Abstract—Recent shallow-water experiments in sea channels have been performed using two vertical coplanar densely sampled source and receive arrays. Applying a double-beamforming algorithm on the two arrays both on synthetic numerical simulations and on experimental data sets, we extract efficiently source and receive angles as well as travel times for a large number of acoustic rays that propagate and bounce in the shallow-water waveguide. We then investigate how well sound-speed variations in the waveguide are reconstructed using a ray time-delay tomography based on a Bayesian inversion formulation. We introduce both data and model covariance matrices and we discuss on the synthetic numerical example how to choose the a priori information on the sound-speed covariance matrix. We attribute the partial sound-speed reconstruction to the ray-based tomography and we suggest that finite-frequency effects should be considered as the vertical and horizontal size of the Fresnel zone significantly spreads in the waveguide. Finally, the contribution of a different set of ray angles for tomography goal is also presented.

Index Terms—Array signal processing, shallow-water tomography, underwater acoustics.

I. INTRODUCTION

OCEAN acoustic tomography was introduced by Munk and Wunsch [1] to provide large scale images of the ocean sound-speed fluctuations using low-frequency acoustic waves. Typical parameters of these experiments are a range of hundreds of kilometers, a water depth of a few kilometers, and frequencies around 60 Hz [2]. The method consists in identification and tracking of the different ray arrivals. The variation of the arrival time of these rays is used to solve the inverse problem and estimate sound-speed fluctuations. Results are strongly dependent on: 1) the number of solved ray paths and 2) the uniform spatial coverage of the oceanic waveguide by the ray paths. To identify and use a ray in the tomography process, its arrival time has to be measured accurately but also the ray has to be assigned to a theoretical ray path. A good background model of the environment is then necessary. In practical at-sea experiments in deep or shallow water, an accurate knowledge of the range- and depth-dependent fluctuating environment is not available, and therefore, the number of solved (extracted, separated and identified) ray arrivals is often too small to provide satisfying tomography results. Furthermore, besides experimental limitations, Munk and Wunsch [3] theoretically discussed the number of resolved eigenrays or eigenmodes from a point-to-point deep-water experimental configuration where travel-time separation between low grazing angle rays has to face bandwidth issues. The same analysis was pursued with rays in shallow water showing similar physical limitations of travel-time-based tomography [4]. Indeed, as the scale ratios between propagation range, depth, and wavelength are comparable to large scale experiments, refraction and reflection in shallow-water environment provide multipath propagation as in deep-water configuration. Basically, because of the limited number of resolved rays, shallow-water tomography does not give satisfying results. Furthermore, moored thermistor strings and conductivity–temperature–depth sensors (CTD) bearing gliders are technologies that provide higher resolution and lower variance estimates of the ocean sound speed. However, if all eigenrays between a set of source/receivers could be extracted and identified in shallow water on a kilometer scale, for example, using source and receive arrays, then the unique advantage of the acoustic tomography would be to monitor a slice of the ocean structure in real time providing a spatial–temporal measurement of the dynamics of mixing layers in shallow-water environment.

To avoid the technical limitations of large scale experiments, a series of small scale experiments (ranging from 1 to 8 km, in a 50–100-m-deep waveguide, with a source at \( f = 3 \text{ kHz} \)) have been conducted in the last few years (2003 Focused Acoustic Field (FAF03) [5] and 2005 Focused Acoustic Field (FAF05) [6] experiments carried out around Elba Island, Italy).

Usually, small-scale tomographic experiments have been tested in the ocean between a single source and a receiver or an array of receivers [7]. During FAF03 and FAF05, Roux et al. [5] used a time-reversal experiment data set, which provides signals between two source and receive arrays. They developed a double-beamforming algorithm (simultaneously on the source array and on the receive array), allowing the identification of...
each individual acoustic ray of the multipath signals by its launch angle, receive angle, and arrival time [6].

This double-beamforming approach between the source and receive arrays tackles the problem of resolved/unresolved eigenrays in a point-to-point or point-to-array configuration. Basically, every acoustic ray is now isolated and identified given the diffraction limit of the system that depends on the size, the central frequency, and the bandwidth of the arrays. Moreover, double beamforming naturally increases signal-to-noise ratio (SNR) as an array gain is performed on both source and receive arrays. In their paper [6], Roux et al. discuss the suitability of the observed processed signals after double beamforming as estimators of ray travel times.

