Advanced signal processing tools for dispersive waves

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ABSTRACT

Two field examples are presented, showing the advantages of using multicomponent sensors for surface-wave studies. Multicomponent sensors allow the use of specific signal-processing tools such as the multicomponent singular value decomposition filter and the multicomponent polarization filter, which are both very efficient at separating surface waves from the other waves that comprise a seismic field record.

Firstly, some signal-processing tools for studying surface waves are described. The various filters range from classical to advanced techniques. For processing single-component data, the filters are the $f-k$ filter and filters based on singular value decomposition and on spectral matrix decomposition. For processing multicomponent data, the filters are the 4C-singular value decomposition filter and the classical or high-order polarization filter.

Secondly, processing sequences that can be applied to the field data are described and the single-component processing sequence and the multicomponent processing sequence are compared. Two field examples are presented. The first data set is a land seismic data recording on 2C sensors. The second data set was obtained from a marine acquisition with OBS (4 components). The results obtained illustrate the advantages of using multicomponent filters. The efficiency of the 4C-SVD filter and the high-order statistic polarization filter is demonstrated.

INTRODUCTION

In land surface seismic surveys, the ground roll is composed of surface waves, which are pseudo-Rayleigh waves and Love waves. They propagate directly from the source to the sensors without penetrating deeply into the subsurface. Surface waves are mainly coherent surface noise with large amplitudes. Their reflections may create disturbances which may mask (or interfere with) the signal of interest. For example, when interest is focused on reservoir detection, surface waves are considered as unwanted noise and wavefield separation filters are usually designed to attenuate these waves. However, if near-surface characterization is of interest, then surface waves are used to obtain information about the acoustic parameters of the near surface, which acts as a seismic waveguide. Others waves, such as reflection arrivals and refracted signals, become the undesired noise. In marine surveys, especially in the case of shallow water, seismic-wave propagation is affected by frequency dispersion related to various waveguides, depending on the depth (Pekeris 1948; Jensen et al. 1994; Admundsen and Reitan 1995). These waves are almost unaffected by absorption during their underwater propagation. They propagate at long range and can be used to estimate the geo-acoustical parameters even if the source is far from the receivers. The estimation of velocities relative to the sedimentary layer and of the parameters of dispersion is used to determine the constraints for modelling the propagation environment (Glangeaud et al. 1999; Nicolas et al. 2003).

The analysis of surface waves and the study of their dispersion allow us to evaluate the shear-wave velocity as a function of depth and to determine the shear modulus of the first tens of metres below the ground surface. For any recording, we need a source and at least two geophones. The source can be impulsive, i.e. hammer-type, or it can have a single frequency at every shot. Useful frequencies for the determination of the pseudo-Rayleigh velocity vary from 3 to 200 Hz. Surveys are often carried out over the frequency range from 10 to 100 Hz. Six geophones are generally used, with distances from the source being typically 0.25, 0.5, 1, 2, 3 and 4 m for accurate measurement of phase differences. This allows a phase velocity versus wavelength diagram to be computed. The measurement of phase is carried out in the frequency domain after a Fourier transform of traces recorded by the geophones. When a vibrator is used, one trace is obtained per frequency emitted, the frequency being that for which the amplitude spectrum is maximal. With an impulsive source, phases are measured in the frequency zone corresponding to the maximum of the amplitude spectrum and the coherence function. In this approach, we assume that the surface waves are dominant and the effects of the other waves are negligible. Once the phase velocity versus wavelength diagram is obtained, an inversion process is used to...
transform the previous diagram into a graph giving the shear modulus or stiffness modulus versus depth. The simplest inversion is empirical. It consists of assuming a constant ratio between wavelength and depth. Generally accepted values are 2 when the stiffness is approximately constant with depth or 4 when the stiffness increases significantly with depth. Up to this stage, calculations can be carried out in real time at the acquisition site. Results can be then refined using sophisticated inversion techniques, which by iteration adjust a model defined by mechanical parameters to the field data.

We describe processing methods which can be applied to surface waves in order to separate between these waves from others waves considered as noise (i.e. reflected, refracted, converted waves). After processing, surface wave characteristics can be more clearly identified.

Firstly, we introduce different types of filter, from classical, i.e. the $f-k$ filter (Yilmaz 2002) to advanced filters based on the latest research in signal processing, i.e. the high-order polarization filter (Lacoume et al. 1998; Vrabie et al. 2004; Le Bihan and Ginolhac 2004). The choice of filter depends mainly on the type of survey used to record signals (single-component or multicomponent).

