Robust discrete time control

Design of robust discrete time controllers

Outline

- Pole placement (tracking and regulation)
- Tracking and regulation with independent objectives
- Internal model control (tracking and regulation)
- Pole placement with sensitivity function shaping
- Design examples

Computer control (discrete time controllers)
Possibilities and advantages

- Large choice of strategies for controller design
- Use of more complex algorithms but with better performance than the PID
- Techniques well suited for the control of:
  - *systems with delay (dead time)*
  - *systems characterized by high order dynamic models*
  - *systems with low damped vibration modes*
- Easy combination of control design and system identification

Digital controllers – Design methods

- Pole placement (tracking and regulation)
- Tracking and regulation with independent objectives
- Internal model control (tracking and regulation)
- Pole placement with sensitivity function shaping

Remarks:

- *All the controllers witll have the R-S-T structure (two degrees of freedom controller)*
- *The « memory » (number of parameters) depends upon the complexity of the model used for design*
- *All the design methods can be wiewed as particular cases of the pole placement*
- *The design and tuning of the controllers require the knowledge of a discrete time model of the plant*

Pole placement

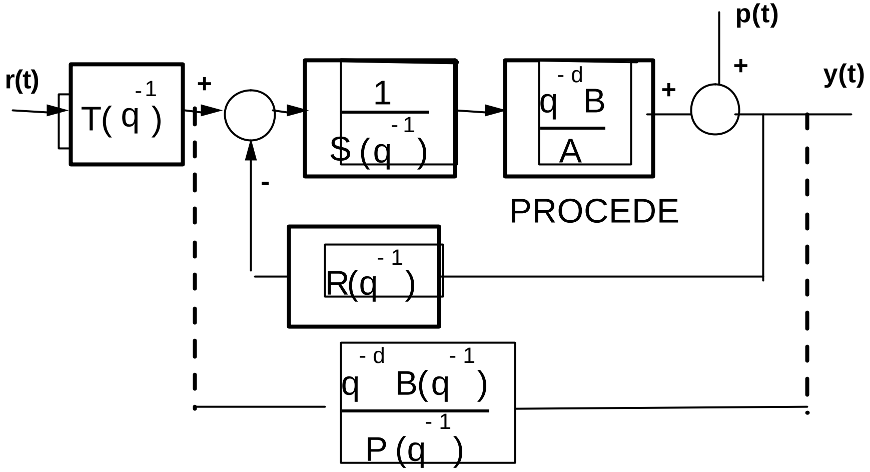
The pole placement allows to design a R-S-T controller for

- stable or unstable systems
- without restriction upon the degrees of *A* and *B* polynomials
- without restrictions upon the plant model zeros (stable or unstable)

It is a method which does not simplify the plant model zeros

The digital PID can be designed using pole placement

Structure



Plant model: $$G(q^{-1}) = H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} \qquad B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n_B} q^{-n_B} = q^{-1} B^*(q^{-1})$$

Pole placement

Closed loop T.F. (r $\longrightarrow$ y) *(reference tracking)*

$$H_{BF}(q^{-1}) = \frac{q^{-d}T(q^{-1})B(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{q^{-d}T(q^{-1})B(q^{-1})}{P(q^{-1})}$$

$$P(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = 1 + p_1 q^{-1} + p_2 q^{-2} + \ldots$$

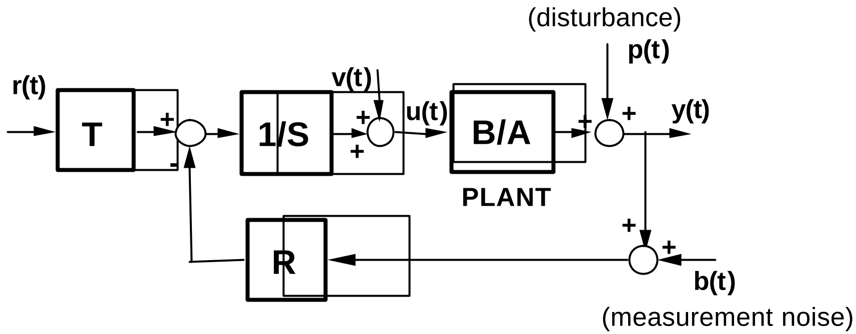
Defines the (desired)closed loop poles

Closed loop T.F.(p $\rightarrow$ y) *(disturbance rejection)*

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})}$$

Output sensitivity function

Digital control in the presence of disturbances and noise



Output sensitivity function
(p ——y)

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Input sensitivity function
(p ——u)

$$S_{up}(z^{-1}) = \frac{-A(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Noise-output sensitivity function
(b—— y)

$$S_{yb}(z^{-1}) = \frac{-B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Input disturbance-output sensitivity function
(v—— y)

$$S_{yv}(z^{-1}) = \frac{B(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

All four sensitivity functions should be stable ! (see book pg.102 - 103)

Choice of desired closed loop poles (polynomial *P*)

$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Dominant poles **Auxiliary poles**

Choice of $P_D(q^{-1})$ (dominant poles)

Specification in continuous time (t_M, M) — 2nd order (ω_0, ζ) —discrétisation, T_e→ $P_D(q^{-1})$

$$0.25 \le w_0 T_e \le 1.5$$
$$0.7 \le z \le 1$$

Auxiliary poles

- *Auxiliary poles are introduced for robustness purposes*
- *They usually are selected to be faster than the dominant poles*

Regulation(computation of *R(q⁻¹)* and *S(q⁻¹)*)

(Bezout) $$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = P(q^{-1}) \quad \textbf{(*)}$$

? ↗ ↖ **?**

$$n_A = \deg A(q^{-1}) \quad n_B = \deg B(q^{-1})$$

A and *B* do not have common factors

unique minimal solution for :

$$n_P = \deg P(q^{-1}) \le n_A + n_B + d - 1$$

$$n_S = \deg S(q^{-1}) = n_B + d - 1 \qquad n_R = \deg R(q^{-1}) = n_A - 1$$

$$S(q^{-1}) = 1 + s_1 q^{-1} + \dots s_{n_S} q^{-n_S} = 1 + q^{-1}S^*(q^{-1})$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \dots r_{n_R} q^{-n_R}$$

Computation of *R(q-1)* and *S(q-1)*

Equation (*) is written as: $\boxed{Mx = p}$ $\longrightarrow$ $x = M^{-1}p$

$$x^T = [1, s_1, \ldots, s_{n_S}, r_0, \ldots, r_{n_R}] \qquad p^T = [1, p_1, \ldots, p_i, \ldots, p_{n_P}, 0, \ldots, 0]$$

$$M = \underbrace{\left[\begin{array}{cccc|cccc} \multicolumn{4}{c|}{\overbrace{\hphantom{000000000}}^{n_B + d}} & \multicolumn{4}{c}{\overbrace{\hphantom{000000000}}^{n_A}} \\ 1 & 0 & \ldots & 0 & 0 & \ldots & \ldots & 0 \\ a_1 & 1 & & . & b'_1 & & & \\ a_2 & & & 0 & b'_2 & & & b'_1 \\ & & & 1 & . & & & b'_2 \\ & & & a_1 & . & & & . \\ a_{n_A} & & & a_2 & b'_{n_B} & & & . \\ 0 & & & . & 0 & . & . & . \\ 0 & \ldots & 0 & a_{n_A} & 0 & 0 & 0 & b'_{n_B} \end{array}\right]}_{n_A + n_B + d} \Bigg\} \; n_A + n_B + d$$

$b'_i = 0$ pour $i = 0, 1 \ldots d$; $b'_i = b_{i-d}$ pour $i > d$

Use of WinReg or *bezoutd.sci(.m)* for solving (*)

Structure of $R(q^{-1})$ and $S(q^{-1})$

R et S include pre-specified fixed parts (ex: intégrator)

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

H_R, H_S, - pre specified polynomials

$$R'(q^{-1}) = r'_0 + r'_1 q^{-1} + \ldots r'_{n_{R'}} q^{-n_{R'}} \quad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_{n_{S'}} q^{-n_{S'}}$$

- The pre specified filters H_R and H_S will allow to impose certain properties of the closed loop.
- They can influence performance and/or robustness

Parties fixes (H_R, H_S). Exemples

Zero steady state error (S_{yp} should be null at certain frequencies)

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})}{P(q^{-1})}$$

Step disturbance : $H_S(q^{-1}) = 1 - q^{-1}$

Harmonic distrubance : $H_S = 1 + aq^{-1} + q^{-2}$; $a = -2\cos wT_e$

Signal blocking (S_{up} should be null at certain frequencies)

$$S_{up}(q^{-1}) = -\frac{A(q^{-1})H_R(q^{-1})R'(q^{-1})}{P(q^{-1})}$$

Harmonic signal: $H_R = 1 + bq^{-1} + q^{-2}$; $b = -2\cos wT_e$

Blocking at 0.5f_{S}: $H_R = (1 + q^{-1})^n$; $n = 1,2$

Solving pole placement whith pre-specified filters in the controller

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

H_R, H_S, - pre specified polynomials

$$R'(q^{-1}) = r'_0 + r'_1 q^{-1} + \ldots r'_{n_{R'}} q^{-n_{R'}} \quad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_{n_{S'}} q^{-n_{S'}}$$

Eq.(*) (transp. 10) becomes:

$$A(q^{-1})S'(q^{-1})H_S(q^{-1}) + q^{-d}B(q^{-1})R'(q^{-1})H_R(q^{-1}) = P(q^{-1}) \quad \textbf{(**)}$$

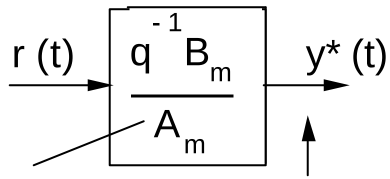
$$n_P = \deg P(q^{-1}) \leq n_A + n_{HS} + n_B + n_{HR} + d - 1$$

$$n_{S'} = \deg S'(q^{-1}) = n_B + n_{HR} + d - 1 \qquad n_{R'} = \deg R'(q^{-1}) = n_A + n_{HS} - 1$$

Use of WinReg or *bezoutd.sci(.m)* for solving (**)
with $A' = AH_S$, $B' = BH_R$

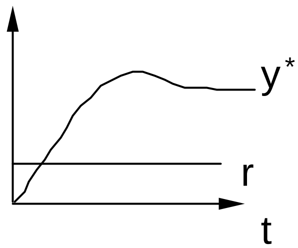
Tracking (computation of $T(q^{-1})$)

Ideal case



Tracking reference model (H_m)

desired trajectory for y (t)



$$H_m(q^{-1}) = \frac{q^{-1}B_m(q^{-1})}{A_m(q^{-1})}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots$$

$$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots$$

Specification in continuous time (t_M, M) —— 2^{nd} order (ω_0, ζ) $\xrightarrow[T_e]{\text{discrertization}}$ $H_m(q^{-1})$

$$0.25 \le w_0 T_e \le 1.5$$
$$0.7 \le z \le 1$$

The ideal case can not be attained (delay, plant zeros)

Objective : to approach y(t)*

$$y^*(t) = \frac{q^{-(d+1)}B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Tracking (computation of $T(q^{-1})$)

Build: $$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Choice of $T(q^{-1})$:

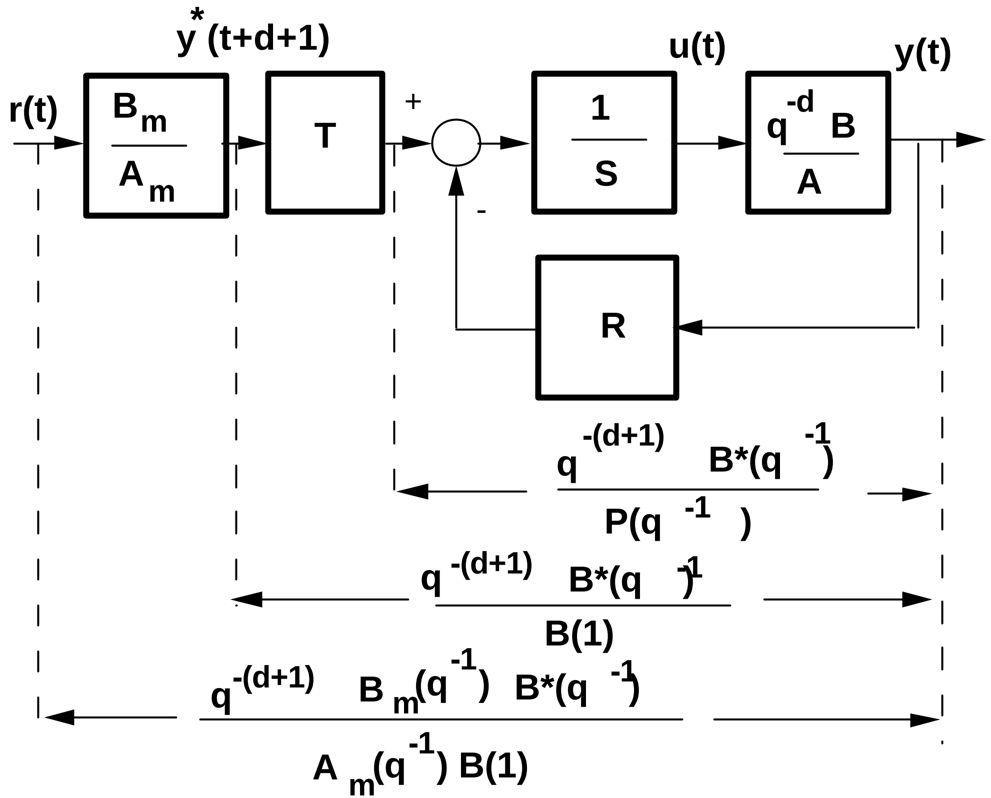
- Imposing unit static gain between y^* and y
- Compensation of regulation dynamics $P(q^{-1})$

$$T(q^{-1}) = GP(q^{-1}) \qquad G = \begin{cases} 1/B(1) & si \quad B(1) \neq 0 \\ 1 & si \quad B(1) = 0 \end{cases}$$

F.T. $r \longrightarrow y$: $$H_{BF}(q^{-1}) = \frac{q^{-(d+1)} B_m(q^{-1})}{A_m(q^{-1})} \cdot \frac{B^*(q^{-1})}{B(1)}$$

$$Particular\ case : P = A_m \qquad T(q^{-1}) = G = \begin{cases} \dfrac{P(1)}{B(1)} & si \quad B(1) \neq 0 \\ 1 & si \quad B(1) = 0 \end{cases}$$

Pole placement. Tracking and regulation



$$S(q^{-1})u(t)+R(q^{-1})y(t)=T(q^{-1})y^*(t+d+1)$$

Pole placement. Control law

$$u(t) = \frac{T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$S(q^{-1})u(t) + R(q^{-1})y(t) = GP(q^{-1})y^*(t+d+1) = T(q^{-1})y^*(t+d+1)$$

$$S(q^{-1}) = 1 + q^{-1}S^*(q^{-1})$$

$$\boxed{u(t) = P(q^{-1})Gy^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t)}$$

$$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})}r(t)$$

$$A_m(q^{-1}) = 1 + q^{-1}A_m^*(q^{-1})$$

$$\boxed{y^*(t+d+1) = -A_m^*(q^{-1})y(t+d) + B_m(q^{-1})r(t)}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots \qquad A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots$$

Pole placement. Example

Plant : $d=0$

$B(q^{-1}) = 0.1\ q^{-1} + 0.2\ q^{-2}$

$A(q^{-1}) = 1 - 1.3\ q^{-1} + 0.42\ q^{-2}$

$Bm(q^{-1}) = 0.0927 + 0.0687\ q^{-1}$

Tracking dynamics -->

$Am\ (q^{-1}) = 1 - 1.2451 q^{-1} + 0.4066\ q^{-2}$

Te = 1s , w_0 = 0.5 rad/s, z = 0.9

Regulation dynamics --> $P\ (q^{-1}) = 1 - 1.3741\ q^{-1} + 0.4867\ q^{-2}$

Te = 1s , w_0 = 0.4 rad/s, z = 0.9

Pre-specifications : Integrator

***** CONTROL LAW *****

$S\ (q^{-1})\ u(t) + R\ (q^{-1})\ y(t) = T\ (q^{-1})\ y^*(t+d+1)$

$y^*(t+d+1) = [Bm(q^{-1})/Am(q^{-1})]\ r(t)$

Controller : $R(q^{-1}) = 3 - 3.94\ q^{-1} + 1.3141\ q^{-2}$

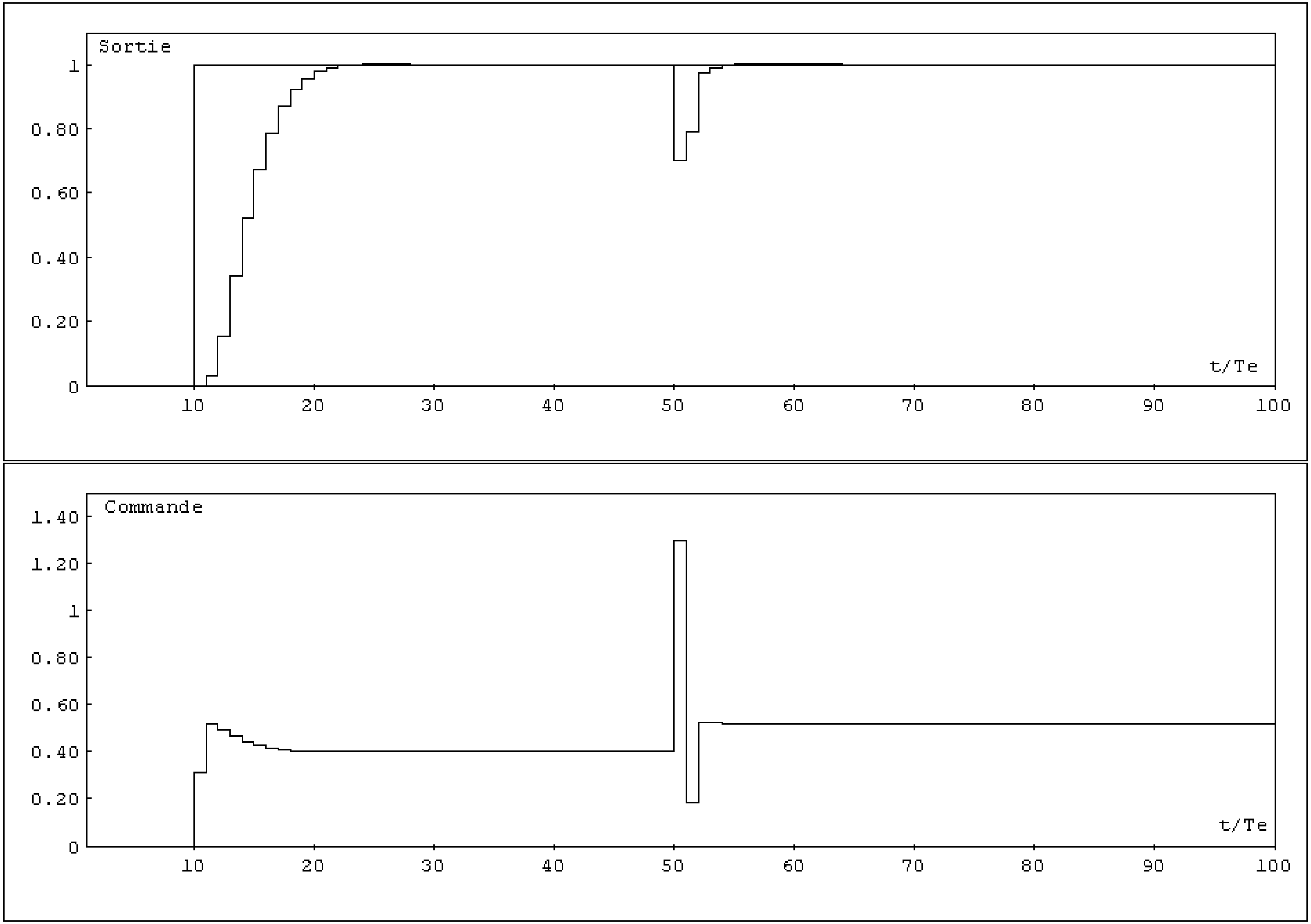
$S(q^{-1}) = 1 - 0.3742\ q^{-1} - 0.6258\ q^{-2}$

