Shape signatures of fuzzy star-shaped sets based on distance from the centroid

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Abstract

We extend the shape signature based on the distance of the boundary points from the shape centroid, to the case of fuzzy sets. The analysis of the transition from crisp to fuzzy shape descriptor is first given in the continuous case. This is followed by a study of the specific issues induced by the discrete representation of the objects in a computer.

We analyze two methods for calculating the signature of a fuzzy shape, derived from two ways of defining a fuzzy set: first, by its membership function, and second, as a stack of its $\alpha$-cuts. The first approach is based on measuring the length of a fuzzy straight line by integration of the fuzzy membership function, while in the second one we use averaging of the shape signatures obtained for the individual $\alpha$-cuts of the fuzzy set. The two methods, equivalent in the continuous case for the studied class of fuzzy shapes, produce different results when adjusted to the discrete case. A statistical study, aiming at characterizing the performances of each method in the discrete case, is done. Both methods are shown to provide more precise descriptions than their corresponding crisp versions. The second method (based on averaged Euclidean distance over the $\alpha$-cuts) outperforms the others.

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1. Introduction

For the purpose of pattern recognition, a quantitative shape analysis is often performed. One example of application is in the area of content based image retrieval useful in multimedia databases (Yoshitaka and Ichikawa, 1999; Zhang, 1999).
Shape properties should be chosen in a way so that they capture essential differences between objects, while maintaining invariance to position, size, and rotation. Numerous properties are available to represent and describe the shape of objects in binary images (Gonzalez and Woods, 2002).

The traditional way of addressing a pattern recognition problem consists in, first, performing a crisp segmentation of the image into object and background, and then describing the segmented object. However, this approach induces an irreversible loss of data. This is illustrated by Fig. 1: a crisp continuous disk, whose border corresponds to the superimposed white circle, is digitized at a low resolution. It gives the fuzzy object presented on the left picture. When such an image is segmented to get a crisp object (in this case, this is achieved with a threshold at intensity level 0.5), important information is removed. The crisp segmentation has a negative effect on the visual appearance of the object as well: the binary “disk” in the right picture has lost much of its visual roundness as compared to the fuzzy pre-segmented object.

Fuzzy segmentation methods have been developed in order to reduce the negative effects of this crisp representation (Udupa and Samarasekera, 1996). In real applications, fuzziness of the studied objects can arise from various reasons, such as limited acquisition conditions, as illustrated by Fig. 1, but also as an intrinsic property of the object of study, which may have uncertain and/or imprecise borders.

Hence, to reduce the described loss of information, and achieve an improved precision, shape analysis should be performed directly on the grey-level image or on its corresponding fuzzy segmented image and not on a crisply segmented object. However, discrete fuzzy shape analysis methods (Rosenfeld, 1979, 1998; Pal and Rosenfeld, 1991) are far less developed and their use is rather limited to date, compared to the (considerable) analogous work in the crisp case.

Recently developed methods for estimating some basic quantitative shape properties, i.e., measurements of area, perimeter, and compactness \( P^2/A \), (Sladoje et al., 2003), produce more precise results for a fuzzy segmentation, than for a crisp one. This encourages development of other shape descriptors for fuzzy discrete sets, with the aim to obtain a description of the shape that is invariant to translation within the digitization grid.

In this paper, our approach is to combine boundary and region information, in order to design a fuzzy shape descriptor based on a one-dimensional (1D) functional representation of the two-dimensional (2D) shape boundary. This kind of descriptor is classically called a signature of the shape (Gonzalez and Woods, 2002). The simplest way to generate a signature is to traverse the boundary and plot the distance from the centroid to the boundary, as a function of the angle. We extend this shape signature function to the case of fuzzy shapes.

In Section 2, some preliminary definitions are recalled and the frame of our study is delimited. In Section 3, the case of continuous objects is addressed: the shape signature we are interested in is presented, and two extensions to the fuzzy case are proposed. In Section 4, some specific issues induced by the discretization are discussed. Their practical consequences on the discrete version of the two proposed shape signatures are presented in Section 5. Section 6 presents the results comparing both methods and enlightening the interest of working with fuzzy objects. Section 7 concludes the study.

