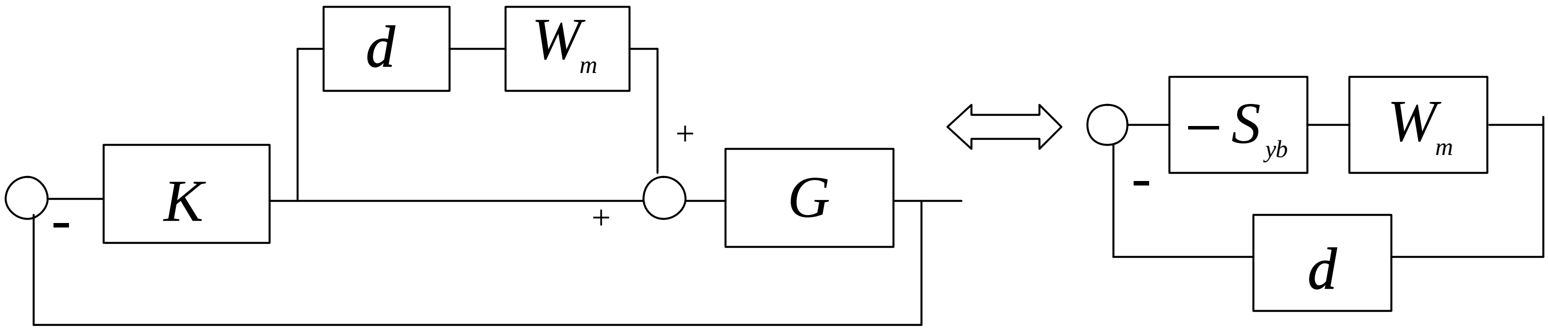
Multiplicative uncertainties

$$G'(z^{-1}) = G(z^{-1})\left[1 + d(z^{-1})W_m(z^{-1})\right]$$

$d(z^{-1})$ any stable transfer function with $\left\| d(z^{-1}) \right\|_\infty \leq 1$

$W_m(z^{-1})$ a stable transfer function

$$W_a(z^{-1}) = H(z^{-1})W_m(z^{-1})$$



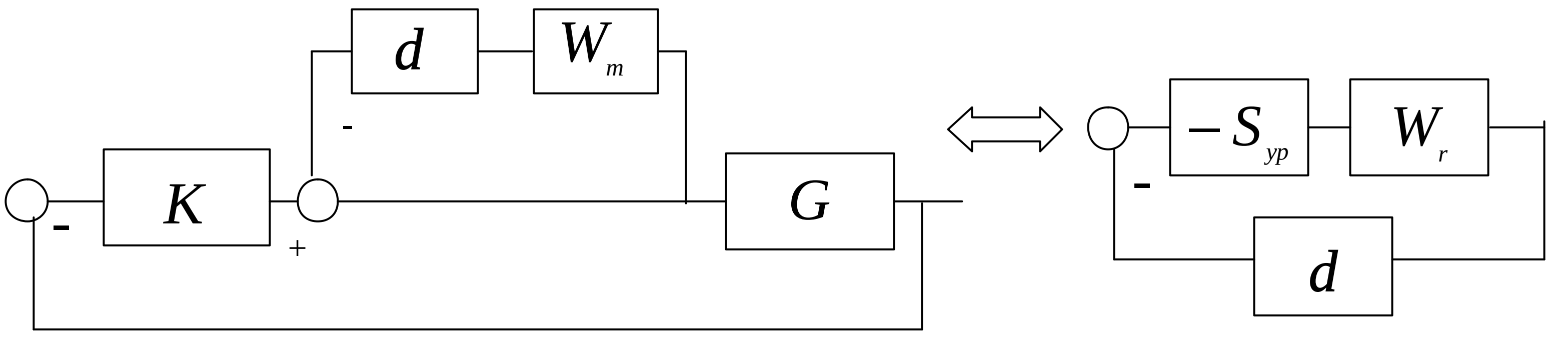
Robust stability condition: $\left\| S_{yb}(z^{-1})W_m(z^{-1}) \right\|_\infty < 1$