$T(q^{-1}) = 3.333 - 4.5806\ q^{-1} + 1.6225\ q^{-2}$

Gain margin : 2.703 Phase margin : 65.4 deg

Modulus margin : 0.618 (- 4.19 dB) Delay margin: 2.1. s

Pole placement. Example



Tracking and regulation with independent objectives

It is a particular case of pole placement
(the closed loop poles contain the palant zeros))

It is a method which simplifies the palnt zeros
Allows exact achievement of imposed performances

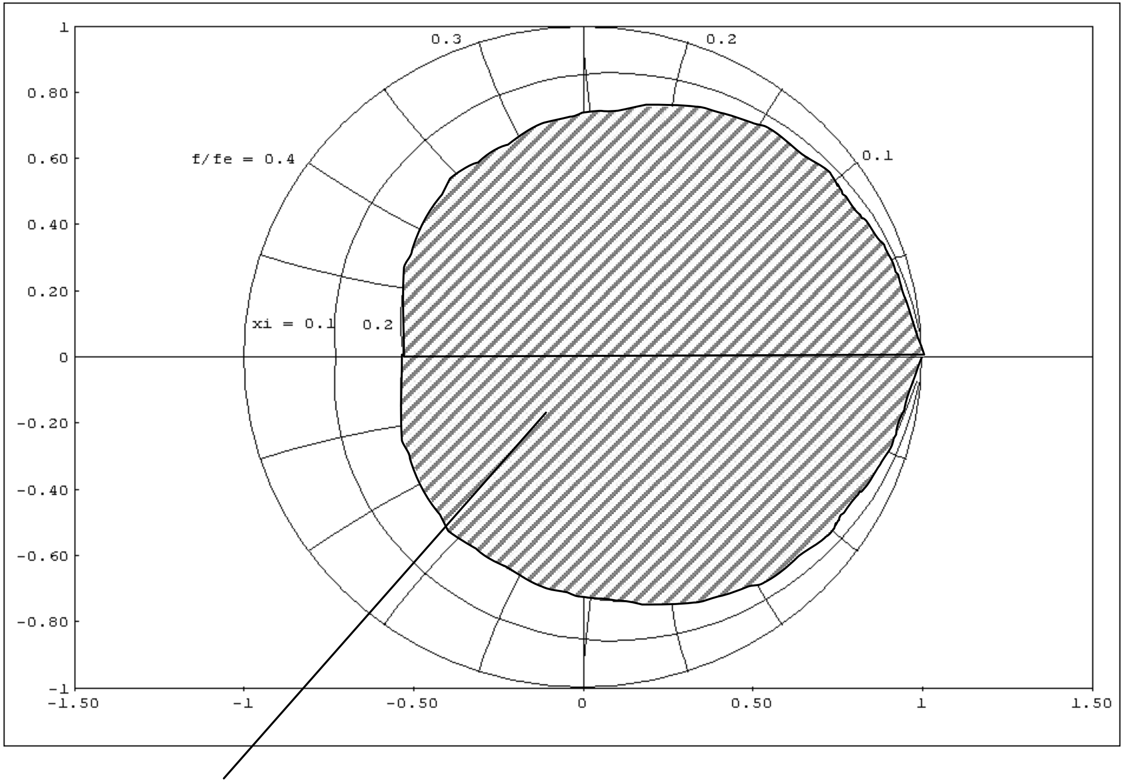
Allows to design a R-S-T controller for:

- stable ou unstables systems
- without restrictions upon the degrees of the polynomials A et B
- without restriction upon the integer delay d of the plant model
- discrete tim plant models with *stable zeros!*

Does not tolerate fractional delay > 0.5 T_S *(unstable zero)*

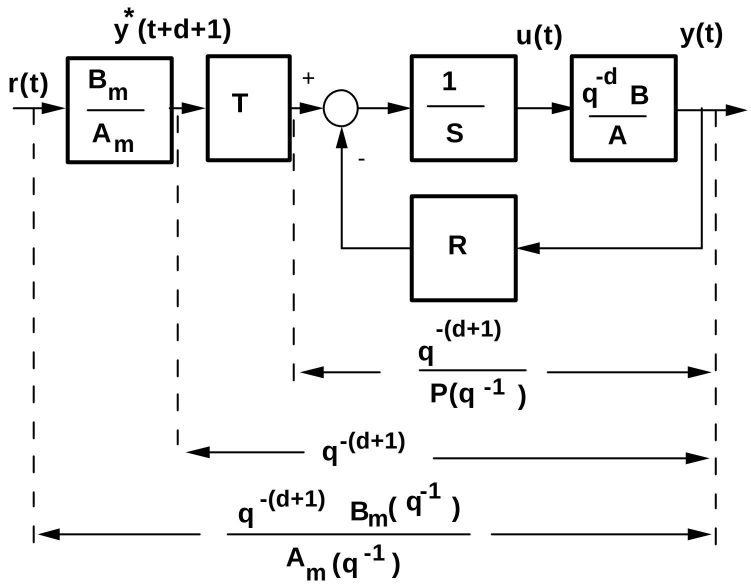
Tracking and regulation with independent objectives

The model zeros should be stable and enough damped



Admissibility domain for the zeros of the discrete time model

Tracking and regulation with independent objectives. Structure



$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Desired closed loop poles are specified as for pole placement

Reference trajectory: (tracking)

$$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Tracking (computation of *R(q⁻¹)* and *S(q⁻¹)*)

Closed loop T.F. without *T:*

$$H_{BF}(q^{-1}) = \frac{q^{-d+1}B^*(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d+1}B^*(q^{-1})R(q^{-1})} = \frac{q^{-d+1}}{P(q^{-1})} = \frac{q^{-d+1}B^*(q^{-1})}{B^*(q^{-1})P(q^{-1})}$$

One needs to solve :

$$A(q^{-1})S(q^{-1}) + q^{-d+1}B^*(q^{-1})R(q^{-1}) = B^*(q^{-1})P(q^{-1}) \quad (*)$$

S should has the form: $S(q^{-1}) = s_0 + s_1 q^{-1} + \ldots + s_{n_S} q^{-n_S} = B^*(q^{-1})S'(q^{-1})$

After simplification by *B**, (*) becomes:

$$\boxed{A(q^{-1})S'(q^{-1}) + q^{-d+1}R(q^{-1}) = P(q^{-1})} \quad (**)$$

Unique solution: $n_P = deg\ P(q^{-1}) = n_A + d$; $deg\ S'(q^{-1}) = d$; $deg\ R(q^{-1}) = n_A - 1$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots r_{n_A-1} q^{-n_A-1} \qquad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_d q^{-d}$$

Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

(**) is written as: $Mx = p \longrightarrow x = M^{-1}p$

$n_A + d + 1$

$$M = \begin{bmatrix} 1 & 0 & & & & & & 0 \\ a_1 & 1 & & & & & & . \\ a_2 & a_1 & & 0 & & & & . \\ : & : & & 1 & & & & . \\ a_d & a_{d-1} & \dots & a_1 & 1 & & & . \\ a_{d+1} & a_d & & & a_1 & 1 & & . \\ a_{d+2} & a_{d+1} & & & a_2 & 0 & & . \\ & & & & . & & & . \\ & & & & . & & & 0 \\ 0 & 0 & \dots & 0 & a_{n_A} & 0 & 0 & 1 \end{bmatrix} \quad \} \; n_A + d + 1$$

(first $d + 1$ columns; last n_A columns)

$$x^T = [1, s'_1, \dots, s'_d, r_0, r_1, \dots, r_{n-1}]$$

$$p^T = [1, p_1, p_2, \dots, p_{n_A}, p_{n_A+1}, \dots, p_{n_A+d}]$$

Use of WinReg or *predisol.sci(.m)* for solving (**)

Insertion of pre specified parts in R and S – same as for pole placement

Tracking (computation of $T(q^{-1})$)

Closed loop T.F.: $r \longrightarrow y$

$$H_{BF}(q^{-1}) = \frac{q^{-(d+1)}B_m(q^{-1})}{A_m(q^{-1})} = \frac{B_m(q^{-1})T(q^{-1})q^{-(d+1)}}{A_m(q^{-1})P(q^{-1})}$$

Desired T.F.

It results : $\boxed{T(q^{-1}) = P(q^{-1})}$

Controller equation:

$$S(q^{-1})u(t) + R(q^{-1})y(t) = P(q^{-1})y^*(t+d+1)$$

$$u(t) = \frac{P(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$u(t) = \frac{1}{b_1}\left[P(q^{-1})y^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t)\right] \qquad (s_0 = b_1)$$

Tracking and regulation with independent obnjectives. Examples

Plant : $d = 0$

$B(q-1) = 0.2\ q-1 + 0.1\ q-2$

$A(q-1) = 1 - 1.3\ q-1 + 0.42\ q-2$

-> $Bm\ (q-1) = 0.0927 + 0.0687\ q-1$

Tracking dynamics -----

-> $Am\ (q-1) = 1 - 1.2451q-1 + 0.4066\ q-2$

$\mathbf{Te = 1s\ ,\ w_0 = 0.5\ rad/s,\ z = 0.9}$

Regulation dynamics ---> $P\ (q-1) = 1 - 1.3741\ q-1 + 0.4867\ q-2$

$\mathbf{Te = 1s\ ,\ w_0 = 0.4\ rad/s,\ z = 0.9}$

Pre-specifications : Integrator

***** CONTROL LAW *****

$S\ (q-1)\ u(t) + R\ (q-1)\ y(t) = T\ (q-1)\ y^*(t+d+1)$

$y^*(t+d+1) = [Bm\ (q-1)/Am\ (q-1)]\ .\ r(t)$

Controller : $R(q-1) = 0.9258 - 1.2332\ q-1 + 0.42\ q-2$

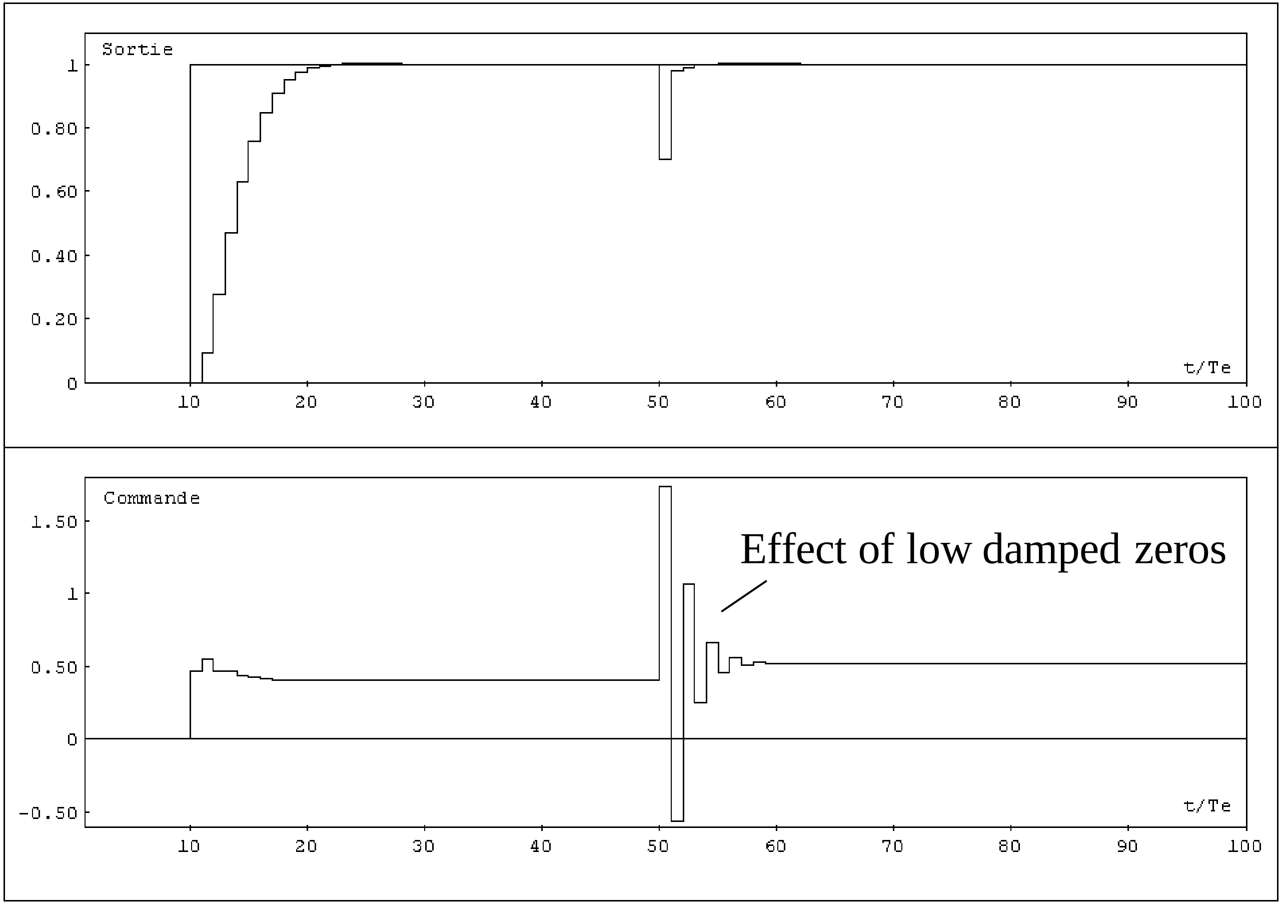
$S(q-1) = 0.2 - 0.1\ q-1 - 0.1\ q-2$

$T(q-1) = P(q-1)$

Gain margin : 2.109 Phase margin : 65.3 deg

Modululus margin : 0.526 (- 5.58 dB) Delay margin : 1.2

Tracking and regulation with independent objectives. (d = 0)



The oscillations on the control input when there are low damped zeros can be reduced by introducing auxiliary poles (see book pg. 169-171)

Tracking and regulation with independent objectives. (d = 3)

Plant : $d = 3$

$B(q^{-1}) = 0.2\, q^{-1} + 0.1\, q^{-2}$

$A(q^{-1}) = 1 - 1.3\, q^{-1} + 0.42\, q^{-2}$

--> $Bm\,(q^{-1}) = 0.0927 + 0.0687\, q^{-1}$

Tracking dynamics -----

--> $Am\,(q^{-1}) = 1 - 1.2451 q^{-1} + 0.4066\, q^{-2}$

$Te = 1s$, $w0 = 0.5\ rad/s$, $z = 0.9$

Regulation dynamics --> $P\,(q^{-1}) = 1 - 1.3741\, q^{-1} + 0.4867\, q^{-2}$

$Te = 1s$, $w0 = 0.4\ rad/s$, $z = 0.9$

Pre-specifications : Intégrator

***** CONTROL LAW *****

$S\,(q^{-1})\, u(t) + R\,(q^{-1})\, y(t) = T\,(q^{-1})\, y^*(t+d+1)$

$y^*(t+d+1) = [Bm\,(q^{-1})/Am\,(q^{-1})\,]\,.ref(t)$

Controller :

$R(q^{-1}) = 0.8914 - 1.1521\, q^{-1} + 0.3732\, q^{-2}$

$S(q^{-1}) = 0.2 + 0.0852\, q^{-1} - 0.0134\, q^{-2} - 0.0045\, q^{-3} - 0.1785\, q^{-4} - 0.0888\, q^{-5}$

