Self-triggered coordination with ternary controllers ⋆

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Abstract: This paper regards coordination of networked systems with ternary controllers. We develop a hybrid coordination system which implements a self-triggered communication policy, based on polling the neighbors upon need. We prove that the proposed scheme ensures finite-time convergence to a neighborhood of the consensus state. We estimate both the size of the error neighborhood, and the time and communication costs, as functions of the sensitivity of the quantizer: our analysis highlights natural trade-offs between accuracy of consensus and costs. In the same hybrid self-triggered framework, we also design a time-varying controller which asymptotically drives the system to consensus.

Keywords: Event-based control, coordination, ternary controllers, hybrid systems

1. INTRODUCTION

The key issue in distributed and networked systems resides in ensuring performance with respect to a given control task (e.g. stability, coordination), in face of possibly severe communication constraints. In practice, although the system may be naturally described by a continuous-time dynamics, the control law is only updated at discrete time instants: these can either be pre-specified (time-scheduled control), or be determined by certain events that are triggered depending on the system's behavior. It is thus natural to ask under which conditions event-based control and communication policies are preferable with respect to time-scheduled ones. In a networked system, controls and triggering events regarding an agent must only depend on the states (or the outputs) of the agent's neighbors. Actually, of special interest in distributed systems are self-triggered policies, in which communication and control actions are decided on the basis of criteria which are specific to each agent alone. Indeed, the implementation of an event-based policy which requires continuous monitoring of a condition depending the agent's neighbors may not be suitable.

Summary and statement of contribution

In this paper we design a self-triggered continuous-time consensus system. At each sampling time, a certain subset of "active" agents poll their neighbors obtaining relative measurements of the consensus variable of interest and, using the available information, update their controls and compute their next update time. Controls are constrained to belong to $\{-1, 0, +1\}$: the assumption of such coarsely quantized controllers is motivated by methodological and opportunity reasons. On one hand, we are interested in demonstrating the effectiveness of ternary controllers for self-triggered coordination. On the other hand, using constrained controllers provides implicit information on the agent dynamics, which can be effectively exploited in designing a self-triggering policy.

Our modeling and design approach naturally leads to an hybrid system which is defined in Section 2. Next, in Section 3 we prove, by a Lyapunov analysis, that the hybrid system converges in finite-time to a condition of "practical consensus": that is, solution are within a neighborhood of the consensus point, and the size of the neighborhood can be made arbitrarily small by decreasing a certain parameter of the quantizer. This parameter, which we denote by ε and is defined in (1) below, represents the *sensitivity* of the quantizer: the smaller ε, the more the system is demanding in terms of communication resources. We thus identify a trade-off between communication and coordination performance. Furthermore, in Section 4 we show that a suitable time-varying controller, which is designed as a modification of the above model, can asymptotically drive the system to a consensus state. x

Related literature

Since the seminal work by Åström and Bernhardsson (2002), the control community has been interested in investigating event-based control policies: recent examples in networked systems include Tabuada (2007), Wang and Lemmon (2011), Mazo and Tabuada (2011), De Persis

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et al. (2011), and Blind and Allgöwer (2011). The work by Ceragioli et al. (2011) is also related, as it presents a hybrid coordination dynamics requiring (quantized) communication only when specific thresholds are met. This last paper (as many others in the literature about coordination) uses uniform quantization: instead, ternary controllers are used to stabilize consensus in our present work and in Cortés (2006). In a centralized setting, the use of ternary controllers in connection with quantized communication has been investigated in De Persis (2009).

Recent relevant work includes the solution of practical consensus problems using self-triggered broadcast communication in Seyboth et al. (2011). Compared to this reference, the present manuscript proposes a different communication policy, which is based on polling the neighbors upon need, instead than on broadcasting to them.

Self-triggered policies have also been used for deployment of robotic networks in Nowzari and Cortés (2010): in this paper, the authors exploit the knowledge of the speed of the deploying robots in order to design the self-triggering policy. A similar idea features in our present work.