Feedback uncertainties on the input

$$G'(z^{-1}) = \frac{G(z^{-1})}{\left[1 + d(z^{-1})W_r(z^{-1})\right]}$$

$d(z^{-1})$ any stable transfer function with $\left\| d(z^{-1}) \right\|_\infty \leq 1$

$W_r(z^{-1})$ a stable transfer function



Robust stability condition: $\left\| S_{yp}(z^{-1})W_r(z^{-1}) \right\|_\infty < 1$

Robust stability conditions

$H, H' \in \mathrm{P}(W, d)$ —— Family (set) of plant models

Robust stability :

The feedback system is asymptotically stable for all the plant models belonging to the family $\mathrm{P}(W, d)$

- Additive uncertainties

$$\left\| S_{up}(z^{-1}) W_a(z^{-1}) \right\|_\infty < 1 \iff \left| S_{up}(e^{-jw}) \right| < \left| W_a(e^{-jw}) \right|^{-1} \quad 0 \le w \le p$$

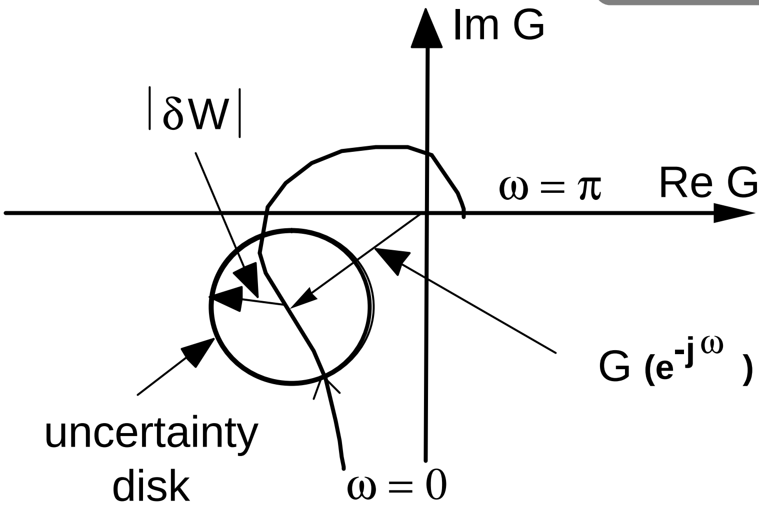
- Multiplicative uncertainties

$$\left\| S_{yb}(z^{-1}) W_m(z^{-1}) \right\|_\infty < 1 \iff \left| S_{yb}(e^{-jw}) \right| < \left| W_m(e^{-jw}) \right|^{-1} \quad 0 \le w \le p$$

- Feedback uncertainties on the input (or output)

$$\left\| S_{yp}(z^{-1}) W_r(z^{-1}) \right\|_\infty < 1 \iff \left| S_{yp}(e^{-jw}) \right| < \left| W_r(e^{-jw}) \right|^{-1} \quad 0 \le w \le p$$

Robust Stability



Family of plant models:

$$G' \in F(G, d, W_{xy})$$

$$G - \textit{nominal model;}\ \|d(z^{-1})\|_\infty \le 1$$

$$W_{xy}(z^{-1}) \text{- } \textit{size of uncertainty}$$

Robust stability condition:

a related sensitivity function

a type of uncertainty

$$\|S_{xy}W_{xy}\|_\infty < 1$$

$$\Downarrow$$

defines the size of the tolerated uncertainty

$$|S_{xy}| < |W_{xy}|^{-1}$$

defines an upper template for the modulus of the sensitivity function

There also lower templates (because of the relationship between various sensitivity fct.)