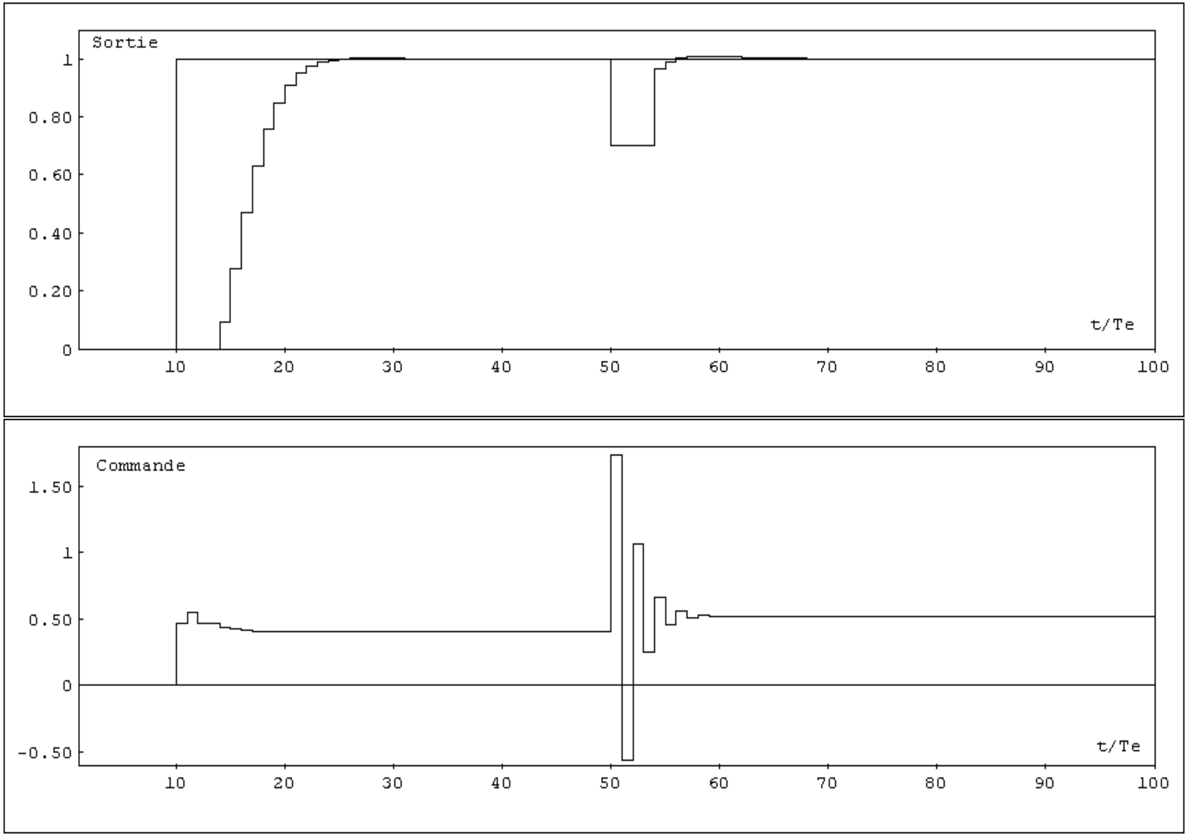
$T(q^{-1}) = P(q^{-1})$

Gain margin: 2.078 Phase margin: 58 deg

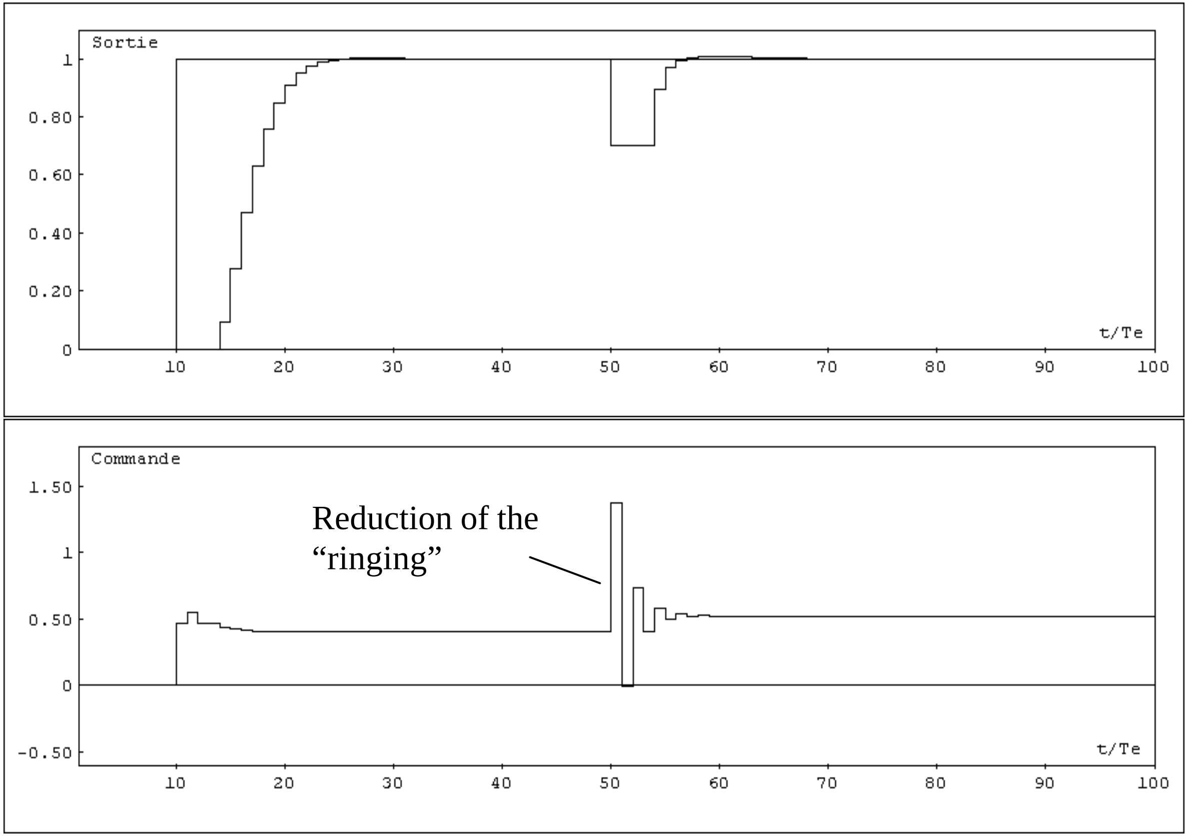
Modulus margin : 0.518 (- 5.71 dB) Dealay margin : 0.7 s

The delay margin can be improved by adding auxiliary poles

Tracking and regulation with independent objectives. (d = 3)



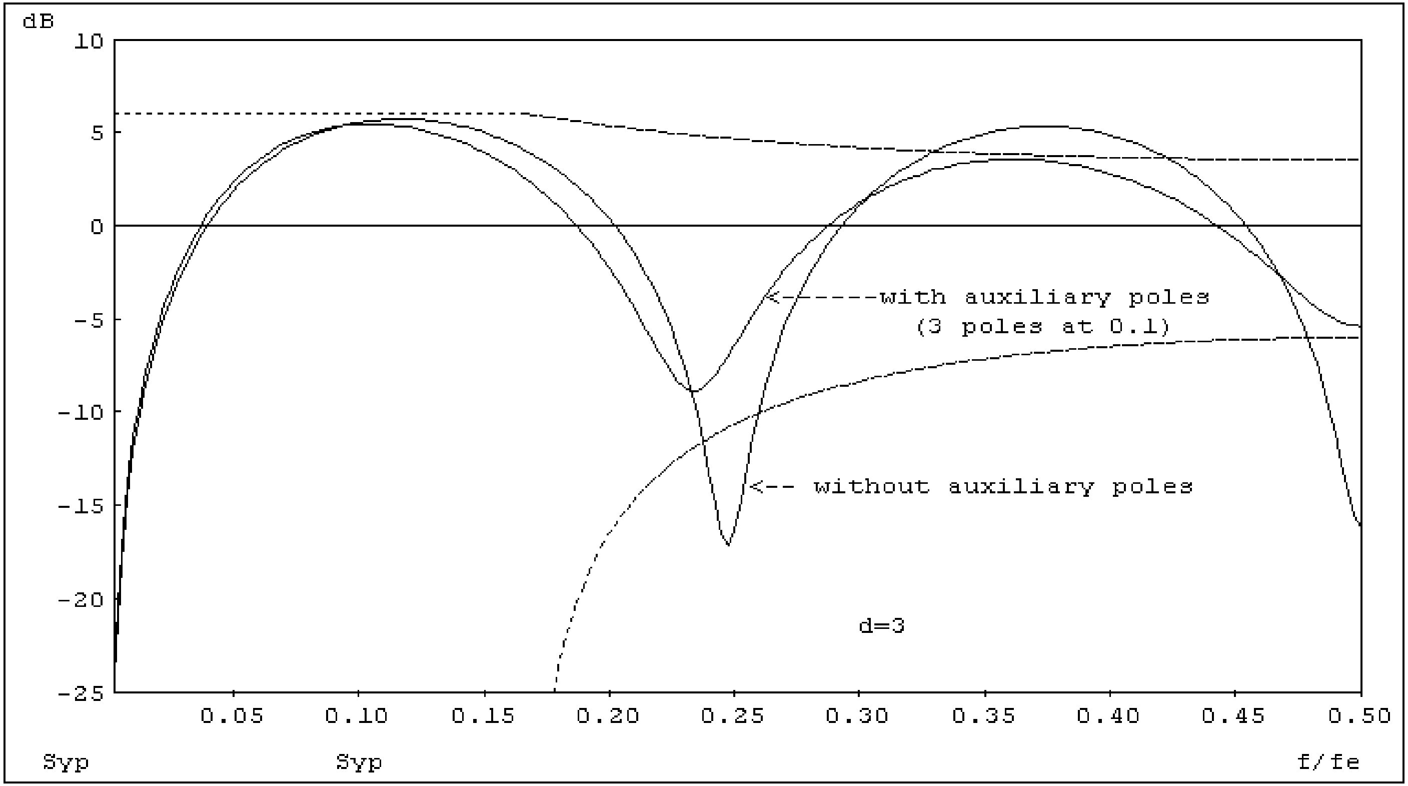
Tracking and regulation with independent objectives. (d = 3)



Three auxiliary poles at *0.1* have been introduced;

Tracking and regulation with independent objectives. (d = 3)

Output sensitivity function : effect of auxiliary poles



Delay margin (without auxiliary poles) : *0.7s*

Delay margin (with auxiliary poles) : *1.19s*

Internal model control -Tracking and regulation

It is a particular caseof the pole placement

The dominant poles are those of the plant model

Does not allow to accelerate the closed loop reponse

Allows to design a R-S-T controller for:

- well damped stable systems
- without restrictions upon the degrees of the polynomial *A* and *B*
- without restrictions upon the delay of the discrete time model

The plant model should be stable and well damped !

Often used for the systems featuring a large delay

Remark: The name is misleading since it has nothing in common with the "internal model principle"

Regulation (computation of $R(q^{-1})$ and $S(q^{-1})$)

$$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = A(q^{-1})P_F(q^{-1}) = P(q^{-1}) \quad (*)$$

Dominant poles

$$P_F(q^{-1}) = \left(1 + aq^{-1}\right)^{n_{P_F}}$$

(typical choice)

R should has the form: $R(q^{-1}) = A(q^{-1}).R'(q^{-1})$

After elimination of the common factor $A(q^{-1})$,(*) devient:

$$S(q^{-1}) + q^{-d}B(q^{-1})R'(q^{-1}) = P_F(q^{-1})$$

Solution for: $S(q^{-1}) = \left(1 - q^{-1}\right)S'(q^{-1})$ (typical choice)

$$R(q^{-1}) = A(q^{-1})\frac{P_F(1)}{B(1)}$$

$$S(q^{-1}) = (1 - q^{-1})S'(q^{-1}) = P_F(q^{-1}) - q^{-d}B(q^{-1})\frac{P_F(1)}{B(1)}$$

For other cases – see book pg.174-175

Tracking (computation of $T(q^{-1})$)

$$T(q^{-1}) = A(q^{-1})P_F(q^{-1}) / B(1)$$

Particular case : $A_m = AP_F$ (tracking dynamics = regulation dynamics)

$$T(q^{-1}) = T(1) = \frac{A(1)P_F(1)}{B(1)}$$ (suppression of the tracking reference model)

Internal Model Control

$H_R(q^{-1}) = 1$

$$R(q^{-1}) = A(q^{-1})\frac{P_F(1)}{B(1)}$$

$$S(q^{-1}) = P_F(q^{-1}) - \frac{P_F(1)}{B(1)}q^{-d}B(q^{-1})$$

$$T(q^{-1}) = A(q^{-1})P_F(q^{-1}) / B(1)$$

$H_R(q^{-1}) \neq 1$

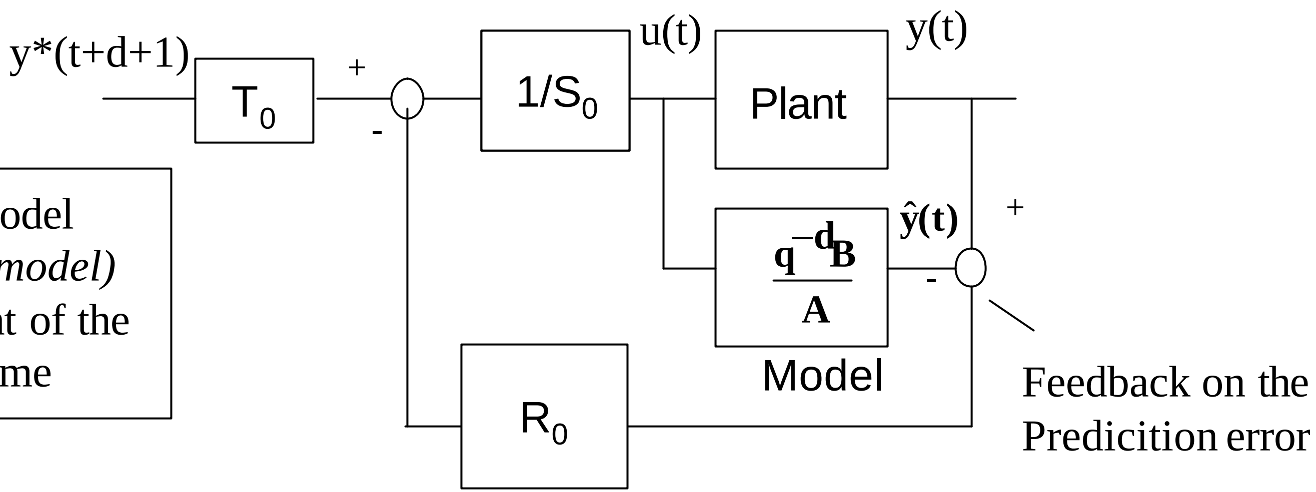
$$R(q^{-1}) = A(q^{-1})H_R(q^{-1})\frac{P_F(1)}{B(1)H_R(1)}$$

$$S(q^{-1}) = P_F(q^{-1}) - q^{-d}B(q^{-1})H_R(q^{-1})\frac{P_F(1)}{B(1)H_R(1)}$$

$$T(q^{-1}) = A(q^{-1})P_F(q^{-1}) / B(1)$$

Interpretation of the internal model control

Equivalent scheme



The plant model *(prediction model)* is an element of the control scheme

$$R_0(q^{-1}) = \frac{P_F(1)}{B(1)} A(q^{-1}) \ \left(for\ H_R(q^{-1}) = 1 \right)$$

$$S_0(q^{-1}) = P_F(q^{-1})$$

$$T_0(q^{-1}) = \frac{1}{B(1)} P(q^{-1}) = \frac{1}{B(1)} A(q^{-1}) P_F(q^{-1})$$

Rem.: *For all the strategies one can show the presence of the plant model in the controller*

Interpretation of the internal model control

$$H_R(q^{-1}) = 1 \qquad S(q^{-1})u(t) = \left[P_F(q^{-1}) - \frac{P_F(1)}{B(1)} q^{-d} B(q^{-1}) \right] u(t) =$$

$$\left[\frac{1}{B(1)} A(q^{-1}) P_F(q^{-1}) y^*(t+d+1) - \frac{P_F(1)}{B(1)} A(q^{-1}) y(t) \right]$$

T R

$$P_F(q^{-1})u(t) = \frac{1}{B(1)} A(q^{-1}) P_F(q^{-1}) y^*(t+d+1) - \frac{P_F(1)}{B(1)} \left[A(q^{-1}) y(t) - q^{-d} B(q^{-1}) u(t) \right]$$

$A(q^{-1})$ *is asymptotically stable*

$$S_0(q^{-1})u(t) = P_F(q^{-1})u(t) = \frac{1}{B(1)} A(q^{-1}) P_F(q^{-1}) y^*(t+d+1) -$$

$$\frac{P_F(1)}{B(1)} A(q^{-1}) \left[y(t) - \frac{q^{-d} B(q^{-1})}{A(q^{-1})} u(t) \right] =$$

$$T_0(q^{-1}) y^*(t+d+1) - R_0(q^{-1}) \left[y(t) - \frac{q^{-d} B(q^{-1})}{A(q^{-1})} u(t) \right]$$

IMC – Sensitivity functions

$$S_{yp}(q^{-1}) = \frac{S(q^{-1})}{P_F(q^{-1})} = 1 - \frac{q^{-d}B(q^{-1})H_R(q^{-1})P_F(1)}{B(1)H_R(1)P_F(q^{-1})}$$

$$S_{yb}(q^{-1}) = -\frac{q^{-d}B(q^{-1})R(q^{-1})}{A(q^{-1})P_F(q^{-1})} = -\frac{q^{-d}B(q^{-1})H_R(q^{-1})P_F(1)}{B(1)H_R(1)P_F(q^{-1})}$$

$$S_{up}(q^{-1}) = -\frac{R(q^{-1})}{P_F(q^{-1})} = -\frac{A(q^{-1})H_R(q^{-1})P_F(1)}{B(1)H_R(1)P_F(q^{-1})}$$

$$S_{yv}(q^{-1}) = \frac{q^{-d}B(q^{-1})S(q^{-1})}{A(q^{-1})P_F(q^{-1})} = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}S_{yp}$$

- The plant model has to be stable
- $H_R(q^{-1}) \big/ P_F(q^{-1})$ directly influences the sensitivity functions

Partial IMC

Motivation:

We would like to modify the dominant poles of the model and leave unchanged the secondary poles (often outside the attenuation band)

$$A(q^{-1}) = A_1(q^{-1})A_2(q^{-1})$$

dominant poles

Bezout equation (pole placement):

$$A_1(q^{-1})A_2(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = P_D(q^{-1})A_2(q^{-1})P_F(q^{-1})$$

But:

$$R(q^{-1}) = A_2(q^{-1})R'(q^{-1})$$

Simplified Bezout equation:

$$A_1(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R'(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Internal model control of a system with large delay

Plant model: $d = 7;\ A = 1 - 0.2q^{-1};\ B = q^{-1};\ T_S = 1$

Objective: $\Delta M \geq 0.5;\ \Delta t \geq 1.T_s$

Using IMC with: $P_F(q^{-1}) = 1;\ H_R(q^{-1}) = 1$

$$S_{yp}(z^{-1}) = 1 - \frac{z^{-d}B(z^{-1})}{B(1)} = 1 - z^{-d-1} = (1 - z^{-1})(1 + z^{-1} + z^{-2} + \ldots + z^{-d})$$

$$S_{yb}(z^{-1}) = -\frac{z^{-d}B(z^{-1})}{B(1)} = -z^{-d-1}$$

$$\left|S_{yp}(e^{-jw})\right|_{\max} \leq 2;\ 0 \leq w \leq p$$

Modulus margin is OK

$$\left|S_{yb}(e^{-jw})\right| \equiv 1;\ 0 \leq w \leq p$$

Above the template for delay margin = 1 over $0.17f_S$

Condition for the delay margin:

$$\left|z^{-d-1}\right| \leq \frac{1}{\left|z^{-1} - 1\right|}$$

Upper template on S_{yb}

Internal model control of a system with large delay

Use of auxiliary poles $(P_F \neq 1;\ H_R = 1)$

$$P_F(q^{-1}) = (1 + a\ q^{-1}) \quad -1 < a < 0$$

Delay margin condition:

$$\left|S_{yb}(z^{-1})\right| = \left|\frac{z^{-d-1}P_F(1)}{P_F(z^{-1})}\right| < \frac{1}{\left|1 - z^{-1}\right|};\ \ z = e^{jw}\ \ 0 \le w \le p$$

Or:

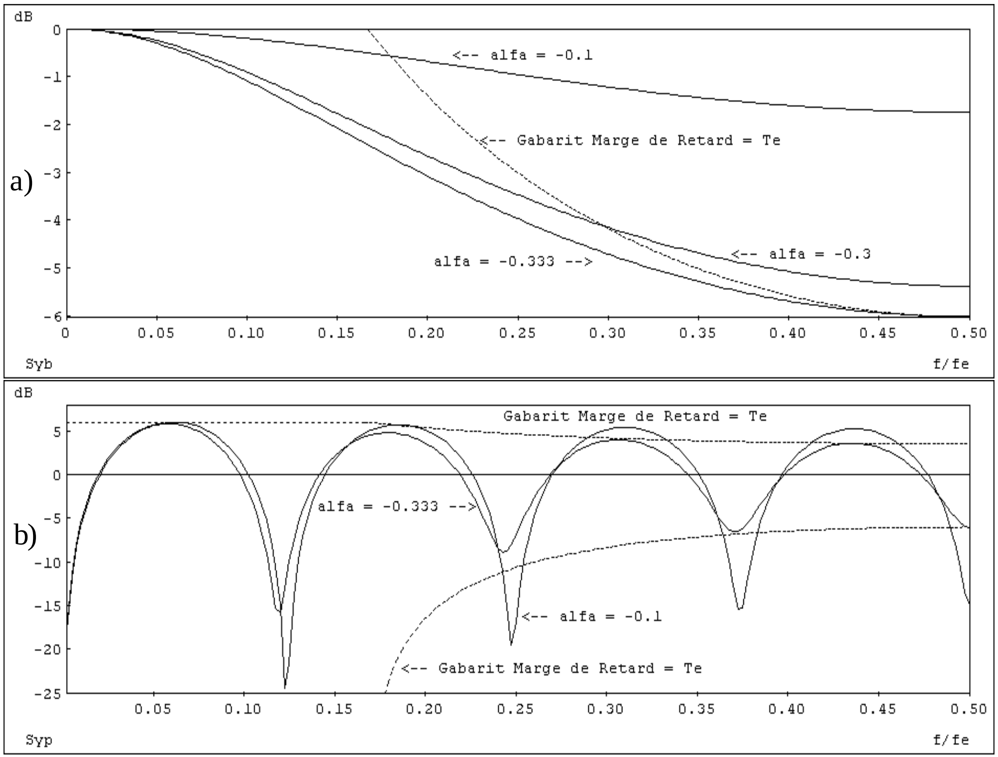
$$\left|\frac{1+a}{1+a\ z^{-1}}\right| < \frac{1}{\left|1 - z^{-1}\right|};\ \ z = e^{jw}\ \ 0 \le w \le p$$

Worst situation: $\left|\dfrac{1+a}{1-a}\right| < 0.5 \Rightarrow \boxed{a \le -0.333}$

Internal model control of a system with large delay

Plant model: $d = 7;\ A = 1 - 0.2q^{-1}\ ;\ B = q^{-1}$

$$P_F(q^{-1}) = (1 + a\ q^{-1}) \qquad -1 < a < 0$$



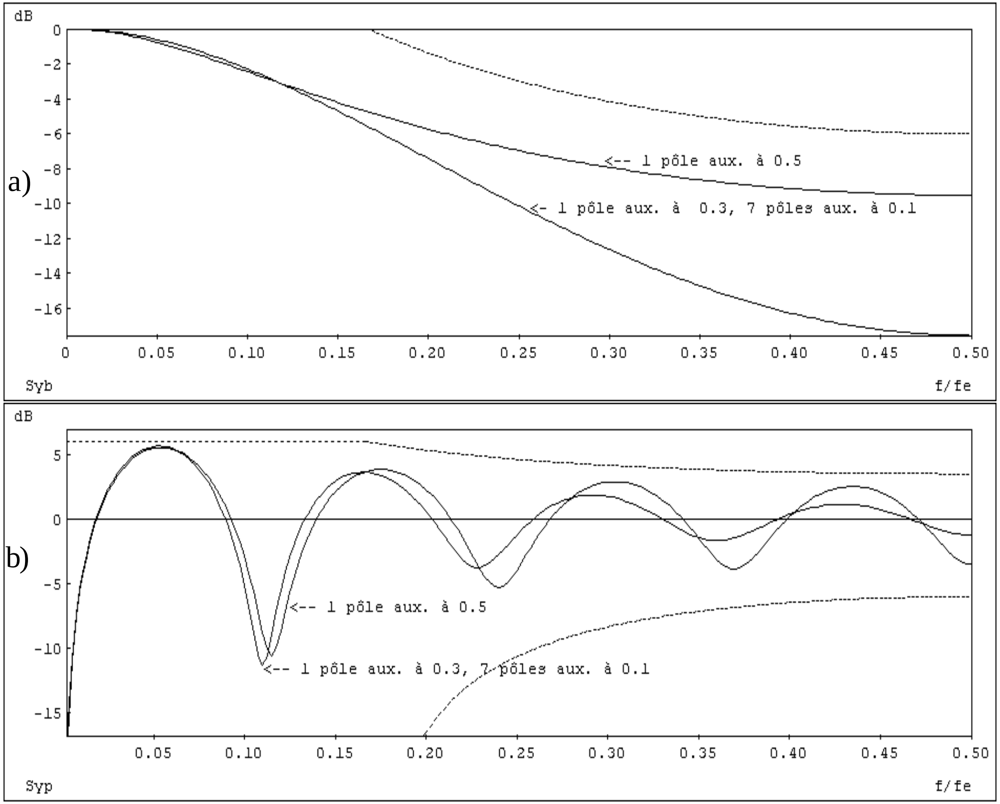
$\alpha = -0.1; -0.2; -0.333$

The good value (can be obtained analytically)

Internal model control of a system with large delay

One can also use:

$$P_F(q^{-1}) = (1 + aq^{-1})(1 + a'q^{-1})^{n_{P_F}-1} \quad -1 < a \leq 0 \qquad -0.25 < a' \leq -0.05 \qquad n_{P_F} \leq n_B + d$$



Internal model control of a system with large delay

Use of: $H_R(P_F = 1; H_R \neq 1)$

$$R(q^{-1}) = A(q^{-1})H_R(q^{-1})\frac{1}{B(1)H_R(1)} \qquad H_R(q^{-1}) = 1 + bq^{-1}$$

$$S(q^{-1}) = 1 - \frac{q^{-d}B(q^{-1})(1+bq^{-1})}{B(1)(1+b)}$$

$$S_{yb}(z^{-1}) = -\frac{z^{-d-1}(1+bz^{-1})}{1+b}$$

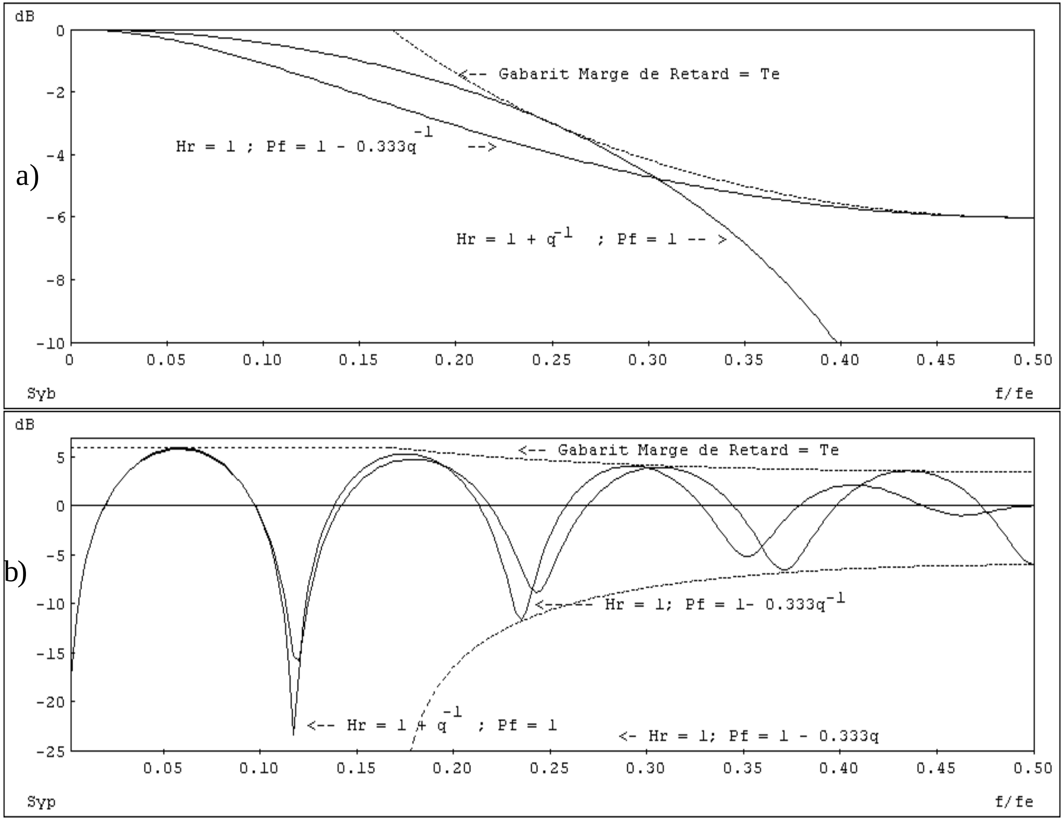
Delay margin condition:

$$\left|\frac{1+b\ z^{-1}}{1+b}\right| < \frac{1}{\left|1-z^{-1}\right|}; \quad z = e^{jw} \quad 0 \leq w \leq p$$

$$\frac{1+b^{2}}{(1+b)^{2}} \leq 0.5 \Rightarrow b = 1$$

$$H_R(q^{-1}) = 1 + q^{-1}$$

Internal model control of a system with large delay

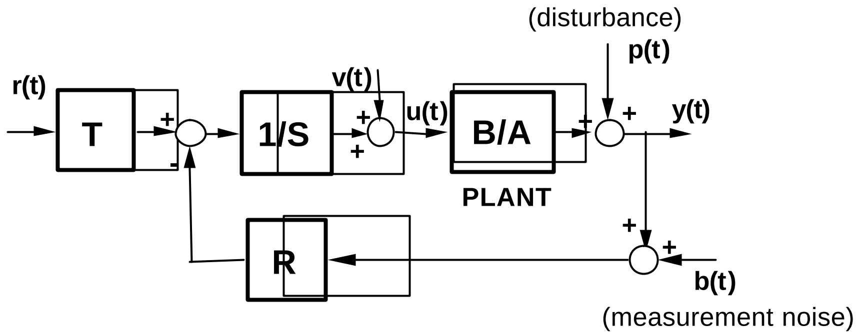


$H_R(q^{-1}) = 1 + q^{-1}$ corresponds to the opening of the loop at $0.5f_S$

See also:

I.D. Landau (1995) : Robust digital control of systems with time delay (the Smith predictor revisited) Int. J. of Control, v.62,no.2 pp 325-347

Digital control in the presence of disturbances and noise



Output sensitivity function
(p —— y)

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Input sensitivity function
(p —— u)

$$S_{up}(z^{-1}) = \frac{-A(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Noise-output sensitivity function
(b —— y)

$$S_{yb}(z^{-1}) = \frac{-B(z^{-1})R(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

Input disturbance-output sensitivity function
(v —— y)

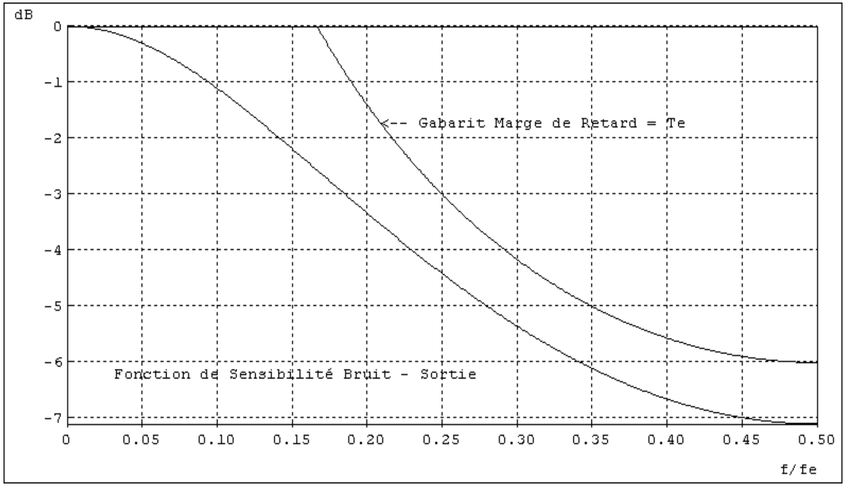
$$S_{yv}(z^{-1}) = \frac{B(z^{-1})S(z^{-1})}{A(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1})}$$

All four sensitivity functions should be stable ! (see book pg.102 - 103)

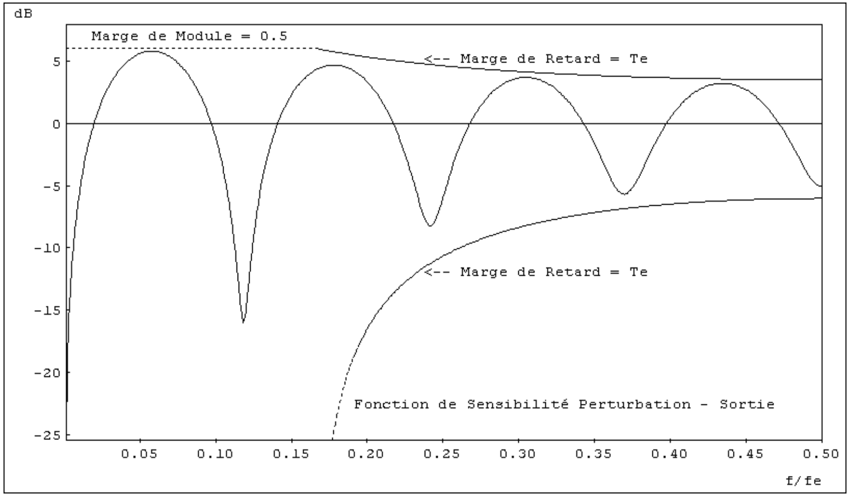
Frequency templates on the sur sensitivity functions

The robust stability conditions allow to define frequency templates on the sensitivity functions which guarantee the delay margin and the modulus margin;

The templates are essential for designing a good controller



Frequency template on the noise-output sensitivity function S_{yb} for $\Delta\tau = T_S$



Frequency template on the output sensitivity function S_{yp} for $\Delta\tau = T_S$ and $\Delta M = 0.5$

Pole placement with sensitivity functions shaping

Performance specification for pole placement :

- Desired dominant poles for the closed loop
- The reference trajectory (tracking reference model)

Questions:

- How to take into account the specifications in certain frequency regions?
- How to guarantee the *robustness* of the controllers ?
- How to take advantage from the degree of freedom for the maximum number of poles which can be assigned ?

Answer:

Shaping the sensitivity functions by:

- introducing auxiliary poles

- introducing filters in the controllers

Sensitivity functions - review

Output sensitivity function:

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})}$$

Input sensitivity function:

$$S_{up}(q^{-1}) = -\frac{A(q^{-1})R(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})}$$

Controller structure :

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

Pre specified parts (filters)

Dominant and auxiliary filters:

$$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = P(q^{-1}) = P_D(q^{-1})P_F(q^{-1})$$

Study of the properties of the sensitivity functions in the frequency domain: $q=z=e^{jw}$

Properties of the sensitivity functions

•It is fundamental to understand and to interpret the behaviour of the sensitivity functions in the frequency domain

•The following slides will explore the properties of the output and input sensitivity functions

For the examples one uses:

Plant model: $A(q^{-1}) = 1 - 0.7q^{-1}; B(q^{-1}) = 0.3q^{-1}; d = 2$

Polynomial ***P***:

Defined by the discretization of a continuous time 2nd order system with:

ω_0 = 0.4; 0.6; 1; ζ =0.9

Properties of the output sensitivity function

P.1- *The modulus of the output sensitivity function at a certain frequency gives the amplification or attenuation factor of the disturbance on the output*

$S_{yp}(w) < 1(0\ dB)$ attenuation

$S_{yp}(w) > 1$ amplification

$S_{yp}(w) = 1$ operation in open loop

P.2

$$\Delta M = \left(\left| S_{yp}(jw) \right|_{\max} \right)^{-1}$$

Modulus margin

Properties of the output sensitivity function

P.3 – *The open loop (KG) being stable one has the property:*

$$\int_0^{0.5 f_S} \log\left|S_{yp}(e^{-j2pf/f_S})\right| df = 0$$

The sum of the areas between the curve of Syp and the axis 0dB taken with their sign is null

↓

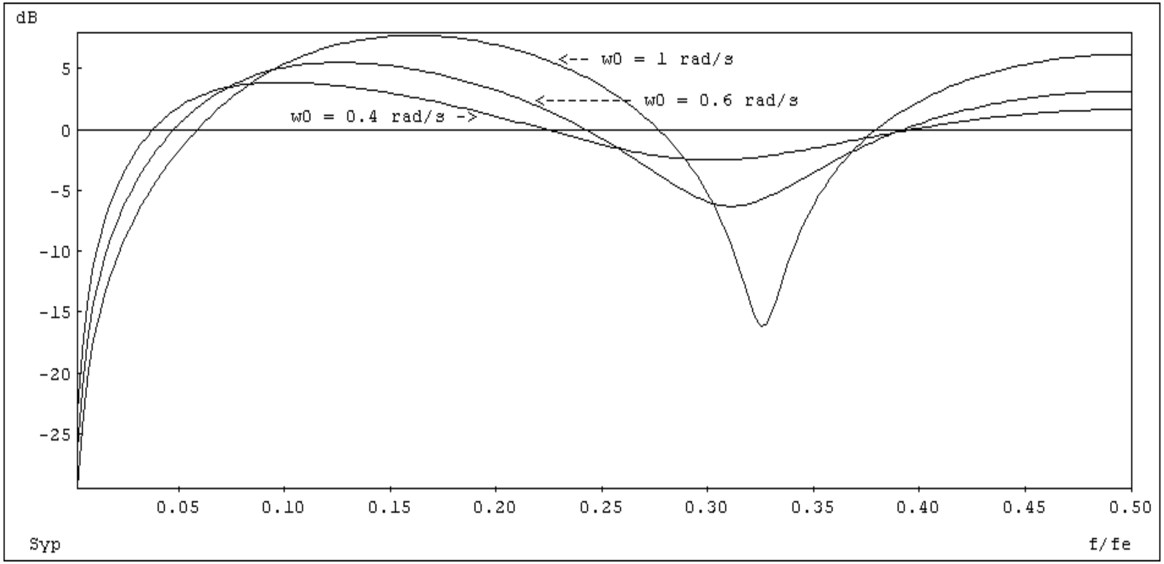
Disturbance attenuation in a frequency region implies amplification of the disturbances in other frequency regions!

