EXTENDING MATHEMATICAL MORPHOLOGY
TO COLOR IMAGE PROCESSING

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ABSTRACT

This paper addresses the problem of the extension of morphological operators to the case of color images. Two main strategies are proposed: the marginal strategy and the vectorial one, which basically comes to the definition of a well suited vector ordering relation. We briefly discuss the possible extensions found in the literature and propose an ordering scheme based on the bit mixing paradigm. Results concerning the different extensions of basic morphological operators are presented and compared.

Keywords: vectorial ordering, color image processing, multivalued morphology.

1. INTRODUCTION

Mathematical morphology theory has now become a widely used non linear technique for image processing. Based on set theory, mathematical morphology definition requires an algebraic structure T (complete lattice) such that:

• T is induced by a (partial) ordering relation “≤”,
• for any family of elements in T, there exists a smallest majorant called the sup (for supremum) and a greatest minorant called the inf (for infimum).

In the case of color images, these two properties are generally missing because there is no natural means for total ordering of multivariate pixels. Then, two main strategies are proposed:

• the first one consists to process each component separately (FIG. 1-a). It is called the Marginal approach or Component-wise approach.
• the second one consists of using a purely vector approach (FIG. 1-b) that processes the different color components all at once. But this strategy requires the definition of a well suited vector lattice structure to work in and, in particular, a vector ordering relation to define the sup and the inf of any family of N-dimensional vectors.

FIG. 1 : The two main strategies to process color images

The definition of a vector ordering relation can be done in different ways:

• by using the ordering relations defined for the vector order filters. Indeed, many extensions of the scalar order filters have been proposed for color images, requiring the use of an ordering relation between color vector pixels. Very often, this order is based on the computation of a distance or a cumulative distance between pixels within the filtering window, yielding the definition of a great diversity of vector order filters [8], [1], [9], [5], [6]. Of course, this approach can also be used to define color morphology. However the obtained vector ordering relation is generally a pre-order relation (antisymmetry is not verified).

• by using a real total order in the color space. Only a few approaches apply this strategy[3], [10], [7].

The aim of this paper is to give a few ideas and comparisons about these different strategies. It is composed of two parts:

• a presentation and analysis of the different strategies,
• the application to some basic morphological operators.
2. PRESENTATION AND ANALYSIS OF THE DIFFERENT STRATEGIES

2.1. Marginal Morphology

Each component of the color image is processed separately. This approach is not fully satisfying for different reasons:

- it does not take into account the inter-component correlation,
- if the image has N different components, N processings are required, leading to time consuming operators,
- the output vector will rarely be one of the input vectors, which yields false color appearance. However, this drawback can be reduced by processing the color image in a specific space and by preserving Hue information: the following scheme gives such a solution. In a Hue, Saturation, Intensity basis, Hue and eventually Saturation are not processed.

![Partial Marginal Strategy in a Hue-Saturation-Intensity Space](image)

Nevertheless, it is preferable to use a vector approach, processing the image all at once. But we then have to define a vector ordering relation to get the sup and the inf of any family of color pixels.

2.2. Vector Morphology based on a distance measure

In vector order filtering, color pixels are generally ordered according to one of the two following ways.

2.2.1. Order with a cumulative distance

Let \( x_1, ..., x_n \) be the color pixels within the filtering window \( W \). If \( \text{dist}(x_i, x_j) \) is a measure of dissimilarity between the two pixels \( x_i \) and \( x_j \), the scalar quantity,

\[
\sum_{j=1}^{n} \text{dist}(x_i, x_j)
\]

is associated to each color pixel \( x_i \). This distance measures the global distance between the pixel \( x_i \) and its neighbours. Then, ordering the \( d_i \)'s implies the same ordering to the corresponding \( x_i \)'s. The functions \( \text{dist} \) which are commonly used are the euclidean distance, absolute distance, angular distance, ... .

So, the sup and the inf of n color pixels \( x_i \) are defined as:

\[
\text{sup}(x_i) = x_{\text{sup}} \text{ so that } d_{x_{\text{sup}}} \geq d_i \quad \forall i = 1, n
\]

\[
\text{inf}(x_i) = x_{\text{inf}} \text{ so that } d_{x_{\text{inf}}} \leq d_i \quad \forall i = 1, n
\]

Then, it is easy to determine basic morphologic operators such as erosion, dilation, opening, closing, ... . However, three main problems arise from these definitions:

- First, the color pixel \( x_{\text{inf}} \) is nothing else than the median of the \( n \) pixels \( x_i \), according to the classical vector median definition [9]. So, it has not the significance which has to be linked to the concept of “lowest pixel”. Therefore, an erosion and a vector median filtering are equivalent.