Following this approach, we have two complementary goals in this paper. First, in Section II, a complete ray solving algorithm based on double beamforming is presented. Its performance to extract a large number of ray arrivals between two source and receive arrays is discussed, using at-sea data from FAF03 experiment. Then, we investigate deeper our capacity of sound-speed reconstruction on a synthetic data set starting from an experimental configuration equivalent to FAF03. We present the ray-based tomographic approach through a Bayesian formulation and we discuss the limitations of the ray-based inversion with respect to a diffraction-based sensitivity kernel approach [8].

II. TRAVEL TIME TOMOGRAPHY USING DOUBLE-BEAMFORMING ALGORITHM

In the FAF03 experiment, two vertical source and receive arrays are used in a shallow-water acoustic waveguide. A broadband acoustic pulse is emitted for each source in a round robin sequence, and the acoustic response of the waveguide is recorded on the receive array [5]. This procedure lasts a short time compared to the sound-speed temporal fluctuations in the medium. Thus, the propagation medium remains stationary during the acquisition and we consider the recorded data set as the acoustic transfer matrix between all sources and receivers. The data matrix takes the form of a pressure field \( p(t, z_r, z_s) \) recorded at a receiver depth \( z_r \) for an emission at a source depth \( z_s \).

A. Double Beamforming

Invoking spatial reciprocity principle, a time-delay beamforming algorithm (similar to the Slant-Stack used in seismic exploration [9], [10]) is performed in two steps, both on the receive array and the source array. In the first step, beamforming is performed on the receive array for each source. The pressure field \( p(t, z_r, z_s) \) becomes \( p(t, \theta_r, z_s) \), where \( \theta_r \) is the receive angle (Fig. 1(a)). By reciprocity principle [11], [12], \( p(t, \theta_r, z_s) \) can be seen as the pressure field recorded at the source array when directional emission at \( \theta_{s0} \) is produced by the receive array. As a result, the second step consists in applying beamforming on the source array for each \( \theta_r \) leading to \( p(t, \theta_r, \theta_s) \) [Fig. 1(b)], where \( \theta_s \) is the launch angle.

Equation (1) presents the mathematical formulation of this processing called double beamforming, which is illustrated in Fig. 1

\[
p(t, \theta_r, \theta_s) = \frac{1}{N_r N_s} \sum_{i=1}^{N_r} \sum_{j=1}^{N_s} a_{ij} p(t + \tau(\theta_r, z_{ri}) + \tau(\theta_s, z_{sj}), z_{ri}, z_{sj})
\]

where \( z_{ri} \) and \( z_{sj} \) are the receiver and source depths, respectively. \( \tau(\theta, z) \) corresponds to the time delay to be applied to one element of an array at depth \( z \) to beamform in a direction \( \theta \). Coefficients \( a_{ij} \) are the amplitude weights of the array elements to reduce secondary sidelobes of the beamforming process.

If sound speed is uniform \((c(z) = c)\) along the array, plane-wave beamforming is obtained by

\[
\tau(\theta, z) = (z - z_0) \frac{\sin \theta}{c}.
\]  

When the sound-speed profile \( c(z) \) significantly changes over depth along the array, optimal time-delay beamforming is obtained by the turning-point filter [13]

\[
\tau(\theta, z) = \int_{z_0}^{z} \frac{1}{c^2(z)} \frac{c_0^2 \theta^2}{c_{\text{min}}^2} dz.
\]  

In (2) and (3), \( z_0 \) is the chosen reference depth on the array. In (3), \( c_{\text{min}} \) is the minimum sound speed along the array.

Consequently, the double beamforming provides the eigenrays propagating between the reference source at \( z_{s0} \) and the reference receiver at \( z_{r0} \) by their propagation time, receive angle, and launch angle (in the \([t, \theta_r, \theta_s]\) domain). The goal of the double-beamforming process is then to extract from the recorded pressure field data matrix the acoustic contribution of
a given eigenray propagating between the reference source and the reference receiver \([z_{s0}, z_{r0}]\), and defined by its launch/receive angles \([\theta_s, \theta_r]\).

**B. Beamforming Resolution and Subarray Choice**

Our goal is to perform a shallow-water tomography using identified ray paths between sources and receivers. As explained above, double beamforming provides isolated ray arrivals between a reference source/receiver couple \([z_{s0}, z_{r0}]\). To resolve more eigenrays, the propagation between several source/receiver references is analyzed. For each couple \([z_{s0}, z_{r0}]\), the subarrays on which the double beamforming will be performed has to be chosen appropriately. In this paper, choice is conducted by two criteria, justified in this section: 1) the number of sensors used in each subarray should be as large as possible; and 2) the subarrays must be centered on the reference source \(z_{s0}\) and the reference receiver \(z_{r0}\) (see Fig. 2).

