AURALIZATION OF COUPLED SPACES BASED ON A DIFFUSION EQUATION MODEL

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ABSTRACT

Auralization of room acoustics consists in audio rendering based on the sound characteristics of a virtual space. It is defined by Vorländer [1] as “the creation of audible acoustic sceneries from computer-generated data”, as the auditory equivalent of visualization techniques. Auralization is obtained by convolving a room impulse response with an anechoic recording, adding room presence to the reverberation-free excitation signal, providing subjective immersion in the considered space. Since acoustically coupled spaces are encountered in various venues such as large stairways distributing corridors or rooms, naves and side galleries in churches, even crossing streets in dense cities, it becomes interesting to produce accurate auralization in these types of venues. Such coupled room impulse responses can be synthesized using a recently proposed sound energy decay model based on a diffusion equation and adapted to coupled spaces. This paper presents the parametric model of sound energy decay and describes the impulse response synthesis process leading to auralization of coupled spaces.

1. INTRODUCTION

The term auralization has been used since the early twentieth century in the musical community in the sense of “pre-hearing” according to Summers [2]. It was later defined for the room acoustics community by Kleiner et al. [3] as “the process of rendering audible, by physical or mathematical modeling, the sound field of a source in a space”. Thus the process of auralization is artificial and different than real reverberation experienced by a listener in an acoustic space. However it is interesting to create new acoustical environments or even to recreate lost ones, e.g. based on maps and descriptions of buildings which do not exist anymore. According to Lokki et al. [4], auralization process, to be indistinguishable from the real auditory environment, requires simulation or reproduction of three issues: directivity of sound sources, sound propagation in a 3-D space, and reproduction of spatial sound. The present study focuses on sound propagation to obtain monaural impulse responses, possibly further adapted for spatialized rendering.

Pioneer studies used sound recording in scale models, played back at lower speeds with respect to the scale factor, as performed by Spandöck [6]. Nowadays computer modeling is often used to generate room impulse responses to be further convolved with anechoic recordings. These operations can be either pre-calculated for a given space or real-time convolution can be performed [7, 8], e.g. using the “waveguide” method [9]. A number of different means to generate impulse responses are available with various advantages and drawbacks in terms of sound quality and computation time. Those methods are either based on wave approach (e.g. BEM or FDTD) for small volumes whose acoustics have modal behavior, geometrical acoustics (e.g. ray-marching, radiosity), or statistical acoustics for larger volumes. However, results can present important variations from one method to another when ap-
plied to coupled spaces, as shown by Luizard et al. [10]. Therefore, the choice of the employed simulation method is determinant and depends on characteristics of the venue.

Coupled spaces have particular acoustical characteristics due to the energy exchange between several architectural volumes (Fig. 1). A signature of this sort of system is the curved sound energy decay which can present several decay rates as opposed to most single volume rooms. The early decay, presenting a steeper slope than the late part, contributes to give an important sense of sound clarity while the lower late decay rate induces an impression of reverberation, although these concepts are usually antagonistic in single volume rooms. Therefore, coupled volume acoustics is worth being exploited, particularly for theater and music purpose, and auralization is a relevant means to virtually explore acoustically coupled spaces with various goals, e.g. design or entertainment.

This study first presents the proposed analytical model of sound energy decay and its application to coupled spaces, then the auralization process is described from room impulse response synthesis to final audible rendering. Furthermore, suggestions are proposed to improve auralization quality and listener engagement in the virtual room.

2. PARAMETRIC MODEL OF SOUND ENERGY DECAY

Previous research [11, 12] has been conducted in room acoustics to develop analytical models of sound energy decay in order to predict sound field behavior in various spaces. The present model is based on a diffusion equation under the hypothesis that sound behaves as moving particles in a uniformly scattering medium, as proposed by Ollendorff [13] who introduced the use of diffusion equation to model acoustic phenomena. The diffusion equation (eq. (1)) is expressed in terms of sound energy density $w(r, t)$ and is composed of four terms: a temporal derivative, a spatial derivative (Laplace term), an absorption term, and a source term with acoustical power $F$. Considering source-receiver distance $r$ allows for estimating energy variation throughout the reverberant space.

$$\frac{\partial}{\partial t} w(r, t) - D \nabla^2 w(r, t) + \sigma w(r, t) = F(r, t) \quad (1)$$

Introducing the mean free path between two successive collisions $\lambda = \frac{4V}{c}$ makes it possible to express statistical quantities which influence the behavior of sound field, depending on architectural parameters such as the room volume and surface. Coefficients $D$ (eq. (2)) and $\sigma$ (eq. (3)) are related to sound diffusion and absorption, respectively:

$$D = \frac{\lambda c}{3} = \frac{4V c}{3S}, \quad (2)$$

$$\sigma = \frac{\sigma c}{\lambda} = \frac{\sigma S}{4V}, \quad (3)$$

where $c$ is the speed of sound, $\sigma$ is the mean absorption coefficient, $V$ and $S$ are the volume and surface of the room.