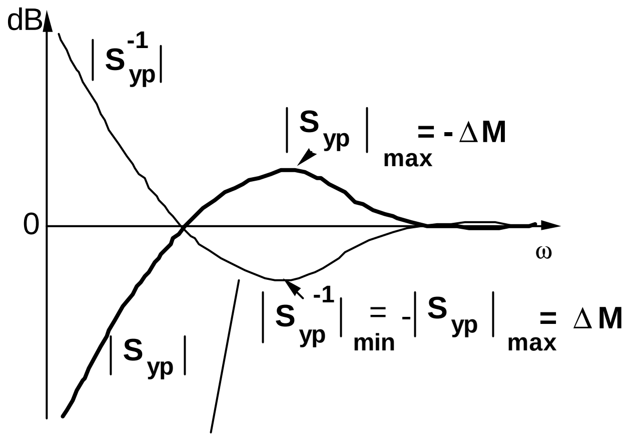
Robust stability and templates for the sensitivity functions

Robust stability condition:

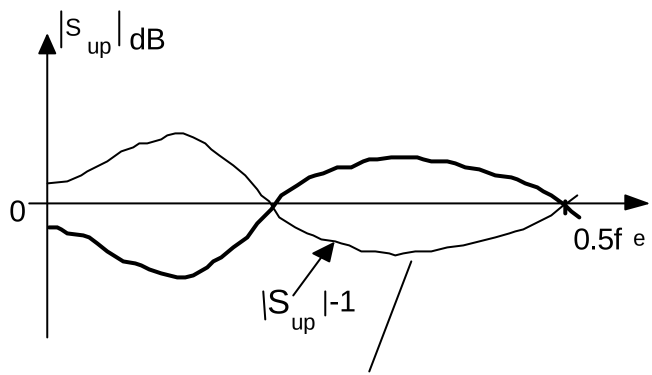
$$\left|S_{xy}(e^{-jw})\right| < \left|W_z(e^{-jw})\right|^{-1} \quad 0 \le w \le p$$

- *The functions* $\left|W(z^{-1})\right|^{-1}$ *(the inverse of the size of the uncertainties) define an "upper" template for the sensitivity functions*

- *Conversely the frequency profile of* $\left|S_{xy}(e^{-jw})\right|$ *can be interpreted in terms of tolerated uncertainties*