2. SYSTEM DEFINITION AND MAIN RESULTS

We assume to have a set of nodes $I = \{1, \ldots, n\}$ and an undirected[1] graph $G = (I, E)$ with $E \subset I \times I$. We denote by L the Laplacian matrix of G, which is a symmetric matrix. For each node $i \in I$, we denote its degree by d_i and the set of its neighbors by $\mathcal{N}_i$.

We consider the following hybrid dynamics on a triplet of n-dimensional variables involving the *consensus variable* x, the *controls* u, and the *local time* variables θ. All these variables are defined for time $t \geq 0$. Controls are assumed to belong to $\{-1, 0, +1\}$. The specific quantizer of choice is $\mathrm{sign}_\varepsilon : \mathbb{R} \to \{-1, 0, 1\}$, defined according to

$$\mathrm{sign}_\varepsilon(z) = \begin{cases} \mathrm{sign}(z) & \text{if } |z| \geq \varepsilon \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $\varepsilon > 0$ is a *sensitivity* parameter.

The system $(x, u, \theta) \in \mathbb{R}^{3n}$ satisfies the following continuous evolution

$$\begin{cases} \dot{x}_i = u_i \\ \dot{u}_i = 0 \\ \dot{\theta}_i = 0 \end{cases} \tag{2}$$

except for every t such that the set $\mathcal{I}(\theta, t) = \{i \in I \;:\; \theta_i = t\}$ is non-empty. At such time instants the system satisfies the following discrete evolution

$$\begin{cases} x_i(t^+) = x_i(t) \quad \forall i \in I \\ u_i(t^+) = \begin{cases} \mathrm{sign}_\varepsilon\,(\mathrm{ave}_i(t)) & \text{if } i \in \mathcal{I}(\theta, t) \\ u_i(t) & \text{otherwise} \end{cases} \\ \theta_i(t^+) = \begin{cases} \theta_i(t) + f_i(x(t)) & \text{if } i \in \mathcal{I}(\theta, t) \\ \theta_i(t) & \text{otherwise} \end{cases} \end{cases} \tag{3}$$

where for every $i \in I$ the map $f_i : \mathbb{R}^n \to \mathbb{R}_{>0}$ is defined by

$$f_i(x) = \begin{cases} \dfrac{|\sum_{j \in \mathcal{N}_i} (x_j - x_i)|}{4d_i} & \text{if } |\sum_{j \in \mathcal{N}_i} (x_j - x_i)| \geq \varepsilon \\ \dfrac{\varepsilon}{4d_i} & \text{otherwise} \end{cases} \tag{4}$$

and for brevity of notation we let

$$\mathrm{ave}_i(t) = \sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)).$$

We note that each variable θ_i naturally defines a sequence of local switching times $\{t^i_k\}_{k \in \mathbb{Z}_{\geq 0}}$. Initial conditions can be chosen as $x(0) = \bar{x} \in \mathbb{R}^n$, $u(0) = 0$, $\theta(0) = 0$. With this choice of initial conditions, we note that $\mathcal{I}(0, 0) = I$, that is, every agent undergoes a discrete update at the initial time: $t^i_0 = 0$ for every $i \in I$. The model (2)-(3) describes the following protocol, which is implemented by each agent i to collect information and compute the control law:

Protocol A

initialization: for all $i \in I$, set $t^i_0 = 0$ and $u_i(0) = 0$;
for all $k \in \mathbb{Z}_{\geq 0}$ **do**
  for all $i \in I$ **do**
    if $t = t^i_k$ **then**
      i polls each neighbor $j \in \mathcal{N}_i$ and collects the information $x_j(t^i_k) - x_i(t^i_k)$;
      i computes $\mathrm{ave}_i(t^i_k)$;
      i computes the next sampling time
      $$t^i_{k+1} = t^i_k + f_i(x(t^i_k));$$
      i computes the next control value $u_i(t^{i+}_k)$ by (3);
    end if
    while $t \in (t^i_k, t^i_{k+1})$ **do**
      i applies the control $u_i(t^{i+}_k)$;
    end while
  end for
end for

In the next section we shall prove the following convergence result:

Theorem 1. (Practical consensus). For every initial condition $\bar{x}$, let $x(t)$ be the solution to (2)-(3) such that $x(0) = \bar{x}$. Then $x(t)$ converges in finite time to a point x^* belonging to the set

$$\mathcal{E} = \{x \in \mathbb{R}^n \;:\; |\sum_{j \in \mathcal{N}_i} (x_j - x_i)| < \varepsilon \;\forall\, i \in I\}.$$

This result can be seen as a "practical consensus" result, as the size of the consensus error can be be made as small as needed by choosing ε. Moreover, we can estimate the time and communication costs of the system, as follows:

Proposition 2. (Time and communication costs). Let $x(\cdot)$ be the solution to system (2)-(3). Define the *time cost* $T = \inf\{t \geq 0 \;:\; x(t) \in \mathcal{E}\}$ and the *communication cost* $C = \max_{i \in I} \max\{k \;:\; t^i_k \leq T\}$. Then,

$$T \leq \frac{2(1 + d_{\max})}{\varepsilon} \sum_{(i,j) \in E} (\bar{x}_i - \bar{x}_j)^2$$

and

$$C \leq \frac{8 d_{\max}(1 + d_{\max})}{\varepsilon^2} \sum_{(i,j) \in E} (\bar{x}_i - \bar{x}_j)^2,$$

where $\bar{x} \in \mathbb{R}^n$ is the initial condition.

[1] We note that this assumption entails communication in both directions between pairs of connected nodes. However, our communication protocol –described below– does not require synchronous bidirectional communication.

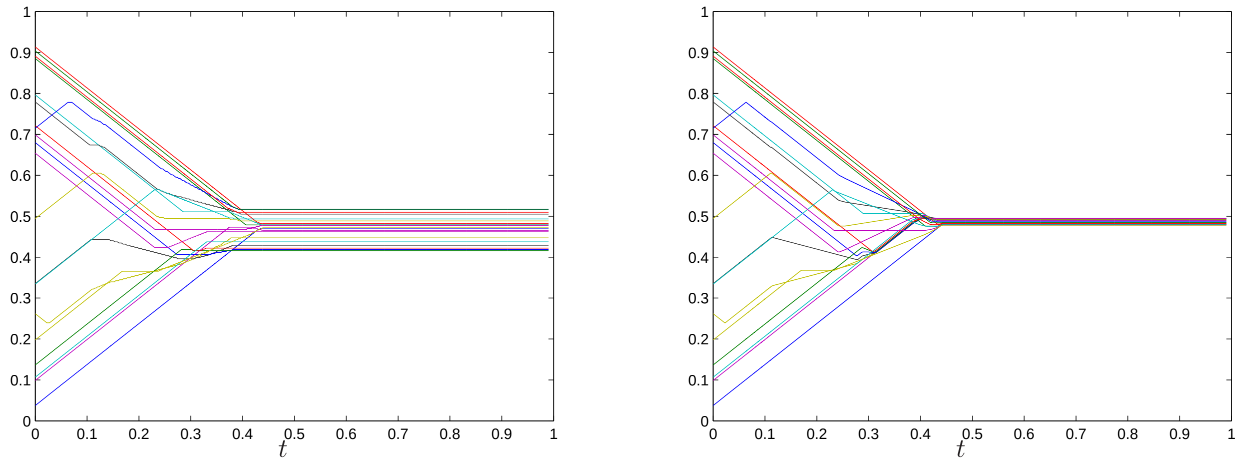

Fig. 1. Two sample evolutions of the states x in (2)-(3) starting from the same initial condition and on the same graph (a ring graph with $n = 20$ nodes). Left plot assumes $\varepsilon = 0.01$, right plot assumes $\varepsilon = 0.001$.

