Identification of a Block-Structured Model with Several Sources of Nonlinearity

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***Abstract*— This paper focuses on a state-space based approach for the identification of a rather general nonlinear block-structured model. The model has several Single-Input Single-Output (SISO) static polynomial nonlinearities connected to a Multiple-Input Multiple-Output (MIMO) dynamic part. The presented method is an extension and improvement of prior work, where at most two nonlinearities could be identified. The location of the nonlinearities or their relation to other parts of the model does not have to be known beforehand: the method is a black-box approach, in which no states, internal signals or structural properties need to be measured or known. The first step is to estimate a partly structured polynomial (nonlinear) state-space model from input-output measurements. Secondly, an algebraic approach is used to split the dynamics and the nonlinearities by decomposing the multivariate polynomial coefficients.**

I. INTRODUCTION

Linear system identification is already a well-established discipline, with several well-known and regularly used reference works [5], [11]. But the demands of engineers are increasing and companies want to further decrease costs, increase productivity or improve their products. To push the limits further than the current state of the art, nonlinear distortions (or simply nonlinear behaviour) can no longer be ignored. Whereas for linear systems, a unifying framework exists, nonlinear systems come in all sorts of colours and flavours, and so do the models and modelling techniques. For instance, there exist neural networks and machine learning methods [14], [7], [13], NARMAX models [3], [12], nonlinear state-space models [18], [6] and block-structured models [1], [2]. The latter are interconnections between linear dynamic blocks and static nonlinearities (SNLs).

The method described in this paper somehow bridges two worlds: nonlinear state-space models and block-structured models. At first, a partly structured state-space model is constructed. Secondly, nonlinearities are extracted and separated from the linear dynamics in the model through an algebraic approach to decompose multivariate polynomial coefficients. The resulting block structure, called nonlinear LFR (linear fractional representation) model, is surprisingly general compared to the more regular block structures that have been and are being investigated up to now. The model is able to adequately represent any block-oriented system, consisting of a given number of SNLs. It can model nonlinear feedback phenomena, which is particularly useful in e.g. electro-mechanical engineering. On the other hand, it also encompasses feedforward structures such as Wiener, Hammerstein, Wiener-Hammerstein, Hammerstein-Wiener, as well as the more general types with several parallel branches [10].

The nonlinear LFR model can be seen as a very structured nonlinear state-space model, which contains less parameters. This reduces the noise sensitivity of the model.

Previous work already obtained results for nonlinear LFR models with one [16], [17] or two [15] static nonlinearities (SNLs). This paper aims at extending the number of nonlinearities to a number r, possibly higher than two. The new method is also more elegant and user-friendly than the method in [15]. The approach in [17] is completely different: it needs two experiments at different settings (e.g. different operating point or input amplitude). It combines two linear approximations to initialise the linear and nonlinear blocks. One advantage of this approach is the speed, another is the fact that the type of nonlinearity need not be specified beforehand. But, up to now, this approach was limited to one Single-Input Single-Output (SISO) SNL.

The remainder of this paper is organised as follows. Section II presents the model structure and describes the problem. The problem is rewritten as a rank-one combination problem in Section III. The solution is given in Section IV, in the form of an algorithm. Section V studies the robustness of the solution with respect to noise. A simulation result of the method, applied to the nonlinear model, is presented in Section VI and the conclusions of this paper follow in Section VII.

II. THE NONLINEAR LFR MODEL

A. Definition and model equations

The nonlinear LFR model in Figure 1 is a block-oriented model with r decoupled SNLs connected through a general MIMO Linear Time-Invariant (LTI) part with $n_u + r$ inputs and $n_y + r$ outputs. This general interconnection allows to describe any system with r local SNLs. The state-space equations for the linear dynamic part G_{MIMO} are

$$\begin{aligned} x(t+1) &= Ax(t) + B_u u(t) + B_v v(t) && (1)\\ y(t) &= C_y x(t) + D_{yu} u(t) + D_{yv} v(t) && (2)\\ z(t) &= C_z x(t) + D_{zu} u(t) && (3) \end{aligned}$$

with states $x(t) \in \mathbb{R}^n$ (n is the model order), inputs $u(t) \in \mathbb{R}^{n_u}$ and $v(t) \in \mathbb{R}^r$ (n_u is the number of inputs