Conventionally, seismic data sets are recorded using a single-component sensors array (vertical component). In this case, wavefield separation can be perform by several techniques (Mari et al. 1997), classified in three categories: template methods, such as $f-k$ or $\tau-p$ filters (Yilmaz 2002); inversion methods (Esmersoy 1988); and matrix methods, such as the singular value decomposition filter (Freire and Ulrych 1988) or filters based on spectral matrix estimation (Mars et al. 1987, 2004; Spitz 1991; Mari et al. 1997). Knowledge of wave polarization requires multicomponent sensor surveys using multicomponent filters, i.e. classical and high-order polarization filters (Lacoume et al. 1998) or 4C-singular value decomposition filters (Meunier et al. 2001).

Secondly, we present two field examples. The first one is a land seismic data set recorded on a 2C sensors array. The second data set was obtained from a marine acquisition with OBS (4 components). Using these examples, we illustrate the processing sequence used for surface-wave characterization and guided-wave extraction.

**SOME SIGNAL PROCESSING TOOLS FOR SURFACE-WAVE ESTIMATION**

Filtering is an essential operation in seismic and near-surface geophysics, used to characterize the contribution of each wavefield. We describe various signal processing methods used to separate each surface wave from the body waves. Our goal is to extract the surface wave in order to characterize its dispersion. Ranging from classical to advanced techniques, we describe the following filters:

- the classical $f-k$ filter;
- the classical polarization filter and the high-order polarization filter used respectively on sections recorded by a single-component sensor array and by a multicomponent sensor array;
- the singular value decomposition filter with its extension to multicomponent data sets (4C-singular value decomposition);
- the spectral matrix decomposition filter.

**Model**

Here, we assume that seismic signals are recorded on an array of scalar sensors (single-component sensors) or vectorial sensors (multicomponent sensors). These signals are generally described as the summation of several events related to the different sources (reflected waves, refracted waves, converted waves, surface waves) propagating through the media. The observed scalar or vectorial signal, dependant on time $t$ and distance $x$ (a seismic section), is given by

$$r(t,x) = \sum_i a_i(t)*s_i(t,x),$$  

where $a_i(t)$ is the wavelet from source $i$, $s_i(t,x)$ is the propagation vector of the source and $*$ denotes convolution. The signal $r(t,x)$ can be described in dual domains associated with the time and distance variables as:

- $S(f,x) = FT[\{s(t,x)\}]$, the distance–frequency space representation;
- $S(f,k) = FT[S(f,x)]$, the frequency–wavenumber representation (2D Fourier transforms on time and distance variables).

![FIGURE 1](image_url)  
Non-dispersive wave (model). (a) Time–distance representation; (b) horizontal component.

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Filtering in the frequency–wavenumber domain on single-component acquisitions

A synthetic example demonstrates the use of the $f$–$k$ filter on surfaces waves. A wave propagating in a non-dispersive medium (with constant velocity $V$) can be expressed in the time–distance domain as $r(t,x) = w(t-x/V)$ (Fig. 1a). The modulus of the 2D Fourier transform of $r(t,x)$ (Bracewell 1986) gives the expression for the wave in the $f$–$k$ plane as

$$ R(f,k) = FT_{r,x}[w(t-x/V)] = W(f) \delta(k + f/V). \quad (2) $$

In the $f$–$k$ plane, $\delta(k + f/V)$ represents the straight-line $k + fV = 0$ passing through the origin with slope $-1/V$ (Fig. 1b). In a dispersive medium, the dispersion law is not linear. The wave velocity depends on frequency. The group velocity and phase velocity of the wave can be estimated as $v_g = \frac{df}{dk}$ and $v_p = f/k$, respectively. For a seismic section with one dispersive event, the signal $r(t,x)$ is written in the time–distance domain (Fig. 2a) and in the frequency–wavenumber domain (Fig. 2b), respectively, as

$$ r(t,x) = w(t-x/V)e^{-i\phi} \quad (3) $$

and

$$ R(f,k) = W(f) \delta(k + f/V + \phi/2\pi). \quad (4) $$

In the $f$–$k$ plane, a dispersive wave is characterized by a non-straight line passing through the wavenumber axis at position $k_o$, not equal to zero (Fig. 2b). In order to select dispersive waves by a masking filter in the $f$–$k$ plane (fan filter, strip filter; Fail and Grau 1963), it is necessary to apply a group-velocity correction and a phase-shift correction (i.e. translation of the $k_o$ value on the wavenumber axis) (Yilmaz 2002).