Properties of the output sensitivity function

Augmenting the attenuation or widening the attenuation zone

Higher amplification of disturbances ouside the attenuation zone

Reduction of the robustness (reduction of the modulus margin)

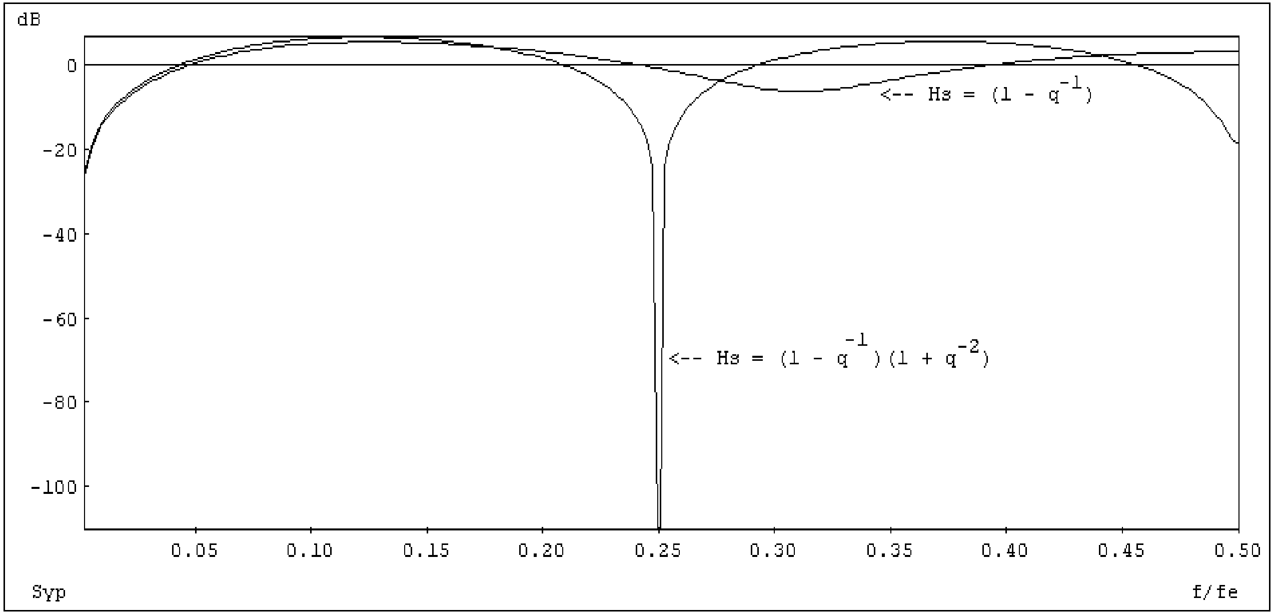


Properties of the output sensitivity function

P.4 – *Cancellation of the disturbance effect at a certain frequency:*

$$A(e^{-jw})S(e^{-jw}) = A(e^{-jw})H_S(e^{-jw})S'(e^{-jw}) = 0 \quad ; \quad w = 2p\ f / f_e$$

Zeros of S_{yp} (under $A(e^{-jw})$) — H_S: Allows introduction of zeros at desired frequencies

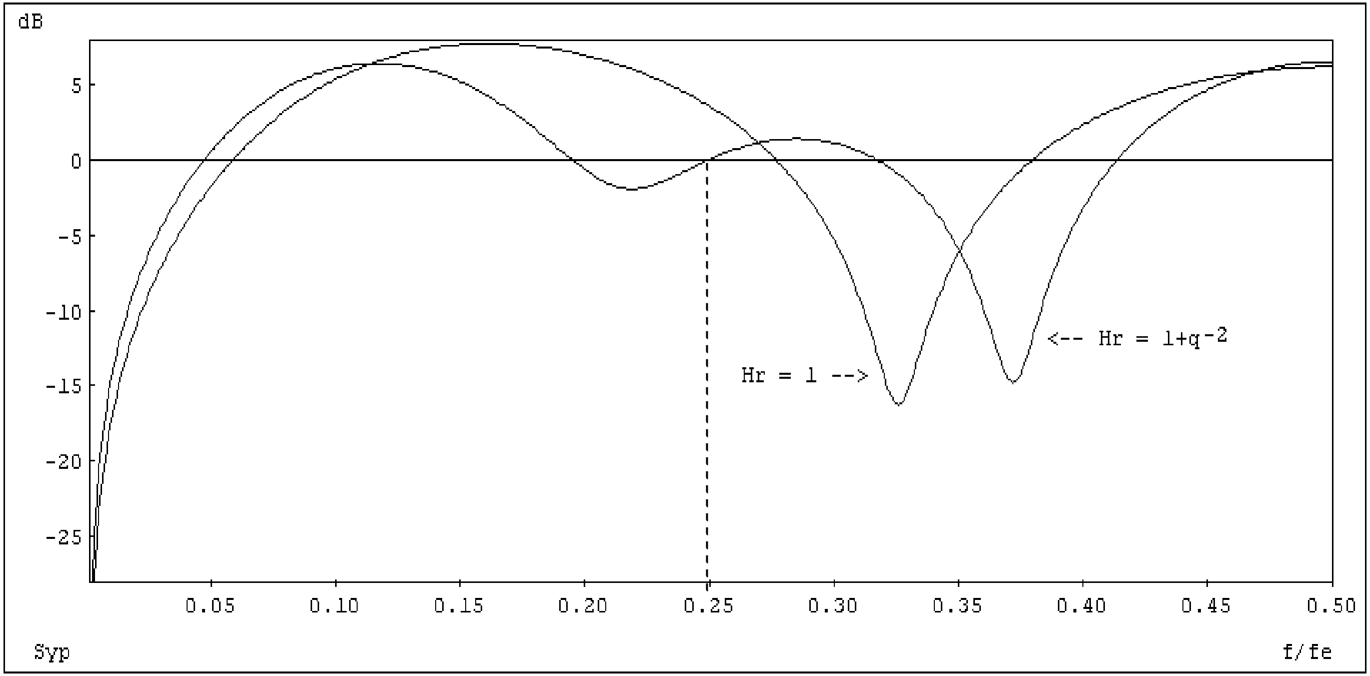


Properties of the output sensitivity function

P.5 - $\left|S_{yp}(jw)\right| = 1\,(0\ dB)$ *at the frequencies where:*

$$B^*(e^{-jw})R(e^{-jw}) = B^*(e^{-jw})H_R(e^{-jw})R'(e^{-jw}) = 0\ ;\ w = 2p\ f / f_e$$

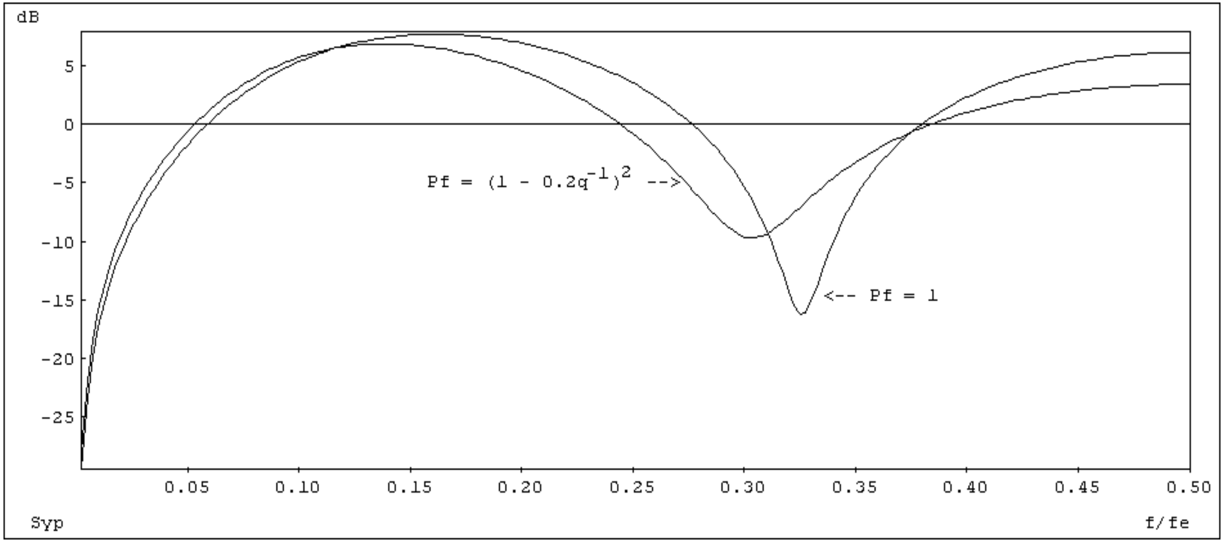
Allows introduction of zeros at desired frequencies



Properties of the output sensitivity function

P.6 – *Asymptotically stable auxiliary poles (P_F) lead (in general) to the reduction of* $|S_{yp}(jw)|$ *in the attenuation band of $1/P_F$*

$$P_F(q^{-1}) = (1 + p'q^{-1})^{n_{P_F}} \qquad -0.5 \le p' \le -0.05 \qquad n_{P_F} \le n_P - n_{P_D}$$



In many applications, introduction of high frequency auxiliary poles is enough for assuring the required robustness margins

Properties of the output sensitivity function

P.7 – *Simultaneous introduction of a fixed part* H_{Si} *and of a pair of auxiliary poles* P_{Fi} *having the form:*

$$\frac{H_{S_i}(q^{-1})}{P_{F_i}(q^{-1})} = \frac{1 + b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}}$$

resulting from the dicretization of :

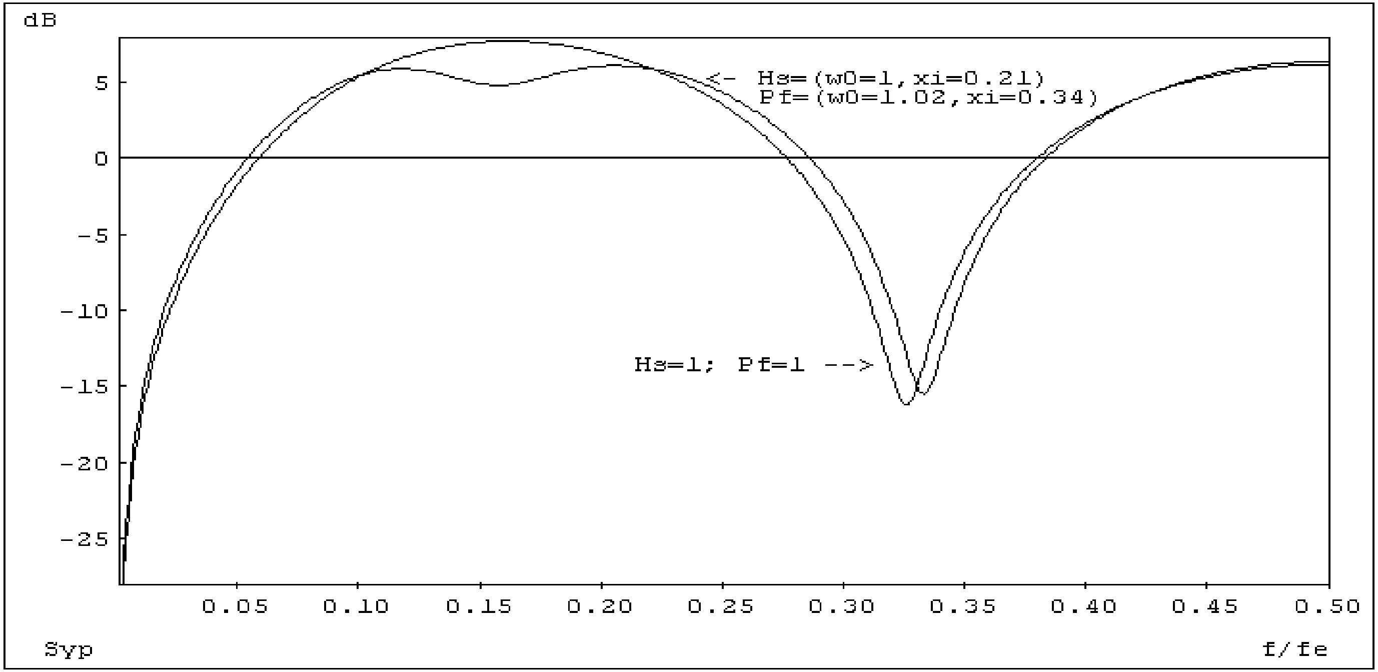
$$F(s) = \frac{s^2 + 2\zeta_{num} \omega_0 s + \omega_0^2}{s^2 + 2\zeta_{den} \omega_0 s + \omega_0^2} \qquad \textit{with:} \qquad s = \frac{2}{T_e} \frac{1 - z^{-1}}{1 + z^{-1}}$$

introduces an attenuation at the normalized discretized frequency:

$$\omega_{disc} = 2\arctan\left(\frac{\omega_0 T_e}{2}\right) \quad \textit{with the attenuation: } M_t = 20\log\left(\frac{\zeta_{num}}{\zeta_{den}}\right) \quad (\zeta_{num} < \zeta_{den})$$

and with negligible effect at $f \ll f_{disc}$ *and at* $f \gg f_{disc}$

Properties of the output sensitivity function



For computation details see book pg.194-197.

Effective computation with the function: *filter22.sci (.m)*

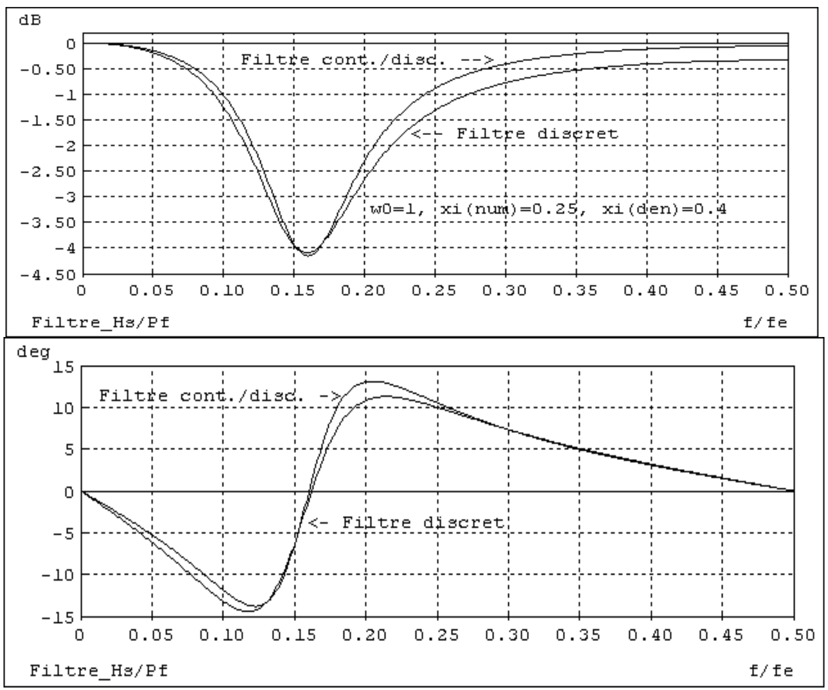
Computation of H_S/P_F for $f < 0.17 f_S$

For the frequencies below $0.17 f_s$ the computations can be done directly in discrete time with a very good precision.

In this case: $w_0 = w_{0den} = w_{0num} = w_{disc} \qquad z_{num} = 10^{M_t/20} z_{den}$

M_t : desired attenuation at the frequency w_{disc}

$$H_S = disc(w_0, z_{num}); P_F = disc(w_0, z_{den}); (z_{den})_{\min} \geq 0.3$$



$$w_0 = 1 rad/s \; (f_0 = 0.159 f_s)$$

Computation of H_S/P_F (general formula)

Central frequency for attenuation f_{disc} $(w_{disc} = 2p\ f_{disc})$

Desired attenuation at f_{disc} : M_t

Minimum damping for P_F : $(z_{den})_{\min} \geq 0.3$

Step I : computation of the analog filter

$$w_0 = \frac{2}{T_e} \tan\left(\frac{w_{disc}}{2}\right) \qquad 0 \leq w_{disc} \leq p \qquad z_{num} = 10^{M_t/20} z_{den}$$

$$F(s) = \frac{s^2 + 2z_{num} w_0 s + w_0^2}{s^2 + 2z_{den} w_0 s + w_0^2}$$

Step II : computation of the digital filter using the bilinear transformation

$$s = \frac{2}{T_e} \frac{1 - z^{-1}}{1 + z^{-1}}$$

For details of formulas see "Commande des systèmes" (Landau 2002)

For effective computation use *filter22.sci(.m)* or *ppmaster*

Properties of the input sensitivity function

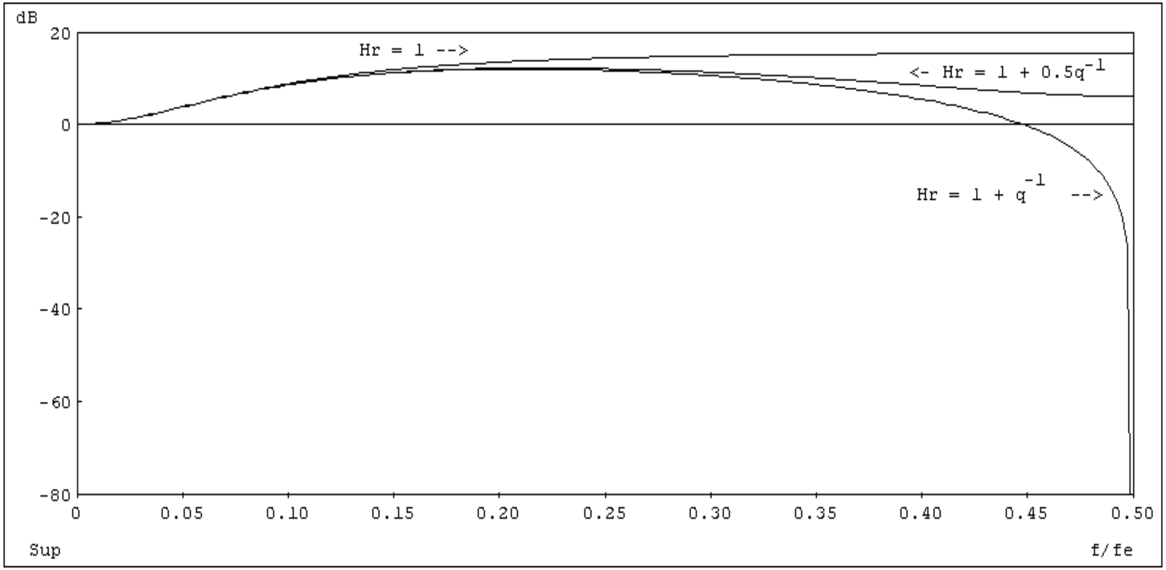
P.1 – *Cancellation of the disturbance effect on the input at a certain fréquency ($S_{up} = 0$):*

$$A(e^{-jw})H_R(e^{-jw})R'(e^{-jw}) = 0 \quad ; \quad w = 2p\ f / f_e$$

Allows introduction of zeros at desired frequencies

$$H_R(q^{-1}) = 1 + bq^{-1} \qquad 0 < b \leq 1$$

(active at $0.5f_S$)



Rem: The system operate in open loop at this frequency

Properties of the input sensitivity function

P.2 – *At the frequencies where:*

$$A(e^{-jw})H_S(e^{-jw})S'(e^{-jw}) = 0 \quad ; \quad w = 2p\ f / f_e$$

One has:

$$\left|S_{yp}(jw)\right| = 0 \qquad \left|S_{up}(e^{-jw})\right| = \left|\frac{A(e^{-jw})}{B(e^{-jw})}\right|$$

Inverse of the sytem gain

Consequence : strong attenuation of the disturbances should be done only in the frequency regions where the system gain is enough large (inorder to preserve robusteness and avoid too much stress on the actuator)

Remember: $\left|S_{up}(jw)\right|^{-1}$ gives the tolerance with respect to additive uncertainties on the model (high $\left|S_{up}(jw)\right|$ = weak robustness)