2. Definitions and assumptions

In this section, we recall some definitions that are used in the paper. We also give two assumptions delimiting our study.
Definition 1 (Zadeh, 1965). A fuzzy subset $S$ of a reference set $X$ is a set of ordered pairs $S = \{(x, \mu_S(x)) | x \in X\}$, where $\mu_S : X \rightarrow [0, 1]$ is the membership function of $S$ in $X$.

Definition 2 (Rosenfeld, 1984). For $\alpha \in [0, 1]$, the $\alpha$-cut of $S$, defined by $\mu_S$, is the crisp set $S_\alpha = \{x \in S | \mu_S(x) \geq \alpha\}$.

A fuzzy set can equivalently be interpreted as a stack of its $\alpha$-cuts. If $S$ is given by its $\alpha$-cuts $S_\alpha$, for $\alpha \in [0, 1]$, its membership function $\mu_S$ can be reconstructed by using the fuzzification principle

$$\mu_S(x) = \int_0^1 S_\alpha(x) \, d\alpha.$$ 

Definition 3 (Rosenfeld, 1984). The support of a fuzzy set $S$ is

$$\text{Supp}(S) = \{x \in X | \mu_S(x) > 0\}.$$ 

Definition 4 (Dubois and Prade, 1998). The coordinates $x_c$, $y_c$ of the centroid $X_c$ of a fuzzy set are defined as the averaged coordinates of the points of the fuzzy set weighted by their membership values,

$$x_c = \frac{\int x \cdot \mu(x,y) \, dx \, dy}{\int \mu(x,y) \, dx \, dy},$$

$$y_c = \frac{\int y \cdot \mu(x,y) \, dx \, dy}{\int \mu(x,y) \, dx \, dy}.$$ 

Definition 5 (Diamond, 1990). A set $K \subset \mathbb{R}^n$ is star-shaped with respect to a point $x \in K$ if for each point $y \in K$, the line segment connecting $x$ and $y$ is contained in $K$.

Definition 6 (Diamond, 1990). The kernel $\text{ker} K$ of a set $K$ is the set of all points $x \in K$ such that for each point $y \in K$ a line segment connecting $x$ and $y$ is a subset of $K$.

Definition 7 (Diamond, 1990). A fuzzy set $S$ is fuzzy star-shaped with respect to $y$ if all its $\alpha$-cuts are star-shaped with respect to $y$.

Definition 8 (Diamond, 1990). The kernel of a fuzzy star-shaped set is the intersection of the kernels of all its $\alpha$-cuts.

Note: Definition 8 corresponds to a kernel of a fuzzy set defined as a crisp set. Fuzzy kernel of a fuzzy star-shaped set is not used in this paper; definition can be found in, e.g., (Diamond, 1990).

Definition 9. The core of a fuzzy set is the (crisp) set containing the points with the largest membership value. It is the highest non-empty $\alpha$-cut.

Note: The kernel of a fuzzy star-shaped set is always included in its core, but the reverse inclusion does not necessarily hold; as an example, observe that the core of a crisp non-convex star-shaped object is equal to the set itself, while its kernel is a proper subset of the set.

Assumption 1. We assume that the considered fuzzy set is fuzzy star-shaped.

Assumption 2. We assume that the centroid of the considered fuzzy set is included in its kernel.

The presented work is limited to the above mentioned assumptions.

3. Shape signature for continuous objects

One of the classical shape representation techniques consists in describing the studied object by a shape signature, which is a 1D-function representing a 2D-shape (Fig. 2). Conventional Fourier descriptors can be obtained by applying the Fourier transform to the shape signature. Some examples of shape signatures are listed in (Kindratenko, 2003; Zhang, 2002).

In this section, we analyse a signature of a continuous shape. Starting from the definition of a shape signature commonly used for crisp objects, we propose possible extensions of this signature for fuzzy continuous objects.

