IDENTIFICATION IN CLOSED LOOP
A powerful design tool
(theory, algorithms, applications)

better models, simpler controllers

I.D. Landau
Laboratoire d 'Automatique de Grenoble, (INPG/CNRS), France

Part 5 : Controller reduction by identification in closed loop

Prepared for Marie Curie Action TOK 3092

Sevilla
November 2004

CONTROLLER REDUCTION. Why ?

- Complex Models
- Robust Control Design

High Order Controllers

Example : The Flexible Transmission
(Robust control benchmark, EJC no. 2/1995 and no.2-4/1999)

Model complexity : $G(q^{-1}) = \dfrac{q^{-d}B(q^{-1})}{A(q^{-1})}$ $n_A = 4$; $n_B = 2$; $d = 2$

Fixed controller part : Integrator

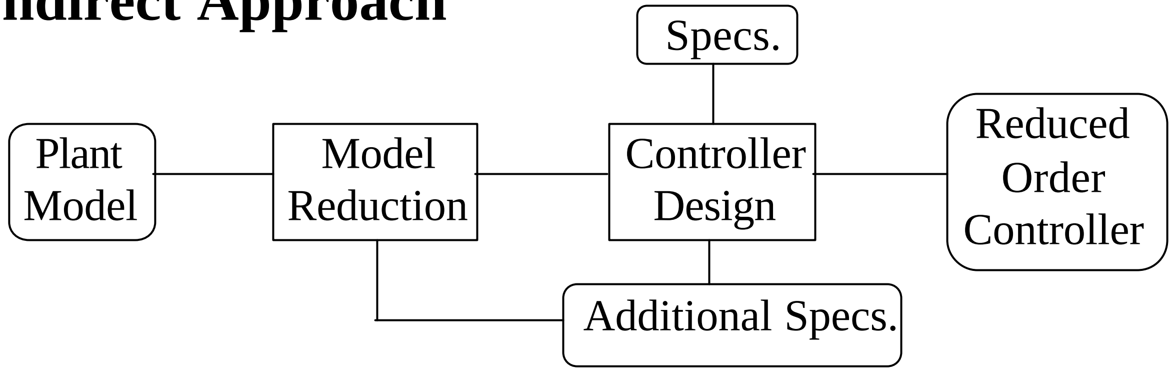
Pole placement design : $K(q^{-1}) = \dfrac{R(q^{-1})}{S(q^{-1})}$ $n_R = 4$; $n_S = 4$

Complexity of controllers achieving 100 % of specifications:

Max : $n_R = 9$; $n_S = 9$ (Nordin) **Min** : $n_R = 7$; $n_S = 7$ (Langer)

Approaches to Controller Reduction

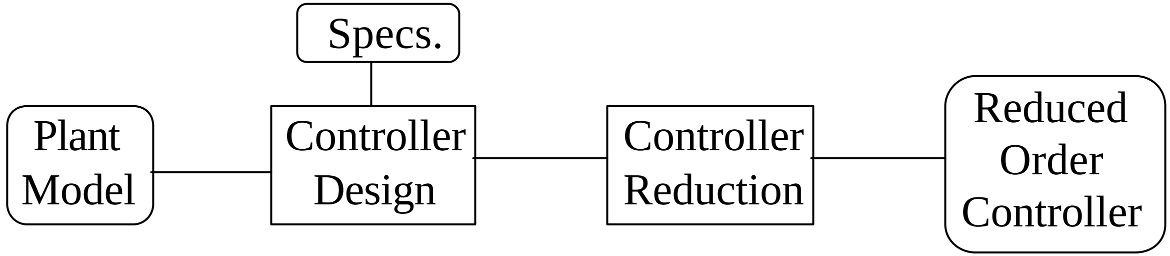
Indirect Approach



-Does not guarantee resulting controllers of desired order

- Propagation of model errors

Direct Approach



- Approximation carried in the final step
- Further controller reduction for "indirect approach"

Controller Reduction

Basic rule :

Controller reduction should be done with the aim to preserve as much as possible the closed loop properties.

Reminder :

Controller reduction without taking into account the closed loop properties can be a disaster !

Some basic references :

- Anderson &Liu : IEEE-TAC, August 1989
- Anderson : IEEE Control Magazine, August 1993

Rem: Direct design of a constrained complexity controllers is still an open problem

Identification in Closed Loop and Controller Reduction

- Identification in closed loop is an efficient tool for *control oriented model order reduction*

- **Closed loop identification techniques can be used (with small changes) for *direct estimation of reduced order controllers***

Duality

Identification of reduced order models in closed loop

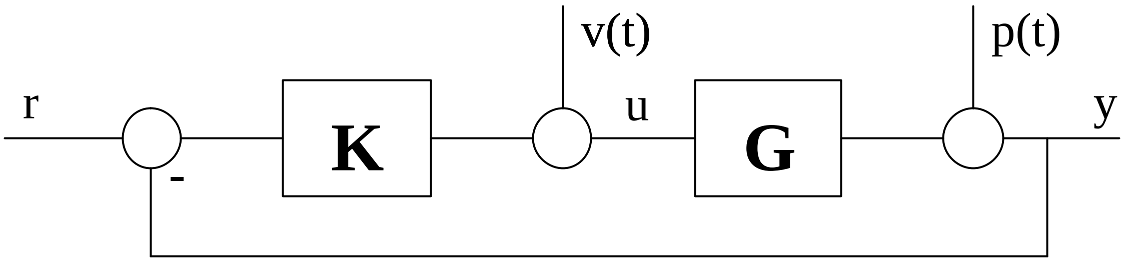
Identification of reduced order controllers in closed loop

- Possibility of using "real data" for controller reduction

Outline

- Introduction
- Notations
- Specific Objectives
- Basic Schemes
- The Daphné Algorithms
- Properties of the algorithms
- Properties of the estimated reduced order controllers
- Validation of reduced order controllers
- Experimental results (Active Suspension Control)
-Practical Hints
- Coherence between controller reduction and closed loop id.
- REDUC – Matlab toolbox for controller reduction
- Conclusions

Notations



$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \qquad K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}$$

Sensitivity functions :

$$S_{yp}(z^{-1}) = \frac{1}{1+KG} \;;\; S_{up}(z^{-1}) = -\frac{K}{1+KG} \;;\; S_{yv}(z^{-1}) = \frac{G}{1+KG} \;;\; S_{yr}(z^{-1}) = \frac{KG}{1+KG}$$

Closed loop poles : $P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$

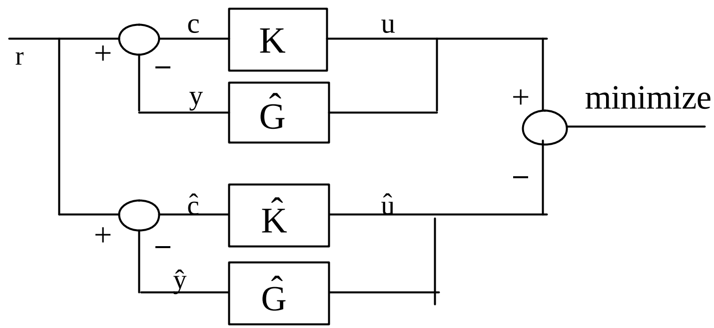
True closed loop system :(K,G), P, S_{xy}

Nominal simulated closed loop : $(K, \hat{G}), \hat{P}, \hat{S}_{xy}$

Simulated C.L. using reduced order controller : $(\hat{K}, \hat{G}), \hat{\hat{P}}, \hat{\hat{S}}_{xy}$

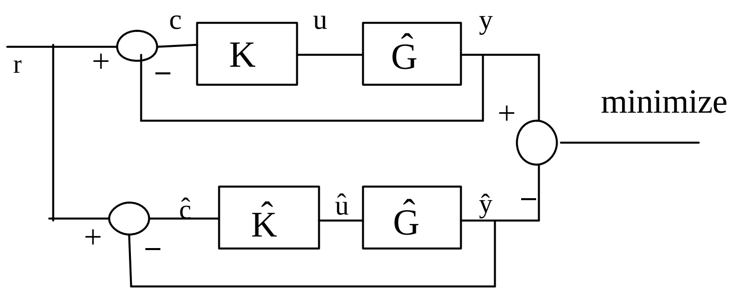
Controller Reduction - Objectives

Input matching



$$\hat{K}^* = \arg\min_{\hat{K}} \left\| \hat{S}_{up} - \hat{\hat{S}}_{up} \right\| = \arg\min_{\hat{K}} \left\| \hat{S}_{yp} (K - \hat{K}) \hat{\hat{S}}_{yp} \right\|$$