Filtering by singular value decomposition on single-component data

After propagation through the media, the received signal $r(t)$ on sensor $i$ is the result of the superposition of $N_s$ (number of sources) waves $[a_{1}(t), \ldots, a_{N_{s}}(t)]$ via the transfer functions $x_{r,i}(t)$, and is given by

$$ r = \{r_{f,j}| f = 1, \ldots, N_{f}; j = 1, \ldots, N_{c}\} \in \mathbb{R}^{N_{f} \times N_{c}}. \quad (5) $$

where $b(t)$ is noise, assumed to be Gaussian, white and centred, and $N_{c}$ is the number of sensors. With signals sampled in time, we write the received signals in a data matrix as

$$ r = \{r_{f,j}| f = 1, \ldots, N_{f}; j = 1, \ldots, N_{c}\} \in \mathbb{R}^{N_{f} \times N_{c}}. \quad (6) $$

The singular value decomposition of the time–space data matrix $r$ provides two orthogonal matrices $u$ and $v$, and one diagonal matrix $\Delta$ made up of singular values (Klema and Laub 1980; Golub and Van Loan 1996; Vrabie et al. 2004). The initial data matrix is expressed as

$$ r = u \Delta v^T = \sum_{i=1}^{N_{r}} \lambda_{i} u_{i} v_{i}^T \quad \text{with} \quad N = \{\text{min}(N_{r}, N_{c})\}, \quad (7) $$

- where $u = [u_{1}, \ldots, u_{N_{c}}]$ is an $N_{r} \times N_{c}$ orthogonal matrix made up of left-hand singular vectors $u_{i}$ giving the amplitude in the real case (amplitude and phase in the complex case), therefore called propagation vectors;
- $v = [v_{1}, \ldots, v_{N_{r}}]$ is an $N_{r} \times N_{c}$ orthogonal matrix made up of right-hand singular vectors $v_{i}$ giving the time dependence, hence called normalized wavelets;
- $\Delta = \text{diag}(\lambda_{1}, \ldots, \lambda_{N_{r}})$ is an $N_{r} \times N_{r}$ diagonal matrix with the diagonal entries ordered $\lambda_{1} \geq \lambda_{2} \geq \ldots \geq \lambda_{N_{r}} \geq 0$.

The product $u_{i} v_{i}^T$ is an $N_{r} \times N_{r}$ unitary rank matrix known as the $i$th singular image of data matrix $r$. Therefore, $r$ is given by the sum of all the $k$th singular images multiplied by their corresponding $k$th singular values $\lambda$. The rank of the matrix $r$ is the number of non-zero singular values in $\Delta$. In the noise-free case, if the recorded signals are linearly dependent (for example, if they are equal to within a scale factor, i.e. one wave with an infinite velocity), the matrix $r$ is of rank one and a perfect reconstruction requires only the first singular image (Freire and Ulrich 1988; Mars et al. 1999; Vrabie et al. 2004). For one dispersive wave,
which has an infinite velocity and is noise-free, two singular images are required. In practice on a real data set, a maximum of five singular values is usually necessary. If the \( N \) recorded signals are linearly independent, the matrix \( r \) is full rank and the perfect reconstruction requires all singular images. Hence, in practice, before performing the SVD filtering, a velocity correction is applied to the initial data to obtain an infinite apparent velocity for the selected wave. This preprocessing enables decomposition with a smaller space (Mari et al. 1997; Glangeaud et al. 1999).

Using the SVD filter, the separation between the signal and the noise subspace is given by

\[
r = r^s + r^{\text{noise}} = \sum_{i=1}^{N_s} \lambda_i u_i v_i^T + \sum_{i=N_s+1}^{N} \lambda_i u_i v_i^T.
\]

(8)

The signal subspace \( r^s \) is characterized by the first \( N_s \) higher singular images (associated with the first \( N_s \) higher singular values). It gives roughly the waveform of the dominant wave, its energy and its amplitude repartition on the sensors. The remainder, subspace \( r^{\text{noise}} \), contains the waves with a low degree of sensor-to-sensor correlation and the noise (Mars et al. 2004; Vrabie et al. 2004).