- Secondly, as the induced order is only a pre-order (antissymetry is not verified), the color pixel \( x_{\text{inf}} \) and \( x_{\text{sup}} \) are not defined in a unique way.

- Finally, whereas the definition of the inf is quite stable, the definition of the sup is highly unstable: a little difference between the \( n \) color pixels can induce a very important change in the \( x_{\text{sup}} \) definition. The following figure (FIG. 3) exhibits this problem in the case of a color space limited to the two Red and Green components. A little modification of the value of one of the color pixels (denoted \( \times \) in the figure) induces a very important change in the \( x_{\text{sup}} \) definition.

![Sensitivity in the \( x_{\text{sup}} \) definition with the use of a cumulative distance](image)

2.2.2. Reduced ordering

The color pixels are now ordered according to their distance to a measure centre \( a \) [2]. The scalar value asso-
associated to each pixel \(x_i\) within the filtering window \(W\) is now \(d_i = \text{dist}(x_i, a)\), and ordering the \(d_i\)'s implies the ordering on the color pixels \(x_i\)'s.

Of course, the resulting ordering relation is depending of the choice of the measure centre \(a\). If \(a\) is the average vector or the median vector of the color pixels \(x_i\), the situation is very close to the situation obtained with a cumulative distance (§ 2.2.1), and the definitions of the \(\sup\) and the \(\inf\) involve the same problems. If the centre is the origin of the color space, despite the order is always a pre-order, the meanings associated to the definition of the \(\sup\) and the \(\inf\) are relevant.

Using the previous example, it can now be noted (FIG. 4) that the determination of the \(\sup\) and the \(\inf\) are more robust and relevant than with the use of a cumulative distance.

![FIG. 4: Sensitivity in the \(x_{\sup}\) definition with the use of a reduced order](image)

The extension of basic morphologic operators such as erosion, dilation, opening, closing, ... can also be performed according to these definitions of the \(\sup\) and \(\inf\) [4].

### 2.3. Vector Morphology with a total order

The only way to avoid the previous difficulties is to define a **total order** on the three-dimensional color space \(\mathbb{R}^3\). This can be done by defining a **bijective** transform \(h\):

\[
h : \mathbb{R}^3 \rightarrow \mathbb{R}
\]

\[
x \rightarrow h(x)
\]

Then, the order in the color space is defined by the natural order on \(\mathbb{R}\):

\[
\forall (x_i, x_j) \in \mathbb{R}^3 \times \mathbb{R}^3, x_i \leq x_j \Leftrightarrow h(x_i) \leq h(x_j).
\]

As \(h\) is bijective, the \(\sup\) and the \(\inf\) are now uniquely defined:

\[
x_{\inf} = h^{-1}[\text{minimum} (h(x_i))] \quad (\text{Eq. 1})
\]

\[
x_{\sup} = h^{-1}[\text{maximum} (h(x_i))] \quad (\text{Eq. 2})
\]

In [3], a transform \(h\), based on a bit-mixing paradigm is proposed. The principle is to mix the 8 bits of the three R-G-B components to get a 24-bit scalar data. FIG. 5 presents the coding and the decoding of such a color pixel.

![FIG. 5: Coding \((h)\) and decoding \((h^{-1})\) of one color pixel](image)

With such a transform, the \(\sup\) and the \(\inf\) fit well the concept of “greatest pixel” and “lowest pixel”. This transform \(h\) has been chosen to minimize the unavoidable asymmetry inherent to this kind of coding: approximately the same weight is attached to each component of the image. This can be expressed in terms of “space filling curve”. A space filling curve [11] is a curve that goes through each point of the space one single time. It gives a mono-dimensional representation of a multi-dimensional space. Therefore, there is a double equivalence [3]:

\[
\text{total order on } \mathbb{R}^3 \leftrightarrow \text{bijective application } h : \mathbb{R}^3 \rightarrow \mathbb{R}
\]

\[
\text{space filling curve in } \mathbb{R}^3
\]

Such curves are presented on in the case of a bi-dimensional space (Red-Green for instance) where each component is coded on 4 bits (from 0 to 15). Regazzoni used the curve shown on the figure 6-a to propose a vector median filter [10] and to extend morphological operators to color images [12]. Figure 6-b shows the curve associated with the classical lexicographic order. The curve corresponding to the order induced by the bit mixing paradigm is present-
Then, the corresponding color morphologic operators are the grey-level morphologic operators defined according to (Eq. 1) and (Eq. 2) and following the scheme represented below (FIG. 7):

FIG. 7 : Synopsis of a Processing based on a total order.