Ouput matching



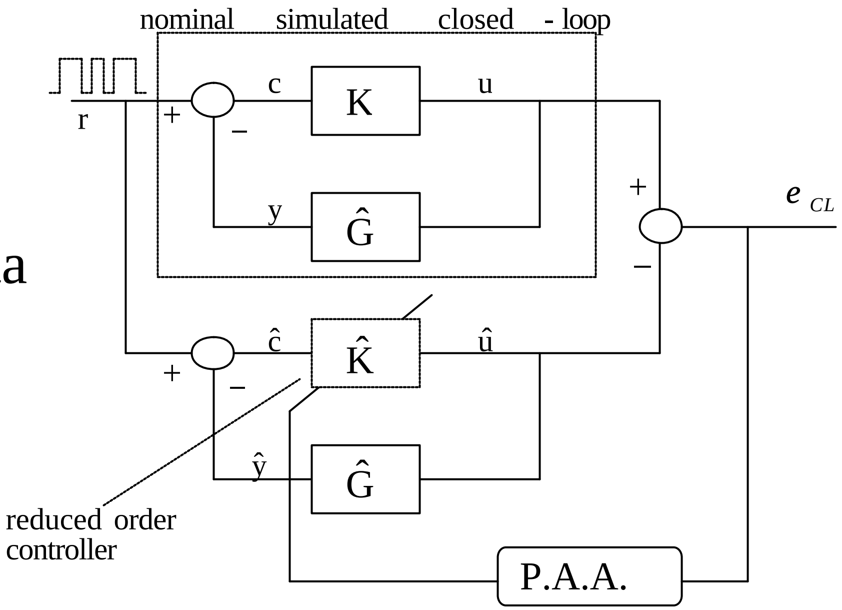
$$\hat{K}^* = \arg\min_{\hat{K}} \left\| \hat{S}_{yr} - \hat{\hat{S}}_{yr} \right\| = \arg\min_{\hat{K}} \left\| \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right\| = \arg\min_{\hat{K}} \left\| \hat{S}_{yp} (K - \hat{K}) \hat{\hat{S}}_{yv} \right\|$$

Identification of Reduced Order Controllers

Closed Loop Input Matching (CLIM)

with external excitation added to the controller input

Use of simulated data

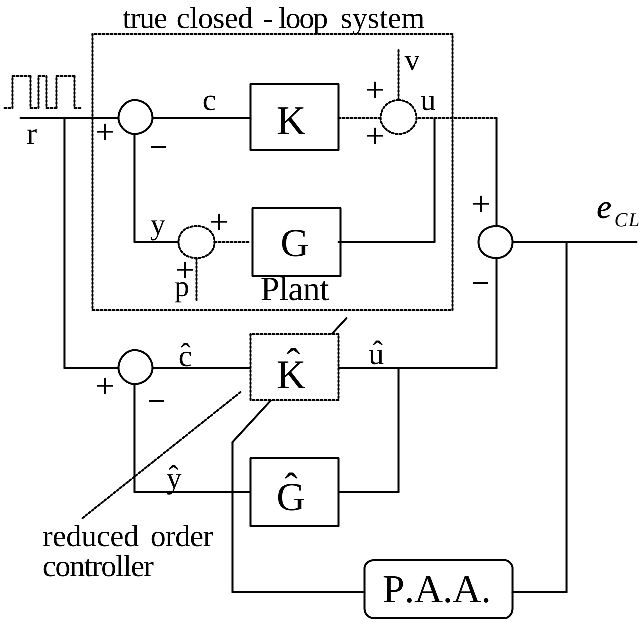


Identification of Reduced Order Controllers

Closed Loop Input Matching (CLIM)

with external excitation added to the controller input

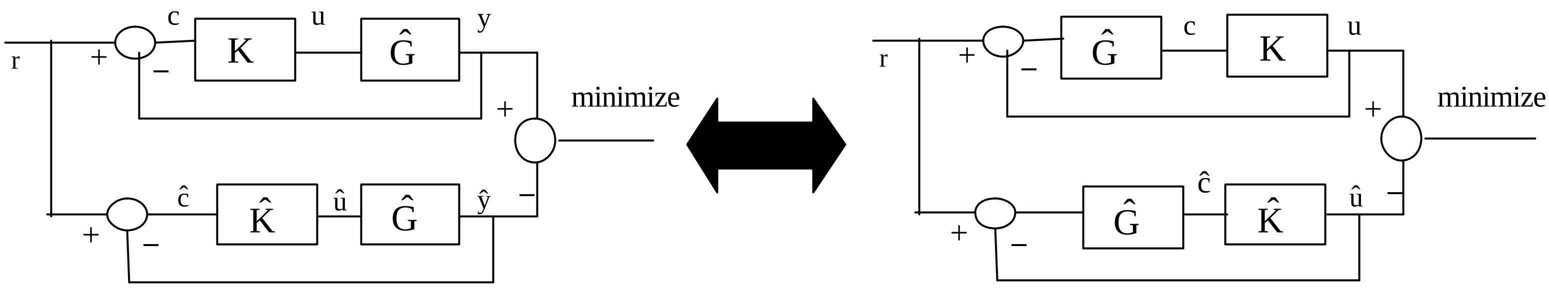
Use of real data



Remarks:

- new $\hat{G}$ identified in closed loop can be used (can be better than the design model)
- $\hat{K}$ will try to minimize the discrepancy between the two loops (will take into account $(G-\hat{G})$)

Relationship between CLIM and CLOM



Closed loop output matching (CLOM) with excitation added to the controller input

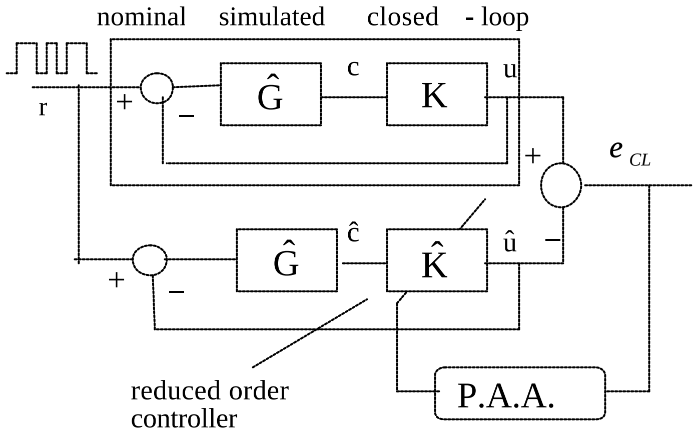
Closed loop input matching (CLIM) with excitation added to the plant input

- These two configurations are equivalent
- CLIM with excitation added to the plant input leads to a simpler algorithm

In defining a configuration one has to specify how the error is generated and where the excitation is added

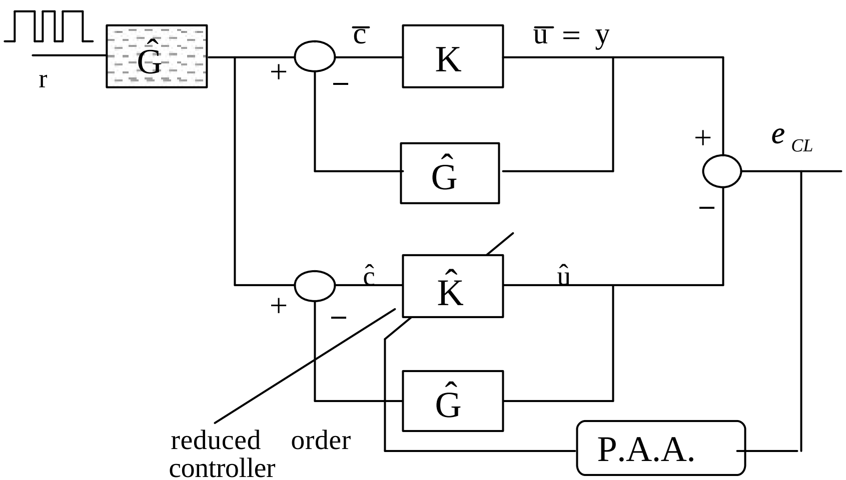
Identification of Reduced Order Controllers

CLIM with excitation added to the plant input



-Equivalent to CLOM with excitation added to the controller input

An alternative realization :



-CLIM algorithm with excitation added to the controller input but using a filtered excitation

- Same asymptotic steady state properties

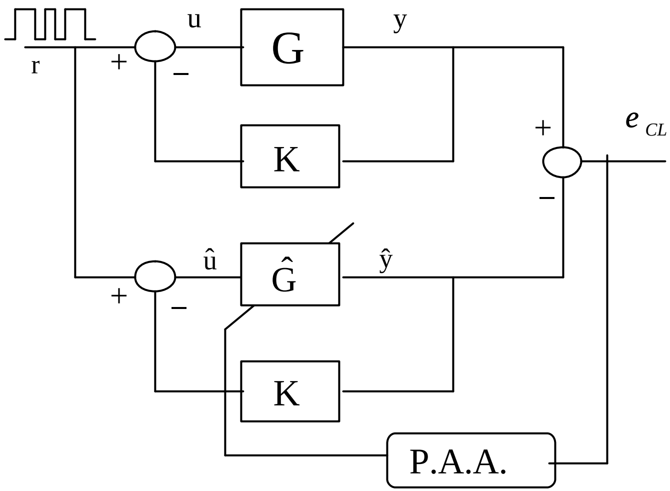
Notation convention

In order to simplify the writing we will use the following convention:

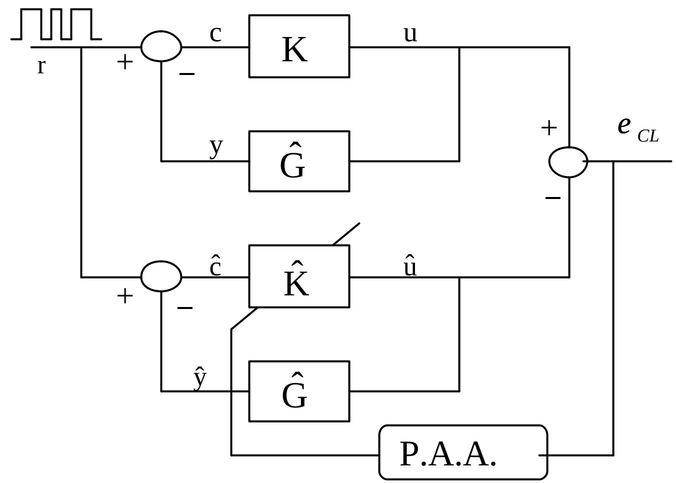
Algorithm		*Shortened name*
CLIM algorithm with excitation added to the controller input	→	**CLIM**
CLIM algorithm with excitation added to the plant input (equivalent to CLOM algorithm with excitation added to the controller input)	→	**CLOM**

Duality

CLOE



Daphné (CLIM)



$$G \rightarrow K$$
$$\hat{G} \rightarrow \hat{K}$$
$$K \rightarrow \hat{G}$$

Plant model identification in closed loop

Reduced order controller identification in closed loop

Remark : one should take care of the structure of R and B

CLIM Algorithm

CLIM with excitation added to the controller input

Controller output

$$u(t+1) = -S^*(q^{-1})u(t) + R(q^{-1})c(t+1)\ ;\ \ (S(q^{-1}) = 1 + q^{-1}S^*(q^{-1}))$$

Controller input

$$c(t+1) = r(t+1) - y(t+1)$$

Estimated controller output

$$\hat{u}^0(t+1) = -\hat{S}^*(t,q^{-1})\hat{u}(t) + \hat{R}(t,q^{-1})\hat{c}(t+1) = \hat{q}^T(t)f(t)$$

Estimated controller input

$$\hat{c}(t+1) = r(t+1) - \hat{y}(t+1) = r(t+1) + \hat{A}^*\hat{y}(t) - \hat{B}^*u(t-d)$$

$$\hat{q}^T(t) = \left[\hat{s}_1(t),\ldots\hat{s}_{n_{\hat{S}}}(t),\hat{r}_0(t),\ldots\hat{r}_{n_{\hat{R}}}(t)\right]$$ Estimated controller parameters

$$f^T(t) = \left[-\hat{u}(t),..-\hat{u}(t-n_{\hat{S}}+1),\hat{c}(t+1),..\hat{c}(t-n_{\hat{R}}+1)\right]$$

CLIM Algorithm

CLIM with excitation added to the controller input

Parameter adaption algorithm

$$e_{CL}^{0}(t+1)=u(t+1)-\hat{u}^{0}(t+1)$$

$$\hat{q}(t+1)=\hat{q}(t)+F(t+1)\Phi(t)e_{CL}^{0}(t+1)$$

$$F^{-1}(t+1)=l_{1}(t)F^{-1}(t)+l_{2}(t)\Phi(t)\Phi^{T}(t)$$

$$0<l_{1}(t)\leq 1;\ 0\leq l_{2}(t)<2$$

$$\textit{Choice of } \Phi(t):$$

$$CLIM:\Phi(t)=f(t) \qquad F-CLIM:\Phi(t)=\frac{\hat{A}(q^{-1})}{\hat{P}(q^{-1})}f(t)$$

CLOM Algorithm

(CLIM with excitation added to the plant input)

Controller output

$$u(t+1) = -S^*(q^{-1})u(t) + R(q^{-1})c(t+1)\,;\ \ (S(q^{-1}) = 1 + q^{-1}S^*(q^{-1}))$$

Controller input

$$c(t+1) = G(q^{-1})[r(t+1) - u(t+1)]$$

Estimated controller output

$$\hat{u}^0(t+1) = -\hat{S}^*(t, q^{-1})\hat{u}(t) + \hat{R}(t, q^{-1})\hat{c}(t+1) = \hat{q}^T(t)f(t)$$

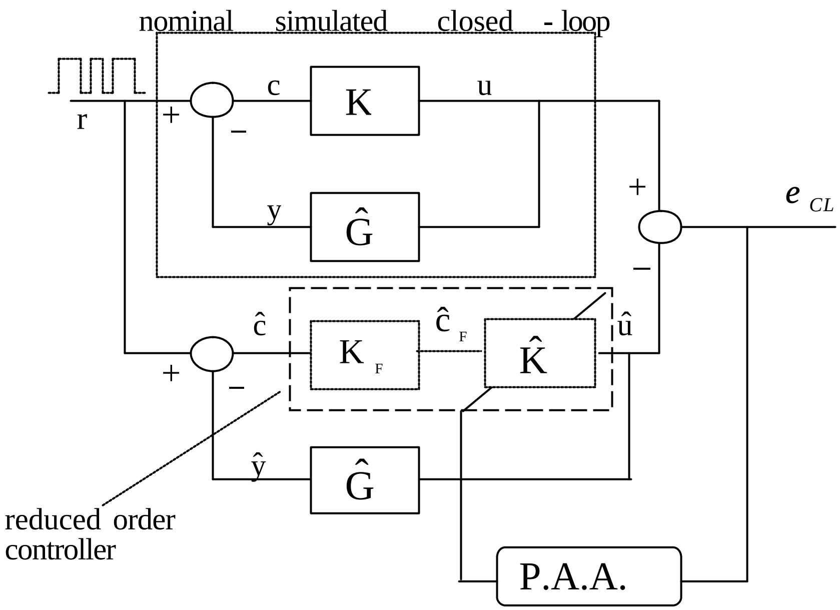
Estimated controller output

$$\hat{c}(t+1) = \hat{G}(q^{-1})[r(t+1) - \hat{u}(t+1)]$$

Same algorithm as CLIM, but the defintion of $\hat{c}(t+1)$ *is different (see previous slide for details)*

Forcing fixed parts in the reduced order controller

We would like that the "reduced order" controller maintains certain components of the "nominal" controller (ex: integrator, opening of the loop, etc)



$$\hat{K} = K_F \hat{K}' , \quad K_F \text{ is known}$$

Same algorithm but $\hat{c}$ *is replaced by* $\hat{c}_F = K_F \hat{c}$

Stability Analysis

A) $n_{\hat{R}} = n_R \; ; \; n_{\hat{S}} = n_S$

$$\lim_{t\to\infty} e_{CL}(t+1) = \lim_{t\to\infty} e^{0}_{CL}(t+1) = 0$$

if:

$$(*) \quad H'(z^{-1}) = H(z^{-1}) - \frac{l}{2}; \;\; \max_t l_2(t) \le l < 2$$

is a *strictly positive real transfer function* where:

$$H = \begin{cases} \hat{A}/\hat{P} & for\ CLIM \\ 1 & for\ F-CLIM \end{cases}$$

B) $n_{\hat{R}} < n_R \; ; \; n_{\hat{S}} < n_S$

Hypotheses:

A stabilizing controller with orders $n_{\hat{R}}$ and $n_{\hat{S}}$ exists

$$u(t+1) = -\hat{S}^*(q^{-1})u(t) + \hat{R}(q^{-1})c(t+1) + h(t+1)$$

$$r(t), h(t) = norm\ bounded$$

All signals are norm bounded under the passivity condition (*)

Asymptotic Properties of the Estimated Controller

CLIM

(CLIM with excitation added to the controller input)

Simulated data

$\hat{\theta}^*$ - *vector of the estimated controller parameters*

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \hat{S}_{up} - \hat{\hat{S}}_{up} \right|^2 \phi_r(\omega) d\omega = \arg\min_{\theta} \int_{-\pi}^{\pi} \left| \hat{S}_{yp} \right|^2 \left| K - \hat{K} \right|^2 \left| \hat{\hat{S}}_{yp} \right|^2 \phi_r(\omega) d\omega$$

- $\left\| \hat{S}_{up} - \hat{\hat{S}}_{up} \right\|_2$ is minimized if *r(t)* is white noise
- The frequency distribution of $\left| K - \hat{K} \right|^2$ is weighted by the output sensitivity functions for the nominal and for the reduced order controller
- The frequency distribution of $\left| K - \hat{K} \right|^2$ can be tuned by the choice of *r(t)*

Asymptotic Properties of the Estimated Controller

CLIM

(CLIM with excitation added to the controller input)

Use of Real Data

$$\hat{q}^{*} = \arg\min_{q} \int_{-p}^{p} \left\{ \left| S_{up} - \hat{\hat{S}}_{up} \right|^{2} f_{r}(w) + \left| S_{yp} \right|^{2} f_{v'}(w) \right\} dw$$