Since each polling action involves polling at most $d_{\max}$ neighbors, we also conclude that the total number of messages to be exchanged in the whole network in order to achieve (practical) consensus is not larger than

$$\frac{8d_{\max}^2(1+d_{\max})n}{\varepsilon^2}\sum_{(i,j)\in E}(\bar{x}_i-\bar{x}_j)^2.$$

Our theoretical results suggest that, by choosing the sensitivity ε, we are trading between precision and cost, both in terms of time and of communication effort. However, simulations indicate that the role of ε in controlling the speed of convergence is limited, as long as $x(t)$ is far from $\mathcal{E}$. Before approaching the limit set, solutions are qualitatively similar to the solutions of consensus dynamics with controls in $\{-1,+1\}$: this remark is confirmed if we compare Fig. 1 with (Cortés, 2006, Fig. 1 (rightmost)). Consistently, Fig. 2 demonstrates that the state trajectories "brake", and the controls switch between zero and non-zero, as the states approach the region of convergence. Once this is reached (in finite time), the controls stop switching and remain constantly to zero, as the analysis in the next section shows.

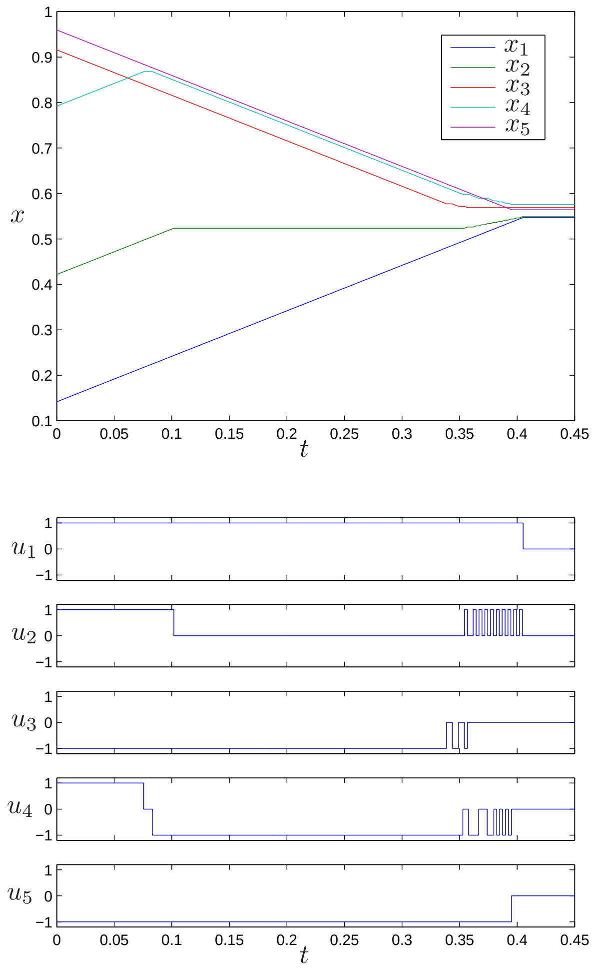

Fig. 2. Sample evolutions of states x and corresponding controls u in (2)-(3) on a ring with $n = 5$ nodes, $\varepsilon = 0.02$.

3. CONVERGENCE ANALYSIS

This section is devoted to the proof of Theorem 1, while the proof of Proposition 2 is omitted.

First of all, we remark that $t^i_{k+1} - t^i_k = f_i(x(t^i_k))$ and $f_i(x) \geq \frac{\varepsilon}{4d_i}$ for every $x \in \mathbb{R}^n$. Then, we immediately argue that, for every $i \in I$, the sequence of local switching times $\{t^i_k\}_{k\in\mathbb{Z}_{\geq 0}}$ has the following "dwell time" property: for every $k \geq 0$,

$$t^i_{k+1} - t^i_k \geq \frac{\varepsilon}{4d_{\max}}. \tag{5}$$

The inequality (5) implies that there exists a positive dwell time between subsequent switches and this fact in turn implies that for each initial condition, the system has a unique solution $x(\cdot)$, which is an absolutely continuous function of its time argument. Furthermore, solutions are bounded, since one can show that for all $t > 0$ it holds that $\min_i x_i(0) \leq \min_i x_i(t)$. We are thus interested in studying the convergence properties of such solutions. For every $t \geq 0$, we let