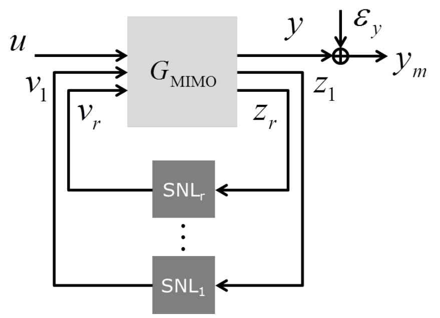

Fig. 1. block structure with r SISO static nonlinearities. The input of the system is u, the noiseless output y and the noisy, measured output (after adding n_y) is y_m.

of the nonlinear LFR model), outputs $y(t) \in \mathbb{R}^{n_y}$ and $z(t) \in \mathbb{R}^r$ (n_y is the number of outputs of the nonlinear LFR model) and t the discrete-time index. Note that, similar to previous work [16], [15], it is assumed that there is no direct feedthrough term D_{zv} between the outputs and inputs of the SNLs (to avoid nonlinear algebraic loops when simulating the model output). The internal signals $v_j(t) \in \mathbb{R}$ depend nonlinearly on $z_j(t) \in \mathbb{R}$ through a degree-d polynomial as in

$$v_j(t) \quad = \quad \sum_{s=2}^{d} \alpha_{js} z_j^s(t) \qquad j = 1, \ldots, r. \tag{4}$$

In what follows, for convenience, we denote this signal as

$$v(z, \alpha) = \left[\begin{array}{ccc} v_1 & \cdots & v_r \end{array} \right]^T, \tag{5}$$

where v_j is given by (4) and $\alpha^\top = [\alpha_{12} \cdots \alpha_{1d} \cdots \alpha_{r2} \cdots \alpha_{rd}]$ contains the polynomial coefficients of all the SNLs.

B. Structured nonlinear state-space representation

After elimination of $z(t)$ and $v(t)$ from (1)–(4), this can be rewritten more compactly as a structured nonlinear state-space model of the form

$$\left[\begin{array}{c} x(t+1) \\ y(t) \end{array} \right] = \left[\begin{array}{cc} A & B_u \\ C_y & D_{yu} \end{array} \right] w(t) + M\zeta\left(w(t)\right) \tag{6}$$

with

$$w = \left[\begin{array}{c} x \\ u \end{array} \right] \tag{7}$$

of dimension $n_w = n + n_u$ and monomial vector $\zeta(w) \in \mathbb{R}^{n_\zeta}$ defined as

$$\zeta^\top = \left[w_1^2 \quad w_1 w_2 \quad \cdots \quad w_{n_w}^2 \quad w_1^3 \quad \cdots \quad w_{n_w}^d \right] \tag{8}$$

Hereafter, time index t is omitted for simplicity. The product of monomial coefficient matrix $M \in \mathbb{R}^{(n+n_y) \times n_\zeta}$ with $\zeta(w)$ can generate arbitrary multivariate polynomials in w in every state and output equation. However, in the case under study, equations (1)–(4), M is structured as visualised in Figure 2, in which L and R are linear static mappings, defined as

$$L = \left[\begin{array}{cc} C_z & D_{zu} \end{array} \right] \in \mathbb{R}^{r \times n_w} \tag{9}$$

$$R = \left[\begin{array}{c} B_v \\ D_{yv} \end{array} \right] \in \mathbb{R}^{(n+n_y) \times r} \tag{10}$$

and

$$M\zeta(w) = Rv(z, \alpha) = Rv(Lw, \alpha). \tag{11}$$

After introducing $M^{(v)}(L, \alpha)$ similar to M, such that

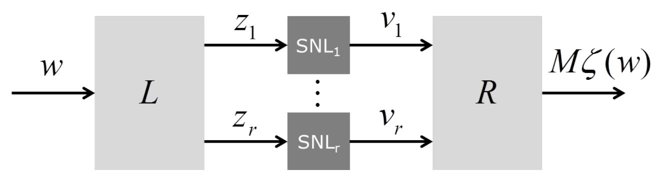

Fig. 2. Relation between $M\zeta(w)$ and w. Note that signal $M\zeta(w)$ has dimension $n + n_y$.