Properties of the input sensitivity function

P.3 – *Simultaneous introduction of a fixed part H_{Ri} and of a pair of auxiliary poles P_{Fi} having the form:*

$$\frac{H_{R_i}(q^{-1})}{P_{F_i}(q^{-1})} = \frac{1 + b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}}$$

resulting from the dicretization of :

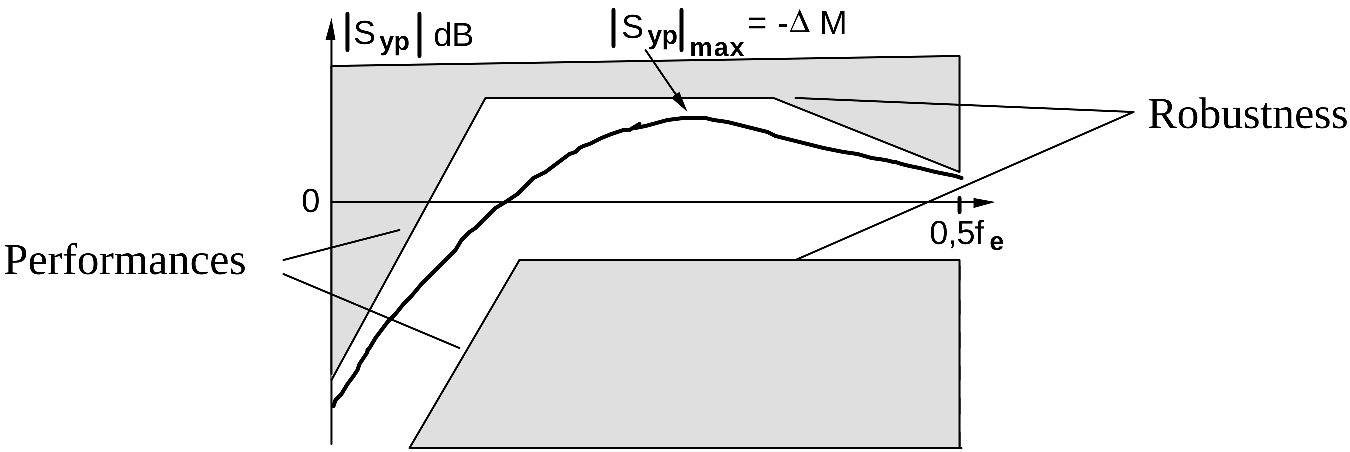
$$F(s) = \frac{s^2 + 2z_{num} w_0 s + w_0^2}{s^2 + 2z_{den} w_0 s + w_0^2}$$ with: $$s = \frac{2}{T_e}\frac{1 - z^{-1}}{1 + z^{-1}}$$

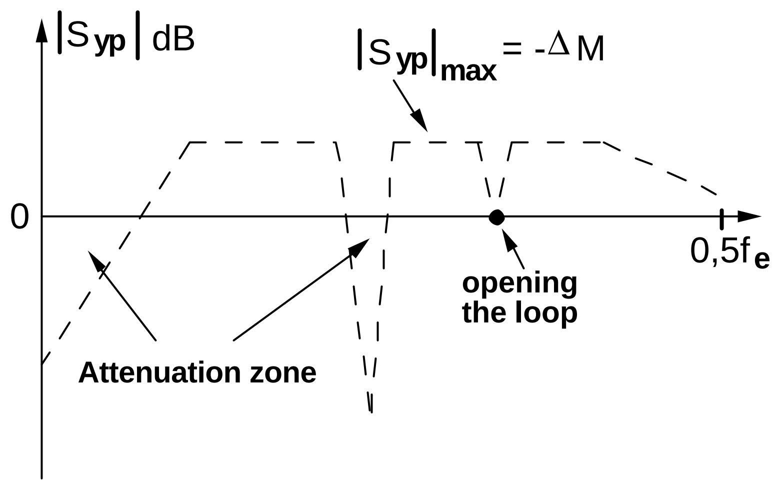
introduces an attenuation at the normalized discretized frequency:

$$w_{disc} = 2\arctan\left(\frac{w_0 T_e}{2}\right)$$ wih the attenuation: $$M_t = 20\log\left(\frac{z_{num}}{z_{den}}\right) \quad (z_{num} < z_{den})$$

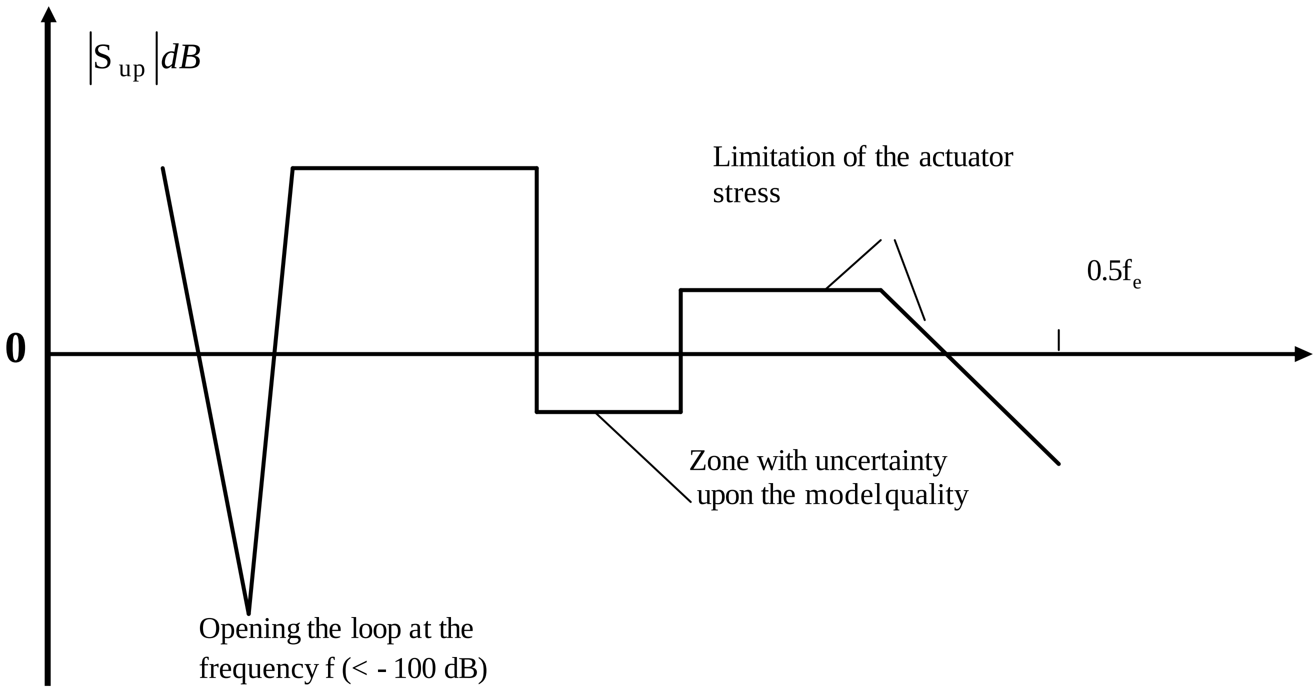
and with negligible effect at $f << f_{disc}$ and at $f >> f_{disc}$

Templates for the output sensitivity functions S_{yp}





Templates for the input sensitivity function S_{up}



Shaping the sensitivity functions

1. Choice of the dominants et auxiliary poles of the closed loop
2. Choice of the fixed part of the controller (H_S and H_R)
3. Simultaneous choice of the fixed parts and the auxiliary poles

Procedure:

Basic shaping : use 1 and 2
Fine shaping: use 3

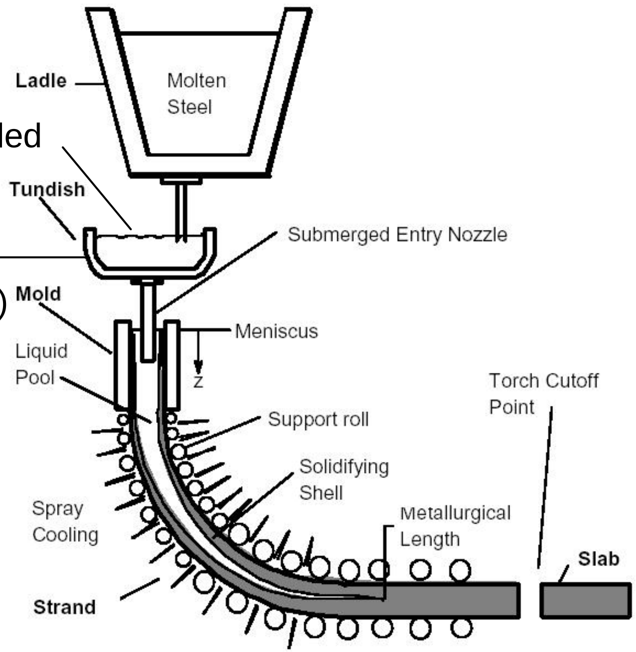
Tools for sensitivity shaping: WinReg (Adaptech) and *ppmaster.m*

There exist also tools for automatic sensitivity function shaping based on convex optimization (Optreg from Adaptech)

Continuous steel casting

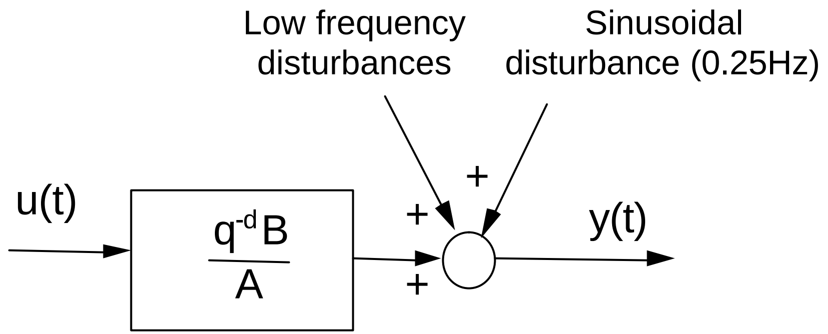
Level to be controlled

Shaking
(to prevent adherence)



Continuos steel casting

Plant (integrator): $A = 1 - q^{-1}$; $B = 0.5q^{-1}$; $d = 2$; $T_s = 1s$



Specifications:

1. No attenuation of the sinusoidal disturbance (0.25 Hz)
2. Attenuation band in low frequencies: 0 à 0.03 Hz
3. Disturbance amplification at 0.07 Hz: < 3dB
4. Modulus margin > -6 dB and Delay margin > T_S
5. No integrator in the controller

Shaping the sensitivity functions

- Synthesis of the fixed parts: $H_R = 1 + q^{-2}$; $H_S = 1$

Opening the loop at *0.25 Hz*

- Dominant poles: discretization of 2nd order: ω_0 = 0.628 rad/s, ζ = 0.9

Controller A : specs.à 0.07 Hz are not satisfied (see 71)

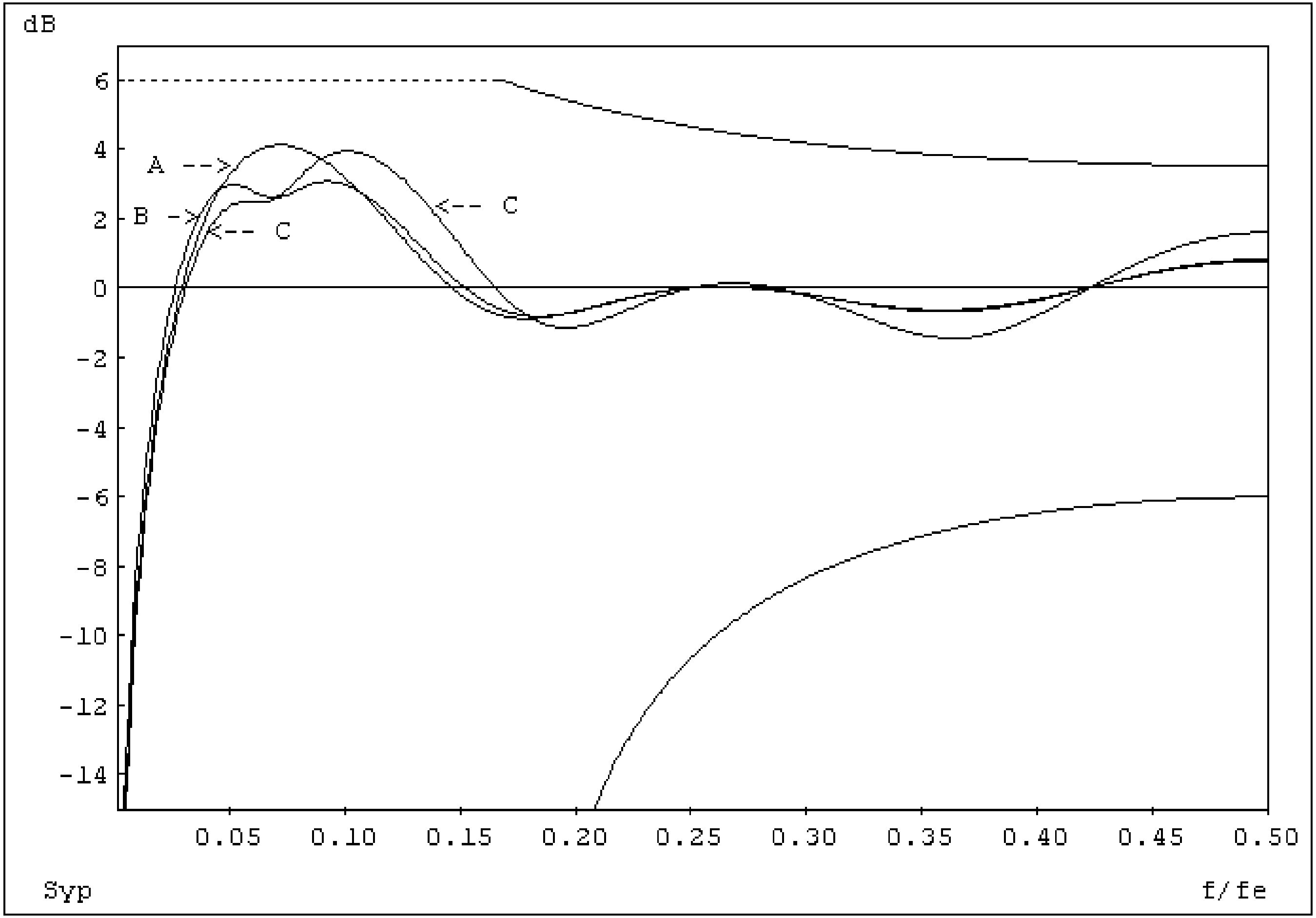
- insertion of a dipole H_S/P_F centered at ω_0 = 0.44 rad/s

Controller B : Attenuation band smaller than specs.

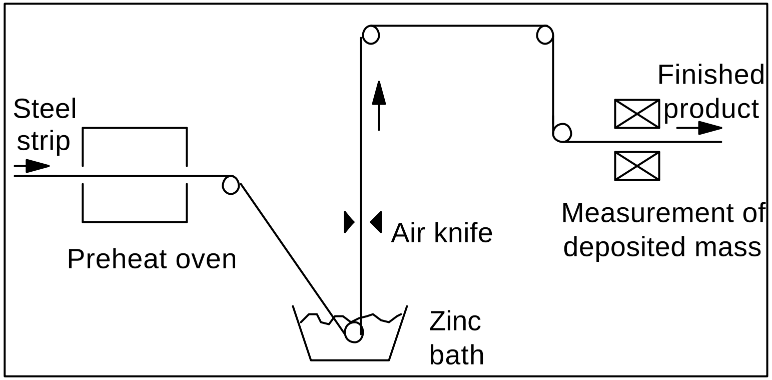
- acceleration the dominant poles: ω_0 = 0.9 rad/s

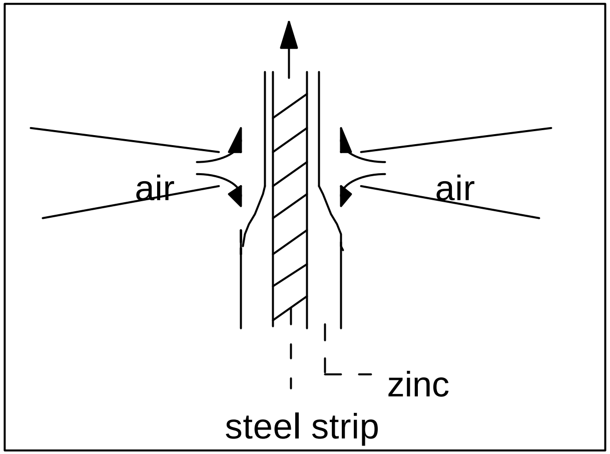
Controller C : Correct (see 71)

Output sensitivity functions – Continuous casting

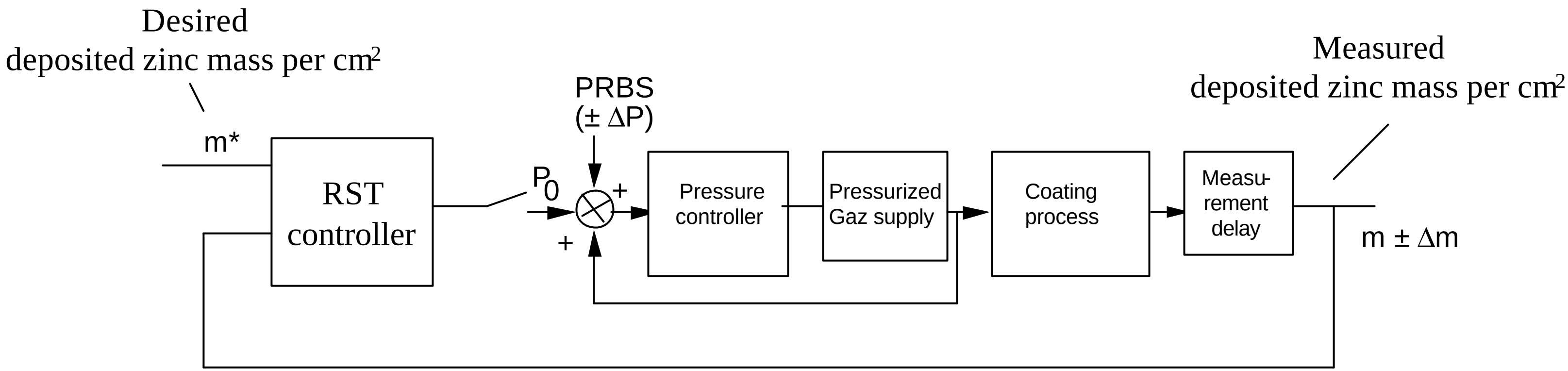


Hot dip galvanizing. Control of the deposited zinc





Hot dip galvanizing. The control loops



- important time delay with respect to process dynamics
- time delay depends upon the steel strip speed
- sampling frequency tied to the steel strip speed
- constant integer delay in discrete time
- parameter variations of the process as a function of the type of product