$v'(t) = v(t) - Kp(t)$: equivalent input noise

- The noise does not affect estimation of controller parameters

- When using real data, the closed loop system with reduced order controller approximates the real closed loop system (instead of the *nominal simulated system*)

Asymptotic Properties of the Estimated Controller

CLOM

(CLIM with excitation added to the plant input)

Simulated Data

$$\hat{q}^{*} = \arg\min_{q} \int_{-p}^{p} \left| \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right|^{2} f_{r}(w)dw = \arg\min_{q} \int_{-p}^{p} \left| \hat{S}_{yp} \right|^{2} \left| K - \hat{K} \right|^{2} \left| \hat{\hat{S}}_{yv} \right|^{2} f_{r}(w)dw$$

- $\left\| \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right\|_{2}$ is minimized if r(t) is white noise

-The frequency distribution of $\left| K - \hat{K} \right|^{2}$ is weighted by $\hat{S}_{yp}$ and $\hat{\hat{S}}_{yv}$

- The frequency distribution of $\left| K - \hat{K} \right|^{2}$ can be tuned by the choice of *r(t)*

Asymptotic Properties of the Estimated Controller

CLOM

(CLIM with excitation added to the plant input)

Use of Real Data

$$\hat{q}^* = \arg\min_q \int_{-p}^{p} \left\{ \left| \hat{\hat{S}}_{up}(G-\hat{G})S_{yp} - \hat{\hat{S}}_{yp}(K-\hat{K})S_{yv} \right|^2 f_r(w) + \left| S_{yp} \right|^2 f_{v'}(w) \right\} dw = \arg\min_q \int_{-p}^{p} \left\{ \left| S_{yp} - \hat{\hat{S}}_{yp} \right|^2 f_r(w) + \left| S_{yp} \right|^2 f_{v'}(w) \right\} dw$$

- The noise does not affect estimation of controller parameters

-Minimization of $\left| K - \hat{K} \right|^2$ in the frequency regions where the $\left| S_{yp} \right|$ and $\left| \hat{\hat{S}}_{yv} \right|$ are high

- Minimization of the gain of $\hat{\hat{S}}_{up}$ at the frequencies where important additive modeling errors exist and the gain of the estimated model is low

Validation of Estimated Reduced Order Controllers

Simulated Data

-The reduced order controller should **stabilize** the nominal model

- The (reduced) sensitivity functions should be **close** to the nominal ones in the critical regions for performance and robustness

-The (Vinnicombe) generalized stability margin for the reduced order system should be **close** to the nominal one

Validation tools

–ν-gap between "nominal" and "reduced order" sensitivity fct.
(Vinnicombe distance)

$$\delta_\nu\left(S,\hat{S}\right)=\left\|(1+S'S)^{-1/2}(S-\hat{S})(1+\hat{S}'\hat{S})^{-1/2}\right\|_\infty<1$$

(+ winding number condition. S' denotes complex conjugate of S)

The ν-gap should be small

- Visual comparison of the sensitivity functions.

One assumes: $\hat{G}=G$ *! (as everybody in reduction business)*

- Closeness of the generalized stability margin
(for the reduced and nominal controller)

Normalized distance between two transfer functions (G_1, G_2)

The winding number:

$$wno(G) = n_{z_i}(G) - n_{p_i}(G)$$

Unstable zeros Unstable poles

wno(G)>0 wno(G)<0

$wno(G)$ = number of encirclements of the origin (winding number)
(+ : counter clock wise , - : clock wise)

One can compares transfer functions satisfying :

$$wno(1 + G_2^* G_1) + n_{p_i}(G_1) - n_{p_i}(G_2) - n_{P_1}(G_2) = 0 \qquad \{w\}$$

G^* = complex conjugate of G $\qquad$ $n_{P_1}(G_2)$ = number of poles on the unit circle

Normalized distance between two transfer functions (G_1, G_2)

One assumes that {w} is satisfied.

Normalized difference :

$$\Psi\left[G_1(jw), G_2(jw)\right] = \frac{G_1(jw) - G_2(jw)}{\left(1 + \left|G_1(jw)\right|^2\right)^{1/2}\left(1 + \left|G_2(jw)\right|^2\right)^{1/2}}$$

Normalized distance (Vinnicombe distance or ν-gap) :

$$\boxed{d_n(G_1, G_2) = \left|\Psi\left[G_1(jw), G_2(jw)\right]\right|_{\max_w} = \left\|\Psi\left[G_1(jw), G_2(jw)\right]\right\|_\infty \quad \text{for } w = 0 \text{ à } p\ f_e}$$

$$0 \le d_n(G_1, G_2) < 1$$

If {w} is not satisfied : $d_n(G_1, G_2) = 1$

Vinnicombe Stability Margin [b(K,G)]

$$b(K,G) = \begin{cases} \left\| T(K,G) \right\|_{\infty}^{-1} & if \;\; (K,G) \; is \; stable \\ 0 & otherwise \end{cases}$$

$$T(K,G) = \begin{bmatrix} S_{yr} & S_{yv} \\ -S_{up} & S_{yp} \end{bmatrix}$$

Vinnicombe Robust Stability Test

The controller K which stabilizes plant model G_1 will stabilize also G_2 if :

$$\boxed{d_n(G_1, G_2) \leq b(K, G_1)}$$

Initial robust design

We would like to have for the reduced order controller:

$$d_n(G_1, G_2) \leq b(\hat{K}, G_1) \qquad \textit{(preservation of robustness)}$$

Validation test:

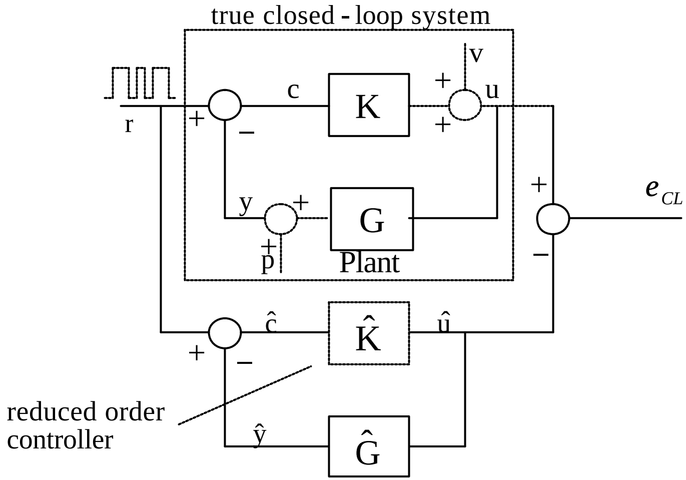
$$\boxed{\left| b(K, G_1) - b(\hat{K}, G_1) \right| < e \; ; e > 0}$$

Validation of Estimated Reduced Order Controllers

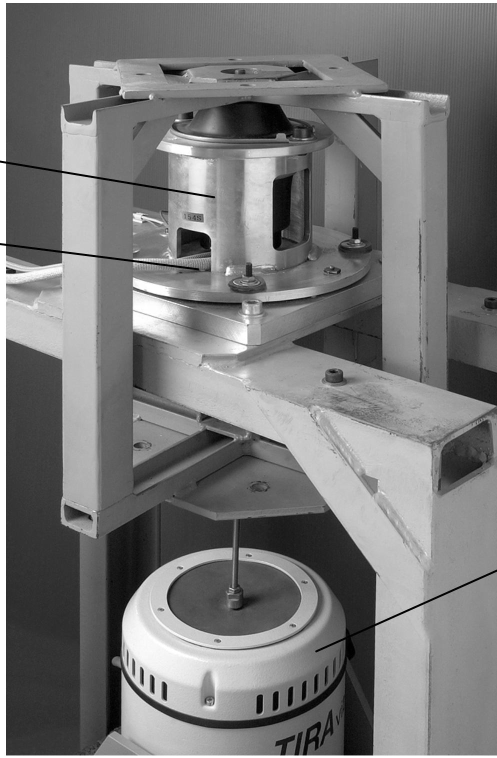
Use of Real Data

- Statistical tests (like in closed loop identification)
  - *variance of residual closed loop error*
  - *cross-correlations* (ε_{CL} / $\hat{u}$)
- Vinnicombe gap between :