**Filtering by singular value decomposition on multicomponent data (4C-SVD)**

For a multicomponent section (3C or 4C), the objective of the filter is to obtain the polarization state of the waves. In order to characterize it, we apply the singular value decomposition filter for each 4-component trace (4C-SVD) (De Franco and Mussachio 2001; Meunier et al. 2001). Each section we consider is composed of each individual sensor of the four components contained in a matrix \( r \in \mathbb{R}^{4 \times N} \). As no time delay appears between the components of each sensor, SVD processing is applied under optimal conditions (no velocity correction is required in this case). The 4C-SVD filtering on each sensor enables decomposition with four singular images as

\[
r = \sum_{i=1}^{N_s} \lambda_i u_i v_i^T.
\]

(9)

The singular vector \( u_i \) describes a wavelet associated with a propagation mode. The singular vector \( v_i \) expresses the amplitude variations of a wavelet (versus distance). The singular value \( \lambda_i \) characterizes amplitudes relating to the section of rank \( i \). For each sensor (4-component vectors, \( X, Y, Z \) and hydrophone), we only keep the first singular image (equal to \( \lambda_i u_i v_i^T \)). At each sensor, the components \( v_x, v_y, v_z, v_h \) of the first singular vector \( v_i \) are used to calculate the rotation angles in order to line up the seismic sensor. The angle \( \alpha = \tan(v_y/v_x) \) is used to realign the sensor in the horizontal plane to obtain in-line and cross-line sections.

**Filtering by spectral matrix filtering on single-component data**

On a single-component section, surface-wave separation and/or enhancement of the signal-to-noise ratio can be performed by applying an eigendecomposition filter in the frequency domain (Mars et al. 1987). In the frequency domain, the received signal resulting from the superposition of \( N \) waves is given by

\[
R(f) = \sum_{i=1}^{N_s} a_i S_i(f) + b(f),
\]

where \( a_i \) is the amplitude (at frequency \( f \)) of the wave \( i \), \( S_i(f) \) is the vector of the wave \( i \) describing the propagation between sensors and \( b(f) \) is Gaussian, white, centred noise. The spectral matrix of the received signal is defined as

\[
\text{SM}(f) = E \left[ R(f) R(f)^\dagger \right],
\]

where \( ^\dagger \) denotes the transpose conjugate operation and E (the mathematical expectation) is a smoothing operator applied in the frequency domain, in the distance domain or by experience according to the experimental context (Shan et al. 1985; Mari et al. 1997). Thus we obtain an \( N_s \times N_s \) spectral matrix at each frequency bin \( f \). Under the assumption of white noise, the spectral matrix is expressed as

\[
\text{SM}(f) = S(f) \text{SM}_f(f) S(f)^\dagger + \gamma I,
\]

(12)

where \( \gamma \) is the noise power spectral density and

\[
\text{SM}_f(f) = E[ A(f) \cdot A(f)^\dagger].
\]

The rank of \( S(f) \text{SM}_f(f) S(f)^\dagger \) is equal to \( N_s \) (the number of sources).

The eigendecomposition of \( \text{SM}(f) \) gives

\[
\text{SM}(f) = \sum_{i=1}^{N_s} \lambda_i V_i(f) V_i^T,
\]

(13)

where \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{N_s} > 0 \) are the eigenvalues and \( V_1, V_2, \ldots, V_{N_s} \) are their corresponding vectors. The number \( N_s - N_n \) of minimal eigenvalues of \( \text{SM}(f) \) equal to \( \gamma \) is the number of waves. The eigenvectors associated with these minimal eigenvalues are orthonormal to the columns of the matrix \( S(f) \). The subspace generated by the \( N_s - N_n \) eigenvalues could be referred to as the ‘noise space’ and its orthogonal complement as the ‘signal space’. For each frequency bin, the subspace decomposition is written as

\[
\text{SM}(f) = \sum_{i=1}^{N_s} \lambda_i V_i(f) V_i^T + \sum_{j=N_s+1}^{N} \lambda_j Y_j(f) Y_j^T.
\]

(14)

In the case of white noise, the number of dominant eigenvalues gives the number of waves. Projection between the eigenvectors associated with the dominant eigenvalues and the initial data gives the signal subspace. In some specific cases, the waves can be identified through the spectral matrix when

