Reset Unknown Input Observer for Robust Actuator and Sensor Fault Estimation Based on a Descriptor system Approach

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Abstract

This paper proposes a Reset Unknown Input Observer (R-UIO) using descriptor approach. The proposed R-UIO can be employed for estimating actuator and sensor faults in linear systems which are subject to external disturbances. In the R-UIO, states of the observer are reset to the after-reset value based on an appropriate reset law in order to decrease the sum of estimation squared error. By introducing a new state, the sensor fault is converted to an actuator fault and the actuator fault is considered as an auxiliary state in the descriptor system. To estimate the fault, the structure of the R-UIO for the descriptor system is introduced and the stability conditions of estimation error dynamics are developed through LMI optimization. Finally, the capabilities of the R-UIO-based fault estimation strategy are demonstrated by applying it to a practical model. It is shown that fault estimation is carried out more rapidly and accurately by the proposed R-UIO observer.

Keywords— Reset Theory, Unknown Input Observer, Fault Estimation

1 Introduction

Safety and reliability are among the most important engineering aspects considered in design and development of the new systems (e.g. [1–5]). Diagnosing the occurred faults is one of the necessary requirements of various systems such as airplanes, power plants, robots, chemical and nuclear reactors to ensure the system performance and safety (e.g. [6–11]). In such areas, a minor fault regardless of the type (actuator, sensor or system plant fault) should be detected and handled as soon as possible. Otherwise, it may lead to unpredictable consequences and cause damage to equipment and/or humans. Hence, much effort must be paid to fault diagnosis. Detection and isolation are the two main steps in fault diagnosis. This step is the critical step for fault-tolerant control problems. For example in [12], a novel adaptive observer is designed to handle the nonlinear systems.

Started in the early 70's, the model-based fault diagnosis has significantly progressed. Successful implementation in industrial processes and automatic systems demonstrates its efficiency in detecting faults [13]. Nowadays, development of model-based fault diagnosis techniques especially for safety-critical systems are accelerated and these methods are an integrated part of different systems, such as; robots, vehicle control, transport, power, manufacturing processes, process control (see [14–23]). [24] was the first observer-based approach proposed for this fault diagnosis, which considers the estimation of instrument fault as a special case of a sensor fault. In [25], a strong foundation for the observer-based fault estimation is developed based on the Luenberger observer [26] which reconstructs the state variables under deterministic hypotheses.

In model-based fault-diagnosis problems, output estimation is based on residual signal which is computed through comparing the estimated system outputs and their measured or expected values. On the other hand, any system may include simultaneously unknown inputs (UIs) such as actuator fault [27], external disturbance [27, 28] and parameter perturbation [29] which can degrade the its performance. To distinguish between fault and disturbance (or uncertainty) in disturbed (or uncertain) systems, robust fault detection is mandatory for fault estimation. Hence, generating robust residual is critical. To make the residuals insensitive to the UI, decoupling it by algebraic and geometric methods is proposed. Employing Unknown Input Observers (UIO) is also helpful for decoupling the UI from residual [30].

UIO is widely employed in observer-based control and fault diagnosis for estimating system states when unknown disturbance and input exist (*e.g.*[31–35]). Meanwhile, the problem of state and fault estimation of systems with UI is still an open problem. Harmonic disturbance is observed and controlled using proper observer in nonlinear [36] and stochastic systems [37]. In [38] and [39] a full-order observer for linear systems subject to UIs is developed. The existence of a UIO is investigated in [14, 40], and its necessary and sufficient conditions are presented. Besides, the reduced order UIO can be designed using a systematic procedure [41]. The capabilities of the UIOs for state and fault estimation in the presence of uncertainty and disturbance are demonstrated in [42, 43] and different approaches for designing an UIO have been developed. In [44], Linear Matrix Inequalities (LMIs) are used to design a full-order nonlinear UIO for a class of nonlinear Lipschitz systems with unknown input. Moreover, a reduced order UIO for the one-sided nonlinear Lipschitz system is proposed in [45]. A robust UIO for fault detection using linear parameter varying model with uncertainties is presented in [46]. A UIO design for a class of nonlinear systems which can be represented by Takagi-Sugeno fuzzy bilinear system is proposed in [47].

Various observer based fault reconstruction approaches are suggested based on descriptor approach (DA). In the context of systems theory and control engineering, a descriptor system, also known as a singular or differential-algebraic system, is a mathematical model that describes the behavior of a dynamic system. Unlike ordinary differential equation systems, which only involve derivatives of the system variables, descriptor systems involve both derivatives and algebraic equations. These algebraic equations typically arise from constraints or relations within the system. Descriptor systems are employed to describe various systems such as robots, gas turbines, and electrical networks (see [48–51]). Using this method, all static and dynamic governing equations of a system are represented in a unified framework. By realizing an integrated formulation for the system through DA, observer-based fault diagnosis is more feasible. Hence, a growing attention in recent years has been attracted to this field [52–55]. In [56], a descriptor observer is designed for multi-variable linear system. Fault detection for time delayed and discrete-time descriptor systems are reported in [57] and [58]. A state augmentation approach to interval fault estimation for descriptor systems in suggested in [59]. Moreover, fault estimation and fault-tolerant control for disturbed and uncertain nonlinear descriptor systems is developed in [60] and [61] respectively. Fault estimation for descriptor switched systems is also studied in multiple works such as [52, 62].

A reset observer (RO) is a nonlinear observer consisting of a base observer and a reset law. Using RO, the states of the observer are reset to a predefined value when some reset conditions are satisfied. Hence, a traditional observer can change to a RO by utilizing the reset mechanism. In [63], a new type of adaptive observer is proposed by applying the reset to the observer states. An optimization problem is solved to obtain an optimal reset adaptive observer in [64]. Moreover, application of the reset strategy to a Proportional-Integral observer for time-varying systems and fault estimation problem is investigated in [20] and [65]. Finally, a reset UIO (R-UIO) for state estimation is also developed in [66] and is used for fault estimation as well in [67]. One can find in the aforementioned works that the reset approach improves the estimation error behavior including settling time, overshoot, and rise time. On the other hand, this method is not employed through the descriptor approach which facilitates discussing a wide range of practical systems in a unified structure.

To the best of our knowledge, the application of the RO in fault estimation is not investigated. In this paper, a R-UIO for fault estimation problem using descriptor approach is suggested. In this regard, proposing a new state definition, the sensor fault is considered as an actuator fault. Then, the actuator fault can be regarded as a new state to form a descriptor system, and the R-UIO is designed for this system. By employing LMI, it has been shown that the fault estimation error converges asymptotically to zero and the efficiency of the proposed method is demonstrated by exploiting an aircraft model as a practical example. The results which are compared with the conventional UIO (C-UIO) show a significant improvement in the fault estimation in the sense of accuracy and rapidity.

The paper contents are organized as follows: in Section 2, a conventional approach to design the base augmented UIO for fault estimation is investigated. In Section 3, the descriptor approach augmented R-UIO with full and partial measurement is developed and the reset law is obtained. In Section 4, the proposed method is validated through a practical model and the results are compared with literature to verify the performance of the estimation strategy. Finally, the concluding remarks are provided in Section 5.

2 Preliminaries

Consider the system:

$$\left\{\begin{array}{lcl}\dot{x} & = & Ax + Bu + Dv + E_f f \\ y & = & Cx\end{array}\right. \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $v \in \mathbb{R}^d$, $y \in \mathbb{R}^p$ and $f \in \mathbb{R}^r$ and are the state vector, the known input vector, the unknown input vector, the output of the system and the actuator fault respectively. A, B, C, D, and E_f are known matrices with appropriate dimension. In (1), the number of output channels is greater than or equal to the number of fault inputs i.e. $p \geq r$. Without loss of generality, we assume that D is of full column rank [14].

The goal is to design a UIO to estimate faults in the system. If constant fault occur ($\dot{f} = 0$),(1) can be re-written as

$$\left\{\begin{array}{lcl}\dot{\bar{x}} & = & \bar{A}\bar{x} + \bar{B}u + \bar{D}v \\ y & = & \bar{C}\bar{x}\end{array}\right. \tag{2}$$

where

$$\bar{x} = \begin{bmatrix} x \\ f \end{bmatrix}, \bar{A} = \begin{bmatrix} A & E_f \\ 0_{r\times n} & 0_{r\times r} \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0_{r\times m} \end{bmatrix}, \bar{D} = \begin{bmatrix} D \\ 0_{r\times d} \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0_{p\times r} \end{bmatrix}. \tag{3}$$

Zero matrix is denoted by 0 with specified dimension.