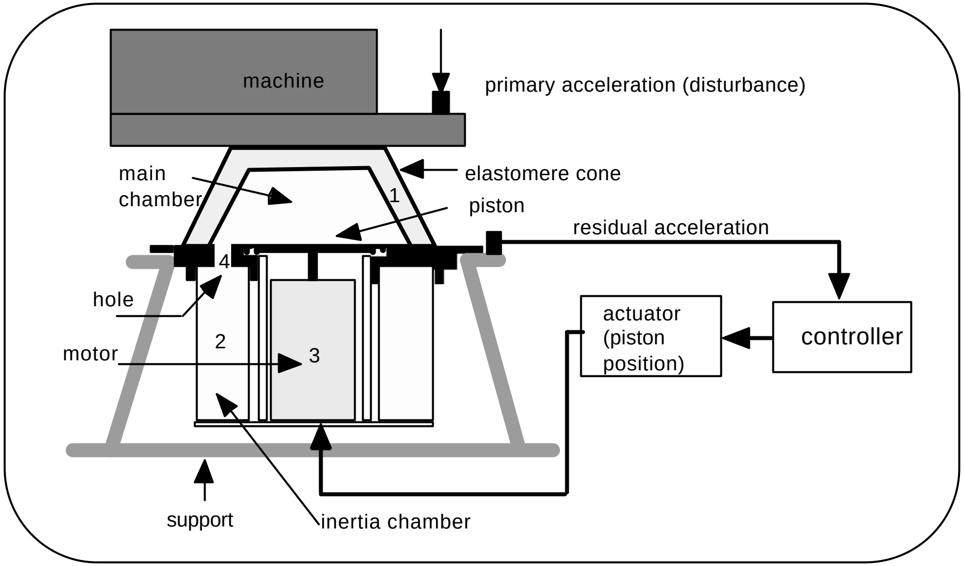
Identified T.F. of the true nominal closed loop **and** *Computed T.F. of the simulated closed loop with reduced order controller*



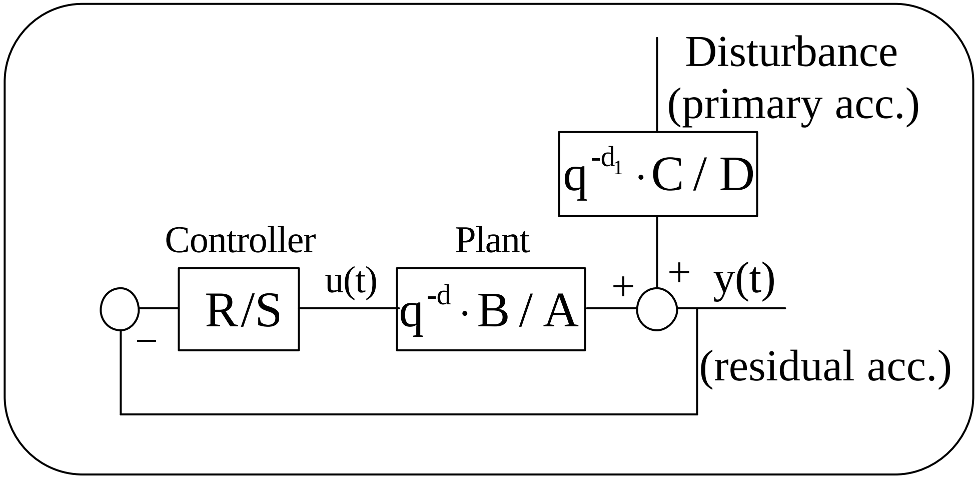
The Active Suspension



Experimental Results - Control of an Active Suspension



- controller: PC
- sampling freq.: 800 Hz



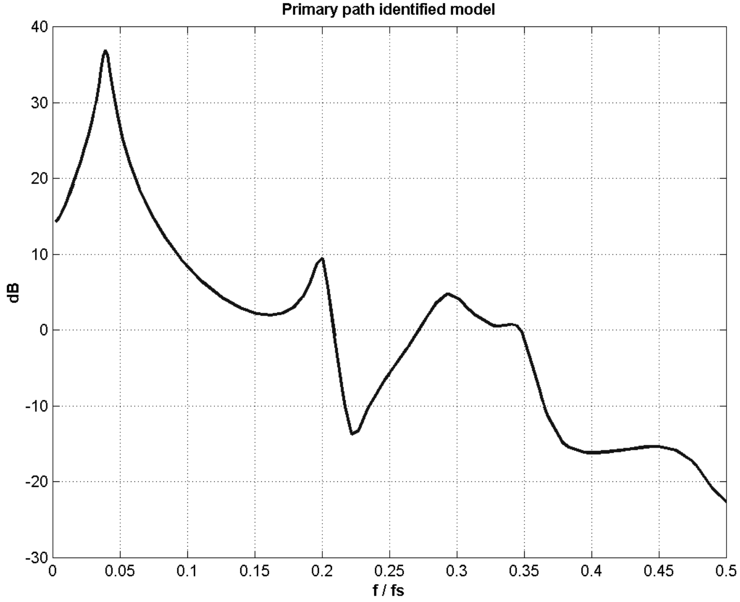
Interesting frequency range for vibration attenuation: 0 - 200 Hz

(Wide band attenuation pb.)

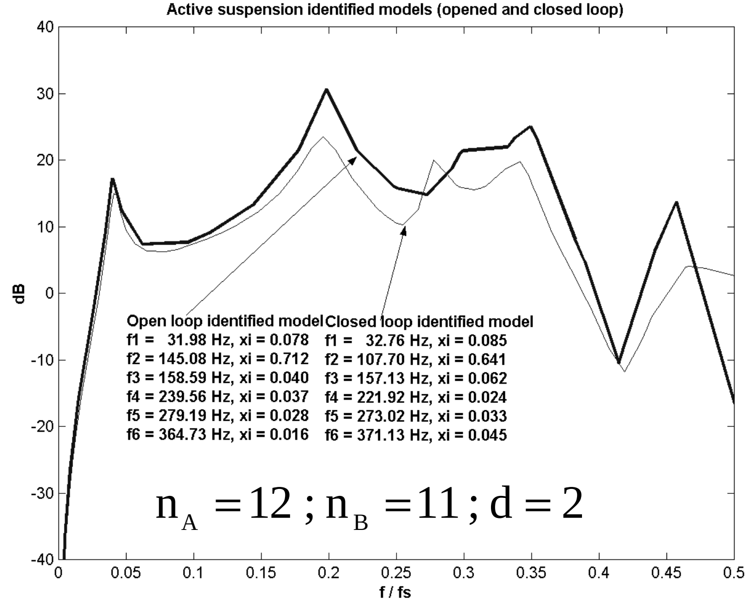
Active Suspension

Frequency Characteristics of the Identified Model

Primary path



Secondary path



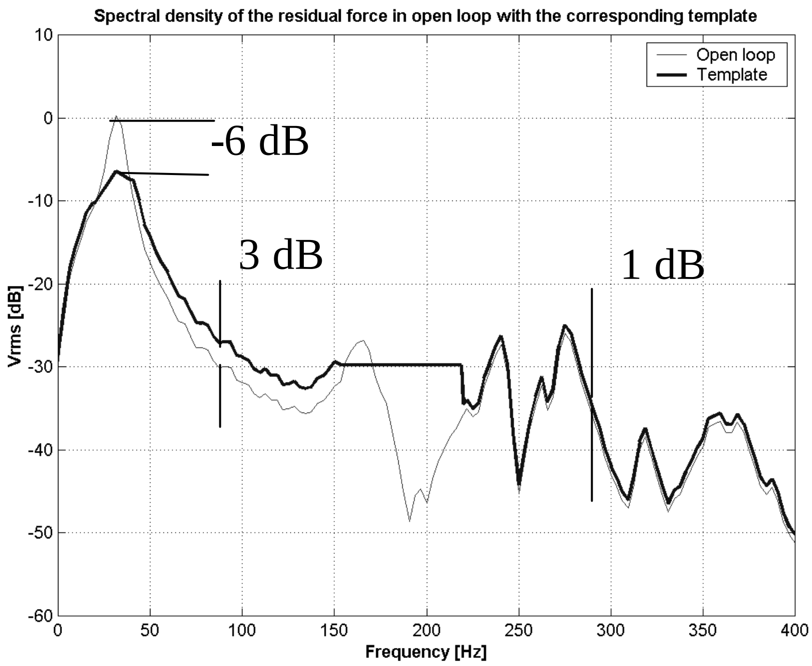
Control objectives :

- Minimize residual acc. around first vibration mode
- Distribute amplification of disturb. over high frequency region

- Open loop identified model (design model)
- Closed loop identified model used for controller reduction (better C.L. validation)

Control objectives (wide band problem)

- Attenuate residual force (acc.) around first vibration mode (32 Hz)
- Distribute amplification of disturbance over high frequency region
- Operate almost in open loop close to the Nyquist frequency



The Nominal Controller

Design method: Pole placement with sensitivity shaping using convex optimization

Dominant poles : first vibration mode with ξ=0.8 (instead of 0.078)