- only one source is recorded on the data; the dominant eigenvector gives the wave vector of this source;
- one dominant and energetic wave is present; eigenvectors associated with the highest eigenvalues provide a good estimator of the dominant wave.
For the case of one dominant surface wave, after eigendecomposition of the estimated spectral matrix \( \mathbf{SM}(f) \), we obtain the dominant eigenvector \( \mathbf{Y}_i(f) \) associated with the highest eigenvalue. By projection of the initial data on to this eigenvector, we can describe the dominant surface wave:

\[
\mathbf{S}_i(f) = \mathbf{Y}_i(f) \mathbf{R}(f) > \mathbf{Y}_i(f).
\] (15)

Using the inverse Fourier transform of \( \mathbf{S}_i(f) \), we obtain the dominant wave in time and distance and the residual waves are given on \( \mathbf{r}_{\text{noise}} = \mathbf{r} - \mathbf{S}_i \).

**Filtering by polarization filter on multicomponent data**

When seismic sections are recorded on multicomponent sensors, the objective of the filter is to determine the polarization state of the waves (Achenbach 1973; Waters 1978). Polarization filters enable the separation of two-component signals into a new domain or into the signal domain, where we find all the waves having a given polarization ‘pol’, such that in the complementary domain (the so-called noise domain) none of the waves are present. In this complementary domain, all the other waves have an orthogonal polarization. In order to distinguish pseudo-Rayleigh waves from other waves, the polarization filter (Mars et al. 1999) is designed by successive operations as follows:

- A phase shift is applied to linearize polarization figure (ellipse).
- This operation is followed by a rotation to bring the orientation of the major axis of the ellipse on to a particular component (for instance, the vertical component). At this step, the selected wave is now entirely localized on the vertical component \( V \). The \( V \)-component is chosen as the signal domain. Once this operation has been carried out, the second wave is repartitioned over the two components as its polarization is not orthogonal to the selected pseudo-Rayleigh wave.
- The phase of the horizontal component \( H \) of the second wave must be changed to obtain a rectilinear polarization (polarization filter has been formulated in the high-order statistics domain). Over the last 10 years, filter formulations have been extended to include high-order statistics in order to handle the limitations imposed by order-2 statistics (correlation and spectra). The polarization filter must be identified on each sensor. In the case of two waves impinging on a 2-component sensor, we can expressed the recorded signals as

\[
\mathbf{r}(t) = \mathbf{a} \cdot \mathbf{z}(t) + \mathbf{b}(t),
\] (17)

where \( \mathbf{a} \) is a mixing matrix of sensor \( i \) describing the polarization parameters and the wave amplitudes (Lacoume et al. 1998). This matrix can be decomposed by singular value decomposition as

\[
\mathbf{a} = \mathbf{u} \mathbf{A}^{1/2} \mathbf{v}^T.
\] (18)

The filtering consists of two steps. The first step, using second-order statistics, is based on the eigendecomposition of the covariance matrix at each sensor \( i \) as

\[
\text{cov} = \mathbb{E}[\mathbf{r}(t)\mathbf{r}^T(t)] = \mathbf{u} \mathbf{A} \mathbf{u}^T.
\] (19)

As \( \mathbb{E}[\mathbf{z}(t)\mathbf{z}^T(t)] = \mathbf{I} \) (sources are assumed non-correlated),

\[
\text{cov} = \mathbf{u} \mathbf{A}^{1/2} \mathbf{v}^T.
\] (20)

At this step, \( \mathbf{u} \) and \( \mathbf{A} \) are estimated by second-order processing but \( \mathbf{v} \) (see equation (18)) is still missing. Using \( \mathbf{u} \) and \( \mathbf{A} \), we can compute the whitened observations as

\[
\mathbf{I}(t) = \mathbf{A}^{1/2} \mathbf{u}^T \mathbf{z}(t) = \mathbf{w}(t),
\] (21)

where spectral matrix filtering to select linear polarization events. Picheral et al. (2001) proposed a shift invariance method to estimate apparent velocity and polarization jointly. Using time–frequency tools, Roueff et al. (2002) designed an efficient oblique polarization filter using a cross scalogram.
The second step of this high-order polarization filter consists of estimating the rotation matrix $\mathbf{v}$ by using, for example, the JADE (jointly approximated diagonalization of eigenelements) algorithm, based on the diagonalization of matrices built with 4th-order cross-cumulants (Cardoso and Souloumiac 1993; Le Bihan and Mars 2000).