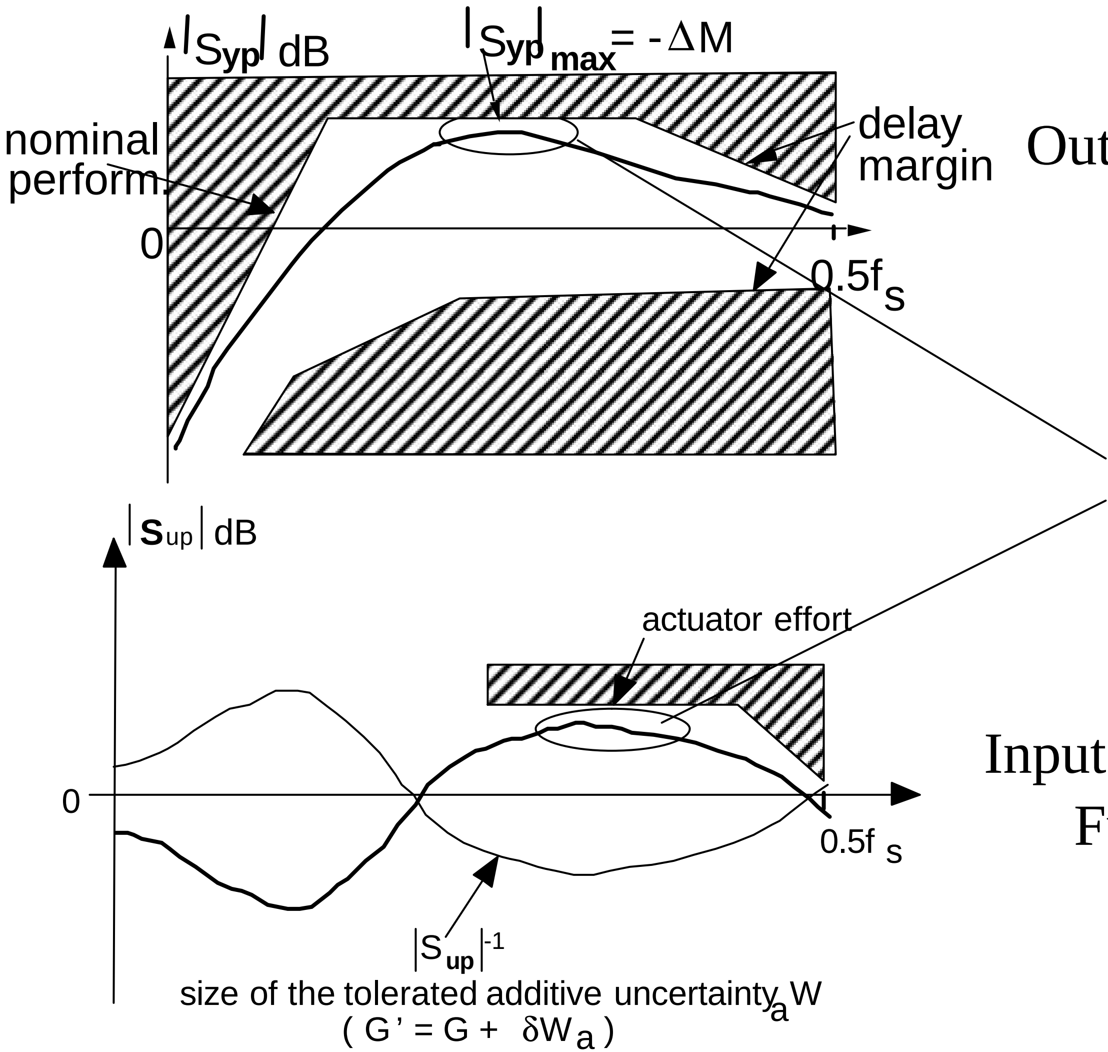


Tolerated feedback uncertainty on the input



Tolerated additive uncertainty

Templates for the Sensitivity Functions



Output Sensitivity Function

Dangerous zones.

Need for good models in these regions

Input Sensitivity Function

Robust Controller Design

Pole placement with sensitivity functions shaping

Nominal performance: P_D *and part of* H_R *and* H_S

$P = P_D P_F$

$R = R' H_R$

$S = S' H_S$

Allow to shape the sensitivity functions

Several approaches to design :

-Iterative

Choosing P_F *and using band stop filters* H_{Ri} / P_{Fi} , H_{Sj} / P_{Fj}

(matlab toolbox « ppmaster »)