State estimation of (2) can be accomplished through a full-order UIO which is known as conventional UIO (C-UIO) as

$$\left\{\begin{array}{lcl}\dot{\zeta} & = & N\zeta + Gu + Ly \\ \hat{\bar{x}} & = & \zeta - \bar{E}y \\ \hat{y} & = & \bar{C}\hat{\bar{x}}\end{array}\right. \tag{4}$$

where $\zeta \in \mathbb{R}^{(n+r)}$ is the state of this full-order observer, $\hat{\bar{x}} \in \mathbb{R}^{(n+r)}$ is the estimated state vector, $N \in \mathbb{R}^{(n+r)\times(n+r)}, G \in \mathbb{R}^{(n+r)\times m}, L \in \mathbb{R}^{(n+r)\times p}, \bar{E} \in \mathbb{R}^{(n+r)\times p}$ are design matrices for unknown input decoupling goal. Parameters of the C-UIO observer can be obtained using [44]:

$$\left\{\begin{array}{lcl}N & = & \bar{M}\bar{A} - K\bar{C} \\ G & = & \bar{M}\bar{B} \\ L & = & K(I + \bar{C}\bar{E}) - \bar{M}\bar{A}\bar{E} \\ \bar{M} & = & I + \bar{E}\bar{C} \\ \bar{M}\bar{D} & = & 0\end{array}\right. \tag{5}$$

The two conditions for C-UIO existence are $rank(\bar{C}\bar{D}) = rank(\bar{D})$ and $(\bar{C}, \bar{M}\bar{A})$ is detectable. Using the last equation in (5) and after some simple algebraic calculation, a general solution can be found as

$$E = -D(CD)^{+} + Y(I - (CD)(CD)^{+})$$

in which, $(CD)^{+}$ is defined as $(CD)^{+} = ((CD)^{T}(CD))^{-1}(CD)^{T}$, K is a chosen such that N is Hurwitz, Y is an arbitrary matrix and [14]

$$\bar{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}.$$

Defining the estimation error as $\bar{e} = \hat{\bar{x}} - \bar{x}$ and using the system equation (2) and the observer (4) the continuous error dynamics can be obtained as:

$$\dot{\bar{e}} = \dot{\hat{\bar{x}}} - \dot{\bar{x}} = N\bar{e}.$$

Since N is Hurwitz the above error dynamics indicates that the estimation error converges asymptotically to zero and thus $\hat{\bar{x}} \longrightarrow \bar{x}$.

Meanwhile, there are many cases in which the system fault is time-varying ($\dot{f} \neq 0$). To estimate these faults, (1) can be rewritten in higher dimensions. For example for ramp fault the following form can be obtained:

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{f} \\ \ddot{f} \end{bmatrix} = \begin{bmatrix} A & E_f & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ f \\ \dot{f} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} v, \\ y = \begin{bmatrix} C & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ f \\ \dot{f} \end{bmatrix}. \end{cases} \tag{6}$$

Let

$$\bar{x} = \begin{bmatrix} x \\ f \\ \dot{f} \end{bmatrix}, \bar{A} = \begin{bmatrix} A & E_f & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \tag{7}$$

$$\bar{D} = \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0 & 0 \end{bmatrix}, \tag{8}$$

then one can realize (2) and similar method as the one given for constant fault can be applied. It is readily concluded that this fault estimation observer design can be extended to a large class of typical faults, i.e. $f^{(i)}(t) = 0$.

On the other hand, a fault may happen in the sensor rather than the actuator. Therefore, the system (1) can be rewritten as follows:

$$\left\{ \begin{array}{rcl} \dot{x} & = & Ax + Bu + Dv, \\ y & = & Cx + E_f f. \end{array} \right. \tag{9}$$

Now, using a simple transformation, sensor fault can be treated as an actuator fault. Regarding this, define a new state $z = \int_0^t y(\tau)d\tau$ such that

$$\dot{z} = Cx + E_f f. \tag{10}$$

Equations (9) and (10) can be combined to form an augmented system as:

$$\left\{ \begin{array}{rcl} \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} & = & \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} D \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ E_f \end{bmatrix} f, \\ y & = & \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + E_f f. \end{array} \right. \tag{11}$$

Considering the fault as a new state and z as an auxiliary output, the augmented system (11) subject to a constant fault can be written as:

$$\left\{ \begin{array}{rcl} \begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{f} \end{bmatrix} & = & \begin{bmatrix} A & 0 & 0 \\ C & 0 & E_f \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ f \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} v, \\ \begin{bmatrix} y \\ z \end{bmatrix} & = & \begin{bmatrix} C & 0 & E_f \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ f \end{bmatrix}. \end{array} \right. \tag{12}$$

Let

$$\bar{x} = \begin{bmatrix} x \\ z \\ f \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 & 0 \\ C & 0 & E_f \\ 0 & 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \bar{D} = \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0 & E_f \\ 0 & I & 0 \end{bmatrix}, \tag{13}$$

one can rewrite (9) in the form of (2) and the UIO defined in (4) can be designed for all of these systems. In other words, through computing (5) by the defined parameters in (13) instead of (3), C-UIO can be employed for systems with sensor fault.

Remark 1. *As long as the observability considtion is not violated by the augmented state coffecient matrix ($\bar{A}$) this column is out of concern.*

3 Descriptor Approach R-UIO Design for Fault Estimation

In this section, the novel observer design through descriptor approach is addressed. Moreover, by using the aforementioned transformation, an augmented system can be realized. Hence, the suggested R-UIO is general and can be employed for accurate estimation of both sensor and actuator fault. The design steps are divided into two cases. In the first case which is called R-UIO with full-state measurement (or ideal case), it is assumed that all the system states can be measured and it is just useful for the derivation of main results. Then this case is extended to the main approach named R-UIO with partial state measurement (or non-ideal case), in which only the outputs are available.

3.1 Descriptor Approach UIO for Fault Estimation

The method employing the descriptor approach in order to design the fault estimator is illustrated first. Consider the system dynamic model

$$\left\{ \begin{array}{rcl} \bar{T}\dot{\bar{x}} & = & \bar{A}\bar{x} + \bar{B}u + \bar{D}v, \\ y & = & \bar{C}\bar{x}, \end{array} \right. \tag{14}$$

where $\bar{T} \in \mathbb{R}^{(n+r)\times(n+r)}$ is a constant square matrix that may have rank deficiency. $\bar{x}$ is the augmenetd states containong the state x, the fault f and its derevatives if necessary. The matrcies $\bar{A}, \bar{B}, \bar{C}, \bar{D}$ are the augmented matrices in equation (7) and (13) for actuator and sensor fault respectively. Employing state estimation formulation similar to (4), UIO parameter in the descriptor approach should be computed through Proposition 1.

Proposition 1. *If the UIO parameters in descriptor approach satisfy the following relations*

$$\left\{ \begin{array}{ccl} \bar{M}\bar{T} & = & \bar{E}\bar{C} + I_{(n+r)\times(n+r)} \\ G & = & \bar{M}\bar{B} \\ N\bar{M}\bar{T} + L\bar{C} - \bar{M}\bar{A} & = & 0 \\ \bar{M}\bar{D} & = & 0 \end{array} \right. \tag{15}$$

then

$$\dot{\bar{e}} = N\bar{e}$$

which ensures accurate estimation.