3.1. Shape signature for crisp objects

A commonly used signature is the centroid distance function, $CD = CD(t)$, corresponding to the...
Euclidean distance between each boundary point \( X(t) = (x(t), y(t)) \) and the centroid \( X_c = (x_c, y_c) \) of the shape:

\[
CD(t) = \sqrt{(x(t) - x_c)^2 + (y(t) - y_c)^2}.
\]  

(1)

Fig. 2 presents a binary shape and the corresponding signature.

Our focus is on this shape signature function, and its extension to fuzzy objects.

3.2. Shape signatures for fuzzy objects

The two given interpretations of a fuzzy set lead to two approaches in defining fuzzy operations by generalizing their binary counterparts. If a fuzzy set is understood as a function, binary operators are generalized using their functional counterparts; if it is considered as a stack of binary sets, fuzzy operations are obtained by “stacking” binary operators, i.e., by applying the fuzzification principle. We consider both approaches.

The shape signature based on Eq. (1) relies on the boundary points of the object. In the case of a fuzzy object, boundary points are not strictly defined; there is a progressive transition of the membership values from the background to the core. Therefore, some adjustments are needed to provide a suitable description in the fuzzy case. In the following, two equivalent extensions are presented.

3.2.1. Radial integral of the membership function

In our first approach, the fuzzy set is defined in terms of its membership function. We assume that the observed fuzzy set has a bounded support, i.e., \( \mu_S = 0 \) outside a bounded region. To derive the centroid distance shape signature, we need the centroid, the boundary, and a (pseudo-)distance function. The proposed extension addresses these notions as follows:

- The centroid of the fuzzy shape is computed according to Definition 4.
- The boundary points of a fuzzy shape are taken to be the boundary points of the support of the fuzzy set.
- The Euclidean distance between a boundary point and the centroid is replaced by the integral of the membership function along the corresponding straight (spatial) path between them. If this path is parametrized by \( \rho = \rho(t) \), we get:

\[
CD_{fuzzy1}(t) = \int_{X_c}^{X_t} \mu_S(x(\rho), y(\rho)) \, d\rho.
\]

Since a crisp set can be seen as a special case of a fuzzy set, Eqs. (1) and (2) should produce equal results when applied to a crisp set. This is guaranteed by Assumptions 1 and 2. Otherwise, the straight path between \( X_c \) and \( X_t \) is not necessarily included in the shape; the external part of the path is taken into account by Eq. (1), but not by Eq. (2).

3.2.2. Average signature obtained from the \( \alpha \)-cuts

In our second approach, the fuzzy set is defined in terms of the stack of its \( \alpha \)-cuts. For each \( \alpha \)-cut, a signature \( CD_\alpha = CD_\alpha(t) \) is calculated by using Eq. (1). The signature for the fuzzy shape is obtained by averaging the results over \( \alpha \). Since the considered set is fuzzy star-shaped, all the boundaries...
of its $z$-cuts can be jointly indexed by the same parameter $t$ in a way that, for any given $t$, the boundary points, indexed by $t$, in all $z$-cuts, lie on a straight line segment. If that line segment is parametrized by $\rho = \rho(t)$, the signature of a fuzzy set is, according to the fuzzification principle,

$$CD_{\text{fuzzy}2}(t) = \int_0^1 CD_z(t) \, dz.$$  

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Under Assumptions 1 and 2, the membership function $\mu(\rho)$ is monotonous for any $t$, hence it is bijective, i.e., its inverse function exists. It is easy to notice that, for each $t$, $z \rightarrow CD_z(t)$ is an inverse mapping of the membership function along $\rho = \rho(t)$.

Consequently, using intuitive simplified notations, the following equality holds:

$$\int_{X_c} \mu(\rho) \, d\rho = \int_0^1 \rho_{\mu}(t) \, d\mu.$$  

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Eqs. (2) and (3) being equivalent, the two proposed methods are equivalent:

$$CD_{\text{fuzzy}1}(t) = CD_{\text{fuzzy}2}(t).$$  

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4. Specific issues induced by the discretization

In the previous section, we presented methods for the shape signature computation, designed for the continuous case. However, data analysed in practical cases are discrete. Our main goal is to propose shape signature computation methods for discrete fuzzy shapes. That requires adjustments regarding the definition of the centroid, the tracking of a boundary, and the approximation of a straight line.