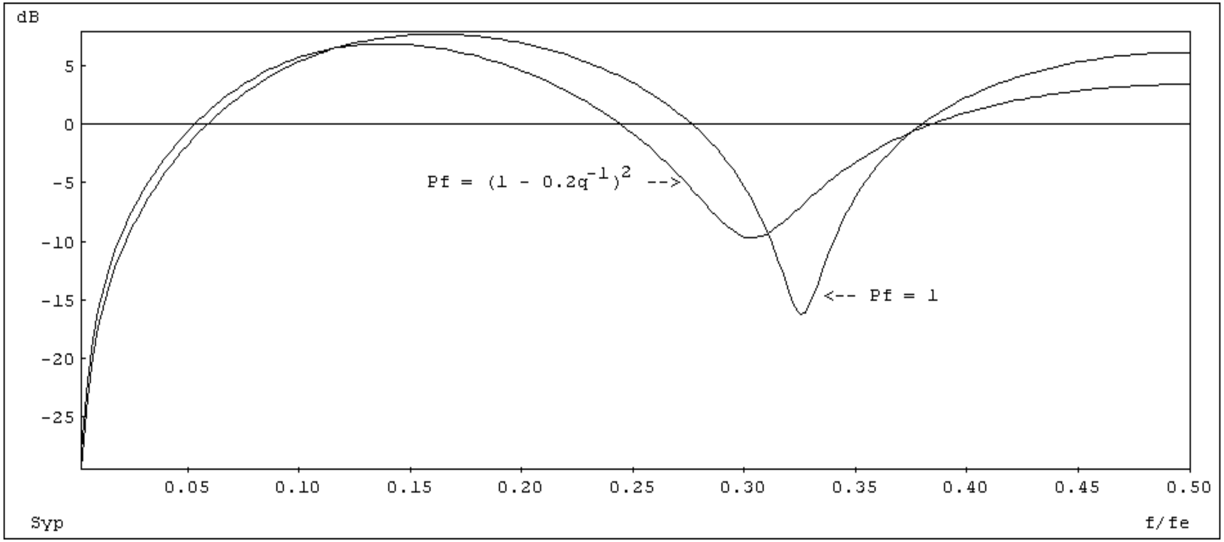
-Convex optimization

(see Langer, Landau, Automatica, June99, *Optreg* (Adaptech))

Properties of the output sensitivity function

The asymptotically stable auxiliary poles (P_F) lead in general to the reduction of $\left|S_{yp}(jw)\right|$ in the frequency regions corresponding to the attenuation regions for $1/P_F$

$$P_F(q^{-1}) = (1 + p'q^{-1})^{n_{P_F}} \qquad -0.5 \le p' \le -0.05 \qquad n_{P_F} \le n_P - n_{P_D}$$



In many applications the introduction of damped high frequency auxiliary poles is enough for assuring the required robustness margins

Properties of the output sensitivity function

Simultaneous introduction of a fixed part H_{Si} and of a pair of auxiliary poles P_{Fi} of the form:

$$\frac{H_{S_i}(q^{-1})}{P_{F_i}(q^{-1})} = \frac{1 + b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}}$$

Obtained by the discretization of :

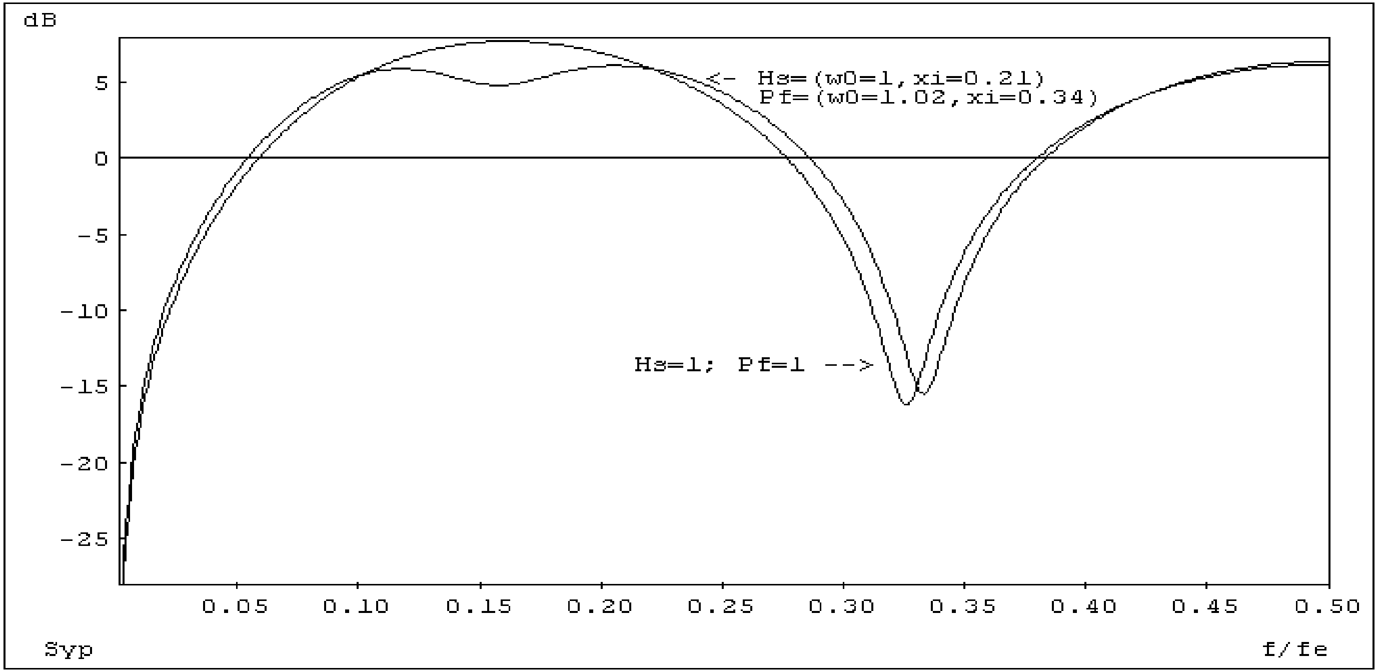
$$F(s) = \frac{s^2 + 2z_{num} w_0 s + w_0^2}{s^2 + 2z_{den} w_0 s + w_0^2} \qquad \text{with:} \qquad s = \frac{2}{T_e}\frac{1 - z^{-1}}{1 + z^{-1}}$$