$$V(t) = \frac{1}{2} x^T(t) L x(t).$$

We note that $V(t) \geq 0$ and we consider the evolution of $\dot{V}(t)$ along the solution. Since L is symmetric, and letting $t_k^i = \max\{t_j^i : t_j^i < t, j \in \mathbb{N}\}$, we have

$$\begin{aligned}\dot{V}(t) =& x^T(t) L u(t) \\ =& -\sum_{i=1}^{n} \left(\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \right) \mathrm{sign}_\varepsilon \left(\mathrm{ave}_i(t_k^i) \right) \\ =& -\sum_{i:|\,\mathrm{ave}_i(t_k^i)| \geq \varepsilon} \left(\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \right) \mathrm{sign}_\varepsilon \left(\mathrm{ave}_i(t_k^i) \right).\end{aligned}$$

We note that

$$t_{k+1}^i = t_k^i + \begin{cases} \dfrac{|\mathrm{ave}_i(t_k^i)|}{4d_i} & \text{if } |\mathrm{ave}_i(x(t_k^i))| \geq \varepsilon \\ \dfrac{\varepsilon}{4d_i} & \text{if } |\mathrm{ave}_i(x(t_k^i))| < \varepsilon. \end{cases} \tag{6}$$

This fact implies that for $t \in [t_k^i, t_{k+1}^i]$, if $\sum_{j \in \mathcal{N}_i} (x_j(t_k^i) - x_i(t_k^i)) \geq \varepsilon$, then

$$\begin{aligned}\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \geq& \sum_{j \in \mathcal{N}_i} (x_j(t_k^i) - x_i(t_k^i)) - 2d_i(t - t_k^i) \\ \geq& \frac{\mathrm{ave}_i(t_k^i)}{2}\end{aligned} \tag{7}$$

Similarly, if $\sum_{j \in \mathcal{N}_i} (x_j(t_k^i) - x_i(t_k^i)) \leq -\varepsilon$, then

$$\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \leq \frac{\mathrm{ave}_i(t_k^i)}{2}.$$

We conclude that, if $|\,\mathrm{ave}_i(t_k^i)| \geq \varepsilon$, then the inequalities above imply that $\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t))$ preserves the sign during continuous flow by continuity of $x(t)$, and moreover

$$\begin{aligned}\left| \sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \right| \geq& \left| \sum_{j \in \mathcal{N}_i} (x_j(t_k^i) - x_i(t_k^i)) \right| - 2d_i(t - t_k^i) \\ \geq& \frac{|\mathrm{ave}_i(t_k^i)|}{2}\end{aligned} \tag{8}$$

On the other hand, if $|\,\mathrm{ave}_i(t_k^i)| \geq \varepsilon$, then also

$$\begin{aligned}\left| \sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \right| \leq& \left| \sum_{j \in \mathcal{N}_i} (x_j(t_k^i) - x_i(t_k^i)) \right| + 2d_i(t - t_k^i) \\ =& \frac{3}{2} |\mathrm{ave}_i(t_k^i)|.\end{aligned} \tag{9}$$

As $\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t))$ preserves the sign during continuous flow, if $|\,\mathrm{ave}_i(t_k^i)| \geq \varepsilon$, then

$$\begin{aligned}& \left(\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \right) \mathrm{sign}_\varepsilon (\underbrace{\sum_{j \in \mathcal{N}_i} (x_j(t_k^i) - x_i(t_k^i))}_{\mathrm{ave}_i(t_k^i)}) \\ =& \left(\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \right) \mathrm{sign} (\sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t))) \\ =& \left| \sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t)) \right|.\end{aligned}$$

Hence, using this last equality and Eq. (8) we deduce

$$\dot{V}(t) \leq -\sum_{i:|\,\mathrm{ave}_i(t_k^i)| \geq \varepsilon} \frac{|\mathrm{ave}_i(t_k^i)|}{2} \leq -\sum_{i:|\,\mathrm{ave}_i(t_k^i)| \geq \varepsilon} \frac{\varepsilon}{2}. \tag{10}$$