A LAND EXAMPLE

Pseudo-Rayleigh waves: 2-component data set

Seismic data were obtained using explosive sources and a line of 2-component receivers (47 sensors). The distance between adjacent geophones is 10 m. The vibration axis of the horizontal component $H$ is located in the plane through the source point and the seismic line (in-line). The second component $V$ is a geophone with a vertical axis. The offset is 50 m. Data are sampled every 16 ms and the recording duration is limited to 4 s. Figure 3 shows the vertical and horizontal components of the initial data recorded. On the vertical component, we can identify a refracted wave, a reflected arrival with a strong amplitude associated with a deep reflector, and two dispersive pseudo-Rayleigh waves characterized by two low apparent velocities and a low-frequency content. Figure 4 shows the 2D amplitude spectrum of the vertical seismic recording. The spectrum is presented with (Fig. 4a) and without (Fig. 4b) normalization of wavenumber $k$. Without normalization of $k$, the two branches associated with the two dispersed surface waves are clearly visible at low frequencies (under 16 Hz). After normalization in the $k$-domain, the refracted wave appears at frequencies over 17 Hz, and the reflected waves are concentrated near the null wavenumber ($k=0$). The first branch of the pseudo-Rayleigh mode is situated in the frequency range 3–10 Hz and the second branch is in the range 10–20 Hz. The purpose of the study is to select the slowest pseudo-Rayleigh wave for analysis. Several types of filter can be applied to select this slow dispersive pseudo-Rayleigh wave: the $f$–$k$ filter, the polarization filter, the SVD filter, the SMF filter, and the high-order polarization filter.

$f$–$k$ filtering

A filter designed in the $f$–$k$ domain allows the extraction of the slowest pseudo-Rayleigh waves after flattening of this wave using a group-velocity correction of 189 m/s. This filter is equivalent to a fan velocity filter. The filtered wave is then subtracted from the initial data. The remaining residuals are used to evaluate the efficiency of such filtering. Filtered data and the residual section are respectively presented in the $x$–$t$ plane in Figs 5(a) and 5(c) and in the $f$–$k$ plane in Figs 5(b) and 5(d). The 2D amplitude spectrum associated with the residual section mainly shows the fast pseudo-Rayleigh wave (Fig. 5b).

To evaluate the benefits of the filtering processing, we compute the phase velocities before and after processing. Figure 6(a) shows the initial data of the vertical component windowed in

$$
\mathbf{v} = \begin{pmatrix}
\cos \theta & \sin \theta e^{-j\phi} \\
\sin \theta & \cos \theta e^{j\phi}
\end{pmatrix},
$$

(22)

FIGURE 3

Land seismic example: raw data showing vertical component ($V$), horizontal component ($H$).

FIGURE 4

Land seismic example: 2D amplitude spectrum of vertical component. (a) Without wavenumber amplitude normalization; (b) with wavenumber amplitude normalization.
time and after a rough group-velocity correction. We can see that the pseudo-Rayleigh wave interferes with other waves along the offset axis. Interference is especially strong in the short offset (between 50 and 200 m). After filtering by an \( f-k \) filter, the pseudo-Rayleigh wave is well isolated without any interference (Fig. 6b), and at this step, we can obtain the geophysical parameters of this pseudo-Rayleigh wave. We observe two branches along the distance axis. The first one, the 'short offset', presents a well-aligned pseudo-Rayleigh wave (correct velocity and phase-shift corrections). The second one, the 'long offset', shows a pseudo-Rayleigh wave with a non-infinite group velocity. Computing the phase velocity for each branch (Fig. 6c) enables us to characterize a change in behaviour in the propagation versus distance plot (probably due to a change of petrophysical parameters). These phase velocities are frequency dependent. Computation of the phase velocity using the initial wave (with interference) gives incorrect information on the propagation.

Polarization filtering
The polarization parameters of the slow pseudo-Rayleigh wave are the phase shift and the rotation, which will make the polarization linear and vertical. The rotation and the phase shift values have to be refined so that the slow pseudo-Rayleigh wave is only present on one component. Phase shift (\(-85^\circ\)) and rotation (\(-45^\circ\)) parameters are estimated from sensor #23 where the pseudo-Rayleigh is fairly visible from 1.5 s onwards. In the field case studied, as the parameters of the polarization of the wave are sta-
tionary in distance, this wave was well isolated. After the filtering process, the extracted wave and the residual wavefield are shown in Figs 7(a) and 7(b).