Proof. Let

$$\bar{e} = \hat{\bar{x}} - \bar{x}, \tag{16}$$

by using (4) one obtains

$$\dot{\bar{e}} = \dot{\zeta} - (\bar{E}\bar{C} + I)\dot{\bar{x}}. \tag{17}$$

Considering (14) and

$$\bar{M}\bar{T} = \bar{E}\bar{C} + I \tag{18}$$

the estimation error dynamic is

$$\dot{\bar{e}} = N\bar{e} + (G - \bar{M}\bar{B})u + (N\bar{M}\bar{T} + L\bar{C} - M\bar{T}\bar{A})\bar{x} - \bar{M}\bar{D}v. \tag{19}$$

Hence, the desired UIO behavior can be obtained if

$$G - \bar{M}\bar{B} = 0,\ N\bar{M}\bar{T} + L\bar{C} - \bar{M}\bar{A} = 0,\ \bar{M}\bar{D} = 0. \tag{20}$$

■

Herein, similar to the well-known results obtained for the classical full-order UIO observer [39], L can be defined as

$$L = K(I_{p\times p} + \bar{C}\bar{E}) - \bar{M}\bar{A}\bar{E}, \tag{21}$$

where

$$\begin{cases} N &= \bar{M}\bar{A} - K\bar{C} \\ K &= L + N\bar{E}. \end{cases} \tag{22}$$

Moreover, considering (15) one can write

$$\Theta\Xi = \Delta, \tag{23}$$

with

$$\Theta = \begin{bmatrix} \bar{M} & -\bar{E} \end{bmatrix}, \Xi = \begin{bmatrix} \bar{T} & \bar{D} \\ \bar{C} & 0_{p\times d} \end{bmatrix}, \Delta = \begin{bmatrix} I_{(n+r)\times(n+r)} & 0_{(n+r)\times d} \end{bmatrix},$$

and a general solution for descriptor approach UIO matrices can be found through

$$\Theta = \Delta\Xi^{\dagger} + \Phi\Pi. \tag{24}$$

If

$$Rank(\Xi) = n + r + Rank(\bar{D}),$$

holds, then (24) has solution and

$$\Xi^{\dagger} = \Xi^{T}(\Xi\Xi^{T})^{-1}, \tag{25}$$

$$\Pi = I_{(n+r+p)\times(n+r+p)} - \Xi\Xi^{\dagger}. \tag{26}$$

$\Xi^{\dagger} \in \mathbb{R}^{(n+r+d)\times(n+r+p)}$ is the right pseudo-inverse matrix which is used for non-square matrices with independent columns. Matrix Π is the null-space projection matrix and $\Phi \in \mathbb{R}^{(n+r+p)\times(n+r)}$ is an arbitrary matrix which can be employed to obtain the desired observer performance.

3.2 R-UIO Design with Full-State Measurement

In this part, a reset action is added to the UIO to improve its performance. The R-UIO can be formulated as:

$$\begin{array}{l} \left.\left\{\begin{array}{lcl} \dot{\zeta} & = & N\zeta + Gu + Ly \\ \hat{\bar{x}} & = & \zeta - \bar{E}y \\ \hat{y} & = & C\hat{\bar{x}} \end{array}\right.\right\} \text{if } \bar{e} \in \mathcal{F} \\ \left.\left\{\begin{array}{lcl} \zeta^{+} & = & (\bar{M}\bar{T} - A_R\bar{E}\bar{C})\zeta - (I - A_R)\bar{M}\bar{T}\bar{E}y \\ \hat{\bar{x}}^{+} & = & \zeta^{+} - \bar{E}y \end{array}\right.\right\} \text{if } \bar{e} \in \mathcal{J} \end{array} \tag{27}$$

in which A_R is the after-reset matrix, $\mathcal{F} = \{\bar{e} \in \mathbb{R}^{n+r} | \bar{e}^T F \bar{e} > 0\}$ is the flow set and $\mathcal{J} = \{\bar{e} \in \mathbb{R}^{n+r} | \bar{e}^T F \bar{e} \leq 0\}$ is the jump set and as soon as $\bar{e} \in \mathcal{J}$ jump will happen. Matrices F and A_R satisfies conditions given hereafter.

For the discrete error dynamics one gets:

$$\begin{aligned} \bar{e}^{+} &= \hat{\bar{x}}^{+} - \bar{x} \\ &= \zeta^{+} - \bar{E}y - \bar{x} = \zeta^{+} - (I + \bar{E}\bar{C})\bar{x}. \end{aligned} \tag{28}$$

Substituting ζ^+ and ζ from the second and fourth relation in (27) and adding and subtracting $A_R e$, $\bar{e}^+$ is

$$\begin{aligned} \bar{e}^+ &= \bar{M}\bar{T}\bar{e} - A_R(I + \bar{E}\bar{C})\bar{e} + A_R\bar{e} \\ &= (A_R - A_R\bar{M}\bar{T} + \bar{M}\bar{T})\bar{e}, \end{aligned} \tag{29}$$

therefore, defining $H = A_R - A_R\bar{M} + \bar{M}\bar{T}$, the error dynamics can be written as:

$$\left\{ \begin{array}{lcll} \dot{\bar{e}} & = & N\bar{e} & \text{if} \quad \bar{e} \in \mathcal{F} \\ \bar{e}^+ & = & H\bar{e} & \text{if} \quad \bar{e} \in \mathcal{J} \end{array} \right. \tag{30}$$

Based on the reset error dynamics the following theorem on the convergence of R-UIO can be stated:

Theorem 1. *For the faulty system (14), if there exist symmetric matrices $\Gamma > 0$, F and matrix Ω and constants $\lambda_f, \tau_f, \tau_j, \tau_w > 0$ and $0 < \lambda_j \leq 1$ such that*

$$N^T\Gamma + \Gamma N + \lambda_f\Gamma + \tau_f F < 0 \tag{31a}$$

$$\begin{bmatrix} \lambda_j\Gamma + \tau_j F & (\Omega - \Omega\bar{M}\bar{T} + \Gamma\bar{M}\bar{T})^T \\ \Omega - \Omega\bar{M}\bar{T} + \Gamma\bar{M}\bar{T} & \Gamma \end{bmatrix} \geq 0 \tag{31b}$$

$$H^T F H + \tau_w F > 0 \tag{31c}$$

then the error dynamics (30) *is well-posed and the R-UIO given by (27) can estimate the fault and makes the error converges to zero asymptotically for any initial condition.*

Proof. Let the quadratic function

$$V(\bar{e}) \quad = \bar{e}^T\Gamma\bar{e} \tag{32}$$

as a Lyapunov function where $\Gamma = \Gamma^T > 0$. Herein, stability condition corresponds to both continuos and discrete time domains. According to reset systems stability conditions [68], asymptotic stability of (30) is guaranteed if:

$$\left\{ \begin{array}{lcll} \dot{V}(\bar{e}) & < & -\lambda_f V(\bar{e}) & \text{if} \quad \bar{e}^T F\bar{e} > 0 \\ V(\bar{e}^+) & \leq & \lambda_j V(\bar{e}) & \text{if} \quad \bar{e}^T F\bar{e} \leq 0. \end{array} \right. \tag{33}$$

Considering (30) along (33), one can realize

$$\begin{cases} \dot{\bar{e}}^T\Gamma\bar{e} + \bar{e}^T\Gamma\dot{\bar{e}} + \lambda_f\bar{e}^T\Gamma\bar{e} &= \bar{e}^T(N^T\Gamma + \Gamma N + \lambda_f\Gamma)\bar{e} \quad \leq 0 \quad \text{if} \quad \bar{e}^TF\bar{e} > 0 \\ V(\bar{e}^+) - \lambda_j V(\bar{e}) &= \bar{e}^T(H^T\Gamma H - \lambda_j\Gamma)\bar{e} \quad \leq 0 \quad \text{if} \quad \bar{e}^TF\bar{e} \leq 0. \end{cases} \tag{34}$$

By employing the S-procedure, the conditions

$$N^T\Gamma + \Gamma N + \lambda_f\Gamma + \tau_f F < 0, \tag{35}$$

and

$$H^T\Gamma H - \lambda_j\Gamma - \tau_j F \leq 0, \tag{36}$$

are obtained which should be satisfied when $\tau_f \geq 0$ and $\tau_j \geq 0$. Herein, (36) can be rewritten through Schur complement lemma as

$$\begin{bmatrix} \lambda_j\Gamma + \tau_j F & H^T \\ H & \Gamma^{-1} \end{bmatrix} \geq 0 \tag{37}$$

which, by pre and post multiplying to $diag(I, \Gamma)$, gives

$$\begin{bmatrix} \lambda_j\Gamma + \tau_j F & H^T\Gamma \\ \Gamma H & \Gamma \end{bmatrix} \geq 0. \tag{38}$$