This inequality implies there exists a finite time $\bar{t}$ such that $|\mathrm{ave}_i(t_k^i)| < \varepsilon$ for all $i \in I$ and all k such that $t_k^i \geq \bar{t}$. Indeed, otherwise there would be an infinite number of time intervals on which $\dot{V}(t) \leq -\frac{\varepsilon}{2}$, contradicting the positivity of V. For all $i \in I$, let $\bar{k}_i = \min\{k \geq 0 \ : \ t_k^i \geq \bar{t}\}$ and define

$$\hat{t} = \inf\{t \geq 0 \ : \ t > t_{\bar{k}_i}^i \text{ for all } i \in I\}.$$

Note that $\hat{t} \geq \bar{t}$ and thus $|\mathrm{ave}_i(t_k^i)| < \varepsilon$ if $t_k^i \geq \hat{t}$. Moreover, by definition of $\hat{t}$, for $t \geq \hat{t}$ and for all $i = 1, 2, \ldots, n$, the controls $u_i(t)$ are zero and the states $x_i(t)$ are constant and such that $|\mathrm{ave}_i(t)| < \varepsilon$ for all $i \in I$.

We conclude that there exists a point $x^* \in \mathbb{R}^n$ such that $x(t) = x^*$ for $t \geq \hat{t}$, and

$$x^* \in \{x \in \mathbb{R}^n : |\sum_{j \in \mathcal{N}_i} (x_j - x_i)| < \varepsilon, \forall i \in I\}.$$

4. ASYMPTOTICAL CONSENSUS

In this section we propose a modification of system (2)-(3), which drives the system to asymptotical consensus. The key idea is to decrease the sensitivity threshold with time and concurrently introduce a decreasing gain in the control loop.

Let $\varepsilon : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ and $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ be non-increasing functions such that

$$\lim_{t \to +\infty} \varepsilon(t) = \lim_{t \to +\infty} \gamma(t) = 0.$$

We consider the system $(x, u, \theta) \in \mathbb{R}^{3n}$ which satisfies the following continuous evolution

$$\begin{cases} \dot{x}_i = \gamma\, u_i \\ \dot{u}_i = 0 \\ \dot{\theta}_i = 0 \end{cases} \tag{11}$$

except for every t such that the set $\mathcal{I}(\theta, t) = \{i \in I \ : \ \theta_i = t\}$ is non-empty. At such time instants the system satisfies the following discrete evolution

$$\begin{cases} x_i(t^+) = x_i(t) \quad \forall i \in I \\ u_i(t^+) = \begin{cases} \mathrm{sign}_{\varepsilon(t)} (\mathrm{ave}_i(t)) & \text{if } i \in \mathcal{I}(\theta, t) \\ u_i(t) \quad \text{otherwise} \end{cases} \\ \theta_i(t^+) = \begin{cases} \theta_i(t) + \dfrac{1}{\gamma(t)} f_i(x(t)) & \text{if } i \in \mathcal{I}(\theta, t) \\ \theta_i(t) \quad \text{otherwise} \end{cases} \end{cases} \tag{12}$$

where for every $i \in I$ the maps $\text{ave}_i(t)$ and $f_i(x)$ are the same maps defined earlier in the paper. We also adopt the same initial conditions as before, namely $x(0) = \bar{x} \in \mathbb{R}^n$, $u(0) = 0$, $\theta(0) = 0$. As a result $t_k^i = 0$ for every $i \in \{1, 2, \ldots, n\}$.

The corresponding protocol is the following:

Protocol B

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initialization: for all $i \in I$, set $t_0^i = 0$ and $u_i(0) = 0$;
for all $k \in \mathbb{Z}_{\geq 0}$ **do**
  for all $i \in I$ **do**
    if $t = t_k^i$ **then**
      i polls each neighbor $j \in \mathcal{N}_i$ and collects the information $x_j(t_k^i) - x_i(t_k^i)$;
      i computes $\text{ave}_i(t_k^i)$;
      i computes the next sampling time
$$t_{k+1}^i = t_k^i + \frac{1}{\gamma(t_k^i)} f_i(x(t_k^i));$$
      i computes the value $u_i(t_k^{i+})$ by (12);
    end if
    while $t \in (t_k^i, t_{k+1}^i)$ **do**
      i applies the control $\gamma(t) u_i(t_k^{i+})$;
    end while
  end for
end for