4.1. Computation of the centroid

In the case of a discrete shape, the definition of the centroid can be derived from Definition 4 in the following straightforward way:

**Definition 10.** The coordinates $x_c$, $y_c$ of the centroid $X_c$ of a discrete fuzzy set are the averaged coordinates of the pixels in the image, weighted by their membership values,

$$x_c = \frac{\sum \sum f \cdot \mu(x,y)}{\sum \sum \mu(x,y)}$$

$$y_c = \frac{\sum \sum g \cdot \mu(x,y)}{\sum \sum \mu(x,y)}.$$  

Using 10 or 10' has different influence on the shape description. In particular, being less precise, the centroid coordinates obtained by Definition 10' may lead to unbalanced shape descriptions, as illustrated in Fig. 3, where we consider a digitized crisp disk. The integer-valued centroid (Definition 10') is the black pixel indicated inside the object. An effect is visible in the shape signature computed as the Euclidean distance between the centroid and the boundary pixels. The distance (ideally, a constant value equal to the radius of the disk) is either over- or under-estimated. Beyond the variations induced by the discrete representation of the boundary, additional perturbation is clearly visible. Since the coordinates of the centroid used to compute the distance to the border are rounded, this distance is successively over-(case of the top left part of the object in the presented example) and under-(bottom right part) estimated. In the worse case, the global range of this perturbation has a $\sqrt{2}$ magnitude, corresponding to the length of one pixel diagonal.

4.2. Boundary tracking

In order to follow the inner boundary of a binary discrete object, we use an algorithm that can be found, e.g., in (Sonka and Hlavac, 1999). We modify it only regarding the choice of the starting point; instead of taking the first pixel of the object reached when the image is scanned line by line, starting from the top left corner, we take the right-most object point in the horizontal direction from the centroid. This approach provides meaningful alignment of the signatures, later in the averaging process. When the starting point is determined, the contour is tracked iteratively, pixel by pixel. Two different versions of this algorithm
can be implemented, depending whether the 8-connectivity or the 4-connectivity is used to define the object. The difference between the two versions is illustrated by Fig. 4. In this figure, we consider a simple crisp disk (center = (10, 10), radius = 5) and the boundaries respectively obtained using the 4- and the 8-connectivity. The corresponding shape signature, in terms of Euclidean distance between the centroid and the boundary pixels is respectively shown on figures (b) and (d). From these curves, we can make the following comments:

- some more “inner” pixels are added to the 8-connectivity contour-pixels by the 4-connectivity representation, leading to a thicker contour (since only isothetic moves are allowed).
- one consequence is that the 4-connectivity boundary is “longer” (in terms of number of pixels); the corresponding signature has more samples.
- another consequence is that the 4-connectivity signature is more noisy (see Fig. 4, where the signatures are represented with the same scale for a better visual inspection); the distances to centroid of the most “inner” contour pixels are under-estimated (see the “negative” peaks on figure (b)).

To give a quantitative evaluation of this phenomenon, we computed a signal to noise ratio (SNR). Assuming that the ideal signature would be a constant equal to the true radius of a disk, the SNR is defined as:

$$\text{SNR} = 10 \cdot \log_{10} \left( \frac{\text{Radius}^2}{\frac{1}{N_{\text{pix}}} \sum_i (\text{signature}(i) - \text{Radius})^2} \right) \text{ dB}$$  (6)

where $N_{\text{pix}}$ is the number of pixels constituting the boundary and the signature is indexed by $i$ ranging

![Fig. 3. Influence of the definition of the centroid. (a) Original discrete disk and its integer-valued centroid. (b) Shape signature using Definition 10’. (c) Shape signature using Definition 10.](image)

![Fig. 4. Influence of the choice of 4- or 8-connectivity. Boundary and shape signature: (a) and (b) for 4-connectivity, (c) and (d) for 8-connectivity.](image)
from 0 to $N_{\text{pix}} - 1$. We obtain the following results:

- using 4-connectivity: SNR = 19.66 dB,
- using 8-connectivity: SNR = 23.97 dB.