Replacing H in the (38) results in

$$\begin{bmatrix} \lambda_j\Gamma + \tau_j F & A_R^T\Gamma - M^T A_R^T\Gamma + M^T\Gamma \\ \Gamma A_R - \Gamma A_R M + \Gamma M & \Gamma \end{bmatrix} \geq 0 \tag{39}$$

To ensure linearity of the relations and well-posedness of the system, two other steps are required. While (35) is a linear inequality, unknown parameters are multiplied together in (39). Hence, the variable change $\Omega = \Gamma A_R$ is applied as

$$\begin{bmatrix} \lambda_j\Gamma + \tau_j F & (\Omega - \Omega M + \Gamma M)^T \\ \Omega - \Omega M + \Gamma M & \Gamma \end{bmatrix} \geq 0 \tag{40}$$

Moreover, well-posedness of the system is guaranteed by

$$(\bar{e}^+)^T F(\bar{e}^+) > 0 \quad \text{if} \quad \bar{e}^TF\bar{e} \leq 0 \tag{41}$$

which can be rewritten as

$$H^T F H + \tau_w F > 0 \ , \tag{42}$$

using the S-procedure where $\tau_w \geq 0$. It is noteworthy that (41) states that the error trajectory leaves the jump set after a jump. Consequently, by establishing (35), (40) and (42) the R-UIO estimates the faulty system accurately. ■

As it has been mentioned before, the ideal case is considered to design the matrices F, Γ and A_R. It means that if all the states are available the mentioned matrices can be obtained by solving the LMIs (31a) and (31b). But the problem with the designed R-UIO in (27) is that some of the augmented system states are not available (*e. g.* fault). Hence, the flow and jump sets, which depend on the estimation error, $\bar{e}$ are not available in general. Moreover, in this observer, the inequality (31c) should be checked a posteriori and it may not be satisfied in some cases.

3.3 Fault Estimation with R-UIO

As discussed before, in practice only some of the states can be measured and an observer must be designed to estimate the unmeasured states. To cope with this problem, it is assumed that instead of the exact error, the error bounds are available and are employed to decide about jump instants.

Suppose that a bound is known for $\bar{e}(t_0)$. It is possible to find a polytope $\mathcal{S}$ includes the boundary set for $\bar{e}(t_0)$ which $\bar{e}_{v_i}$ states its vertices. Note that N_v is the number of vertices and $i = 1, .., N_v$. Hence, $\mathcal{S} \subset \mathbb{R}^n$ is known such that $\bar{e}(t_0) \in \mathcal{S}$. Therefore, $\bar{e}(0)$ is the convex combination of known $\bar{e}_{v_i}$.

Given a vertex as the initial condition and the set of reset instants, the trajectory $(\bar{e}_{v_i}(t))$ can be computed. Looking for a criterion to guarantee the stability of the observer during reset, the convergence of $\bar{e}_{v_i}(t)$ to zero while construct a convex hull such that $\bar{e}(t) \subseteq \mathbf{conv}\{\bar{e}_{v_i}(t)\}$ for $t \in \mathbb{R}^+$ is mandatory. Hence, asymptotic stability of $\bar{e}(t)$ depends on the convergence of the $\bar{e}_{v_i}(t)$. These issues are considered in the following theorems.

Theorem 2. *Consider the observer* (27)*, the augmented dynamics of the faulty system (14) and*

$$T_R \in \{\{t_k\}_{k=0}^{\mathcal{N}} : t_k > t_{k-1}, \mathcal{N} \in \mathbb{N} \cup \{\infty\}\} \tag{43}$$

as the reset times sequence. So, the error dynamics is

$$\left\{\begin{array}{lcl} \dot{\bar{e}}(t) & = & N\bar{e}(t) \quad \textit{if} \quad t \notin T_R \\ \bar{e}(t^+) & = & H\bar{e}(t) \quad \textit{if} \quad t \in T_R \end{array}\right. \tag{44}$$

If $V(\bar{e}) = \bar{e}^T\Gamma\bar{e}$ is such that Γ satisfies (31a) and (31b), and

$$V(\bar{e}(t_k^-)) \leq (1-\epsilon)V(\bar{e}(\tau_k)) \quad \forall t_k \in T_R \tag{45}$$

where $\epsilon \in (0,1)$ and

$$\tau_k = \min\{t \in \mathbb{R}^+ | \bar{e}(t)^T F \bar{e}(t) \leq 0, \;\; t \geq t_{k-1}\}, \tag{46}$$

then asymptomatic stability of (44) can be ensured whenever $\bar{e}(t_k^-)F\bar{e}(t_k^-) \leq 0$ for all $t_k \in T_R$.

Proof. Error dynamics stability can be shown through

$$V(\bar{e}(t)) \leq \beta V(\bar{e}(t_{k-1}^+)) \qquad \forall t \in (t_{k-1}, t_k) \tag{47}$$

where

$$\beta = \max_{\bar{e}\in\varepsilon_n(\gamma)} V(\bar{e}), \tag{48}$$

$$\gamma = \max_{\bar{e}\in\varepsilon(1)} V_n(\bar{e}), \tag{49}$$

$$\varepsilon(\alpha) = \{\bar{e} \in \mathbb{R}^n | V(\bar{e}) \leq \alpha\}, \tag{50}$$

$$\varepsilon_n(\alpha) = \{\bar{e} \in \mathbb{R}^n | V_n(\bar{e}) \leq \alpha\}, \tag{51}$$

and $V_n(e) = \bar{e}(t)^T\Gamma_n\bar{e}(t)$ is a Lyapunov function for the nominal system and $\dot{V}_n \leq -\lambda_n V_n$ holds for $\lambda_n \geq 0$. Moreover, if $V(\bar{e}(t))$ satisfies

$$V(\bar{e}(t_k^+)) \leq \lambda_j(1-\epsilon)V(\bar{e}(t_{k-1}^+) \tag{52}$$

asymptotic stability is ensured.

Considering $\varepsilon(\beta) \geq \varepsilon_n(\gamma) \geq \varepsilon(1)$ consistent with (48- 51) and $\dot{V}_n \leq -\lambda_n V_n$, any trajectory starting in $\varepsilon_n(\gamma)$ at $t = t_0$ stays in $\varepsilon_n(\gamma)$ for all $t \geq t_0$ while flowing.

To prove (47), note that from the definition of β and γ it can be inferred that $\varepsilon(\beta) \geq \varepsilon_n(\gamma) \geq \varepsilon(1)$. Since $\dot{V}_n \leq -\lambda_n V_n$, any trajectory starting in $\varepsilon_n(\gamma)$ at $t = t_0$ stays in $\varepsilon_n(\gamma)$ for all $t \geq t_0$ while flowing. Therefore, it remains in $\epsilon(\beta)$ too and between two jumps cannot leave it. As a result, the function V may increase β times during the flow but not more.

After a jump, because of the (31b), V is decreasing again, and consequently, the error cannot go further than $\varepsilon(\beta)$ and remains bounded when starting in $\varepsilon(1)$. Due to the homogeneity, this reasoning can be extended to other level sets leading to (47) when flowing.
If the error trajectory is in the flow set after the jump, one can obtain

$$V(\bar{e}(\tau_k)) \leq \bar{e}^{-\lambda_f(\tau_k - t_{k-1})} V(\bar{e}(t_{k-1}^+)), \tag{53}$$

through (33) and (46) in $t \in (t_{k-1}^+, \tau_k)$. On the other hand,

$$V(\bar{e}(t_k^+)) \leq \lambda_j V(\bar{e}(t_k^-)) \qquad t \in (\tau_k, t_k^-) \tag{54}$$

and

$$V(\bar{e}(t_k^+)) \leq \lambda_j(1-\epsilon) V(\bar{e}(\tau_k)) \tag{55}$$

can be obtained respectively by considering (31b), (33) and (45). Finally, putting together (55) and (53)

$$\begin{aligned} V(\bar{e}(t_k^+)) &\leq \lambda_j(1-\epsilon) V(\bar{e}(\tau_k)) \leq \lambda_j(1-\epsilon)\bar{e}^{-\lambda_f(\tau_k - t_{k-1})} V(\bar{e}(t_{k-1}^+)) \\ &\leq \lambda_j(1-\epsilon) V(\bar{e}(t_{k-1}^+)) \end{aligned} \tag{56}$$

can be realized since $\lambda_f > 0$.