$$v(Lw, \alpha) = M^{(v)}(L, \alpha)\zeta(w) \tag{12}$$

and

$$M = RM^{(v)}(L, \alpha), \tag{13}$$

it can be seen that the rank of M is limited by the number of SNLs:

$$\operatorname{rank} M \leqslant \min\left(\operatorname{rank} R, \operatorname{rank} M^{(v)}(L, \alpha)\right) \leqslant r. \tag{14}$$

Problem description

First, data $(u(t), y_m(t))_{t=0}^{N-1}$ are collected, assuming $y_m(t) = y(t) + \varepsilon_y(t)$, with $\varepsilon_y(t)$ stationary noise with finite variance: this is an output error approach (weighted least-squares). Note that neither $v(t)$, $z(t)$ or the SNL need to be known.

Next, the approach in [15] is used to identify a state-space model (see (6)) with rank limitation on M (partially structured model, see (14)). In this approach, M is explicitly parametrised as a rank r matrix

$$M = U_M V_M^\top \tag{15}$$

with $U_M \in \mathbb{R}^{(n+n_y) \times r}$ and $V_M \in \mathbb{R}^{n_\zeta \times r}$. This identification leads to an estimate of M, which can be factorised as in (13), but in this case, $M^{(v)}$ would not have the right structure (corresponding to the left part of Figure (2)). The additional structure in $M^{(v)}$ is necessary to find the blocks of the nonlinear LFR model. More information about it follows in Section III. Hence, the method starts with the identification of an overparameterised model (15), and only afterwards, the additional structure is imposed.

The goal can be formulated as: given the multivariate polynomial coefficients in the nonlinear state-space model (i.e. matrix M in (6)), find L, R and α such that (11) holds.

III. A RANK-ONE COMBINATION PROBLEM

A. Definition

In order to rephrase the polynomial decomposition problem as a matrix decomposition problem, we define a linear mapping

$$\Psi : \mathbb{R}^{n_\zeta} \rightarrow \mathbb{R}^{\delta \times n_w}, \tag{16}$$

where $\delta = \sum_{s=2}^{d} \delta_s$, $\delta_s = n_w^{s-1}$. The linear mapping $\Psi(P)$ is constructed as follows. Each polynomial $P\zeta$ can be uniquely represented as the sum

$$\begin{aligned} P\zeta(w) = w^\top \Psi^{(2)} w + \Psi^{(3)} \overline{\times}_1 w \overline{\times}_2 w \overline{\times}_3 w + \cdots \\ + \Psi^{(d)} \overline{\times}_1 w \cdots \overline{\times}_d w, \end{aligned} \tag{17}$$

where $\Psi^{(2)}$ is a $n_w \times n_w$ symmetric matrix, $\Psi^{(s)}$ is a s-way $n_w \times \cdots \times n_w$ symmetric tensor [4], and $\overline{\times}_j$ is the j-mode product of a tensor [4, Sec. 2.5]. Each term $\Psi^{(s)} \overline{\times}_1 w \cdots \overline{\times}_s w$ contains only the monomials of degree s.

Then the matrix $\Psi(P)$ is defined as a concatenation of unfoldings of tensors $\Psi^{(s)}$

$$\Psi(P) := \begin{bmatrix} \Psi^{(2)} & \Psi^{(3)}_{(1)} & \cdots & \Psi^{(d)}_{(1)} \end{bmatrix}^\top,$$

where $\Psi^{(s)}_{(1)}$ is an unfolding of the tensor $\Psi^{(s)}$ with respect to the first mode, described in [4, Sec. 2.4]. Each unfolding $\Psi^{(s)}_{(1)}$ has dimensions $n_w \times n_w^{s-1}$. The matrix Ψ is exactly the matrix Γ constructed in the appendix of [16].