As it is well known from array processing, with a uniform sound speed and \(N\)-equally spaced elements without amplitude shading \((a_{ij} = 1, \forall i, j)\), an analytic formulation of the single-beamforming pattern (SB) in the frequency domain [14]–[16] is given by

\[
SB(\theta_{obs}, \theta, \nu) = S(\nu) \frac{\sin \left( \frac{\Pi N d \nu}{c} (\sin \theta_{obs} - \sin \theta) \right)}{\sin \left( \frac{\Pi d \nu}{c} (\sin \theta_{obs} - \sin \theta) \right)} \times \exp \left( j \phi_e(\theta_{obs}, \theta, \nu, z_0) \right)
\]

(4)

where \(S(\nu)\) is the spectrum of the signal, \(d\) is the interelement distance on the array, \(z_0\) is the position of the reference point on the array, and \(\theta\) and \(\theta_{obs}\) are the actual arrival angle of the ray and the estimated one, respectively. In (4), the fraction and the exponential part express the amplitude gain of the beamforming filter and its phase, respectively.

The choice of the largest subarray is due to the constraint of an angle separation of different ray arrivals, and to the array SNR gain provided by large arrays in the double-beamforming algorithm. From (4), the angular resolution of the beamforming is given by

\[
\Delta \theta \approx \frac{\lambda}{N d}
\]

(5)

where \(\lambda = c / \nu\) is the wavelength, and \(L = Nd\) is the length of the array. Because (5) is written in the frequency domain at the central frequency, it remains a good approximation of the broadband resolution. It follows that the angular resolution is directly related to the length \(L\) of the array, implying the use of the largest array possible. In a depth-dependent ocean environment, subarray length is limited by refraction that makes the wavefront arrival incoherent over the whole array. This is not the case in our simulated waveguide where acoustic propagation is dominated by boundary reflections.

To justify that the subarray must be centered, the phase term of (4) has to be analyzed. The phase term at frequency \(\nu\) is given by

\[
\phi_e(\theta_{obs}, \theta, \nu, z_0) = 2\Pi r \frac{1}{c} (\sin \theta - \sin \theta_{obs}) \left( d \frac{N - 1}{2} + z_1 - z_0 \right)
\]

(6)

where \(z_1\) is the depth of the first receiver. It corresponds to a time delay \(\tau_e = \phi_e / (2\Pi \nu)\). If the array is centered on the reference point \(z_0\), then \(z_0 = d (N - 1) / 2 + z_1\) and the time delay \(\tau_e\) is zero. But if this condition is not respected, and if the estimation of the arrival angle is not exactly equal to the actual angle \((\theta_{obs} \neq \theta)\), a time-delayed version of the ray arrival is extracted after beamforming. The consequence of this time-delay error is important in differential time tomography, because the error could be significant compared to the small arrival time variations due to sound-speed fluctuations. To completely avoid this problem, beamforming is always performed with a reference point \(z_0\) at the center of the subarray.

**C. Interest of the Double Beamforming**

Advantages of using beamforming on a receive array (also called single beamforming in the following) have been summarized in [17]. In this section, the advantages of source/receive array configuration are discussed. The analysis of the data in the \([t, \theta_r, \theta_s]\) domain provided by double beamforming, compared to an analysis in the initial \([t, z_r, z_s]\) domain or in the single beamforming \([t, \theta_r, z_s]\) domain, has three principal advantages: a better separation of different ray arrivals; a better identification of ray arrivals to theoretical ray paths; and finally, a higher SNR.

1) Ray Arrival Separation: The ability of double beamforming to separate ray arrivals is analyzed in the \([t, a_r, a_s]\) domain with \(a_r = \sin \theta_r\) and \(a_s = \sin \theta_s\). In this 3-D domain, each ray arrival spans an elliptical volume of the space. The three diameters of this volume are given by \(\Delta t\), \(\Delta \theta_r\), and \(\Delta \theta_s\), which are the time expansion \(\Delta t\) of the signal (related to the bandwidth), the receive angle expansion of the ray arrival \(\Delta \theta_r = \lambda / (N d \nu)\); and the launch angle expansion of the ray arrivals \(\Delta \theta_s = \lambda / (N d \nu)\), respectively. From a geometrical analysis, two ray arrivals defined by \([t_1, a_{r1}, a_{s1}]\) and \([t_2, a_{r2}, a_{s2}]\) are separated (they do not superset at all) if the following condition is respected:

\[
\frac{(t_2 - t_1)^2}{(\Delta t)^2} + \left( \frac{a_{r2} - a_{r1}}{\Delta \theta_r} \right)^2 + \left( \frac{a_{s2} - a_{s1}}{\Delta \theta_s} \right)^2 > 1.
\]