As a consequence, in the following, we use 8-connectivity for the boundaries, unless otherwise stated.

4.3. Discretization of a straight line

The definition of a discrete straight line is a classical problem in the field of discrete geometry, (Rosenfeld, 1974). The standard grid-intersection digitization scheme (Freeman, 1961), produces an 8-connected digital line; given a coordinate grid superimposed on the curve, then whenever the curve crosses a grid line, the grid point nearest to the crossing becomes the point of the curve's digitization.

We apply this digitization scheme to each of the line segments connecting the centroid of a shape with the points on the shape boundary, and thus we obtain corresponding digital line segments.

5. Shape signature for discrete fuzzy shapes

Similarly as in the continuous case, an extension of the shape signature for the discrete fuzzy shape can be derived in two ways, depending on the interpretation of a fuzzy set.

For each of them, we propose an algorithm for computing discrete shape signature for a fuzzy set, taking into account the effect of the discretization. In the discrete case, the two approaches lead to different results, which is opposite from their behaviour in the continuous case.

5.1. Method 1: Radial summation of the membership values

The discrete version of the method described in Section 3.2.1 for the continuous case consists in the following steps:

- Compute the centroid coordinates. Since this method works pixel-wise, Definition 10’ is used, with the previously mentioned drawback.
- Detect the inner boundary of the lowest $\alpha$-cut using 8-connectivity.
- Compute the signature using length estimation based on local steps for the corresponding discrete straight line.

The signature of a discrete fuzzy shape, calculated using the (pseudo)-distance between the boundary points and the centroid, is given by:

$$CD_{\text{fuzzy discrete}}(k) = \mu(X_c) + \sum_{j=1}^{N_k} \gamma_k(j) \cdot \mu(x_k(j), y_k(j)), \quad (7)$$

where $X_k = (x_k, y_k)$ is the $k$th pixel of the shape boundary, $(x_k(j), y_k(j))$ for $j = 0, \ldots, N_k$ are the pixels belonging to the digital line segment connecting $X_c$ and $X_k$, and $\gamma_k(j)$ is the (spatial) length of a step between the $j$th and the $(j - 1)$th pixel, with $X_c$ corresponding to $j = 0$. The values 0.948, and 1.343, are taken for an isothetic move, and for a diagonal move along the path, respectively (Dorst and Smeulders, 1987).

Eq. (7) leads to an over-estimated signature. One way to reduce the over-estimation is to weight the membership value of the centroid pixel and to share its contribution between the different directions, but for the sake of simplicity, this is not analyzed here.

Since Eq. (7) is a discrete counterpart of Eq. (2), other approaches to estimate the integral in Eq. (2) can be applied, as well. They may produce different results mostly depending on how the continuous membership function is approximated in the discrete case (e.g., interpolation, or spline-functions can be used for that). The approach we suggest is chosen because of its simplicity, and relatively good estimation result.

5.2. Method 2: Average signature obtained from the $\alpha$-cuts

The discrete version of the method described in Section 3.2.2 for the continuous case, consists in the following steps:
• Compute the centroid coordinates using Definition 10.
• For each $x$ (if the data are quantized using 8 bits per pixel, the total number of $x$-cuts is $\gamma_{\text{total}} = 255$).
  – Compute the corresponding $x$-cut.
  – Detect the inner boundary of the $x$-cut.
  – Compute the shape signature of the $x$-cut by using Eq. (1) for each contour pixel $X_x(k)$.
• Resample all the signatures to the number of pixels of the longest obtained signature; this does not necessary correspond to the signature of the lowest $x$-cut. (In this study, the resampling is based on the in-built \textit{resample} Matlab function, using a polyphase implementation.)
• Average the resampled signatures.
\[
CD_{\text{fuzzy2discrete}}(k) = \frac{1}{\gamma_{\text{total}}} \sum_x CD_{x\text{-resampled}}(k),
\]
where $CD_{x\text{-resampled}}(k)$ is the $k$th sample of the resampled signature obtained for one $x$-cut.