B. Properties of $M^{(v)}_{j,:}$ and proof

The row $M^{(v)}_{j,:}$ of the matrix $M^{(v)}$ corresponds to the SNL $v_j(z,\alpha)$ defined in (4), therefore $\Psi(M^{(v)}_{j,:})$ is expressed as

$$\Psi(M^{(v)}_{j,:}) = \begin{bmatrix} \alpha_{j2} L_{j,:}^\top \\ \alpha_{j3} \mathrm{vec}\left(L_{j,:}^\top \circ L_{j,:}^\top\right) \\ \vdots \\ \alpha_{jd} \mathrm{vec}\left(\underbrace{L_{j,:}^\top \circ \cdots \circ L_{j,:}^\top}_{d-1}\right) \end{bmatrix} L_{j,:}, \tag{18}$$

where $\circ$ is the vector outer product [4, Sec. 2.1] and vec is the vectorisation. Indeed, consider a degree-s term of the polynomial $v_j(z,\alpha)$, namely :

$$\begin{aligned} \alpha_{js} z_j^s &= \alpha_{js} z_j \cdots z_j \\ &= \alpha_{js} (L_{j,:} w) \cdots (L_{j,:} w) \\ &= \alpha_{js} \sum_{i_1 \ldots i_s} L_{ji_1} \cdots L_{ji_s} w_{i_1} \cdots w_{i_s} \\ &= \alpha_{js} \left(\underbrace{L_{j,:}^\top \circ \cdots \circ L_{j,:}^\top}_{s}\right) \overline{\times}_1 w \cdots \overline{\times}_s w. \end{aligned} \tag{19}$$

Since the unfolding (along the first mode) of the tensor $\left(\underbrace{L_{j,:}^\top \circ \cdots \circ L_{j,:}^\top}_{s}\right)$ is $L_{j,:}^\top \mathrm{vec}^\top \left(\underbrace{L_{j,:}^\top \circ \cdots \circ L_{j,:}^\top}_{s-1}\right)$, the equality (18) holds true. Similar derivations can be found in the appendix of [16].

C. Properties of $M_{i,:}$ and algebraic problem formulation

The only problem is that $M^{(v)}$ was not identified, but rather M. If $M = M^{(v)}$ were true, then each $L_{j,:}$ could be easily recovered since $\Psi(M^{(v)}_{j,:})$ would have rank one. However, as can be seen from (13), the ith row of M is

$$\begin{aligned} M_{i,:} &= R_{i,:} M^{(v)}(L,\alpha) \\ &= \sum_{j=1}^{r} R_{ij} M^{(v)}_{j,:}(L,\alpha). \end{aligned} \tag{20}$$

Applying the linear transformation Ψ, it follows that

$$\Psi(M_{i,:}) = \sum_{j=1}^{r} R_{ij} \Psi(M^{(v)}_{j,:}). \tag{21}$$

Let $\Psi(M_{i,:})$ be denoted by $Q_i \in \mathbb{R}^{\delta \times n_w}$, $U_{:,j}$ and $V_{:,j}$ denote the left and right factors in (18). Then (21) results in

$$Q_i = \sum_{j=1}^{r} R_{ij} U_{:,j} V_{:,j}^\top. \tag{22}$$

Assume we are able to find R, U and V, given all Q_i. Then we can choose $L = V^\top$ and recover α from U and L (via least-squares fitting). The solution to the decomposition problem (22) is given in the following section.

IV. SOLUTION: SIMULTANEOUS MATRIX DECOMPOSITION

Problem 1: Given matrices $Q_1, \ldots, Q_m \in \mathbb{R}^{p\times q}$, $m \geq 2$, and r satisfying $r \leq p$, $r \leq q$, find $R \in \mathbb{R}^{m\times r}$ and full rank $U \in \mathbb{R}^{p\times r}$ and $V \in \mathbb{R}^{q\times r}$, such that

$$Q_k = U \,\mathrm{diag}(R_{k,:}) V^\top = \sum_{j=1}^{r} R_{kj} U_{:,j} V_{:,j}^\top, \quad k = 1, \ldots, m. \tag{23}$$

We assume that Q_k admit a decomposition (23), and impose an additional assumption on R:

$$\mathrm{rank} \begin{bmatrix} R_{:,k} & R_{:,l} \end{bmatrix} = 2 \text{ for any } k \neq l, \tag{24}$$

which means that the columns of R are pairwise non-collinear (PNC).

Algorithm 1: **Input:** $Q_k \in \mathbb{R}^{p\times q}$, $k = 1, \ldots, m$ and r.