(7)
For the single beamforming, the equivalent condition is

$$\left( \frac{t_2 - t_1}{\Delta t} \right)^2 + \left( \frac{a_{r2} - a_{r1}}{\Delta a_r} \right)^2 > 1$$

(8)

and for the point-to-point configuration

$$\left( \frac{t_2 - t_1}{\Delta t} \right)^2 > 1.$$  

(9)

Equations (7)–(9) show that the separation ability of the double-beamforming process is better than the single-beamforming and the point-to-point one, because the sum of new positive terms on the left-hand side of the inequality always helps in respecting it.

So, from the previous theoretical analysis, the separation of the independent ray arrivals is obtained thanks to the number of discriminant parameters provided by double beamforming. In the $[t_r, a_r, z_s]$ domain, a ray arrival spreads over the whole water column. Indeed, if two different ray paths at a given $z_r$ and $z_s$ do not respect (9), the two ray arrivals produce interferences on the recorded data. A way to partially avoid these interferences is to compute beamforming on the receive array. Indeed, each ray arrival is then localized around its receive angle $\theta_r$ and its travel time $t$. If the two ray paths have the receive angles different enough to respect (8), the two arrivals do not interfere. However, if the receive angles are close, the arrivals cannot be separated (Fig. 3). For example, from the FAF03 data set [5], we extract two ray paths shown in Fig. 3(a) with close travel times and receive angles. In Fig. 3, the black crosses indicate the theoretically predicted travel time/receive angle positions of the two ray paths. The single beamforming gives only one spot, which is actually an interfering spot between two ray arrivals. On the other hand, double beamforming yields separation as launch angles of the two ray paths are very different [Fig. 3(c)].

2) Identification: An obvious advantage of double beamforming is that it provides a robust identification of a ray arrival with its theoretical ray path. In the point-to-point configuration, the only parameter that enables to associate each arrival to a theoretical ray path is its travel time $t$. Single beamforming provides an additional parameter ($\theta_r$) to perform the identification. But ambiguity problem remains when two ray paths have the travel time and the receive angles close to each other. Furthermore, the ambiguity is reinforced, as shown by Roux et al. [6], by the fact that the two parameters may vary with sound-speed fluctuations. Double beamforming is then more robust for this identification task, since it provides one additional parameter ($\theta_r$) that most of the time solves the ambiguity. The interest of double beamforming to identify ray arrival to its theoretical ray path is illustrated in [6].

To illustrate the resolving ability of double beamforming versus point-to-point configuration, we use synthetic numerical simulations in shallow water that will be further described in Section IV. Comparing point-to-point configuration to double-beamforming process on the same source/receive arrays, the number of unresolved rays varies from 1493 to 156, respectively, for a total number of 6171 eigenrays.

Fig. 3. (a) Separation ability of double beamforming is illustrated with two eigenrays extracted from the FAF03 experiment between a source at depth $z_s = 87.3$ m and a receiver at depth $z_r = 39.3$ m. (b) Normalized single-beamforming results performed on the receive array for a source depth $z_s = 87.3$ m and a receive reference $z_r = 39.3$ m. (c) Normalized double beamforming for $z_s = 87.3$ m and $z_r = 39.3$ m. Crosses indicate the ray prediction.
3) **SNR Gain**: SNR gain obtained by double beamforming is higher than the single-beamforming one. In the case of a uniform weighting of all the sensors \((\alpha_{ij} = 1)\) and of spatially white noise, the SNR gain (in decibels) is as follows:

- \(10 \log_{10}(N_r)\) for single beamforming;
- \(10 \log_{10}(N_r N_s)\) for double beamforming;

where \(N_s\) and \(N_r\) are the number of sources and the number of receivers used in the double beamforming, respectively.

**D. From Ray Extraction to Tomography**

Time-delay tomography requires the association of ray arrival times extracted from the data to ray simulated by a propagation model [18]. To perform this association, for each reference source \(z_{0r}\) and reference receiver \(z_{0o}\), a ray is simulated first computed in a reference model. Approximate travel times and source and receive angles are computed for all possible ray paths identified by a ray code [19]. For every ray, double beamforming is performed on the recorded data in the neighborhood (in terms of \([t, \theta_r, \theta_o]\)) of the simulated ray. We choose the angles \((\theta_{rmn}, \theta_{smn})\) if the double-beamforming algorithm exhibits an intensity spot showing an intensity maximum in the processed signals. The time-domain pressure field \(p(t, \theta_{rmn}, \theta_{smn})\) is then associated with the selected ray path.