produce and attenuation (hole) at the normalized discretized frequency:

$$w_{disc} = 2\arctan\left(\frac{w_0 T_e}{2}\right) \quad \text{with attenuation:} \quad M_t = 20\log\left(\frac{z_{num}}{z_{den}}\right) \quad (z_{num} < z_{den})$$

and has negligible effects at $f << f_{disc}$ and at $f >> f_{disc}$

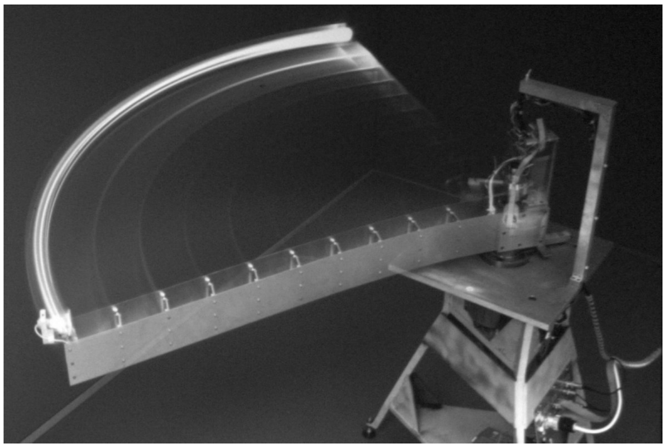
Properties of the output sensitivity function