Hot dip galvanizing. Model and specifications

Plant model for a type of product : $\dfrac{q^{-7}\left(b_1 q^{-1}\right)}{1+a_1 q^{-1}}$

Model: $T_S = 12\,\text{sec}; b_1 = 0.3; a_1 = -0.2(-0.3))$

Specifications (peformance) :

- Modulus margin: $\Delta M \geq 0.5$
- Delay margin: $\Delta t \geq 2T_S$
- Integrator

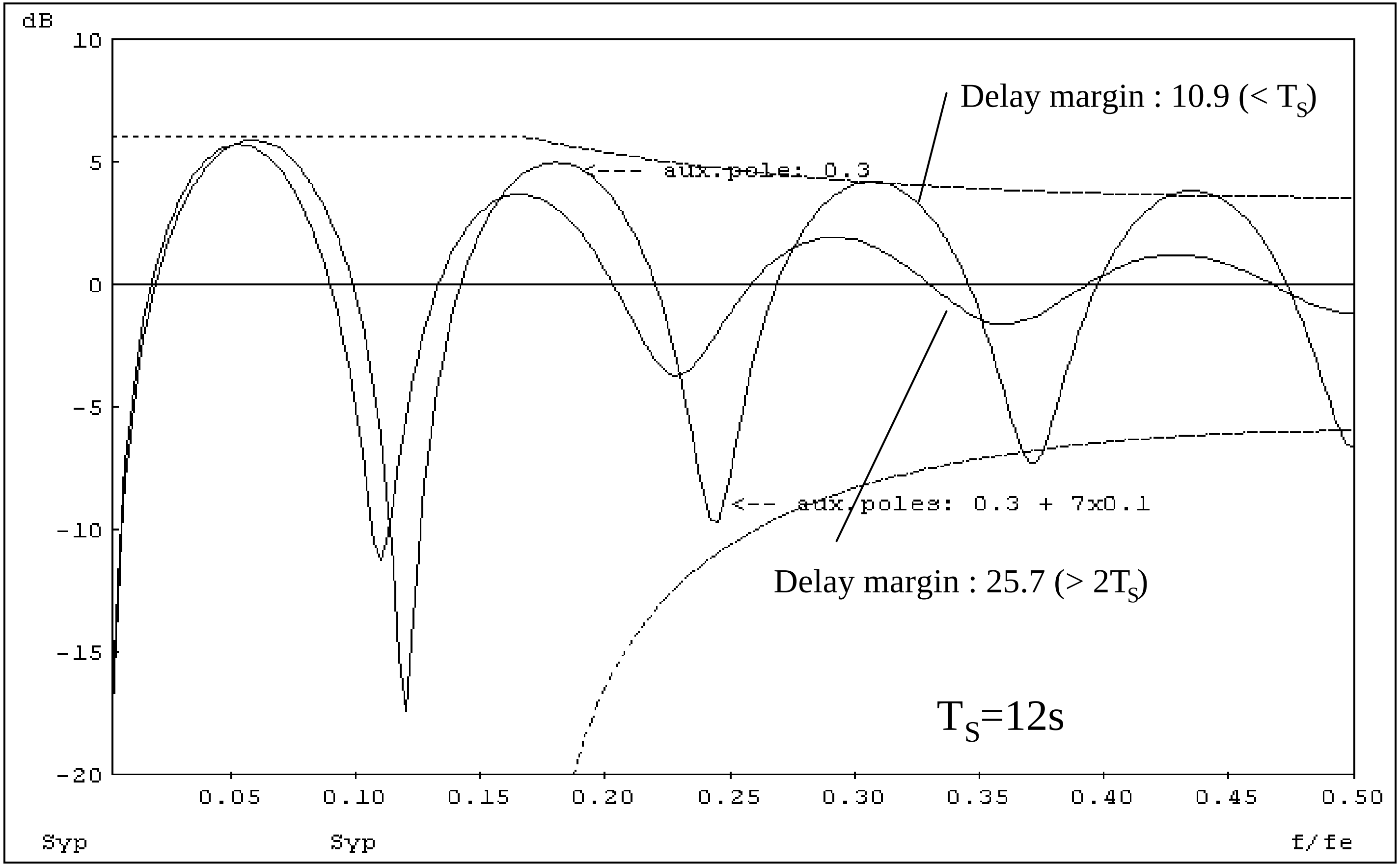
Pole placement (IMC)

$\deg P(q^{-1}) \leq \left(n_A + n_B + d + n_{H_S} - 1\right)$ $\quad n_A + n_B + d + n_{H_S} - 1 = 9$

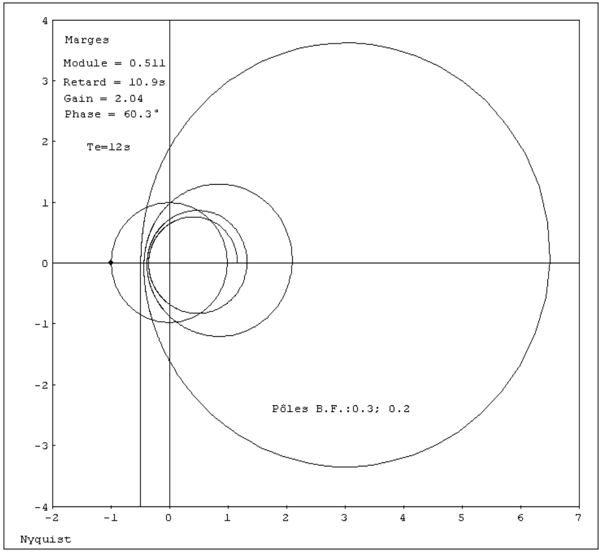
Case I : dominant pole: 0.2 aux. pole:0.3

Case II : dominant pole: 0.2 aux. poles:0.3 + 7x0.1

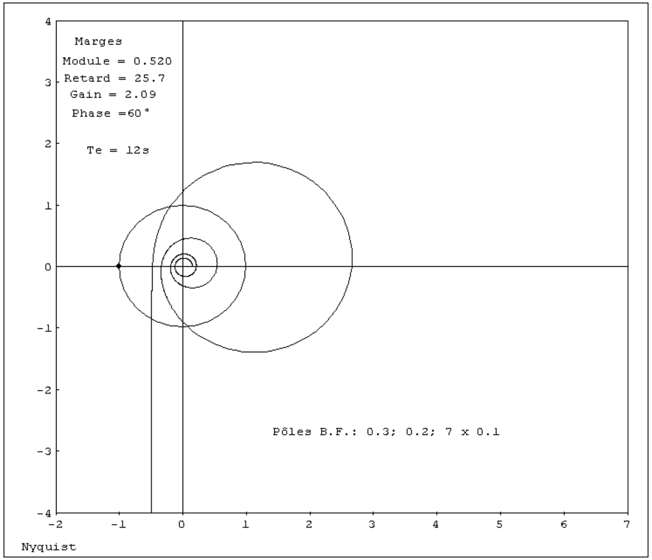
Hot dip galvanizing – output sensitivity



Hot dip galvanizing – Nyquist plot



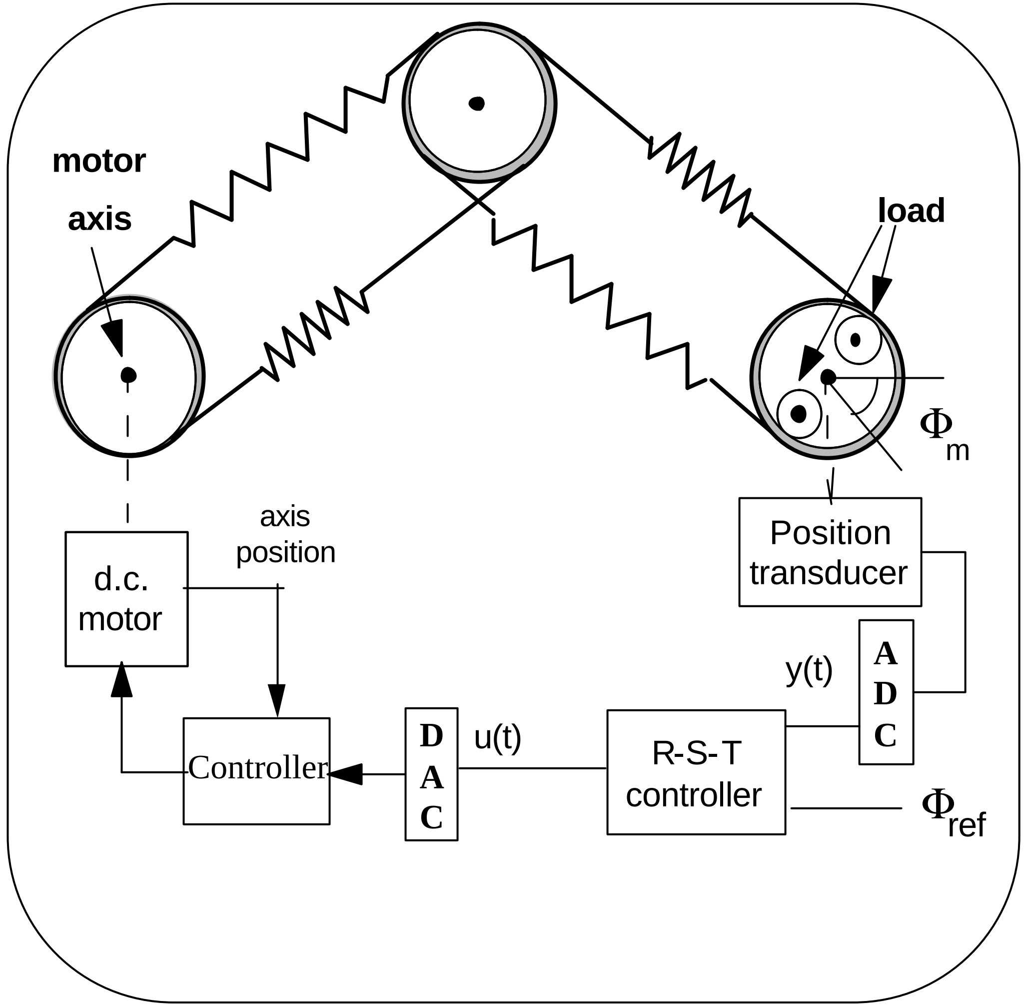
dominant pole: 0.2 aux. pole:0.3



dominant pole: 0.2 aux. poles:0.3 + 7x0.1

Control of a Flexible Transmission

The flexible transmission



Control of a Flexible Transmission

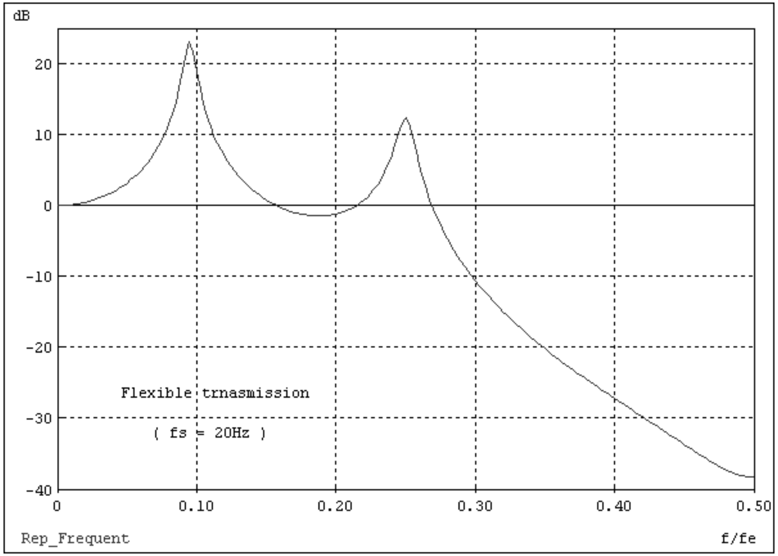
Sampling frequency : *20 Hz*

$$A(q^{-1}) = 1 - 1.609555q^{-1} + 1.87644q^{-2} - 1.49879q^{-3} + 0.88574q^{-4}$$
$$B(q^{-1}) = 0.3053q^{-1} + 0.3943q^{-2}$$
$$d = 2$$

The model

Vibration modes:

$$w_1 = 11.949\, rad/\sec, z_1 = 0.042;\ w_2 = 31.462\, rad/\sec, z_2 = 0.023$$



Specifications:

Tracking: $w_0 = 11.94\, rad/\sec, z = 0.9$

Dominant poles: $w_0 = 11.94\, rad/\sec, z = 0.8$

Robustness margins: $\Delta M \geq 0.5$ $\Delta t \geq 2T_S$

Null static error (integrator)

Constraints on S_{up} :

$$|S_{up}|_{\max} \leq 10\, dB \ \ for\ \ f \geq 0.35 f_S = 7\, Hz$$

Controllers for the Flexible Transmission

Control strategy: Pole placement

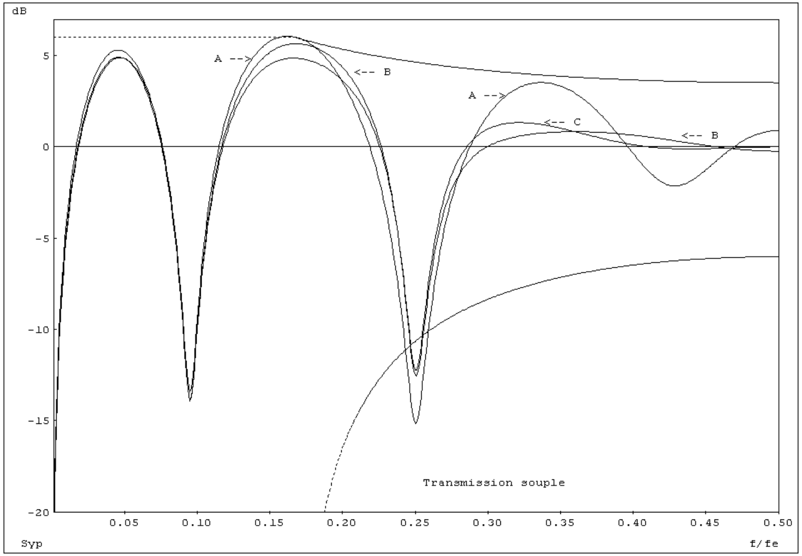
$$n_P = n_A + n_B + n_{H_S} + n_{H_R} + d - 1$$

$$A, B,: n_P = 8 \quad C: n_P = 9$$

	HS(q-1)	HR(q-1)	Closed loop poles		Modulus margin (dB)	Delay margin (s)	Max |Sup| (dB)
			Dominant	Auxiliary			
A	1-q-1		ω0 = 11.94 ζ = 0.8		0.498 (-6.06)	0.043	18.43
B	1-q-1		ω0 = 11.94 ζ = 0.8	ω0 = 31.46 ζ = 0.15 (1-0.2q-1)4	0.522 (-5.65)	0.062	6.24
C	1-q-1	1+q-1	ω0 = 11.94 ζ = 0.8	ω0 = 31.46 ζ = 0.15 (1-0.2q-1)4	0.544 (-5.29)	0.057	1.5

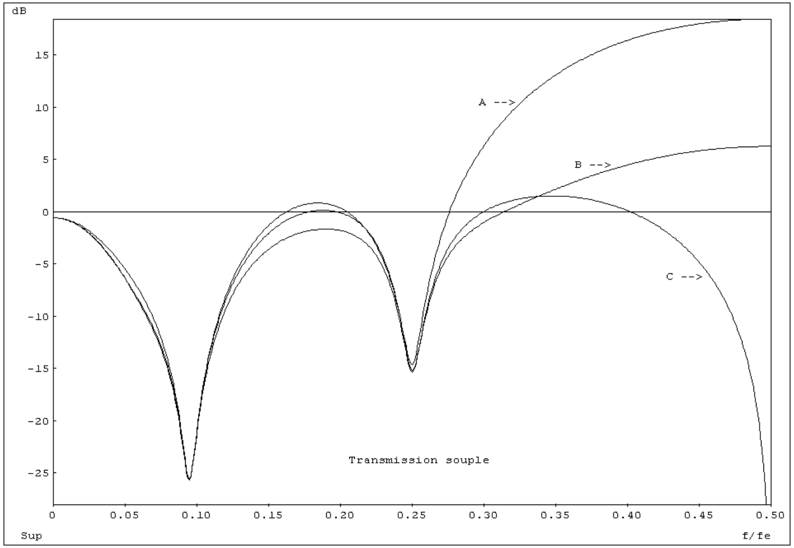
Control of a Flexible Transmission

Output sensitivity function



S_{yp}

Input sensitivity function

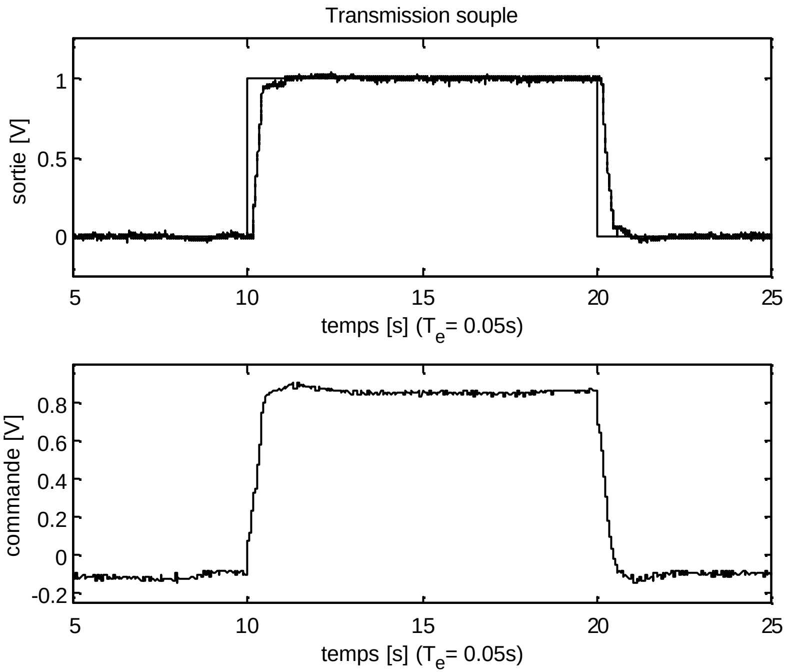


S_{up}

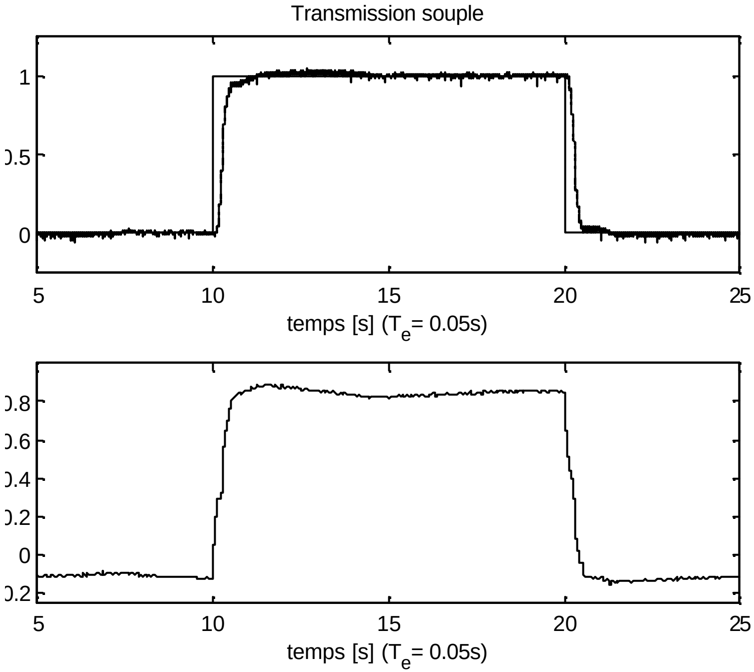
A- without auxiliary poles
B- with auxiliary poles
C- with stop band filter H_R / P_F