Opening of the loop at 0.5f_s : $H_R = 1 + q^{-1}\,;\,(R = H_R R')$

Nominal controller complexity : $n_R = 27\,;\, n_S = 28$

Pole placement complexity : $n_R = 12\,;\, n_S = 13$

Direct Controller Reduction

CLIM algorithm/ simulated data

r(t) = PRBS, L= 4096 , clock = $0.5f_S$, N=10

P.A.A.: *variable forgetting factor*

$$H_R = 1 + q^{-1}\ ;\ (\hat{K} = H_R \hat{K}')$$

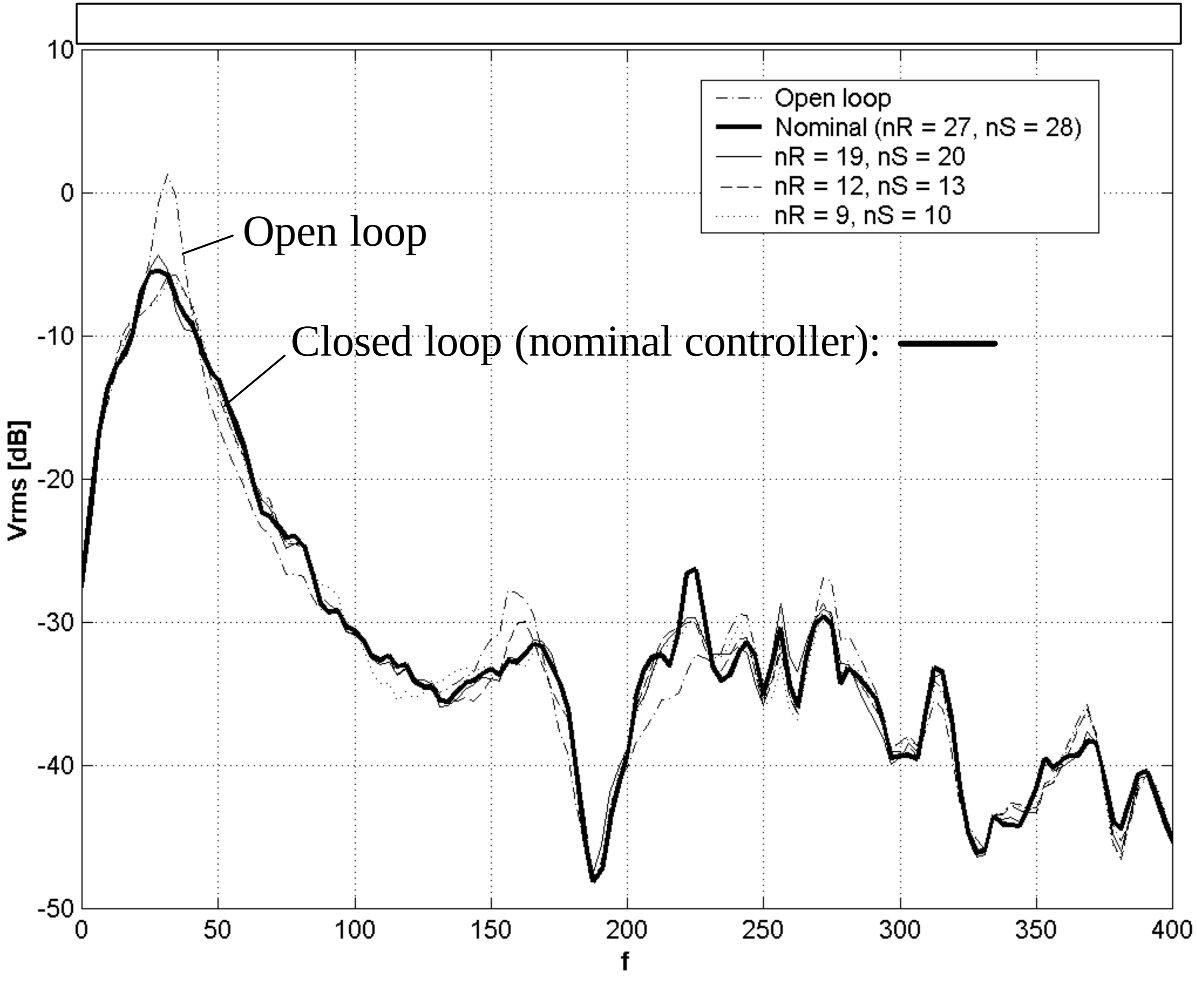
		Controller	K_n $n_R = 27$ $n_S = 28$	K_1 $n_R = 19$ $n_S = 20$	K_2 $n_R = 12$ $n_S = 13$	K_3 $n_R = 9$ $n_S = 10$
		$d_n(K_n, K_i)$	0	0.1810	0.5049	0.5180
		$d_n(S^n_{up}, S^i_{up})$	0	0.1487	0.4388	0.4503
		$d_n(S^n_{yp}, S^i_{yp})$	0	0.0928	0.1206	0.1233
		$b(k)$	0.0800	0.0786	0.0685	0.0810
real time experiments	{	$d_n(CL(K_n), CL(K_i))$	0.1296	0.2461	0.5435	0.5522
		C.L. error variance	0.0023	0.0083	0.0399	0.0398

Performances of the reduced order controllers are very close to those of the nominal controller (see next slide)

Direct Controller Reduction

CLIM algorithm/ simulated data

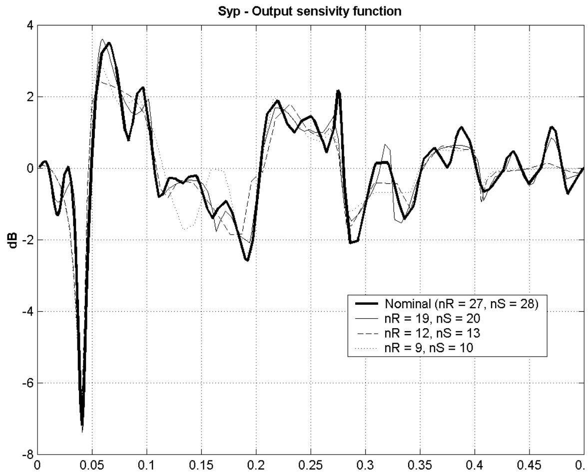
Spectral density of the residual force (performance)

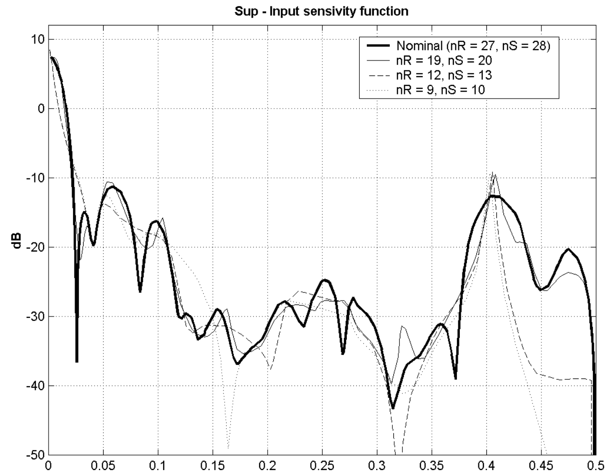


Direct Controller Reduction

CLIM algorithm/ simulated data

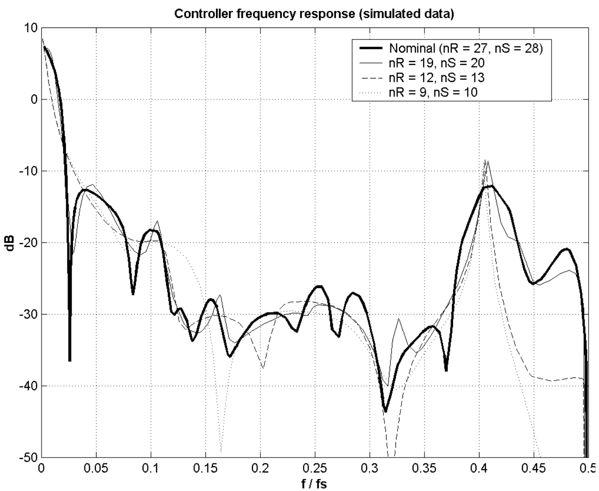
S_{yp}

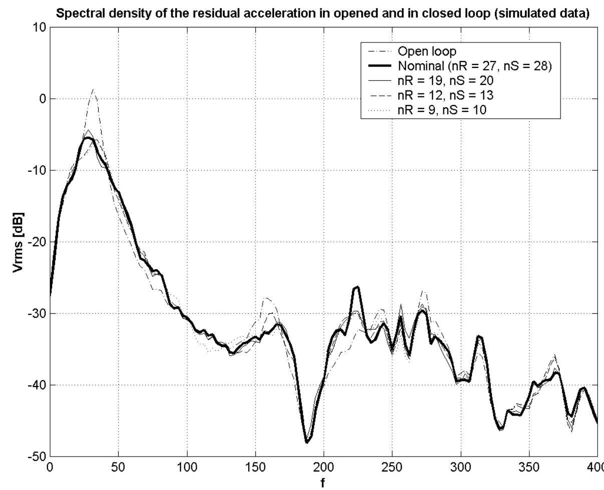




S_{up}

K





Spectral density of residual force (performance)