In some cases $\bar{e}(t)$ remains in the jump sector after the jump and $\tau_k = t_{k-1}^+$. So (56) simplifies to

$$V(\bar{e}(t_k^+)) \leq \lambda_j(1-\epsilon) V(\bar{e}(t_{k-1}^+)) \tag{57}$$

since $\bar{e}^{-\lambda_f(\tau_k - t_{k-1})} = 1$.

It is noteworthy that in the first aforementioned case for the error trajectory, $\bar{e}_{t_k-1}^+$ is in the flow set after the jump and there is no Zeno solution while, in the latter case there could be Zeno solution. However, since t_k^- is greater than τ_k according to (45), Zeno solution does not happen as there is always a flow before the new jump. In other words, inequality (45) guarantees that a positive time interval of flow exists before the next jump which means that there is no Zeno solution and the system is well-posed.

■

Consequently, asymptotic stability of $\bar{e}(t)$ is guaranteed by choosing t_k such that $\bar{e}(t_k^-)\Gamma\bar{e}(t_k^-) \leq 0$ and $V(\bar{e}(t_k^-)) \leq V(\bar{e}(\tau_k))$. Meanwhile, fault estimation is possible through generating trajectories according to their known bounds and initial condition vertices. So, it is necessary to satisfy $e(t) \subseteq$ $\mathbf{conv}\{e_{v_i}(t)\}$ which is addressed as follow.

Theorem 3. *Let V a Lyapunov function satisfying* (31a) *and* (31b) *and considering the known polytope for $\bar{e}(t_0)$, the R-UIO for the faulty system* (44) *is asymptotically stable with the reset times sequence T_R if*

$$V(\bar{e}_{v_i}(t_k^-)) \leq (1-\epsilon)V(\bar{e}_{v_i}(\tau_{k_i})) \quad \forall t_k \in T_R, \epsilon > 0 \quad i = 1, ..., N_v \tag{58}$$

where
$\tau_{k_i} = \min\{t \in \mathbb{R}^+ | \bar{e}_{v_i}(t)^T F \bar{e}_{v_i}(t) \leq 0, \quad t \geq t_{k-1}\}$.

Proof. Since $\bar{e}(0)$ is a convex combination of $\bar{e}_{v_i}$, using the solution of the equation $\dot{\bar{e}}(t) = N\bar{e}(t)$, for $t \in [t_{k-1}, t_k)$ and $t \notin T_R$, $\bar{e}(t)$ is also a convex combination of the error trajectories as

$$\bar{e}(t) = exp(Nt)\bar{e}(t_{k-1}^+) = exp(Nt)(\Sigma_{i=1}^{N_{v_i}}(\alpha_{v_i}\bar{e}_{v_i}(t_{k-1}^+)).$$

and if $t = t_k \in T_R$,

$$\bar{e}(t_k^+) = H\bar{e}(t_k^-) = H(\Sigma_{i=1}^{N_{v_i}}(\alpha_{v_i}(exp(Nt)\bar{e}_{v_i}(t_k^-))).$$

Since $\bar{e}(t)$ is in the convex hull of the $\bar{e}_{v_i}(t)$ which converges to zero according to the Theorem 2, $\bar{e}(t)$ also converges to zero. ∎

Consequently, accurate fault estimation is possible by the proposed R-UIO since all the jumps occurs according to (58) at t_k for all vertex trajectories and the error dynamics is asymptotically stable.

4 Application of R-UIO in fault estimation

In this section, the effectiveness of the presented augmented descriptor approach R-UIO is illustrated through a common aircraft model (see [69]). The parameters of this model according to (14) and (3) are as follows:

$$A = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.49 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},$$

and $E_f = B$. The system states are horizontal velocity V_h, vertical velocity V_v, pitch rate q, pitch angle θ and the inputs are collective pitch control δ_c and longitudinal cyclic pitch control δ_l. It can be verified that the pair (A, C) is observable and (CD) is full column rank.

We consider actuator faults which usually occur in the input channel, so we assume $E = B$. For such system, there are two fault channels defined as $f(t) = [f_1(t),\ f_2(t)]^T$. Meanwhile, since augmented descriptor approach is employed for the proposed R-UIO,

Case I: Constant faults

A constant fault is considered for simulation, that occurs in the first input channel, which is defined as

$$f_1 = \begin{cases} 0, & 0s \leq t \leq 5s \\ 0.8, & 5s \leq t \leq 9s \\ 0.5, & 9s \leq t \leq 15s \end{cases}, f_2 = 0. \tag{59}$$

while

$$\bar{T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using the pole placement method with poles $\{-20 + 0.05j,\ -20 - 0.05j,\ -2.6,\ -4.1,\ -3.2,\ -1.3\}$, the observer gain is obtained as:

$$K = \begin{bmatrix} -2.4424 & -2.7120 & 5.7331 \\ 33.1374 & 17.0394 & -20.9103 \\ -56.8481 & -24.6372 & 24.1962 \\ -19.9793 & -6.9352 & 14.6551 \\ 48.2016 & 8.8524 & -21.1776 \\ -13.2572 & -8.8499 & 8.5896 \end{bmatrix}$$

and by using equations (21-26), the observer parameters can be calculated as:

$$\bar{M} = \begin{bmatrix} 0.6667 & -0.3333 & 0 & -0.3333 & 0 & 0 \\ -0.3333 & 0.6667 & 0 & -0.3333 & 0 & 0 \\ -0.3333 & -0.3333 & 1.0000 & -0.3333 & 0 & 0 \\ -0.3333 & -0.3333 & 0 & 0.6667 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$\bar{E} = \begin{bmatrix} -0.3333 & -0.3333 & -0.3333 \\ -0.3333 & -0.3333 & -0.3333 \\ -0.3333 & -0.3333 & -0.3333 \\ -0.3333 & -0.3333 & -0.3333 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} G = \begin{bmatrix} -0.8867 & 2.6481 \\ 2.2157 & -5.1202 \\ -6.8489 & 6.9620 \\ -1.3289 & 2.4720 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} -21.5780 & 1.2985 & -2.0089 & 5.7634 & -0.8867 & 2.6481 \\ 4.9558 & -14.9601 & 3.2811 & 2.9510 & 2.2157 & -5.1202 \\ 68.8676 & 26.6074 & -6.4696 & -37.0108 & -6.8489 & 6.9620 \\ 5.9066 & 6.2693 & -1.2721 & -8.1923 & -1.3289 & 2.4720 \\ -48.2016 & -8.8524 & 0 & 21.1776 & 0 & 0 \\ 13.2572 & 8.8499 & 0 & -8.5896 & 0 & 0 \end{bmatrix}$$

with these parameters, the design of optimal C-UIO is completed.

Now, to obtain the matrices P, F and A_R, the ideal R-UIO should be designed by solving the inequalities (31a) and (31b) of Theorem 1. It is worth noting that, λ_f, λ_j, τ_f and τ_j are unknown and result in multiplication of parameters. Therefore, to solve these inequalities, a change of variable is used to remove one of them. Consider $\tau_f F = \bar{F}$ thus, $\tau_j F$ can be replaced with $\frac{\tau_j}{\tau_f}\bar{F} = \bar{\tau}_j \bar{F}$. It is the same as letting $\tau_f = 1$ and solving the inequalities. To deal with the other nonlinearities, a grid is considered for λ_f, λ_j and τ_j, then the inequalities are solved at each point of the grid to obtain a feasible solution. Let the $\lambda_f = 2.1$, $\lambda_j = 0.9$,$\tau_j = 1$ and in this case, the related parameters will be obtained as:

$$F = \begin{bmatrix} 0.5663 & -0.6485 & -0.0805 & 0.0516 & 0.6827 & -0.3850 \\ -0.6485 & -0.2223 & -0.1281 & 0.0937 & -0.1544 & -0.4076 \\ -0.0805 & -0.1281 & 0.4233 & 0.0458 & 0.1055 & -0.4454 \\ 0.0516 & 0.0937 & 0.0458 & 2.0360 & -0.0426 & 0.2129 \\ 0.6827 & -0.1544 & 0.1055 & -0.0426 & -0.0120 & -0.3995 \\ -0.3850 & -0.4076 & -0.4454 & 0.2129 & -0.3995 & -0.0164 \end{bmatrix}$$