3. APPLICATION TO SOME BASIC MORPHOLOGICAL OPERATORS

The different strategies are illustrated with two basic operators: the opening and the closing with a flat square structuring element of size 7. This size is voluntarily important to reveal the behaviour of the different methods. The image used to illustrate these behaviours is a painting: “The shout” - Edvard Munch - 1893 - National Gallery - Oslo - FIG. 8-a.

3.1. Marginal approach

Four experiments have been performed. The differences between these experiments come from the color space which is used and from the components which are processed. The first one processes the three RGB components. Secondly, the same marginal strategy is applied in a Hue-Saturation-Intensity space. Thirdly, the marginal processing is only applied on the Saturation and Intensity components. Finally, only the Luminance is processed.

Of course, the four experiments have the same global behavior: the opening tends to eliminate the bright little structures, whereas the closing tends to eliminate the dark little structures. However, some differences between these three results are noticeable.

With the RGB components, the results are satisfying (FIG. 8-b & c), but some false colors may appear. This drawback is shown on the opening result (zoom FIG. 8-b).

It must be noted that the appearance of false colors is not specific to the opening processing. It only depends on the color configuration of the pixels.

Using a Hue-Saturation-Intensity space (FIG. 9) exhibits more false colors, specifically with the closing result (FIG. 9-b).

FIG. 8 : RGB-Marginal color morphology

FIG. 9 : HSI-Marginal color morphology

If the processing is restricted to the Saturation and Intensity (FIG. 10), the false colors appearance is strongly
limited, but still existing. To eliminate the false colors appearance, only the Luminance has to be processed (FIG. 11), but only one component is processed which gives a lower filtering effect.

FIG. 10 : Partial (SI)-Marginal color morphology

FIG. 11 : Partial (I)-Marginal color morphology

3.2. Vector approach

Three experiments have been performed. The first one uses an ordering relation based on a cumulative distance. The second one uses the reduced ordering with the origin as measure centre. The last one is based on the bit mixing paradigm.

Using a cumulative distance (FIG. 12) provides bad results. This is due to the dilation processing which uses an unstable $sup$ definition.

The results obtained with the reduced ordering (euclidian distance) (FIG. 13) and with the bit mixing paradigm (FIG. 14) are quite similar. Thanks to the fact that the output of any processing is always one of the input pixels, no false colors are produced (zoom FIG. 14-b). The counterpart of this property is a slight block effect.

It could be seen that the results obtained with the method proposed by Regazzoni are quite similar to the reduced ordering ones. In fact, the Regazzoni space filling curve is an attempt to approximate the euclidean distance used in the reduced ordering.

FIG. 12 : Color morphology with a cumulative distance

FIG. 13 : Color morphology with the reduced ordering

FIG. 14 : Color morphology with the bit mixing paradigm

The results obtained with the reduced order approach and the bit-mixing approach are compared on a real image (“alloy image” provided by Ugine Savoie company). The
The vector ordering can be a pre-order (reduced ordering) or a total order (bit mixing). Vector ordering based on a cumulative distance, as it is used to extend order filters, is not workable. The performances which are obtained are similar, with a lower complexity for the bit-mixing technique (the processings is a scalar one).

FIG. 15: Color closing on “alloy” image

It can be seen (zoom) that the frontiers of the cells are better defined with the bit-mixing approach. This is probably the consequence of the non-uniqueness of the $\sup$ and $\inf$ definition with the reduced ordering.

4. CONCLUSION

In this paper, different strategies performing color morphology have been investigated.

Generally, marginal approaches induce new (false) colors which can be damaging for the further processing (segmentation processing for example) and which modify the visual perception of the image. The only solution to strictly preserve the Hue information is to use a vector approach which require the definition of a color pixel ordering.

REFERENCES