Obviously, an accurate shallow-water model yields a better match of ray arrivals. However, both the double-beamforming algorithm and the ray tracing code are robust to model mismatch. Furthermore, differential time-delay tomography takes a background of the ocean state at time \(t_0\) and then creates images of the spatial sound-speed fluctuations after \(t_0\). Consequently, we do not need the absolute arrival times of the rays, but only variations of these arrival times, which reduces even more the constraint on the ocean model.

**III. TRAVEL-TIME TOMOGRAPHIC INVERSION: BAYESIAN APPROACH**

Travel-time tomographic inversion is a nonlinear problem when one wants to reconstruct absolute travel times. Up to now, the only successful case is for the depth-dependent investigation [20]. As soon as one wants to consider lateral variations of celerity, we must rely on the travel time delays as the differences between the observed travel times and the travel times in an initial medium [21], which leads to a linear problem [17]. Indeed, considering a reference model for the ocean may help the separation and identification procedure by allowing us a narrow investigation in the double beam space.

For the inversion problem, the depth and range-dependent sound speed is parametrized as the projection of the celerity variations on a global basis of sinusoidal functions. As usually done in ocean acoustic tomography (OAT) [17], the forward problem is expressed through the linear expression

\[
\Delta D = G_0 \Delta M \tag{10}
\]

where \(\Delta D\) represents the time arrival fluctuations between two successive waveguide transfer matrices and \(\Delta M\) represents the celerity variations related to these two waveguides. This system of linear equations defined by the so-called sensitivity matrix \(G_0\), also known as the Fréchet matrix, is built up for the maximum number of well-identified rays.

Bayesian approach for tomography has been largely discussed in seismology [22]–[27] as well as in underwater acoustics [28]–[33]. A reference work on the general inverse problem has been published by Tarantola [34]. In Bayesian approach, the general inverse problem is solved by finding the maximum of the \textit{a posteriori} probability density. The result is called the maximum \textit{a posteriori} (MAP) solution. If uncertainties have Gaussian distributions, the optimization problem is completely defined by second-order statistics: the maximization of the \textit{a posteriori} probability density is equivalent to the minimization of the least squares objective function \(L\) expressed as

\[
L = (\Delta D - G_0 \Delta M)^T C_d^{-1} (\Delta D - G_0 \Delta M) + (\Delta M - \Delta M_{\text{prior}})^T C_m^{-1} (\Delta M - \Delta M_{\text{prior}}) \tag{11}
\]

where the matrix transpose operation is denoted by the superscript \(T\). The covariance matrix for the data space \(C_d\) will be inferred as uncertainties on differential travel time measurements as well as uncertainties related to forward modeling approximations. The covariance matrix for the model space \(C_m\) should be defined as the uncertainties on the \textit{a priori} information that we consider for the tomographic reconstruction \((\Delta M_{\text{prior}})\). Note that the objective function (11) is an adimensional function thanks to the introduction of covariance matrices. The \textit{a priori} differential model \(\Delta M_{\text{prior}}\) will be set to zero as the background oceanic state is expected to be the most probable model. Other choices are possible as an expected low speed zone in a specific area coming from a systematic warming effect for example. The covariance matrix \(C_d = \sigma_d^2 I\) will be taken as a diagonal matrix with a uniform uncertainty for each independent measurement.

We assume that the \textit{a priori} model covariance matrix \(C_{m}\) is a diagonal matrix with Gaussian decrease in spatial frequency. Physically, the decorrelation of different frequency components corresponds to the case where spatial correlation lengths \(\lambda_r\) and \(\lambda_s\) are stationary in the whole medium. The Gaussian decrease corresponds to a Gaussian correlation in the spatial domain. Indeed, for a global uncertainty \(\sigma_m\), the diagonal components of \(C_{m}\) are defined for each of the \([f_{rx}, f_{sj}]\) spatial frequency components as

\[
C_{m \ ij} = \pi R_T D_T \lambda_r \lambda_s \sigma_m^2 \exp \left( - (\pi f_{rx} \lambda_r)^2 + (\pi f_{sj} \lambda_s)^2 \right) \tag{12}
\]

where \(R_T\) and \(D_T\) are the entire range and the entire depth chosen for the construction of the discrete sine and cosine basis. This frequent weighting allows us to stabilize the result of the inversion, diminishing the degrees of freedom of the result (the higher \(\lambda_r\) and \(\lambda_s\) are, the smoother the final result is). On the other hand, this decomposition of the medium of this global base will carry less resolution.