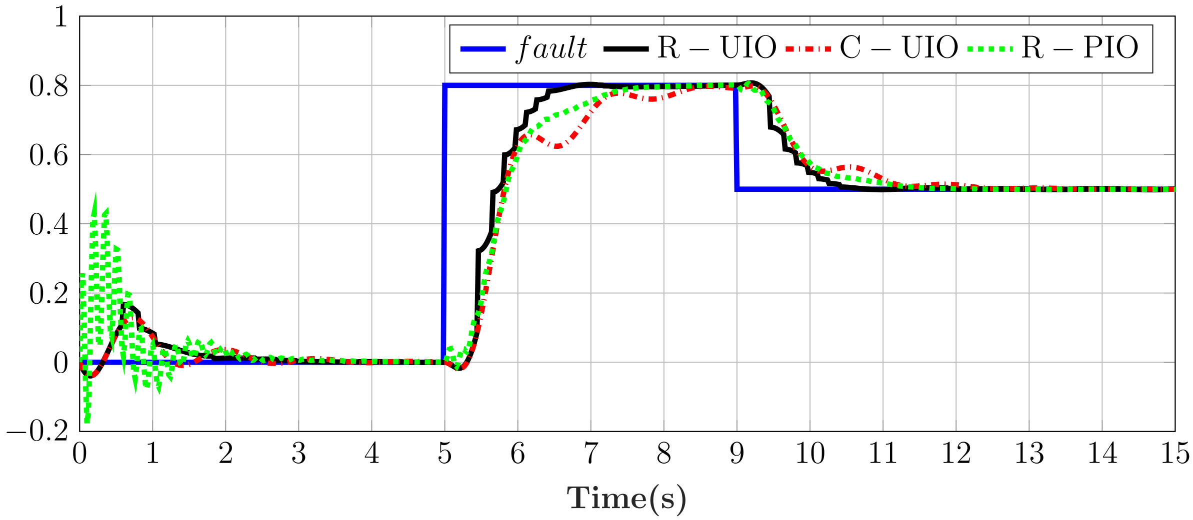
$$P = \begin{bmatrix} 1.7658 & 0.5720 & 0.1069 & -0.8714 & -0.5144 & 0.3856 \\ 0.5720 & 0.5605 & 0.1975 & -0.2376 & 0.1567 & 0.3964 \\ 0.1069 & 0.1975 & 0.2087 & -0.2037 & 0.2503 & 0.0937 \\ -0.8714 & -0.2376 & -0.2037 & 1.2642 & 0.1122 & -0.3370 \\ -0.5144 & 0.1567 & 0.2503 & 0.1122 & 0.6601 & 0.1803 \\ 0.3856 & 0.3964 & 0.0937 & -0.3370 & 0.1803 & 0.7403 \end{bmatrix}$$

$$A_R = \begin{bmatrix} 0.6499 & 0 & 0 & 0 & 0 & 0 \\ -1.3673 & 0 & 0 & 0 & 0 & 0 \\ 1.0401 & 0 & 0 & 0 & 0 & 0 \\ 0.7172 & 0 & 0 & 0 & 0 & 0 \\ -0.0064 & 0 & 0 & 0 & 0 & 0 \\ 0.0136 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

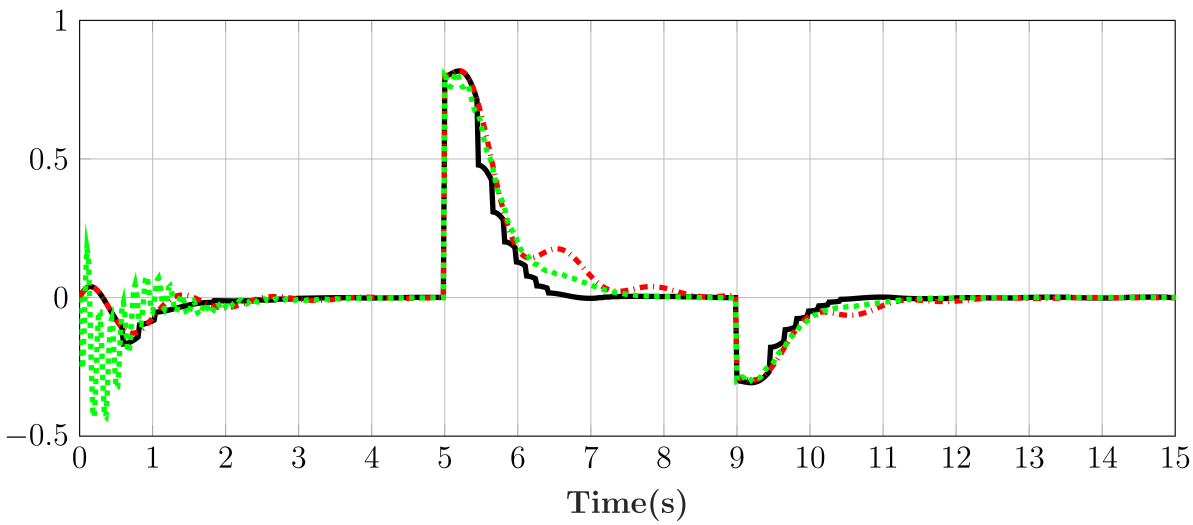
Remark 2. *The matrix F should be chosen such that it is neither positive definite nor negative definite in order to represent a sector.*

The result of estimation with the initial conditions $x_1 = -0.4, x_2 = 0.5, x_3 \in [-0.8, 0.8], x_4 = -0.6$ can be seen in the Figure 1. The result of the proposed method R-UIO is compared with the conventional method C-UIO for fault estimation. It can be seen that, the R-UIO fault estimation outperforms the C-UIO and estimates the fault more rapidly and accurately. Figure 1 shows that R-UIO has considerable better performance in fault estimation than Reset PI observer (R-PIO) suggested in [65].

From the Figure 1 it can be deduced that in the interval $5 \leq t \leq 9$ the R-UIO has a faster estimation and estimates the fault more rapidly than C-UIO. Similarly, in the interval $9 \leq t \leq 15$ R-UIO estimates the fault more quickly and smoothly.



(a) Constant fault estimation



(b) Constant fault estimation error

Figure 1: Constant fault estimation and estimation error

Case II: Ramp faults
In this case, a ramp fault which occurs in the first input channel is con-

sidered as

$$f_1 = \begin{cases} 0, & 0s \le t \le 5s \\ 0.1t, & 5s \le t \le 10s \\ 0.1(t-4), & 10s \le t \le 15s \end{cases}, f_2 = 0. \tag{60}$$

Similarly using pole placement method to place the observer poles at $\{-1+5j, -1-5j, -1.6-1.1, -1.2, -1.3, -1.4, -1.5\}$, the observer parameters can be obtained as:

$$K = \begin{bmatrix} -23.2165 & -1.0153 & 10.2956 \\ 40.8857 & 1.2849 & -18.9845 \\ 1.3113 & 13.5236 & -5.3667 \\ -16.2792 & 1.1306 & 10.0837 \\ 4.1879 & -5.8237 & -4.2790 \\ -0.3987 & -0.0497 & 4.1563 \\ 1.9846 & -3.6026 & -1.7315 \\ 0.1687 & 0.4465 & 2.5425 \end{bmatrix}$$

and using equation (15), the observer parameters can be calculated as:

$$\bar{M} = \begin{bmatrix} 0.6667 & -0.3333 & -0.0000 & -0.3333 & 0 & 0 & 0 & 0 \\ -0.3333 & 0.6667 & -0.0000 & -0.3333 & 0 & 0 & 0 & 0 \\ -0.3333 & -0.3333 & 1.0000 & -0.3333 & 0 & 0 & 0 & 0 \\ -0.3333 & -0.3333 & 0 & 0.6667 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$\bar{E} = \begin{bmatrix} 0.3333 & 0.3333 & 0.3333 \\ 0.3333 & 0.3333 & 0.3333 \\ 0.3333 & 0.3333 & 0.3333 \\ 0.3333 & 0.3333 & 0.3333 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} G = \begin{bmatrix} -0.8867 & 2.6481 \\ 2.2157 & -5.1202 \\ -6.8489 & 6.9620 \\ -1.3289 & 2.4720 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} -0.8039 & -0.3982 & -2.0089 & 1.2009 & -0.8867 & 2.6481 & 0 & 0 \\ -2.7925 & 0.7944 & 3.2811 & 1.0252 & 2.2157 & -5.1202 & 0 & 0 \\ 10.7081 & -11.5533 & -6.4696 & -7.4479 & -6.8489 & 6.9620 & 0 & 0 \\ 2.2064 & -1.7965 & -1.2721 & -3.6209 & -1.3289 & 2.4720 & 0 & 0 \\ -4.1879 & 5.8237 & 0 & 4.2790 & 0 & 0 & 1.0000 & 0 \\ 0.3987 & 0.0497 & 0 & -4.1563 & 0 & 0 & 0 & 1.0000 \\ -1.9846 & 3.6026 & 0 & 1.7315 & 0 & 0 & 0 & 0 \\ -0.1687 & -0.4465 & 0 & -2.5425 & 0 & 0 & 0 & 0 \end{bmatrix}$$

with these parameters, the design of optimal C-UIO is completed.