**SVD filtering**

Singular value decomposition (SVD) is used to separate waves. Applying a group-velocity correction (189 m/s) and a phase-shift correction (22°) reduces the dispersion of the slow pseudo-Rayleigh wave, which has a main frequency of 7 Hz. After these corrections, this wave is flattened and rendered non-dispersive. The pseudo-Rayleigh wave can be characterized by SVD filtering (five singular images are selected to estimate this wave). After filtering, the slow pseudo-Rayleigh wave is returned to its initial position in the x-t plane. The extracted pseudo-Rayleigh wave and the residual section are shown in Figs 8(a) and 8(b). It can be seen that the filtering is effective due to the fact that pseudo-Rayleigh wave is flattened prior to the filtering.

**SVD and SMF filtering**

Considering the vertical component section, flattened by correction of the group velocity and dispersion (phase shift) of the slow pseudo-Rayleigh wave, we want to estimate it by using the SMF filter. As the main frequency of the slow pseudo-Rayleigh wave is 7 Hz, we can estimate the spectral matrix at this specific frequency. Eigendecomposition of this matrix allows us to recover the pseudo-Rayleigh wave characterized by the first eigenvector.
(because a phase and group velocity correction has been applied). Figure 9(a) shows the extracted pseudo-Rayleigh wave after such processing. The residual section is shown in Fig. 9(b). On this section, it is possible to identify the refracted wave, a reflected wave and an aerial wave preceding the fast pseudo-Rayleigh wave (Fig. 9b).

**High-order polarization filtering.**

Figures 10(a) and (b) show respectively the vertical and horizontal components, windowed in time and flattened with a constant-velocity correction. The two waves are clearly mixed on the two sections. The objective now is to extract the slow pseudo-Rayleigh wave using a high-order polarization filter. The result

FIGURE 10

Land seismic example: seismic sections windowed and flattened with constant velocity correction. (a) Vertical component; (b) horizontal component.

FIGURE 11

Land seismic example: high-order polarization filter results. (a) Extracted surface waves on the vertical component (left) and the horizontal component (right); (b) rotation angle and phase parameter for the surface-wave estimation; (c) residual sections on the vertical component (left) and the horizontal component (right).
shows the estimation of the slow pseudo-Rayleigh wave on each component (Fig. 11a), and the residual section on the vertical and horizontal components (Fig. 11c). The polarization parameters were estimated independently for each trace. For the pseudo-Rayleigh wave, the layout of these parameters (rotation angle and phase shift) shows that the polarization varies gradually (Fig. 11b). The fluctuations that are seen are attributed to the inaccuracy of the measurements. Parameters associated with the residual section are not meaningful, due to the interference between several waves. For small source–sensor offsets, this processing is more efficient than the other filtering described above (polarization filter, f–k filter, SVD and SMF). This is due to the fact that this processing does not require previous knowledge as polarization parameters do not change with distance.

**A MARINE EXAMPLE**

**Marine guided waves: four-component data set (OBS-4C)**

A seismic marine survey was provided by the Compagnie Générale de Géophysique (CGG). Data were recorded on a 4-component sensor (one hydrophone and three geophones: OBS-4C) laid on the sea-floor (14 m). We show common-receiver sections with the following parameters: maximal offset 900 m; listening time 1 s; time sampling 2 ms; spatial sampling 50 m. Figure 12 shows the initial 4-component data set. On the hydrophone-, X- and Y-components, respectively shown in Figs 12(a), (b) and (c), guided waves are dominant. The Z-component is quite different (Fig. 12d) due to the fact that the guided wave is strongly aliased. Considering the hydrophone-component, the guided wave is dominant (Fig. 13a). In the f–k domain, the 2D amplitude spectrum, presented without normalization in the wavenumber domain, shows the guided wave with its energy concentrated in the 60–80Hz frequency bandwidth (Fig. 13c). With normalization in the wavenumber domain (Fig. 13e), we see that its bandwidth is much wider (20–220 Hz). After the velocity correction has been applied in the x–t domain (Fig. 13b), we see that the energy associated with the guided wave is not centred on the null wavenumber in the f–k plane (Figs 13d and f) (Mars et al. 1999). This indicates that the guided wave is dispersive.