Finally, for the forward modeling through a linear (10), the minimum of the objective function can be obtained analytically [34] by the inverse operator

\[
G_0^d = \left( G_0^d C_d^{-1} G_0 + C_m^{-1} C_{m0} \right)^{-1} C_{m0} \tag{13}
\]

as long as covariance matrices can be inverted. The MAP solution is given by

\[
\Delta M = G_0^d \Delta D + \Delta M_{\text{prior}}. \tag{14}
\]
The correlation lengths $\lambda_r$ and $\lambda_s$ could be provided through the complementary measurements (such as CTD chain measurements) in underwater acoustics as well as the expected uncertainty $\sigma_{\text{m}}$ on model parameters. They can also be estimated as part of the inversion problem as done in a seismological approach [35]. We have selected the second approach here. We compute the inversion several times, using at each time different correlation lengths $\lambda_r$ and $\lambda_s$ and different global uncertainties $\sigma_{\text{m}}$. At each inversion, the normalized objective function of (11) is minimized and values of the minimum given by

$$L_{\lambda_r,\lambda_s\sigma_{\text{m}}} = L_{\text{d}n,\lambda_s} + L_{m,\lambda_r,\lambda_s}$$

$$= (\Delta D - G_0 \Delta M_{\lambda_r,\lambda_s})^T C_d^{-1} (\Delta D - G_0 \Delta M_{\lambda_r,\lambda_s})$$

$$+ (\Delta M_{\lambda_r,\lambda_s} - \Delta M_{\text{prior}})^T C_{\text{m}}^{-1} (\Delta M_{\lambda_r,\lambda_s} - \Delta M_{\text{prior}})$$

(15)

are recorded. In (15), the celerity perturbation obtained by the inverse procedure using $\lambda_r$, $\lambda_s$, and $\sigma_{\text{m}}$, is denoted by $\Delta M_{\lambda_r,\lambda_s}$. Quantities $L_{\text{d}n,\lambda_r,\lambda_s\sigma_{\text{m}}}$ and $L_{m,\lambda_r,\lambda_s\sigma_{\text{m}}}$ are the contributions of the data space and of the model space to the total objective function, respectively. After performing the inversion for various $\lambda_r$, $\lambda_s$, and $\sigma_{\text{m}}$, the correlation lengths and global uncertainty that gives the smallest objective function value ($L_{\lambda_r,\lambda_s\sigma_{\text{m}}}$) are selected and the corresponding result $\Delta M_{\lambda_r,\lambda_s}$ is chosen as a solution for the inverse problem.

IV. SYNTHETIC DATA EXPERIMENT

To assess the different performances of both double beamforming and travel time tomography, we design a numerical experiment where environmental parameters are close to experimental parameters encountered during the FAF03 experiment. We introduce sound-speed fluctuation between two successive acquisitions of the waveguide transfer matrix. Then, we analyze the quality of the estimated sound-speed fluctuation. The goal of this tomographic simulation is to check for the method performance in an ocean environment similar to shallow water—high-frequency ocean experiments.

The range between the two arrays is 1.5 km, the water depth is 50 m, and the source signal has a central frequency of 2.5 kHz, with a 1-kHz frequency bandwidth. Sound speed in the waveguide is uniform (1500 m/s), and the source and receive arrays are made of 32 equally spaced transducers spanning 46.5-m depth in the water column (between 1.5 and 48 m). The distance between the two elements of an array is $d = 1.5$ m to be compared to a 0.6-m central wavelength $\lambda$. Not respecting the spatial Shanon condition of $d < \lambda/2$ leads to aliasing problems that we take into account in the inversion part by giving a smaller reliability to eigenrays subject to aliasing errors. The bottom is made reflective (bottom sound speed $= 2500$ m/s) to obtain the largest number of ray arrivals with significant amplitude, and we do not introduce noise in the simulation.

We perform two simulations that will be used as “real data.” The first one is performed with a uniform sound-speed profile that defines a reference waveguide. Then, we introduce two local sound-speed variations in the second one with a negative $-0.3$-m/s fluctuation and a positive $0.25$-m/s fluctuation [Fig. 5(a)]. A wide angle parabolic equations algorithm is used to provide the real data in the time domain [36]. Our travel-time tomography goal is to estimate the sound-speed variations between the two experiments, using double beamforming to solve the ray arrivals. A spatial—temporal representation of the field received on the receive array is shown in Fig. 4, for a source depth $z_s = 24$ m.