Now, to obtain reset observer parameters let the $\lambda_f = 2.1$, $\lambda_j = 1$, $\tau_j = 1$ and solve the inequalities (31a) and (31b) of Theorem 1. Thus the unknowns are calculated as:

$$F = \begin{bmatrix} 3.1857 & -0.1499 & -0.8131 & 0.6180 & -1.2404 & 0.7968 & 0.0113 & 0.0268 \\ -0.1499 & -0.0499 & 0.0575 & -0.2176 & 0.0602 & 0.0066 & -0.0156 & 0.0205 \\ -0.8131 & 0.0575 & 0.6536 & -0.6327 & 0.6169 & -0.5705 & 0.0215 & -0.0091 \\ 0.6180 & -0.2176 & -0.6327 & 2.5132 & -0.5321 & 0.0037 & 0.0013 & 0.1421 \\ -1.2404 & 0.0602 & 0.6169 & -0.5321 & 1.1451 & -0.3809 & -0.0564 & -0.0195 \\ 0.7968 & 0.0066 & -0.5705 & 0.0037 & -0.3809 & 1.6119 & -0.0443 & -0.0623 \\ 0.0113 & -0.0156 & 0.0215 & 0.0013 & -0.0564 & -0.0443 & 0.1686 & 0.0491 \\ 0.0268 & 0.0205 & -0.0091 & 0.1421 & -0.0195 & -0.0623 & 0.0491 & 0.2065 \end{bmatrix}$$

$$P = \begin{bmatrix} 1.4798 & -0.1868 & -0.4842 & 0.3129 & -0.6734 & 0.6485 & 0.0278 & -0.2820 \\ -0.1868 & 1.2514 & 0.3614 & 0.5177 & 0.5266 & -0.1797 & 0.1431 & -0.1388 \\ -0.4842 & 0.3614 & 0.4557 & -0.2313 & 0.3915 & -0.4502 & 0.1054 & 0.0913 \\ 0.3129 & 0.5177 & -0.2313 & 2.0030 & -0.0451 & -0.2288 & 0.1326 & -0.7148 \\ -0.6734 & 0.5266 & 0.3915 & -0.0451 & 0.7823 & -0.2760 & -0.2327 & 0.1250 \\ 0.6485 & -0.1797 & -0.4502 & -0.2288 & -0.2760 & 1.3579 & -0.1997 & -0.3342 \\ 0.0278 & 0.1431 & 0.1054 & 0.1326 & -0.2327 & -0.1997 & 0.6182 & -0.0828 \\ -0.2820 & -0.1388 & 0.0913 & -0.7148 & 0.1250 & -0.3342 & -0.0828 & 1.2349 \end{bmatrix}$$

Table 1: Fault estimation SSE and Settling time comparison

Method	constant fault		Ramp fault	
	SSE	T_{stl}	SSE	T_{stl}
R-UIO	20.7728	1.4	22.7273	2.04
C-UIO	24.4217	3.2	44.2249	3.76

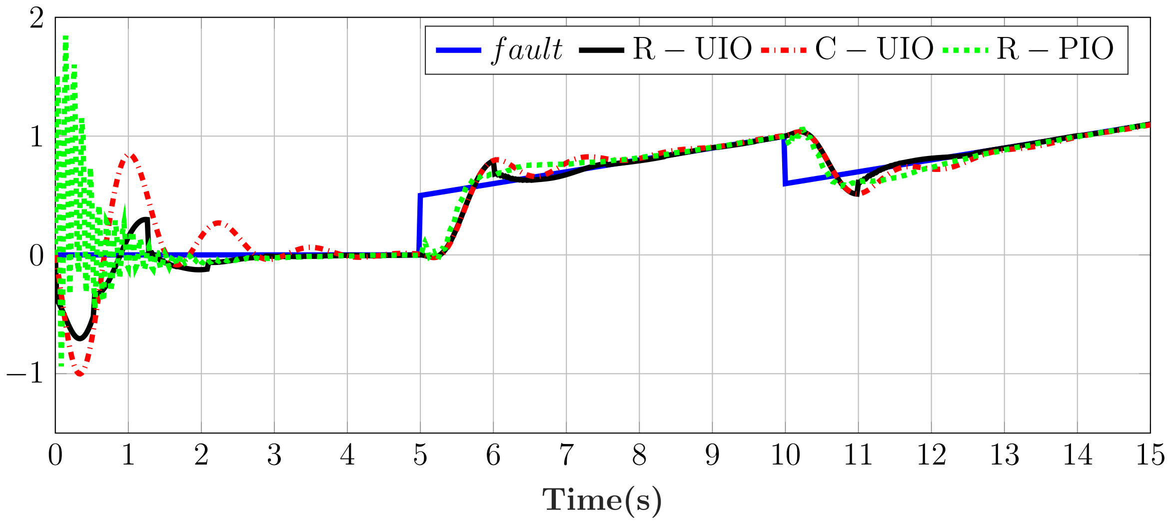
$$A_R = \begin{bmatrix} 0.8589 & 0\,0\,0\,0\,0\,0\,0 \\ -1.7119 & 0\,0\,0\,0\,0\,0\,0 \\ 1.1294 & 0\,0\,0\,0\,0\,0\,0 \\ 0.8402 & 0\,0\,0\,0\,0\,0\,0 \\ -0.0123 & 0\,0\,0\,0\,0\,0\,0 \\ 0.0874 & 0\,0\,0\,0\,0\,0\,0 \\ 0.0021 & 0\,0\,0\,0\,0\,0\,0 \\ -0.0120 & 0\,0\,0\,0\,0\,0\,0 \end{bmatrix}.$$

As can be seen in the Figure 2a, the fault estimation using the proposed method results in a more rapid and accurate estimation of the fault. Moreover, the fault estimation using R-UIO has a better transient response than the other two methods. The estimation error is shown in Figure 2b to depict the efficiency of using R-UIO.