**4C-SVD filtering.**

In order to characterize multicomponent relationships, the SVD method is applied to each 4C trace (4C-SVD). The seismic section we consider is composed of each trace of the four components. In order to apply SVD processing trace-by-trace under good conditions, we checked that the guided wave is in phase on the four components (after velocity correction). The 4C-SVD filtering of each trace (one trace is a 4-component recording) allows
decomposition into four singular sections. For each trace, we keep only the first singular section. Figure 14 shows, from left to right, according to the source-sensor distance: vector \( u_1 \): wavelet on windowed time (0–0.25 s, Fig. 14a); components \( v_x, v_y, v_z, v_{hy} \) of the first singular vector \( \mathbf{v}_1 \) (Fig. 14b); the rotation angle in the horizontal plane defined by \( \alpha = \text{atan}(v_x/v_y) \) (Fig. 14c).

For each trace, the components \( v_x, v_y, v_z, v_{hy} \) of the first singular vector \( \mathbf{v}_1 \) are used to calculate the rotation angles to realign the sensor. The guided wave is entirely contained inside the first singular section associated with each component. The other waves contained in the data are in the complementary space consisting of the sum of the three other sections (from rank 2 to rank 4).

Figure 15 shows the results of the filtering in the 0–1 s initial interval on the different components in the time–distance domain. A correction of inverse velocity has been applied to the data in order to place the waves back in their original positions. For the hydrophone (Fig. 15a top), the filtered section is very near the initial section. On the residual hydrophone component (Fig. 15b top), we can successively observe a refracted wave, a high-frequency guided mode with fast apparent velocity and a 10 Hz dominant frequency mode with slow apparent velocity. For the Z-component, the section filtered by 4C-SVD (Fig. 15a bottom) is of very weak amplitude and does not resemble the initial section because the guided wave is not very energetic. The residual section, on the other hand, is therefore very close to the initial section (Fig. 15b bottom).

The ratio of the guided wave, selected by 4C-SVD, to residual waves, evaluated on the amplitude spectrum, is 20 dB for the hy-, Y- and X-components and –6 dB for the Z-component. We see that the refracted wave is dispersive for this shallow depth and the velocity analysis of the lower layer requires the velocity law to be designed according to the frequency. This multicomponent filtering is limited. We assume that the wave polarization is constant with frequency. In the present case, for dominant frequencies, this proves to be true. On the other hand, it is not true at the limits of the bandwidth. Indeed, for high frequencies, we observe the presence of a fast wave for the filtered hydrophone-component, which precedes the extracted guided wave. For low frequencies and angles close to the critical angle, the energy is high and the wave polarization varies rapidly. In addition, we observe the presence of a second residual wave, slower than the selected guided wave.

CONCLUSION

Seismic data acquisition is currently carried out using single-component sensors. To extract reflected waves from the elementary field records, geophysicists use conventional apparent-velocity filters in order to reject or extract dispersive surface waves. In near-surface geophysics, these dispersive waves are very important because they convey information on the subsurface media. The \( f-k \) filter is currently used, but it is not efficient at low frequencies and requires a large number of traces to give good results. If the number of seismic traces is limited, the singular value decomposition filter (SVD) is a very efficient means of characterizing the dominant wave, which is usually the sur-

FIGURE 14
Marine seismic section: 4C singular value decomposition. (a) Wavelet \( u \) from hydrophone section; (b) amplitude variation versus offset for all component sections; (c) rotation angle versus offset for the hydrophone-component section.

FIGURE 15
Marine seismic section: results after 4C singular value decomposition. (a) Filtered section (from top to bottom: hydrophone-, X-, Y- and Z-sections); (b) residual sections (from top to bottom: hydrophone-, X-, Y- and Z-sections).
face wave associated with ground roll. But the SVD filter requires preprocessing (group-velocity and phase-shift corrections) to flatten the dominant wave before extraction. The two corrections must be accurate. The preprocessing can also be applied to the data in order to select the dispersive wave, using a masking filter in the $f-k$ plane. In the same way, the spectral matrix decomposition filter (SMF) can also be used. However, in this case, only a coarse group-velocity correction is required. Multicomponent data sets allow the use of specific filters for studying and processing dispersive waves observed both on land and marine field data. It has been shown that the SVD filter can be extended to a multicomponent SVD filter (4C-SVD filter). In this case, preprocessing is not required and the results are better than those obtained with the conventional SVD filter. Multicomponent data sets allow the use of polarization filters. It has been demonstrated that the efficiency of conventional polarization filters can be increased by criteria based on high-order statistics (HOS). The results obtained illustrate the advantages of using multicomponent data. The efficiency of the 4C-SVD filter and the high-order statistic polarization filter has been demonstrated. It has been also demonstrated that surface-wave characteristics can be better identified after wave separation.

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REFERENCES