Note that for each $x$-cut, the signature is computed starting from the corresponding points, where corresponding points are those located in the same direction from the centroid. It is essential to use angular sampling of the boundaries, to provide appropriate correspondence between the points on the (parts) of the boundaries having different lengths, when the corresponding boundary subparts of different $x$-cuts are matched.

6. Results

In this section, the two shape descriptors designed for fuzzy discrete objects are evaluated and compared.

6.1. Statistical study of discrete disks

For discrete disks, the shape signature should converge to a constant value corresponding to the radius of the disk. Fig. 5(a) and (b), present the shape signatures obtained by Method 1, when applied to a crisp, and a fuzzy disk, respectively. Due to the difficulties of approximating a straight line on the discrete grid, and estimating its length, the result suffers from large fluctuations. Nevertheless, the signature obtained for the fuzzy shape is a bit smoother.

Fig. 5(c) and (d), present the shape signatures obtained by Method 2 when applied to a crisp, and a fuzzy disk, respectively. The results are significantly improved.

To provide a quantitative analysis, disks with 130 different real-valued radii, ranging from 3 to 40 pixels, have been considered. For each radius, 50 disks with random real-valued center coordinates have been generated. For each continuous disk, we generated its crisp, and fuzzy discrete representation. For the fuzzy object, the membership value of a pixel is computed as a fraction of its area belonging to the original object, where the discrete analogue of the area is used. Area coverage of a pixel is expressed as the number of sub-pixels, within the candidate pixel, each having its centroid inside the object. It is achieved by increasing the resolution of the image, i.e., sub-sampling by a factor 16 ($\gamma_{\text{total}} = 256$). It should be noted that many imaging devices produce images with grey-levels proportional to the area coverage of a pixel.
That gives a practical value to our theoretical study.

Both methods have been applied to every disk in the test-set. Each obtained signature has been quantitatively evaluated by computing the signal to noise ratio defined by:

$$\text{SNR} = 10 \times \log_{10} \left( \frac{\text{Radius}^2}{\frac{1}{N_{\text{pix}}} \sum (\text{signature}(i) - \text{mean} (\text{signature}(i)))^2} \right) \text{dB},$$

where $N_{\text{pix}}$ is the number of pixels on the shape boundary, and $\text{mean} (\text{signature}(i))$ is the mean value of the observed shape signature function.

Note: By subtracting the mean value of the signature in Eq. (9), instead of the radius of the original continuous disk, we focus on the precision, but not the accuracy of the result (i.e., we ignore the bias of the signature). Such an approach is motivated by the fact that the most significant improvement obtained by the use of fuzzy, instead of crisp objects, is expected in terms of precision of the estimates. In addition, as mentioned in Section 5.1, it is possible to reduce the bias by introducing a statistically determined constant factor, or by using more precise area estimation methods.

The SNR values obtained for 50 disks with the same radius are averaged. Fig. 6 presents averaged SNR values, for the increasing radius of a disk. It can be noticed that for both methods, the use of fuzzy, instead of crisp object, improves the description. However, for Method 1, the difference tends to zero when the radius increases. Method 2 greatly outperforms Method 1, both in the crisp and in the fuzzy case. Furthermore, for Method 2 the advantage of using fuzzy objects is obvious and increasing with the increase of the radius of the object.

### 6.2. A more complex shape

To perform another evaluation of the described methods, we observe a more complex shape. The original crisp set, presented on Fig. 7(a), is considered as a “ground truth” object and the corresponding signature (b) is used as a reference. Both this object and its signature are supposed to be unknown and will only be used for qualitative and quantitative evaluation of the proposed descriptors. We suppose that we have access only to the degraded version of the original object presented on Fig. 7(c). Due to (simulated) poor acquisition conditions, the object is fuzzy.