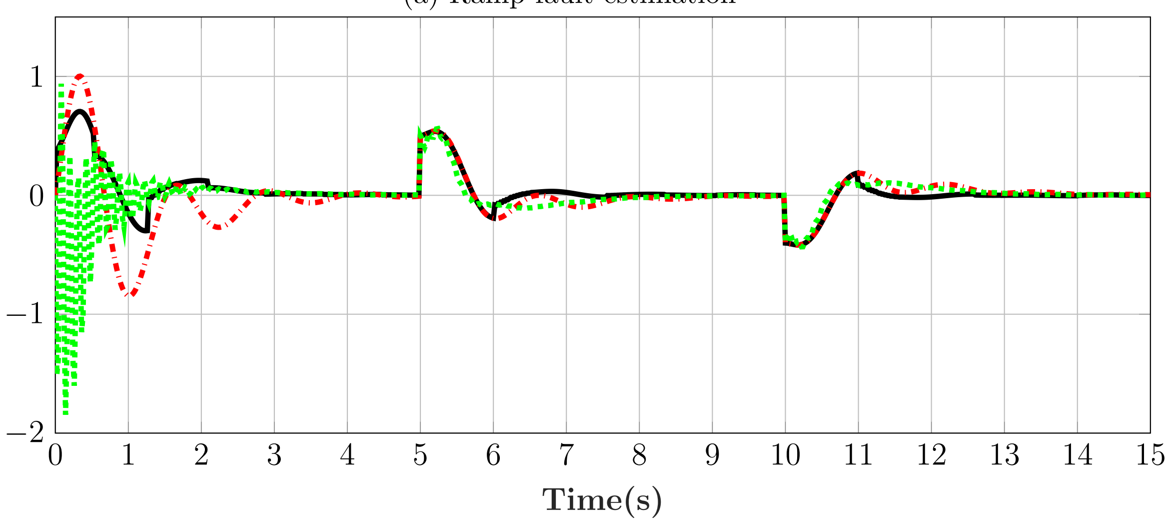
Furthermore, the comparison of the performance measure Sum of Square Error (SSE) $\sum(f-\hat{f})^2$ and the settling time (2%) (The time required for the response curve to reach and stay within a range of 2 percentage of the final value) in both constant and ramp faults are provided in Table 1. From this table, it can be concluded that the R-UIO provides the best fault estimation among the other observers.

Case III: Sensor fault

In this case, the system can be modeled as in equation (9). It is supposed that the previously described aircraft dynamics is subject to a sensor fault as in f_1 in equation (59) with $E_f = [1, 0, 0]^T$. The reset observer (27) for this system is designed and the results can be seen in Figure 3a and 3b. In Figure 3a, the estimation of a constant sensor fault using the proposed method and the conventional method is depicted. As it can be seen, the fault estimation using the designed R-UIO is more accurate and converges more rapidly. Moreover, the estimation error in Figure 3b shows that the estimation obtained from the devised approach is smoother than the C-UIO approach and less oscillating. For the sake of brevity, the obtained parameters are not



(a) Ramp fault estimation



(b) Ramp fault estimation error

Figure 2: Ramp fault estimation and estimation error

Table 2: Sensor fault estimation SSE and Settling time comparison

Method	constant fault	
	SSE	T_{stl}
R-UIO	11.98	0.8
C-UIO	21.55	2.9

shown here.

In Table 2, a numerical comparison between the two approaches is conducted. The Table shows that the there is a significant improvement in both SSE and settling time of estimation error. This shows the capabilities of the proposed method.

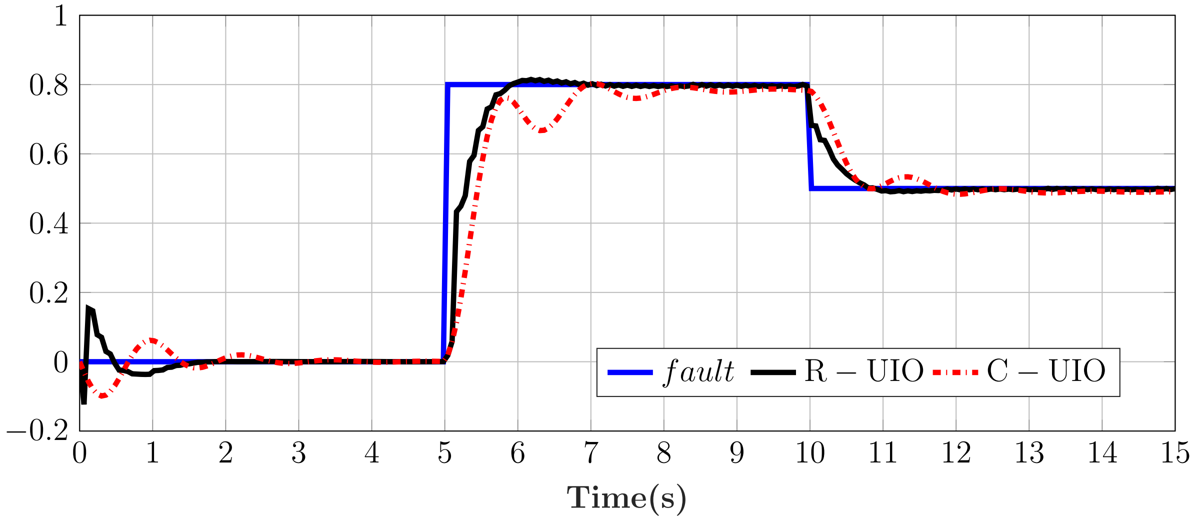
Three important conclusion can be summarized:

a) Resetting action can improve the fault estimation settling time.

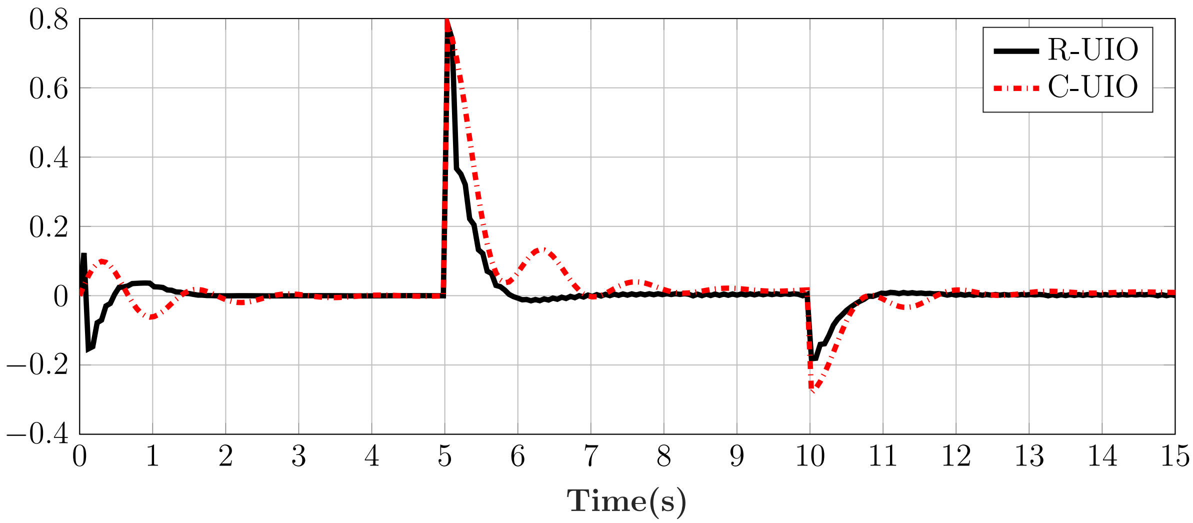
b) R-UIO provides estimation with better transient response with less oscillation than the other method.

c) The improvement in the case of time-varying fault is much more serious. Constant actuator fault estimation using R-UIO in comparison with the C-UIO is about 15% and in the case of the ramp fault is about 50%. The improvement regarding R-PIO is about 12% and 28% for constant and ramp fault respectively.

Remark 3. *Since the R-UIO estimation error converges more rapridly and smoothly, one can expect that the the performance of an observer based controller through R-UIO is more accurate with less control effort in comparison with C-UIO.*



(a) Sensor fault estimation



(b) Sensor fault estimation error

Figure 3: Sensor fault estimation and estimation error

5 Conclusion

In this paper, an application of Reset Unknown Input Observer for fault estimation in an Augmented System with Descriptor modeling is investigated. In the designed observer, the states are reset to a suitable value based on a time-dependent reset law. Both sensor and actuator fault is considered and the augmented system is constructed by considering the fault as an auxiliary state and rewrite the system model in a new representation. Then, the descriptor approach R-UIO based on the augmented system is designed to estimate the faults. Furthermore, we exploited an aircraft model as a simulation example to demonstrates the efficiency of using the R-UIO for fault estimation and reducing the estimation error. The results of the proposed method are compared with a C-UIO and it can be seen that the R-UIO can estimate the fault more rapidly and accurately. Moreover, using the R-UIO leads to an improvement of 15% in actuator constant fault estimation, 50% in actuator ramp fault estimation, and 44% in sensor constant fault estimation. In the future, from theortitical point of view linear parameter varying systems, nonlinear after reset valuse and developing adaptive R-UIO should be investigated. On the other hand, finding the R-UIO tuning parameters such as τ_j, τ_w based on a systematic approach should be followed.

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