Control of a Flexible Transmission

Real time results - Tracking



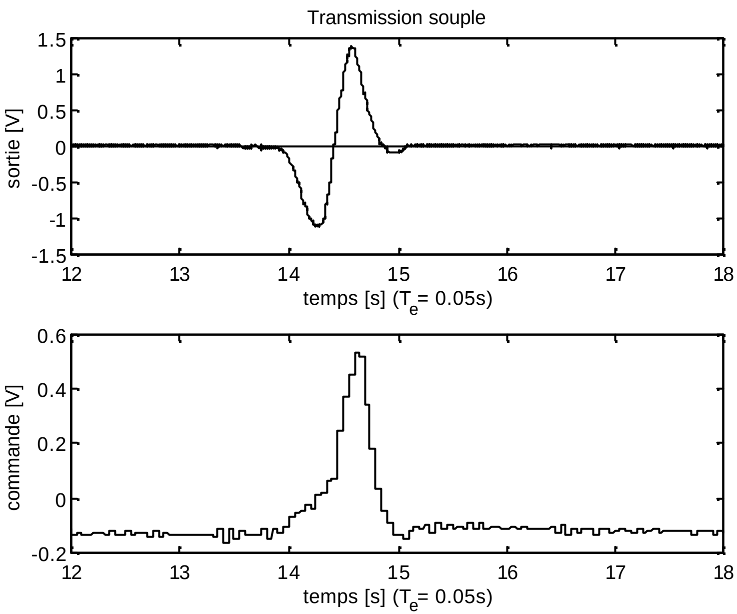
Controller B
(with auxiliary poles)



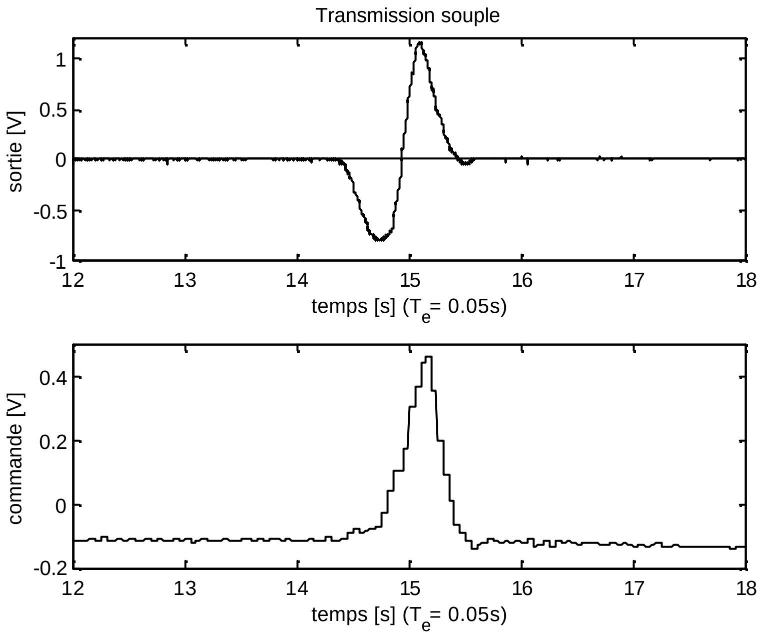
Controller C
(with auxiliary poles and
opening of the loop at $0.5f_S$)

Control of a Flexible Transmission

Real time results - Regulation

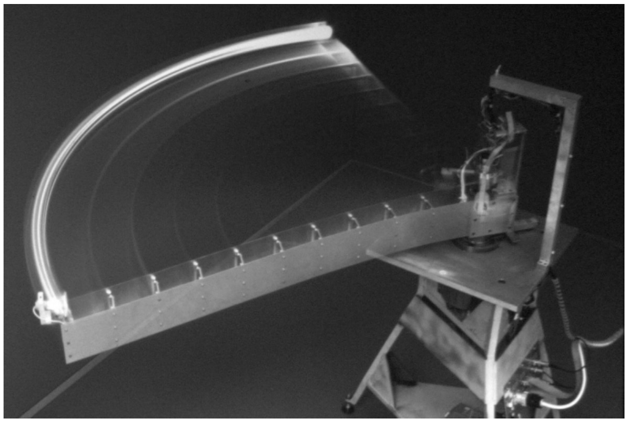


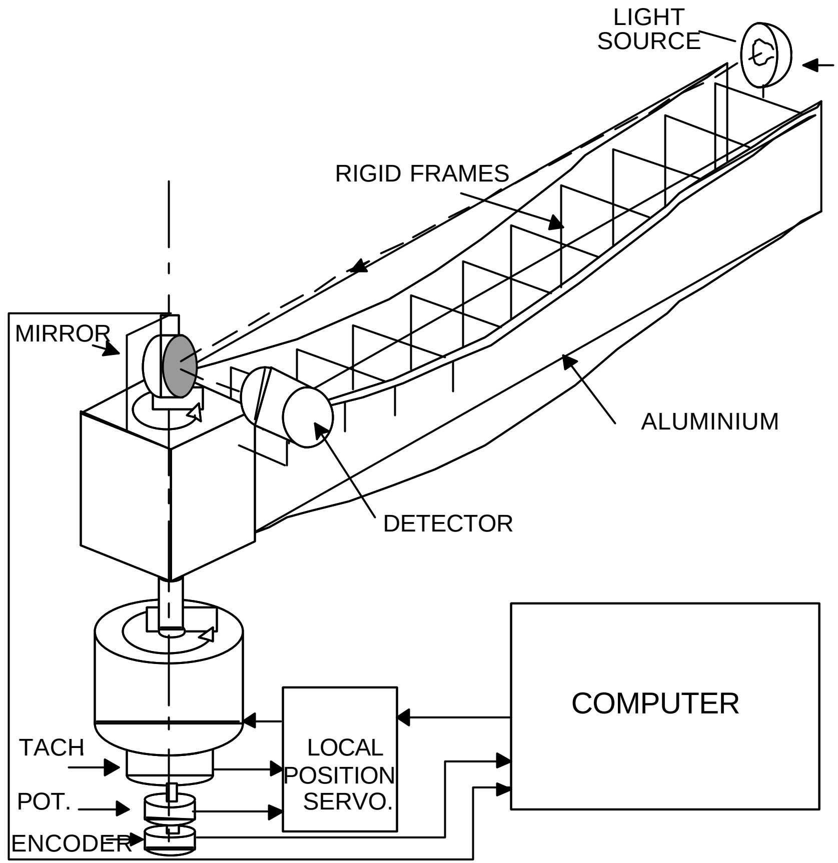
Controller B
(with auxiliary poles)



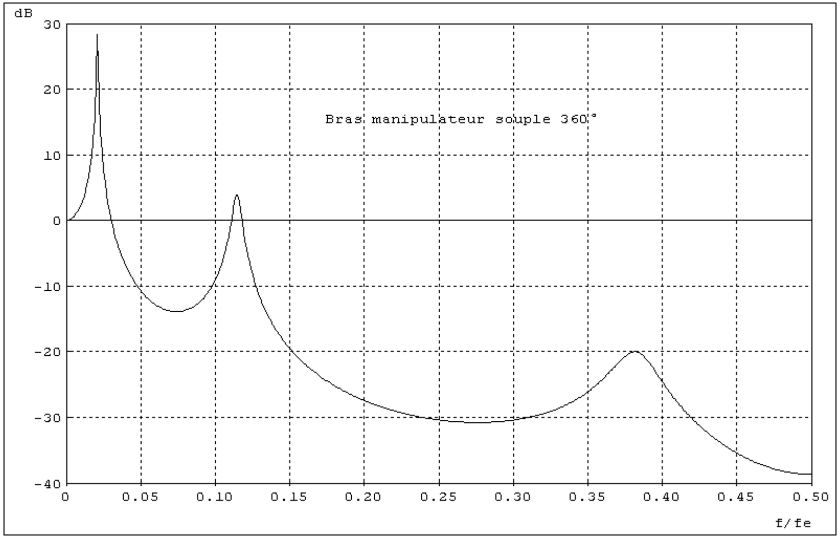
Controller C
(with auxiliary poles and opening of the loop at $0.5f_S$)

360° Flexible Arm

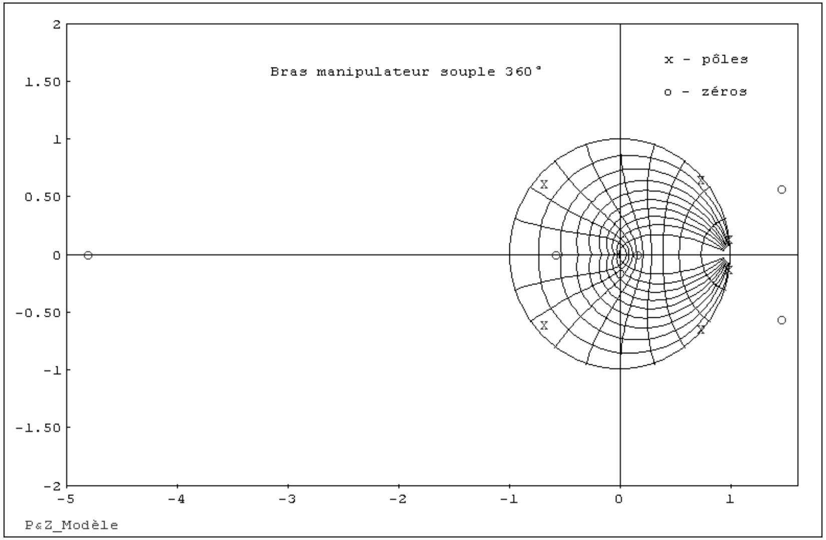




360° Flexible Arm



Frequency characteristics



Poles-Zeros

Unstable zeros !

(Identified Model)

360° Flexible Arm

Sampling frequency : *20 Hz*

Model –

$$A(q^{-1}) = 1 - 2.1049\, q^{-1} + 1.04851\, q^{-2} + 0.33836\, q^{-3} + 0.46\, q^{-4} - 1.5142\, q^{-5} + 0.7987\, q^{-6}$$

$$B(q^{-1}) = 0.0064\, q^{-1} + 0.0146\, q^{-2} - 0.0697\, q^{-3} + 0.044\, q^{-4} + 0.0382\, q^{-5} - 0.007\, q^{-6}$$

$$d = 0$$

Vibration modes:

$$w_1 = 2.617 rad/\sec, z_1 = 0.018;\ w_2 = 14.402 rad/\sec, z_2 = 0.025;\ w_3 = 48.117 rad/\sec, z_3 = 0.038$$

Specifications:

Tracking: $w_0 = 2.6173\, rad/\sec, z = 0.9$

Dominant poles: $w_0 = 2.6173\, rad/\sec, z = 0.8$

Robustness margins: $\Delta M \geq 0.5$ $\Delta t \geq 2T_S$

Zero steady state error (integrator)

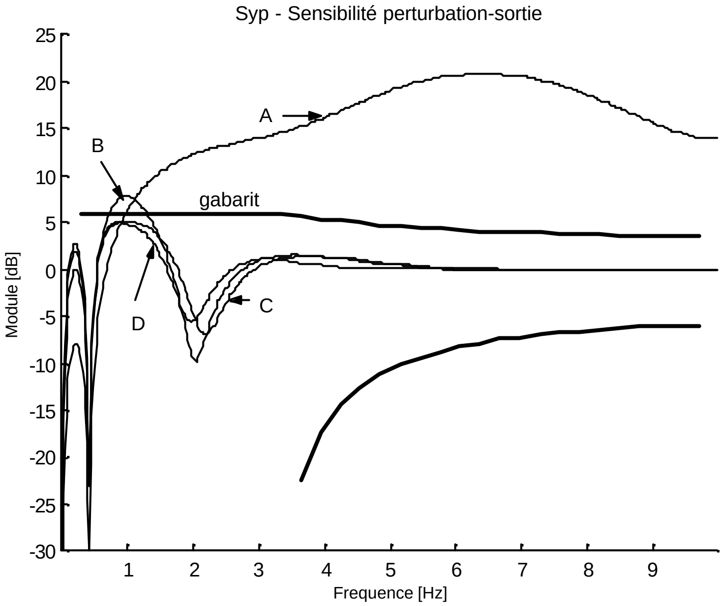
Constraints on S_{up} : $|S_{up}| \leq 15\, dB$ for $f < 4\, Hz$; $|S_{up}| \leq 0\, dB$ for $4.5 \leq f < 6.5\, Hz$;

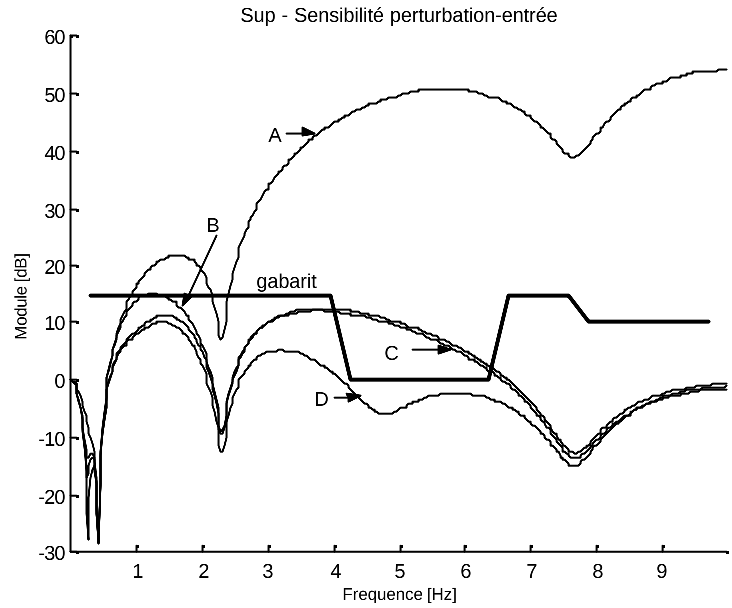
$|S_{up}| < 15\, dB$ for $6.5 \leq f < 8\, Hz$; $|S_{up}| < 10\, dB$ for $8 \leq f \leq 10\, Hz$

360° Flexible Arm - Shaping the Sensitivity Functions

Output Sensitivity Function - S_{yp}

Input Sensitivity Function - S_{up}





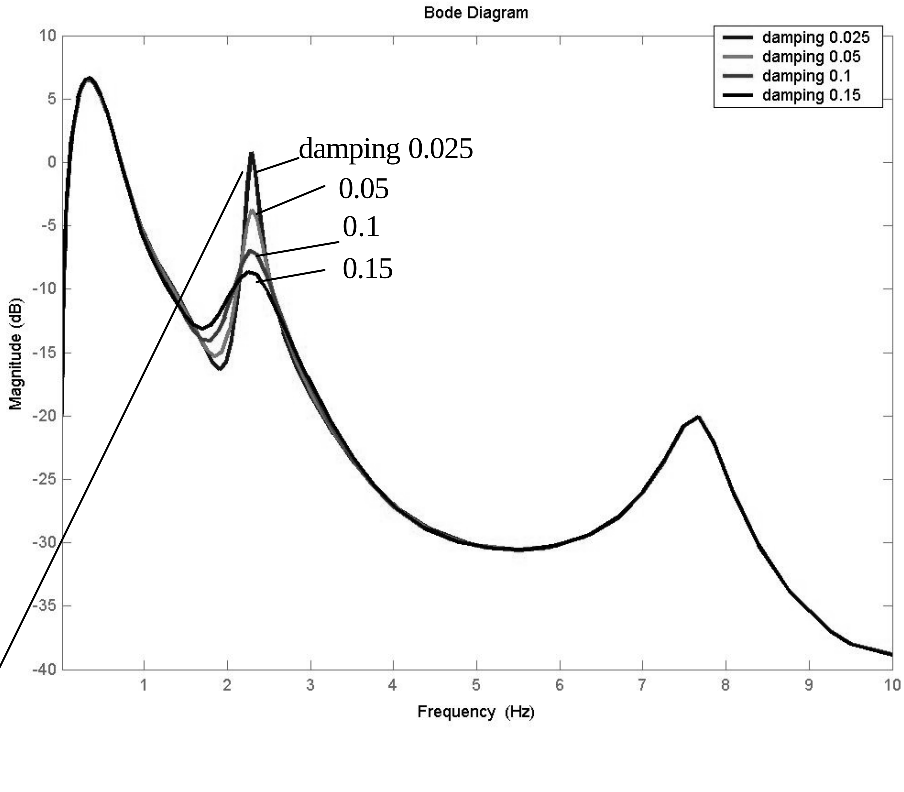
A- with auxiliary poles (2nd, and 3rd vibration modes)
B- with addititionnal auxiliary poles $(1-0.5q^{-1})^{6}$
C- with stop band filter H_{S1}/P_{F1}
D- with stop band filter H_{R2}/P_{F2}

Controllers for the 360° Flexible Arm

	$H_S(q^{-1})$	$H_R(q^{-1})$	Closed loop poles	
			Dominant	Auxiliary
A	$1 - q^{-1}$	-	$\omega_0 =$ 2.6173 $\zeta = 0.8$	2^{nd} and 3^{rd} vibration modes
B	$1 - q^{-1}$	-	$\omega_0 =$ 2.6173 $\zeta = 0.8$	2^{nd} and 3^{rd} vibration modes $(1 - 0.5q^{-1})^6$
C	$1 - q^{-1}$ $\omega_0 = 6.28$ $\zeta = 0.424$	-	$\omega_0 =$ 2.6173 $\zeta = 0.8$	2^{nd} and 3^{rd} vibration modes $(1 - 0.5q^{-1})^6$ $\omega_0 = 6.28$ $\zeta = 0.8$
D	$1 - q^{-1}$ $\omega_0 = 6.28$ $\zeta = 0.424$	$\omega_0 = 29.57$ $\zeta = 0.092$	$\omega_0 =$ 2.6173 $\zeta = 0.8$	2^{nd} and 3^{rd} vibration modes $(1 - 0.5q^{-1})^6$ $\omega_0 = 6.28$ $\zeta = 0.8$ $\omega_0 = 40.1$ $\zeta = 0.74$

360° Flexible Arm

Input disturbance-ouput sensitivity function - S_{yv}



Can be reduced by augmenting the damping of second pair of closed loop poles (frequency of the 2nd vibration mode)