1: Compute $U_S \in \mathbb{R}^{p\times r}$, $V_S \in \mathbb{R}^{q\times r}$ from the truncated SVD of Q_1, i.e. $Q_1 = U_S \Sigma V_S^\top$;
2: Compute $\widetilde{Q}_k = U_S^\top Q_k V_S$;
3: Generate random a, $b \in \mathbb{R}^m$;
4: Set $\breve{Q}_1 = \sum\limits_{k=1}^{m} a_k \widetilde{Q}_k$, $\breve{Q}_2 = \sum\limits_{k=1}^{m} b_k \widetilde{Q}_k$;
5: Compute the matrices W and Z of right (and left) eigenvectors of the pencil $(\breve{Q}_1, \breve{Q}_2)$;
6: Set $U = U_S (Z^\top)^\dagger$, $V = V_S (W^\top)^\dagger$ (superscript $\dagger$ denotes the Moore-Penrose pseudo-inverse);
7: Find R via least-squares fitting, applied to (23).

Output: U, V, R.

If the model (23) is exact, then Algorithm 1 solves Problem 1. When the entries of M have some uncertainty (e.g. due to noise) or when the system does not have the assumed structure, (23) does not hold. In that case, the truncated SVD gives an approximation. The robustness of the algorithm with respect to noise on M is shown in Section V. Below follow some remarks on Algorithm 1.

- Via step 1 and step 2, the problem size is reduced: matrix $\widetilde{Q}_k$ has dimension $r \times r$, whereas matrix Q_k had dimension $p \times q$.
- Steps 3 and 4 ensure that two rank r matrices $\breve{Q}_1$ and $\breve{Q}_2$ are obtained.
- At step 5, the matrices W and Z (with generalised eigenvectors as columns) are computed using the qz routine of MATLAB. The columns of these matrices form a basis for the left and right nullspaces of $\mu_k \breve{Q}_1 - \lambda_k \breve{Q}_2$, with (μ_k, λ_k) the generalised eigenvalues of the pencil.

The simultaneous decomposition (23) is exactly the CP decomposition of a tensor (constructed by stacking the slices Q_i [4, Sec. 3] into a sum of rank-one terms. Algorithm 1 is a modified version of the tensor CP decomposition algorithm initially proposed in [8]. The assumption (24) implies that the k-rank [4, Sec. 3.2] of the matrix R is at least 2. Since the matrices U and V are of full rank, the decomposition (23) is unique (up to scaling of the factors) [4, Sec. 3.2].

V. Robustness study of the decomposition algorithm

To study the robustness of Algorithm 1 with respect to noise, 500 runs were performed with the following settings: $n = 5$, $n_u = n_y = 1$, $r = 3$ and $d = 3$.

First, U, V and R are generated with uniformly distributed elements (ignoring the structure in U, which is the left factor in (18)). Next, the corresponding matrix M is constructed and normally distributed noise is added to this matrix. Fifty (logarithmically spaced) noise levels are considered.

The method is applied, resulting in estimates $\hat{U}$, $\hat{V}$, $\hat{R}$ and hence also $\hat{M}$. The relative error on $\hat{M}$ is computed as $\frac{\|\hat{M}-M\|_\mathrm{F}}{\|M\|_\mathrm{F}}$, where $\|\bullet\|_\mathrm{F}$ denotes the Frobenius norm of $\bullet$. This error is averaged out over the 500 runs and plotted versus the rms of the noise in Figure 3. The figure shows a linear relation. We conclude that, when the noise is reduced by a factor γ, the decomposition error also decreases by a factor γ^κ, with $\kappa = 1$.

VI. Simulation result

The system to be identified is a 3-SNL nonlinear LFR in which A, B_u, C_y and D_{yu} are the state-space parameters of a 5th order lowpass Chebyshev Type 1 filter with 7 dB peak-to-peak ripple in the passband and normalised passband edge frequency 0.21. The matrices B_v, D_{yv}, C_z and D_{zu} are randomly chosen. The SNLs (shown in Figure 4) have nonlinear degree $d = 3$ and randomly generated coefficients.

The system is excited by a random phase multisine [9] with $N = 5000$ points and three realisations (different uniform distributions of the phases). The simulations are in steady-state and noise is added to the simulated output. The signal-to-noise ratio (SNR) is 500.