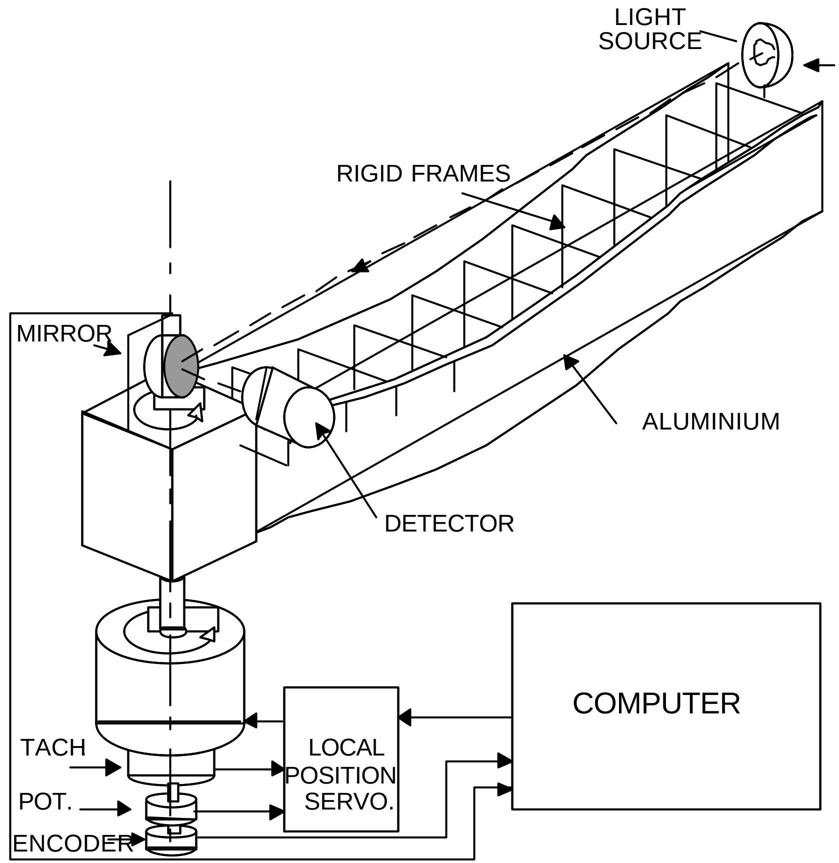


For details see Landau: Commande des Systèmes, Hermes

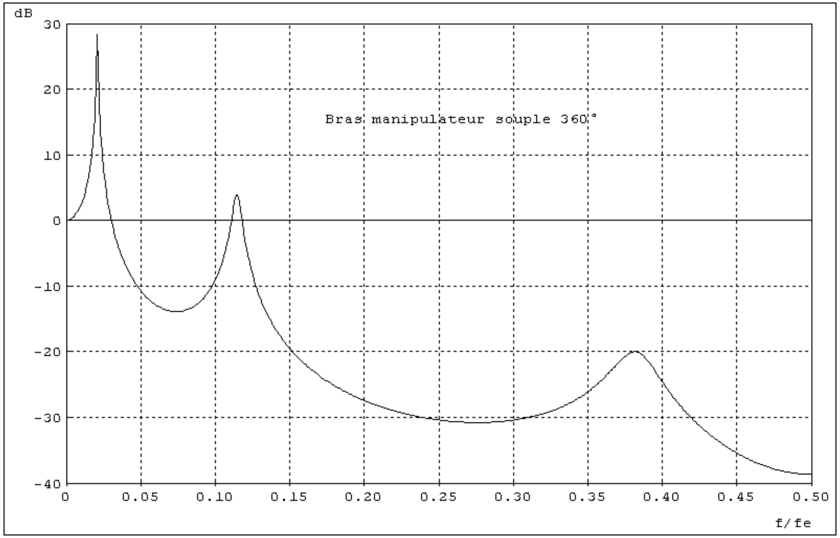
Efective computation using: *filter22.sci (.m)*

