Modeling of gas exchange dynamics using cycle-ergometer tests

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Motivation
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- Propose a model for simulation, estimation or prediction of gas exchange dynamics while cycling.
- Since fatigue can not be measured in practice, the model can be used to establish the metabolic behavior and efficiency of a given cyclist.
- Use three measurable variables: the mechanical power at the pedal level, and volumes per unit time of oxygen and carbon dioxide.

Application

Design metabolic model-based force-assistance systems for bikes.
Gas exchange while cycling

Ways to synthesize ATP:

1. aerobic
   \[ \text{Substrate} + O_2 \rightarrow CO_2 + ATP + H_2O \]

2. lactic anaerobic
   \[ \text{Substrate} \rightarrow CO_2 + ATP + La + H_2O \]

- alactic anaerobic (negligible)
  \[ PCr + ADP \rightarrow Cr + ATP \]

Oxygen consumption and carbon dioxide production during cycling. from (Wasserman, Hansen, Sue, Stringer, & Whipp, 2005) redrawn by (Péronnet & Aguilaniu, 2006).
Gas exchange model scheme

**Measured inputs**
\[
\begin{align*}
\tau_p & = \text{Cyclist torque} \\
\omega_p & = \text{Cadence} \\
P & = \text{Pedal power}
\end{align*}
\]

**Measured outputs**
\[
\begin{align*}
y_1 & = O_2 \\
y_2 & = CO_2 + \rho \varepsilon CO_2
\end{align*}
\]
Structure of the model

In this work, we consider the following discrete-time model:

\[ x_{k+1} = Ax_k + Bu_k + Bw \]  \hspace{1cm} (1)
\[ y_k = C(\rho_k)x_k \]  \hspace{1cm} (2)

where,

\[ \rho_k = 0.5 + 0.5 \tanh \left( \frac{z_k - z_k}{h} \right) \]  \hspace{1cm} (3)

\[ x_k = [O_2, CO_2, \varepsilon CO_2]^T \]
\[ u_k = P \]
\[ w = \text{a constant input} \]

\[ z_k = \delta_{O_2} \dot{V}O_2(k) - \delta_{CO_2} \dot{V}CO_2(k) \]
\[ z_k = y_1(k) - y_2(k) \]  \hspace{1cm} (4)

\[ \delta_{O_2}, \delta_{CO_2} \text{ volumetric mass density of } O_2 \text{ and } CO_2. \]
Optimization problem

The system identification is formulated as an optimization problem:

\[ \text{Find } \mathbf{p} = [\theta, w, z_t, h] \text{ which minimizes} \]

\[ J := \sum_{k=1}^{N} \| (y_k - y_k^{measured}) \|_2 \]  

\[ \text{subject to:} \]

\[ x_{k+1} = A(\theta)x_k + B(\theta)u_k + B(\theta)w \]  

\[ y_k = C(\rho_k)x_k \]  

\[ z_k = H(x_k) \]  

\[ \rho_k = \rho(z_k) \]

\[ \text{for } k = \{1, \cdots, N\} \]
Parameter identification

The steps for identification are:

1. **Aerobic exercise**
   \( (\rho = 0) \).

2. **Anaerobic exercise**
   \( (\rho = 1) \).

3. **Calibration of** \( z_t \) \( \) and \( h \).

\[
z_k = y_1(k) - y_2(k)
\]

\[
\rho(z_k) = \begin{cases} 
0 & \text{for mostly aerobic} \\
1 & \text{for mostly anaerobic} \\
0.5 + 0.5 \tanh \left( \frac{z_t - z_k}{h} \right) & \text{for mixed}
\end{cases}
\]
Parameter identification

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   z_k = y_1(k) - y_2(k)
   \]

   \[
   \rho(z_k) = \begin{cases} 
   0 & \text{for mostly aerobic} \\
   1 & \text{for mostly anaerobic} \\
   0.5 + 0.5 \tanh \left( \frac{z_t - z_k}{h} \right) & \text{for mixed}
   \end{cases}
   \]
Results

- Measured data from an individual was used for the methodology
- Two test were performed in cycloergometer:
  - Incremental Cycling Test
  - Steps with different power
Results

Step 1
Identification of aerobic dynamics
\( \rho = 0 \)

\[ \begin{align*}
A &= \begin{bmatrix} 0.168 & 0.701 \\ 0 & 0.986 \end{bmatrix} \\
B &= \begin{bmatrix} 0.259 \\ 0.259 \end{bmatrix} \times 10^{-3} \\
w &= 14.0130
\end{align*} \]

\[ x = [O_2, CO_2]^T \]
Step 2
Identification of anaerobic dynamics
\( \rho = 1 \)

\[
A = \begin{bmatrix}
0.168 & 0.701 & 0 \\
0 & 0.986 & 0 \\
0 & 0.073 & 0.933
\end{bmatrix} \quad \text{(13)}
\]

\[
B = \begin{bmatrix}
0.259 \\
0.259 \\
-0.670 \\
\end{bmatrix} \times 10^{-3} \quad \text{(14)}
\]

\[
x = [O_2, CO_2, \varepsilon CO_2]^T
\]
**Results**

**Step 3**

**Calibration of transition function**

\[
\rho(z_k) = 0.5 + 0.5 \tanh \left( \frac{z_t - z_k}{h} \right)
\]

\[
z_t = -1.17 \text{ and } h = 0.5054.
\]
Model validation

- There is an initial zone were $O_2$ and $CO_2$ have similar slope.
- There is an overproduction of $CO_2$ at the end of the workout.
Conclusions

- A gas exchange dynamical model is proposed. It depends on the pedal power and describes the consumption of $O_2$ and production of $CO_2$ for a given cyclist.

- For a given cyclist, the model can be calibrated using at least two sequences of data: 1) data from an aerobic scenario and 2) data from an anaerobic scenario.

Perspectives

- Improvement of the model concerning to the recovery dynamics after a high intensity workout.

- Extension of the proposed methodology for more general cycling conditions (not only for using in cycle-ergometers), probably including other measurable variables.

- The use of the model for designing advanced control systems for electrical bikes.
Thank you for your attention