A rank-three Polynomial Nonlinear State-Space (PNLSS) model is fitted via a stepwise approach, as in [15]:

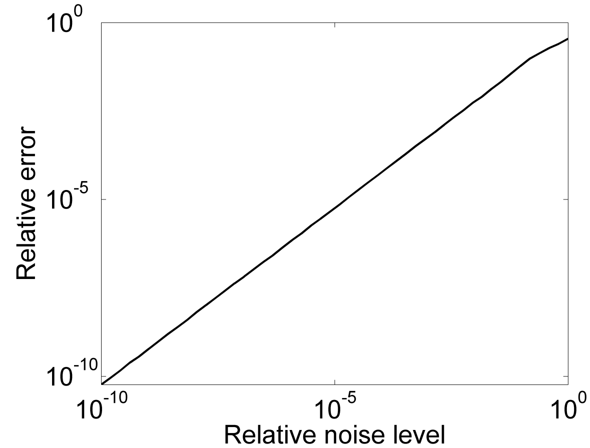

Fig. 3. Relative error on the decomposition of matrix M. The relation between the error and the noise level is linear, with a slight deviation for noise levels above 10^{-2}.

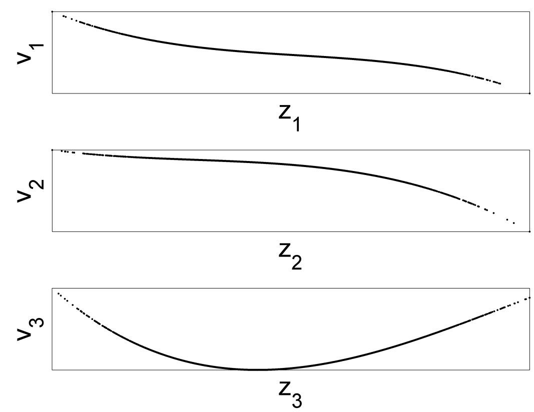

Fig. 4. The shapes of the three polynomial SNLs of the nonlinear LFR system. The ticks are not shown since scaling of z_i or v_i can be compensated via the coefficients in L (9) or R (10).

- estimate the nonparametric Best Linear Approximation (BLA) [9];
- fit a linear state-space model on the nonparametric BLA via a subspace method;
- build up a rank-three PNLSS model by gradually increasing the rank of M (15).

Next, the method of this paper is used:

- construct $Q_i = \Psi(M_{i,:})$ for $i = 1, \ldots, n + n_y$;
- perform Algorithm 1 to retrieve U, V and R;
- deduce L and α out of V and U.

After this, the polynomial LFR model is completely initialised. In practice, when there are modelling errors or if there is noise, a last step should still be performed:

- optimise the initial estimate.

Figure 5 shows the nonparametric BLA together with its standard deviation, a linear state-space fit and the error of this fit. The model error is well in line with the standard deviation.

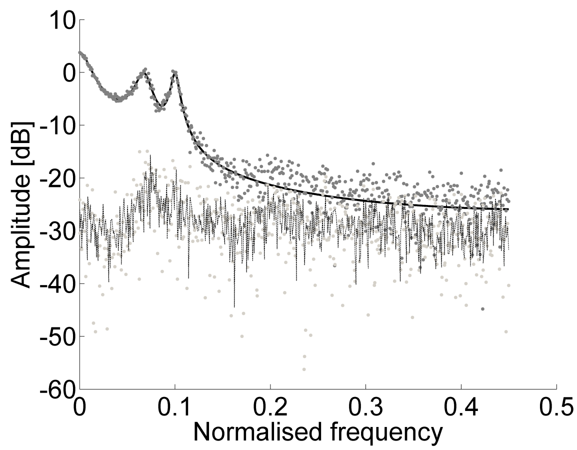

Fig. 5. Nonparametric BLA (dark grey dots), linear state-space fit (black line), linear model error (light grey dots) and standard deviation (dashed line) of the BLA.