In this new protocol we let the parameter ε, which – as established in the previous section – gives a measure of the size of the region of practical convergence, to be time-varying and converging to zero. The obvious underlying rationale is that if the size of the convergence region goes to zero as time elapses, one might be able to establish asymptotic convergence rather than practical. However, letting ε go to zero does not suffice and may induce agents to poll their neighbors infinitely often in a finite interval of time (Zeno phenomenon). To prevent this occurrence we slow down both the process of requesting information to the neighbors and the velocity of the system. The former is achieved via a factor $\frac{1}{\gamma(t)}$ multiplying the map $f_i(x)$, the latter via the factor $\gamma(t)$ which weights the control value $u_i(t)$. It is intuitive that to fulfill the purpose, the function $\gamma(t)$ must be "comparable" with $\varepsilon(t)$. This is achieved assuming that there exists $c > 0$ such that

$$\frac{\varepsilon(t)}{4 d_i \gamma(t)} \geq c \quad \forall i \in I, \, \forall t \geq 0. \tag{13}$$

We can now state the following result.

Theorem 3. (Asymptotical consensus). Let $x(\cdot)$ be the solution to (11)-(12) under condition (13). Then for every initial condition $\bar{x} \in \mathbb{R}^I$ there exists $\alpha \in \mathbb{R}$ such that $\lim_{t \to \infty} x_i(t) = \alpha$ for all $i \in I$ if and only if $\int_0^{+\infty} \gamma(s) ds$ is divergent.

The proof is long and will be presented elsewhere. We instead discuss some illustrative simulation results, collected in Figures 3-4. Simulations show that the evolution of (11)-(12) can be qualitatively divided into two phases. During a first phase of *fast* convergence, ε plays a little role and the behavior of x is reminiscent of Fig. 1. The first phase lasts until the states become close enough to consensus for $\varepsilon(t)$ to be comparable with the differences between the x_is. During the second phase, control actions

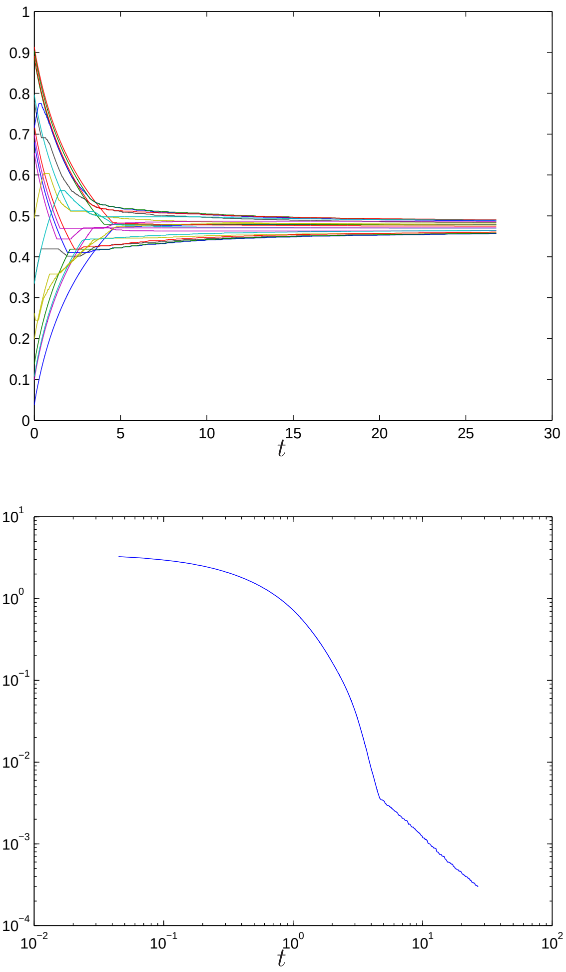

Fig. 3. A sample evolution of (11)-(12) starting from the same initial condition and on the same graph as Fig. 1. Top plot shows the state x, bottom plot shows the Lyapunov function V. Simulation assumes $\varepsilon(t) = \frac{0.05}{1+t}$, $\gamma(t) = \frac{0.25}{1+t}$.