Double beamforming is performed on 11 different source and receive subarrays (so $11 \times 11 = 121$ reference source/receive pairs), placed at $z_{\text{ref}} = [10.5 ; 3 ; 40.5]$ and $z_{\text{ref}} = [10.5 ; 3 ; 40.5]$; 6015 ray arrivals are resolved, which corresponds to 97% of all theoretical ray paths. It provides a complete sensing of the entire waveguide. However, note that for the $[z_{\text{ref}} = 24]$ reference pairs that are close to the waveguide boundaries, the subarray lengths become smaller, and the angular resolution becomes smaller as well. Indeed, the first ray arrivals with close travel time, launch angle, and receive angle cannot be separated, which explains the 3% of unresolved rays.

A. Bayesian Inversion

The generalized inverse matrix is obtained through (13) while the solution is deduced from (14) using ray theory for the estimation of the sensitivity matrix $G_{tr}$ [17]. Forty horizontal frequency components and 20 vertical frequencies components are used to model the celerity variations. As sound-speed fluctuations are modeled on sine and cosine bases, four parameters are used for each frequency ($\sin(\theta), \sin(\phi), \cos(\theta), \cos(\phi)$, and $\cos(\pi/2)$, where subindices $\theta$ and $\phi$ represent the “vertical” and
“horizontal” components, respectively). Indeed, 3200 parameters \((40 \times 20 \times 4)\) are inverted. Uncertainties on travel times are taken as \(\sigma_t = 2 \times 10^{-5}\) s.

The inversion is performed for several \(\lambda_r, \lambda_z\), and \(\sigma_m\) values and the \(L_{\lambda_r, \lambda_z, \sigma_m}\) is recorded. Thanks to the linearized forward problem, the inversion could be performed efficiently as rays are traced only once. We have found that the uncertainty \(\sigma_m = 0.5\) m/s gives the lowest \(L\) value. Similar values for \(L_t, L_m\) show that both data and \textit{a priori} information have contributions in our reconstruction. A cross section \(L_{\lambda_r, \lambda_z}\) is shown in Fig. 6 for our optimal global uncertainty \(\sigma_m = 0.5\) m/s. An increasing value of \(L\) is observed with increasing correlation lengths. Increasing correlation lengths make the model smoother as we remove the short wavelength components of the medium. With these constraints, the data are less fitted and, therefore, the first part of the objective function \(L\) increases. On the other hand, the cost function also increases when correlation lengths become smaller. As we increase the number of degrees of freedom, \(\Delta M\) includes the values with shorter wavelengths, which lead to an increase of second terms of the objective function. Minimal value of the objective function is obtained for \(\lambda_r = 37\) m and \(\lambda_z = 6\) m. The ratio of the vertical correlation length over the entire depth is equal to 10\% while the ratio of the horizontal correlation length over the entire range is equal to 2.5\%. Thus, geometry of acquisition provides a better horizontal resolution than a vertical one.

For the minimum of the objective function (Fig. 6), the celerity variation estimation \(\Delta M_{37\times6}\) is shown in Fig. 5(b). The standard deviation of the data \(\Delta D\) is \(1.6 \times 10^{-6}\) s, and the standard deviation between the data and reconstructed travel time variations \(\Delta D - \Delta D = \Delta D - G_{0} \Delta M_{37\times6}\) is \(1.4 \times 10^{-7}\) s, leading to a reduction down to 8.75\% of the standard deviation of the data. The two sound-speed fluctuations (one positive and one negative) are well localized with a better lateral resolution than a vertical one. However, the recovered fluctuations do not reach the true amplitudes used for computing synthetic data. In fact, we only recover half of the true amplitude in the center of both fluctuations because the reconstructed surface is wider and ghost anomalies are present due to the ray sampling as intrinsic limitations of ray approach [37]. As shown by [38], this partial reconstruction comes from the use of ray theory in the inversion procedure and the fact that the wavefront healing and other diffraction effects are not included in the forward problem.

\section{B. Finite-Frequency Investigation}

The simulated data set has been computed without noise, but the parabolic equation code includes diffraction effects. Double beamforming has been performed with a perfect knowledge of the sound-speed profile leading to optimal travel time measurements. The tomography result [Fig. 5(b)] does not reach the expected sound-speed fluctuations [Fig. 5(a)]: various diffraction effects are not included in the estimation of travel times through ray tracing [38]. Indeed, waves emitted at 2.5 kHz with 1-kHz bandwidth in the 60-m-deep waveguide exhibit features far from those expected by ray theory according to the ratio between the waveguide dimensions and the acoustic wavelength.