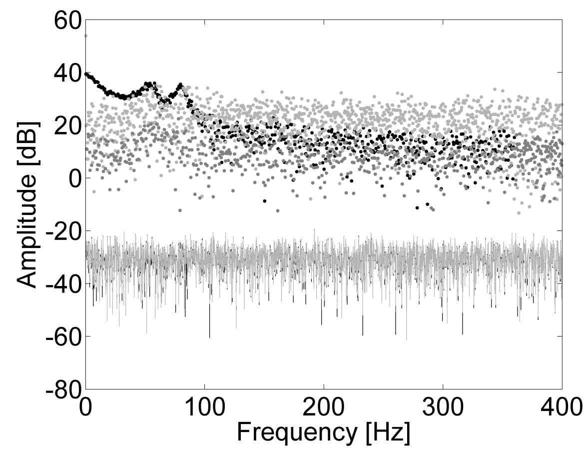

Fig. 6. Output spectrum (black dots: excited lines; light grey dots: even lines; dark grey dots: odd lines), noise level (grey line) and output residuals (black line).

Table I shows the relative output errors of the steps that lead to the final result as well as the final result itself. These values are computed as

$$\frac{\mathrm{rms}(y - y_m)}{\mathrm{rms}(y_m)}$$

i.e. the root mean square (rms) value of the output simulation error relative to the rms-value of the output. These values should be compared with the noise level ($\mathrm{SNR}^{-1} = 0.002$). It can be noted that the final result (a polynomial LFR model with three SNLs) converged and that relatively few iterations were necessary to obtain this result: in total 28. The noise on the data influenced the quality of the initial nonlinear LFR model (obtained directly via the algebraic method in Section IV), but this did not prevent convergence of the last step.

Figure 6 shows the output spectrum (all the dots) and the output error of the final nonlinear block structure (with the noise plotted on top of it). As can be expected from a nonlinear system, the output spectrum looks very noisy. The different colours of the dots show the contributions of odd and even nonlinear distortions. This can be done thanks to the choice of excitation signal, which was a random odd multisine (meaning that the input signal is only present on a selection of odd frequency lines). On the excited frequencies, a combination of the linear behaviour of the system and odd nonlinear distortions is present. On the odd frequency lines, only the odd nonlinear distortions play a role. On the even lines, only the even nonlinear distortions are visible. It can hence be concluded from the output spectrum (even before identification) that the dominant nonlinearities in the system have an even nature. Moreover, if a random noise input signal would have been used, fitting a linear model would have been much more difficult. Thanks to the choice of input signal, the even distortions did not disturb the estimate of the BLA.

Discussion

As could be concluded from the results in Table I, the final model has maximal quality since the noise and output residuals in Figure 6 have the same magnitude. The results are also checked on validation data (not shown in the table). The validation input is white Gaussian noise with linearly increasing amplitude (over time, from zero up to 110% of the standard deviation of the estimation input). Despite the small extrapolation, the validation error also attains the noise level.

	rms($y - y_m$)/rms(y_m)	n_θ
BLA	0.77	11
PNLSS-rank 1	0.08 (7 its.)	94
PNLSS-rank 2	0.021 (8 its.)	177
PNLSS-rank 3	0.0019 (11 its.)	260
Initial nonlinear LFR	0.019	53
Nonlinear LFR	0.002 (2 its.)	53

TABLE I

COMPARISON OF THE RELATIVE RMS ERROR ON THE ESTIMATION DATA, AND NUMBER OF INDEPENDENT PARAMETERS n_θ. THESE VALUES SHOULD BE COMPARED WITH THE RELATIVE VALUE OF THE NOISE, WHICH IS 0.002.

Future work will focus on robustifying the core step, which is the (now deterministic) algebraic decomposition of a multivariate polynomial. The present approach makes no use of the structure in $U_{:,i}$ in (18). A method based on e.g. a symmetric tensor decomposition would make use of this structure and might be less sensitive to noise. The potential benefits still need to be investigated. On the other hand, the presented method is easier to implement and faster.

VII. CONCLUSION

This paper proposes a method to identify a block structure with several sources of nonlinearity, called nonlinear LFR. The method is a black-box approach, in which only input and output data are used (no states, internal signals or structural properties need to be measured or known). In the first step, a (partly structured) polynomial state-space model is estimated. Next, an algebraic approach decomposes the identified multivariate polynomial coefficients such that the nonlinear state-space model can be directly converted into the desired nonlinear LFR model. This nonlinear LFR model, with a user-chosen number of SNLs interconnected via a general linear dynamic block, has few parameters, but good flexibility.

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