become sporadic and convergence depends on the decrease of ε, and is thus *slow*. Indeed, we are assuming that $\varepsilon(t)$ has divergent integral, so that it may not decrease faster than $1/t$. We want to stress that this technical condition is not due to a limitation of our analysis, but is inherent to the system. Indeed, assumption (13) relates ε and γ, so that $\gamma(t)$ may not be larger than (a constant times) $\varepsilon(t)$. In turn, if $\gamma(t)$ had bounded integral, the control would not be large enough to stabilize the system to practical consensus from an arbitrary initial condition.

5. CONCLUSIONS

In this paper we have addressed the problem of achieving consensus in the scenario in which agents collect information from the neighbors only at times which are designed iteratively and independently by each agent on the basis of its current local measurements, a process which following existing literature can be called as self-triggered information collection. As a result we have proposed new algorithms such that (i) practical consensus is achieved in finite time and (ii) agents exchange information with

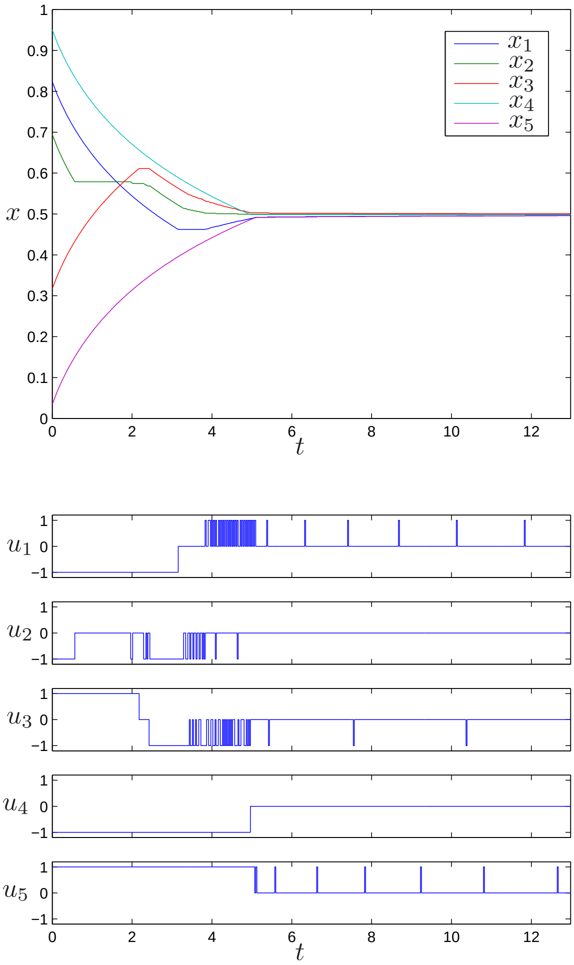

Fig. 4. Sample evolutions of states x and corresponding controls u in (11)-(12) on a ring with $n = 5$ nodes, $\varepsilon(t) = \frac{0.05}{1+t}$, $\gamma(t) = \frac{0.25}{1+t}$.

their neighbors only a finite number of times. The time cost, that is the finite time within which convergence is achieved, and the communication cost, that is the maximum number of times that an agent polls its neighbors in search of information before reaching consensus, depend on how close one wants to get to consensus. There is a clear tradeoff (fully characterized in Proposition 2) between precision and time and communication costs. We have then shown that the algorithm may be suitably modified to achieve *asymptotical* consensus, again using self-triggered collection of information.

The theoretical framework that enabled us to achieve these results is the framework of hybrid control systems and the analysis was mainly based on Lyapunov-like arguments for this class of systems.

Our future research will focus on algorithms in which the agents poll their neighbors singularly and not necessarily at the same time and the information is transmitted in quantized form. We are also interested in investigating the same approach in the case of higher dimensional systems and for more complex coordination tasks.

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