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Fig. 6. SNR of computed shape signatures for disks: comparative study of the two methods on crisp and fuzzy shapes.
The classical approach in shape description assumes crisp segmentation of the object, as the first step. Fig. 8 presents three crisp objects obtained from the fuzzy one by using different threshold values. These sets are typical results of a crisp segmentation. The corresponding shape signatures, computed by Method 2, are given below. Since none of the segmentation results takes into account both of the shape features simultaneously, none of the obtained signatures truly represents the original (fuzzy) shape. This emphasizes the need for a shape descriptor handling the fuzzy shape itself.

The shape descriptions of the fuzzy object, obtained by the two methods proposed in this paper, are presented in Fig. 9. Both methods provide better representation compared to the result obtained for crisply segmented objects: the influence of the two features (peak and valley) is clearly visible on each of the signatures. Nevertheless, the result provided by Method 2 is much better than the one provided by Method 1. As expected, the signature obtained by Method 2 is less noisy (see in particular its “constant” part); this observation confirms the results of the statistical study. The
“valley” of the signature function presented in Fig. 7(b) appears to be too narrow in Fig. 9(b). This is mostly caused by the choice of the boundary for a fuzzy set; in Method 1 it is taken to be the boundary of the lowest \( \alpha \)-cut, while in Method 2 the boundary of each \( \alpha \)-cut is considered.

These results can be quantitatively evaluated by computing the signal to noise ratio (SNR) between the signatures provided by each of the methods and the reference signature (Fig. 7(b)):

\[
\text{SNR} = 10 \times \log_{10} \left( \frac{\sum (\text{signature}_{\text{reference}}(i))^2}{\sum (\text{signature}_{\text{reference}}(i) - \text{signature}_{\text{fuzzy}}(i))^2} \right) \text{ dB.}
\]

For the signature computed with Method 1, SNR = 11.89 dB, while for the one computed with Method 2, SNR = 13.67 dB. Confirming the visual inspection, these results prove that Method 2 outperforms Method 1.

7. Conclusions

In this paper, we consider a shape signature based on the Euclidean distance between boundary points of a shape and the shape centroid. This is a classically used descriptor for crisp objects. Our main goal is to extend it to the case of fuzzy shapes. Using two equivalent representations of fuzzy sets (as a membership function, and as a stack of \( \alpha \)-cuts, respectively), we propose two algorithms for the shape signature calculation. The first method is based on the integration of a membership function over the considered straight paths, i.e., calculation of a length of a fuzzy line segment; the second processes each \( \alpha \)-cut separately and averages the obtained signatures of binary shapes. In the continuous case, these two methods are equivalent, for the class of star-shaped fuzzy object, having a centroid included in the kernel. However, the specific issues induced by the discretization lead to different performances of the proposed methods, when they are applied to discrete shapes. The experiments show that the second method provides better results than the first one. An important conclusion is that the direct handling of fuzzy sets greatly improves the quality of the shape description; the sensitivity of the descriptor to the translation of the object within the digitization grid is highly reduced, compared to the crisp case. This comforts the idea that the segmentation step should be delayed as long as possible in the image processing chain: the loss of information induced at the segmentation stage, which can dramatically harm the shape description, and therefore image understanding, can be avoided, or at least reduced, if fuzzy segmentation methods are followed by fuzzy image analysis techniques. Our result contributes to the fuzzy image analysis tool-box.

The speed of the signature calculation is not significantly decreased by introducing fuzziness; more precisely, computational complexity of Method 1 is the same for both fuzzy and crisp case, while the complexity of Method 2 is \( T_{\text{total}} \) times higher for a fuzzy than for a crisp case. As already mentioned, \( T_{\text{total}} \) is the total number of \( \alpha \)-cuts, and is at most 255 if 8 bits are used for a pixel. However, it is usually sufficient to use only 64, or even 16 \( \alpha \)-cuts, to provide good results.
Our study is restricted to the special case of fuzzy star-shaped fuzzy objects with a centroid included in the kernel of a set. This restriction provides equivalence of the two shape signature calculation methods, in the case of a continuous fuzzy shape. It should be noted that these assumptions are sufficient to ensure satisfactory results; but, if they are not fulfilled, the methods do not necessarily fail to describe a shape, even if they are not equivalent to the hard versions anylongues. Nevertheless, it is of interest to generalize the result by introducing more general shapes.

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