360° Flexible Arm

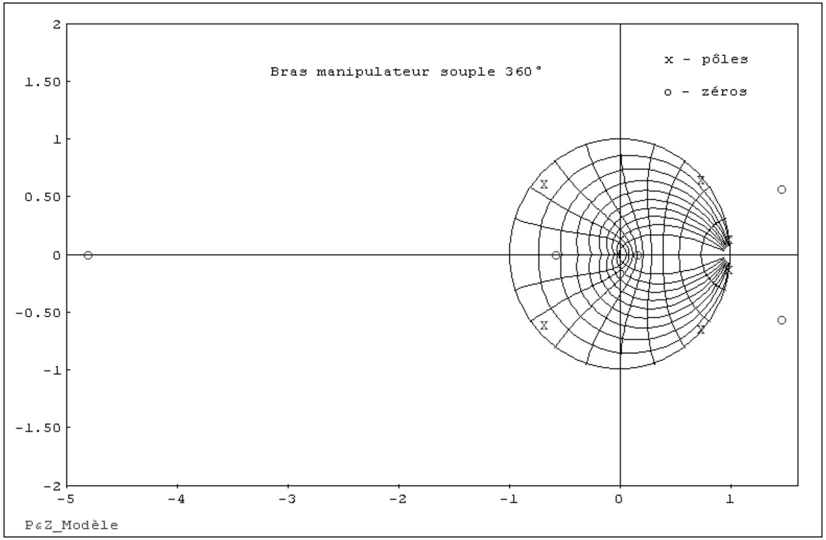




360° Flexible Arm



Frequency characteristics

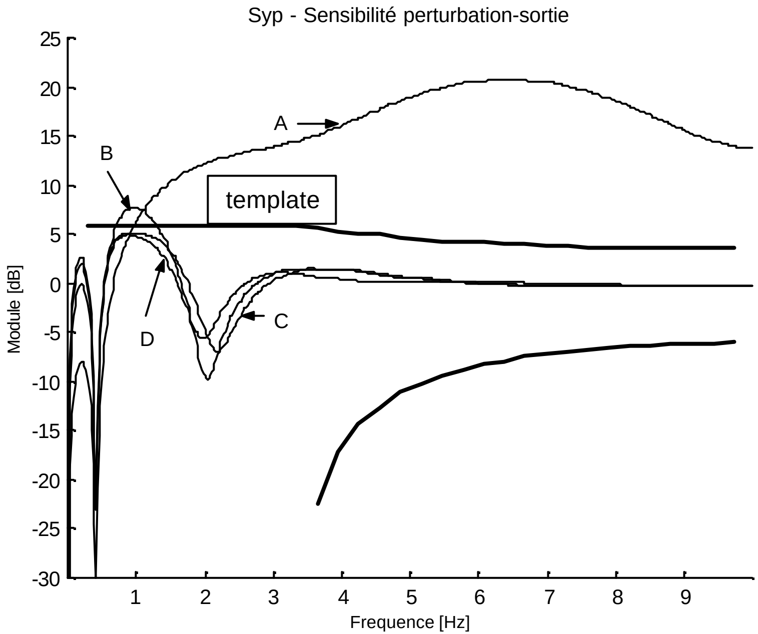


Poles-Zeros

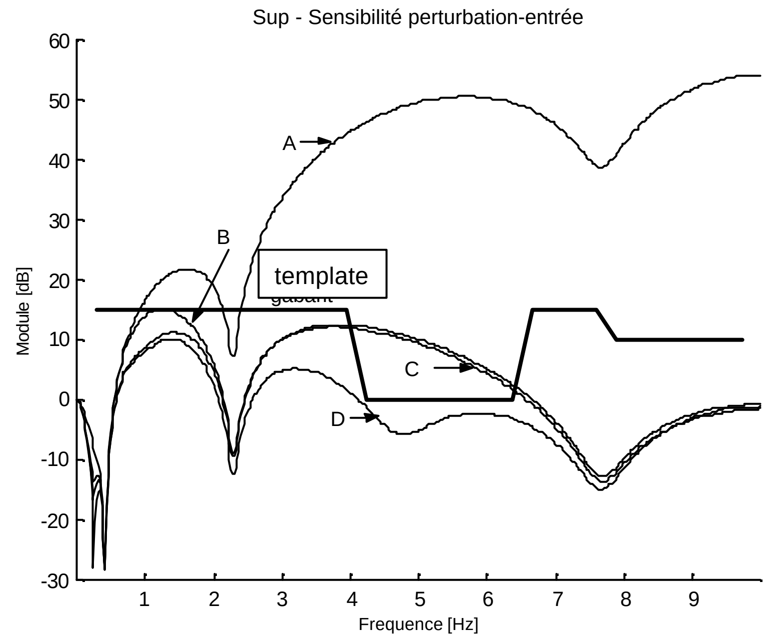
(Identified Model)

Shaping the Sensitivity Functions

Output Sensitivity Function - S_{yp}



Input Sensitivity Function - S_{up}



A- without auxiliary poles
B- with auxiliary poles
C- with stop band filter H_{S1} / P_{F1}
D- with stop band filter H_{R2} / P_{F2}

Robust Discrete Time Controller Design

Some references directly related to the course

More details can be found in :
I.D. Landau: Commande des systèmes – conception, identification, mise en œuvre Hermes, 2002, Paris, chapters 2 and 3 (english translation available)

and

http://landau-bookic.lag.ensieg.inpg.fr

- « Slides » version of the chapters can be downloaded
- Free routines (matlab, scilab) can be downloaded as well as a matlab based software « ppmaster » for pole placement design with sensitivity functions shaping

I.D.Landau, R. Lozano, M. M'Saad « Adaptive Control », Springer, 1997, chap.8

I.D. Landau : A course on « Robust Discrete Time Control », Valencia, April 2004