These effects can be included in the forward modeling through travel time sensitivity kernels (TSKs) developed by [39] and [40] in seismology and by [8] in underwater acoustics. Very little has been said about the shape of these TSKs when considering double beamforming. In Fig. 7, the TSKs are computed in the waveguide for a 15\(^{\circ}\) ray path when increasing the size of the subarray lengths ranging from point-to-point source/receiver pair [Fig. 7(a)] to a 30-m-long source/receive arrays [Fig. 7(d)] on each side of the waveguide. The point-to-point TSKs clearly show the limit of ray theory in the waveguide since the Fresnel zone associated with TSKs strongly spreads in the water column. As expected, we also notice the travel time zero sensitivity along the ray path that corresponds to the "banana-doughnut" shape of the TSKs. On the other hand, the TSKs of the double-beamforming algorithm show an interesting pattern when the size of the source/receive subarrays increases. Two effects are clearly observed. First, the use of large arrays cancels the TSK sidelobes on each
side of the ray path. Second, the TSKs do not exhibit zero sensitivity on the ray path: the classical “banana-doughnut” shape of the Fresnel zone is changed into a “banana-only” shape when double beamforming is performed on finite-length arrays at the source/receiver locations. As such, the TSKs of the double-beamforming algorithm will allow similar inversion procedure as the one based on the ray theory, albeit diffraction effects will be included.

A good illustration of the Fresnel zone influence on the tomography inversion can be seen with rays with different launch angles. In Fig. 8, tomography inversion is performed with inputs consisting of different fans of rays. Fig. 8(a) is the sound-speed fluctuation reconstructed with travel times corresponding to ray angles smaller than 16° only. Similarly, Fig. 8(b) shows the tomography results for ray angles in the [16°, 29°] interval while Fig. 8(c) is obtained with ray angles larger than 29°. The number of ray paths used for each inversion is similar (~2000) and the only difference comes from the different angle intervals linked to the length of each ray and, therefore, to the size of the Fresnel zone.

A good estimation of the sound-speed fluctuations is obtained using low-angle rays only, but ghosts are now more important in Fig. 8(a) when compared to Fig. 5(b). Celerity perturbation amplitudes are partly reconstructed. When using only higher angle ray paths [Fig. 8(b) and (c)], the tomography result is degraded both in shape and amplitude. It appears then that, using ray theory as a forward model, the main part of travel time information is carried out by low angles, while higher angles only help improve the result in amplitude and spatial resolution. The tomography resolution is directly related to the length of the first Fresnel zone [37] equal to $\sqrt{L}$ [37] where $L$ is the ray path length. In our case, high angles correspond to long ray paths while small angles correspond to short ray paths. Then, small angles lead to a better resolution than high angles. The fact that low angles provide a good tomography result is also good news for shallow-water oceanic environments where a 16° critical angle corresponds to a realistic 1560-m/s bottom sound speed.

A singular value decomposition (SVD) analysis of the sensitive matrix $G_0$, for the three angle sets shown in Fig. 8. The solid, dashed, and dashed–dotted lines correspond to small, intermediate, and high launch angles, respectively.

Fig. 9. Singular values of the SVD decompositions of the sensitive matrix $G_0$, for the three angle sets shown in Fig. 8. The solid, dashed, and dashed–dotted lines correspond to small, intermediate, and high launch angles, respectively.

V. CONCLUSION

Performance of shallow-water acoustic tomography has been analyzed using a ray identification algorithm based on double-beamforming processing. This array processing helps in the separation and the identification of the maximum possible number of acoustic rays, thanks to their source/receive angles and travel times between the source array and the receive array in a waveguide. Performing double beamforming on several subarrays permits the extraction of a large number of ray-like arrivals and their travel time measurements. From these ray paths, a uniform coverage of the waveguide by the ray paths is obtained, from which sound-speed fluctuations are reconstructed using a ray-based direct tomography model.

In the shallow-water configuration, ray-based travel time tomography only reconstructs half of the amplitude as diffraction effects (wavefront healings, for example) are not included. Diffraction effects computed through the shape of the TSKs in the waveguide confirm the limitation of ray-based tomography inversion. This influence of the Fresnel zone could also be observed through a selection of launch angles. We have shown that low angles are sufficient to provide satisfactory tomography results while high angles provide locally better range/depth resolution.

From these benchmark results, future work in shallow-water tomography will follow two principal directions. The first task will be to couple double beamforming and a direct model that will use appropriate sensitivity kernels and diffraction-based forward modeling to go beyond the limits of ray approximation and the first Fresnel zone influence. The second goal will be
to perform double beamforming and tomography inversion on the experimental data, which will introduce several challenging difficulties such as complexity of depth-dependent sound-speed profiles and scattering phenomena produced by surface waves.

REFERENCES


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