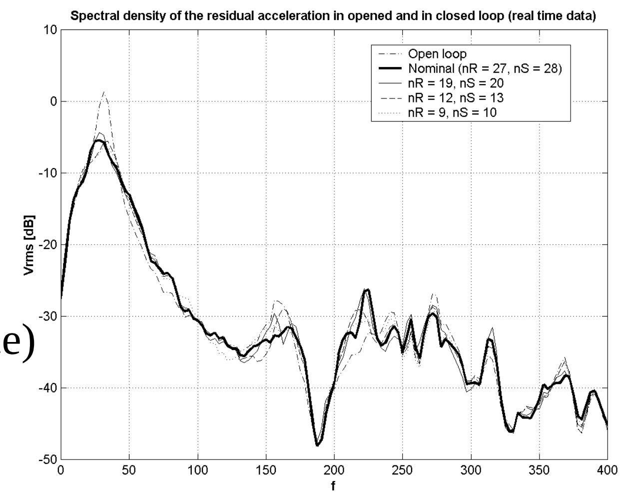
Direct Controller Reduction

CLIM algorithm/ use of real data

Controller	K_n $n_R = 27$ $n_S = 28$	K_1 $n_R = 19$ $n_S = 20$	K_2 $n_R = 12$ $n_S = 13$	K_3 $n_R = 9$ $n_S = 10$
$d_n(K_n, K_i)$	0	0.1500	0.4870	0.5216
$d_n(S_{up}^n, S_{up}^i)$	0	0.1285	0.4197	0.4560
$d_n(S_{yp}^n, S_{yp}^i)$	0	0.1719	0.1639	0.1150
$b(k)$	0.0800	0.0722	0.0605	0.0823
$d_n(CL(K_n), CL(K_i))$	0.1296	0.1959	0.5230	0.5602
C.L. error variance	0.0023	0.0072	0.0359	0.0422

real time experiments (last two rows)

Spectral density of residual force (performance)



Results are very close to those obtained with simulated data

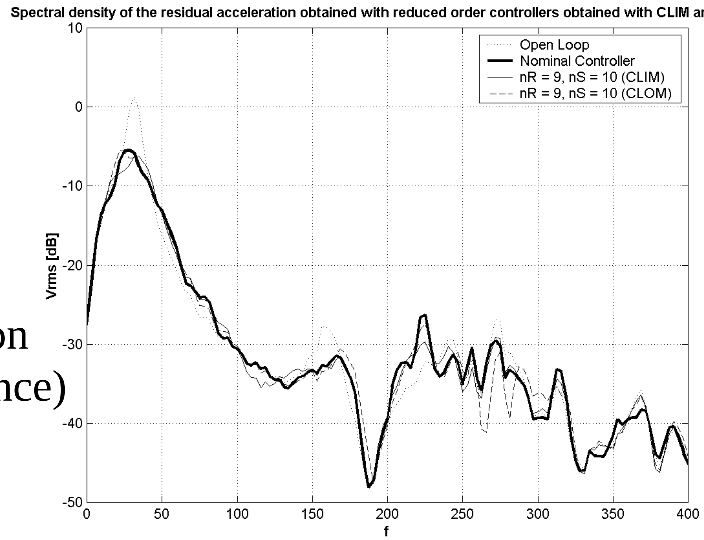
Explanation :
Quality of the model used for controller reduction

Direct Controller Reduction

CLOM algorithm/ simulated data

Controller	K_n $n_R = 27$ $n_S = 28$	K_2 $n_R = 12$ $n_S = 13$	K_3 $n_R = 9$ $n_S = 10$
$d_n(K_n, K_i)$	0	0.7287	0.7743
$d_n(S^n_{up}, S^i_{up})$	0	0.7144	0.7709
$d_n(S^n_{yp}, S^i_{yp})$	0	0.0975	0.1007
$b(k)$	0.0800	0.0786	0.0796

Spectral density of residual acceleration (performance)



– Smaller $\delta_\nu\left(S^n_{yp}, S^i_{yp}\right)$ with CLOM

– Smaller $\delta_\nu\left(S^n_{up}, S^i_{up}\right)$ with CLIM

(coherent with the theory)

CLIM and CLOM provide reduced order controllers with good performances

Practical Hints

A) No access to real-time data

Classical situation for controller reduction techniques

Given : nominal plant model, nominal controller

B) Access to the real system

- Improve the quality of the model by identification in closed loop
- Use also real data for direct controller reduction
- Do real time validation of the reduced order controllers

Controller reduction schemes

Two possibilities for error generation:
-input error
-output error

Two possibilities for applying the external excitation:
-added to the controller input
-added to the plant input

What is in fact important ?

The nominal sensitivity function we would like to approximate

This is related to the control objective
(what is the critical sensitivity fct. for performance and robustness specifications ?)

Selection of controller reduction schemes

Controller reduction criterion	Controller reduction scheme
$\min\left\Vert \hat{S}_{yp} - \hat{\hat{S}}_{yp} \right\Vert$ or $\min\left\Vert \hat{S}_{yr} - \hat{\hat{S}}_{yr} \right\Vert$	**CLIM with external excitation added to the plant input (short name :CLOM)** equivalent to CLOM with external excitation added to the controller input
$\min\left\Vert \hat{S}_{up} - \hat{\hat{S}}_{up} \right\Vert$	**CLIM with external excitation added to the controller input (short name : CLIM)**
$\min\left\Vert \hat{S}_{yv} - \hat{\hat{S}}_{yv} \right\Vert$	**CLOM with external excitation added to the plant input**

COHERENCE

What closed loop plant model identification scheme should be used when a criterion for controller reduction is given ?

Answer: *Same criterion for identification in closed loop and controller reduction*

- ***Tracking and output disturbance rejection (control objective)***

CLOE with excitation added to controller input $\longrightarrow$ Model identification or Model reduction

CLIM with excitation added to plant input $\longrightarrow$ Controller reduction

In both schemes: $\left\| S_{yp} - \hat{S}_{yp} \right\|_2$ **is minimized**

Coherent controller reduction and identification in closed loop

Controller reduction criterion	Controller reduction scheme	Closed loop identification scheme
$\min\left\|\hat{S}_{yp}-\hat{\hat{S}}_{yp}\right\|$ or $\min\left\|\hat{S}_{yr}-\hat{\hat{S}}_{yr}\right\|$	CLIM with external excitation added to the plant input (CLOM)	CLOE with external excitation added to the controller input
$\min\left\|\hat{S}_{up}-\hat{\hat{S}}_{up}\right\|$	CLIM with external excitation added to the Controller input (CLIM)	CLIE with external excitation added to the controller input
$\min\left\|\hat{S}_{yv}-\hat{\hat{S}}_{yv}\right\|$	CLOM with external excitation added to the plant input	CLOE with external excitation added to the plant input

Coherent controller reduction and identification in closed loop

For experimental results on “coherence” of controller reduction and identification in closed loop see :

Landau I.D., Karimi A., (2002): « A unified approach to closed-loop plant identification and direct controller reduction », *European J. of Control*, *vol. 8*, *no.6*

~ 70% improvement in performance of the reduced order controller when coherent algorithms are chosen instead of a non coherent combination

REDUC^{TM}
(Matlab) Toolbox for controller order reduction by closed-loop identification

To be downloaded from the web site:
http//:landau-bookic.lag.ensieg.inpg.fr

- files(.p and.m)
- examples (data files)
- help.htm files (condensed manual)
- manual

REDUC Toolbox

>> help reduc

CONTROLLER ORDER REDUCTION MODULE
by :
ADAPTECH
4 rue du Tour de l'Eau, 38400 Saint Martin d'Heres, France
info@adaptech.com
June 30,1999

List of functions:

conid - Controller Identification Based on Closed Loop Output Error

conidf - Controller Identification Based on Filtered Closed Loop Output Error

conidaf - Controller Identification Based on Adaptive Filtered Closed Loop Output Error

vgap - Vinnicombe's Gap Between Two Discrete Time Loop Output Error

cor - Controller Order Reduction Based on Closed Loop Identification with Simulated Data

cortr - Controller Order Reduction Based on Closed Loop Identification with Real-Time Acquired Data

compcon - Comparison of Reduced Order Controllers Obtained by COR or CORTR Transfer Functions

smarg - Stability Margin Discrete Time Closed Loop Systems

ctod - Discrete Time Polynomial Related to a Damping Factor and Normalized Natural Frequency in Continuous Time

mbode - Magnitude Bode Diagram of a Discrete Transfer Function on a Linear Scale Time Axis

addz - Add Two Polynomials in z-1

Use CONID
(CLIM with excitation
added to the controller input)

>> help cor

COR is a Controller Order Reduction function based on CLOE identification method.

[Rt,St,Table]=cor(r,B,A,R,S,Hr,Hs,Ts,tol,Fin,lam1,lam0)

r is the excitation signal which is added to the input of the controller.
B and A are the numerator and denominator of the plant model.
R and S are the numerator and denominator of the initial controller.
Hr and Hs are the fixed terms on R and S (Robustness filter) with
following default values: Hr=1,Hs=1
Ts is the sampling period in Sec. (default=1)
tol is the tolerance value for vgap computing(default=0.001)
Fin is the initial gain (Fin=1000 by default)

lam1 and lam0 make different adaptation algoritms as follows:
(lam1=0.95,lam0=1)

lam1=1;lam0=1 :decreasing gain
0.95<lam1<1;lam0=1 :decreasing gain with fixed forgetting factor
0.95<lam1,lam0<1 :decreasing gain with variable forgetting factor

Rt and St are the matrices containing the reduced order controllers.

COR - Controller Order Reduction function

>> help cor

Model Polynomials

Fixed filters in the reduced order controller

Tolerance for ν-gap computation

[Rt,St,Table]=cor(r,B,A,R,S,Hr,Hs,Ts,tol,Fin,lam1,lam0)

Reduced order controller

Summary of reduction results

Excitation

Nominal Controller Polynomials

Sampling Period (s)

Initial adaption gain (Fin=1000 by default)

λ_1

λ_0

lam1=1;lam0=1 : decreasing gain (default algorithm)

0.95<lam1<1; lam0=1 : decreasing gain with fixed forgetting factor

0.95<lam1,lam0<1 : decreasing gain with variable forgetting factor

Remark : to start use default values for: *tol,Fin,lam1,lam0*

Examples data files

File ***mods.mat*** : a model of the active suspension ($nA=6$, $nB=8$, $d=0$)
Remark: The delay d is included in B.

Sampling period : *0.00125 s (800 Hz)*

A =

1.0000	-1.6184	1.6617	-1.8469	1.6278	-1.3491	0.7239

B =

0	0	0	0	-0.3149	2.8144	-2.5972	-1.9891	2.0869

File ***reg0.mat*** *:* the nominal controller with $nR= 11$, $nS=13$ and including a fixed part $H_r =1 + q^{-1}$

Remark: the complexity of a simple pole placement design will be: $nR=5$, $nS=7$

File ***excs.mat***: external excitation – PRBS with clock frequency $f_s/4$

We would like to maintain H_r in the reduced order controller

$$\textbf{[Rt,St,Table]=cor(r,B,A,R,S,}\underbrace{\textbf{[1 1]}}_{H_R}\textbf{,}\underbrace{\textbf{1}}_{H_S}\textbf{,0.00125)}$$

[Table] – Summary of the results

```
>> [Table]=cor(r,B,A,R,S,[1,1],1,0.00125)
```

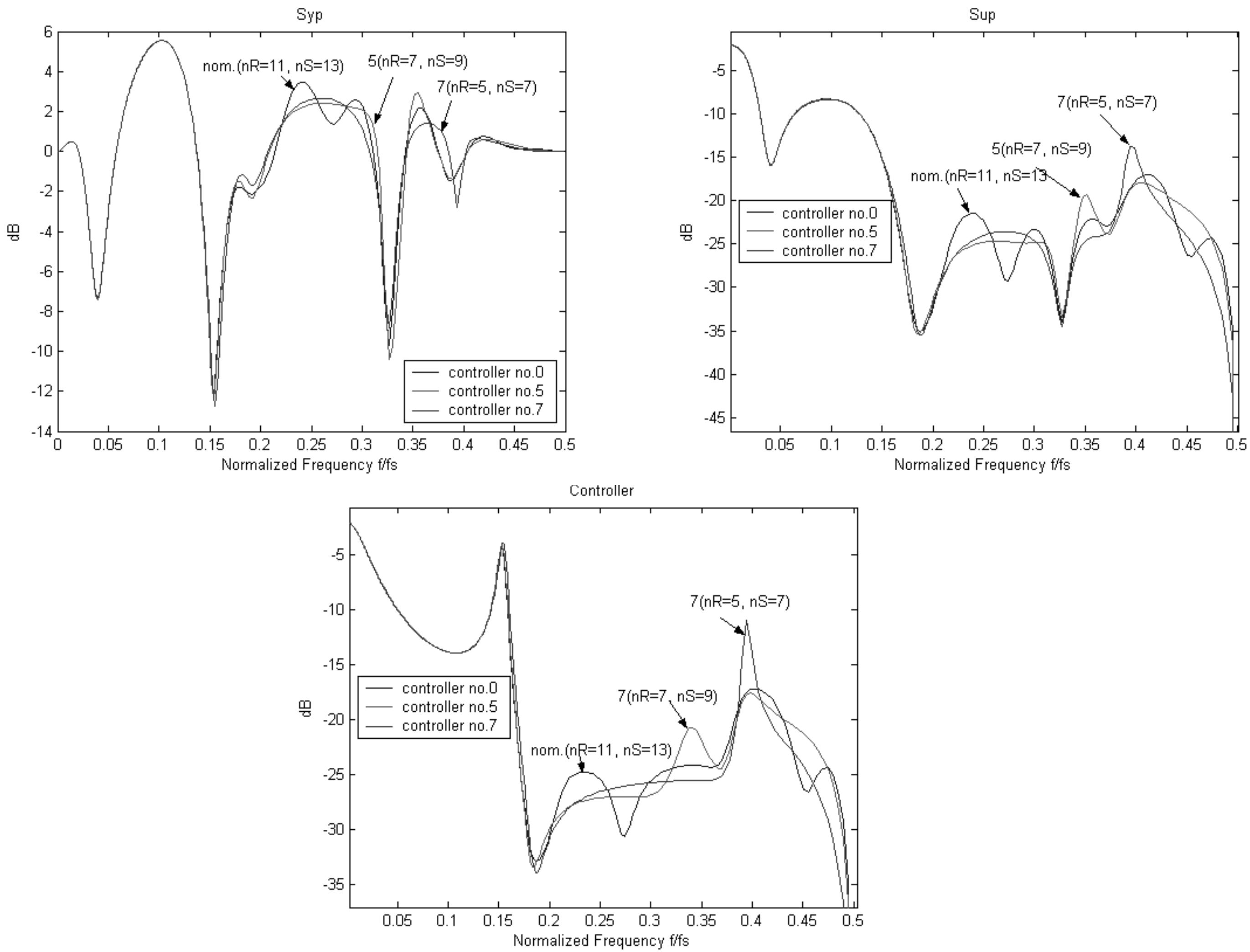
	No.	nR	nS	Vg(R/S)	Vg(Sup)	Vg(Syp)	St-margin	Max(Syp)[fmax]	stable	
Nom. Contr. —	0	11	13	0.0000	0.0000	0.0000	0.0756	5.55[81.90]	1	
Estimated Controllers	1	11	13	0.0352	0.0336	0.0707	0.0763	5.56[81.90]	1	stable
	2	10	12	0.0362	0.0334	0.0678	0.0826	5.56[81.90]	1	
	3	9	11	0.0365	0.0586	0.1578	0.0645	5.56[81.90]	1	
	4	8	10	0.0441	0.0451	0.0838	0.0849	5.56[81.90]	1	
	5	7	9	0.0390	0.0414	0.1221	0.0692	5.55[81.90]	1	
	6	6	8	0.3349	0.0708	0.2239	0.0552	5.62[81.90]	1	
	7	5	7	0.1873	0.1353	0.1191	0.0719	5.55[81.90]	1	
	8	4	6	1.0000	1.0000	1.0000	0.0000	20.74[167.46]	0	unstable
	9	3	5	0.9566	1.0000	0.9843	0.0000	6.38[170.97]	0	
	10	2	4	0.4587	1.0000	1.0000	0.0000	6.51[86.26]	0	
	11	1	3	0.4240	0.3787	0.3685	0.0550	9.60[77.77]	1	
	12	1	2	0.4132	0.3421	0.5279	0.0307	11.79[176.37]	1	
	13	1	1	0.4446	0.1961	0.3991	0.0491	8.71[181.94]	1	
	14	1	0	0.5359	0.5124	0.5915	0.0836	6.30[201.81]	1	

Vg (X) : Vinnicombe distance between nominal X and reduced order $\hat{X}$

St-margin: Vinnicombe stability margin

Comparison of the various controllers

>> compcon(B,A,Rt,St,[0,5,7])



Concluding Remarks

- The Daphné algorithms (CLIM,CLOM) allow to directly estimate reduced order controllers
- The algorithms achieve a two norm minimization between nominal and reduced order sensitivity functions
- They have the unique feature of using also real data (this allows to take in account to a certain extent the modeling error)
- Direct estimation of reduced order controllers can be interpreted as the *dual* of reduced order plant model identification in closed loop
- **Successful use in practice**
- A MATLAB Toolbox is available (REDUC)
- There is an interaction between closed loop identification and direct controller reduction (*coherence*)

References

Anderson B.D.O., Liu Y., (1989) : « Controller reduction : concepts and approaches », *IEEE Trans. on Automatic Control*, vol. 34, no. 8, pp. 802-812.

Anderson B.D.O., (1993) : « Controller reduction : moving from theory to practice », *IEEE Control Magazine*, vol. 13, pp. 16-25.

Landau I.D., Karimi A., Constantinescu A., (2001) : « Direct controller reduction by identification in closed loop », *Automatica*, vol. 37, no. 11, pp. 1689-1702.

Landau I.D., Karimi A., (2002) : « A unified approach to closed-loop plant identification and direct controller reduction », *European Journal of Control, vol. 8, no.6*

A. Constantinescu, I.D. Landau (2003) « Direct controller order reduction by identification in closed loop applied to a benchmark problem », *European J.of Control*, vol.9, no.1

Landau I.D., (2002) *Commande des systèmes – conception, identification et mise en œuvre »* Hermes-Lavoisier, Paris (translation available)

Vinnicombe G., (1993) : « Frequency domain uncertainty and the graph topology », *IEEE Trans. on Automatic Control*, vol. 38, no. 9, pp. 1371-1383.