The proposed solution to eq. (1) is a heuristic approximation which accounts for two different regions defined within the considered space, namely the near and far fields, with a continuous transition from one another. In the neighborhood of the source, the sound energy decays with source-receiver distance (first term of the sum) until being less spatially dependent and becoming homogeneous enough to be associated with the concept of diffuse sound field as defined by Sabine [14] in the classical statistical theory (constant term of the sum). Coefficients defined in eqs (2 & 3) are part of this statistical model. Nevertheless, this expression is exact in steady state condition and for homogeneous energy decay as described by Sabine, asymptotically far from the sound source.

$$w(r, t) = \left( \frac{a}{r} e^{-\sqrt{\pi \tau} r} + b \right) e^{-\sigma t} H(t), \quad t > \frac{r}{c}. \quad (4)$$

Function $H$ is the Heaviside step function representing the fact that sound decay is described from the instant the direct sound reaches the receiver position at distance $r$ from the source.

This model (eq. (4)) can be calibrated with respect to room characteristics by adapting its parameters $a$ and $b$. The latter express the relative importance of spatially decaying sound energy as compared to homogeneous energy through space, governed by $a$ and $b$ respectively.

3. APPLICATION TO COUPLED SPACES

This sound energy decay model can be adapted to coupled spaces in combination with classical statistical theory [15, 16], allowing for simulation of various source-receiver configurations and coupling surface settings, whereas the classical theory does not consider sound level variations within a given subspace. Hence using this model provides finer estimation of sound fields in coupled spaces. First, initial uniform sound levels are estimated in each room for steady state conditions, governed by parameter $b$. The concept of coupling factor $k_i$ is used to estimate the initial sound level in the reverberation chamber such that

$$\begin{cases}
  w_{1o} = \frac{4P}{aS_1(S_1 + S_2)} \\
  w_{2o} = k_2 w_{1o}
\end{cases} \quad (5)$$

with $k_2 = \frac{S_c}{S_2 + S_c}, \quad (6)$

where $P$ is the sound power, $S_c$ is the coupling surface area, and subscripts 1 & 2 refer to the main room and chamber respectively. Then the spatially dependent energy is added, governed by parameter $a$. Finally, the sound energy emitted from the chamber is introduced with respect to the distance between the coupling surface and the receiver, considering the coupling surface as a secondary sound source. This process allows for estimating sound energy density and creating curved energy decays at any receiver position in the main room, according to the characteristics of the rooms.

An example, whose geometry is shown in Fig. 2 and specifics are detailed in Table 1, is performed in quasi-rectangular coupled spaces, the main room being larger but more damped than the reverberation chamber.
such that reverberation time (RT) at mid-frequencies is larger in the latter. Opposite walls are slightly angled in order to avoid flutter echoes. Fig. 3 represents the sound energy density estimated on the ground floor in the main room of coupled volumes. Spatial variations are in the range of 15 dB between the source peak and the lowest energy in the room. The second peak next to a wall corresponds to the energy emitted from the chamber back in the main room.

Considering receivers along a line through the room length, on the axis such that $Y = 12 \text{ m}$ on Fig. 3 with 1 m-step from one another, Fig. 4 shows the temporal energy decays with increasing source-receiver distance. Darker decays stand for receivers nearer the sound source while lighter ones represent distant receivers. The curvature point appears at different levels under the initial level for various receiver positions, such that the further the receiver, the higher the decay curve. This means that the second slope, or late reverberation, appearing earlier and louder, has stronger effects on distant receivers than on ones nearer the sound source. The energy decay given by classical statistical theory, which is the same at every receiver position since no spatial variation is considered, appears as the blue dotted line. Fig. 4 represents normalized decays and distant receivers can provide decay curves with late decay levels above the reference one. A line of receivers different than the one considered above would lead to different results both in terms of total energy variation, as can be imagined from Fig. 3, and in terms of temporal decays because the room configuration is not symmetrical, with the coupling surface on one lateral side. This observation underlines the fact that sound energy decays, and thus impulse responses, generally vary throughout a given space, making it interesting to be able to generate auralization accounting for those differences. Hence using this proposed statistical model which is distance dependent leads to more precise results than the classical statistical model.

### 4. FROM SOUND ENERGY DECAYS TO IMPULSE RESPONSES

Auralization is based on an anechoic sound convolved with an impulse response. The present study deals more specifically with room impulse responses which add reverberation to the dry signal to give it a certain room presence. A room impulse response is the temporal equivalent of a transfer function of the room. It is composed of sound reflected on the walls and received at a specific position.

### Table 1. Architectural and acoustical specifics of the geometry shown in Fig. 2 for each separate room, i.e. without the coupling surface.

<table>
<thead>
<tr>
<th></th>
<th>Main room</th>
<th>Reverberation chamber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td>44</td>
<td>14</td>
</tr>
<tr>
<td>Width (m)</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>Height (m)</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Surface (m²)</td>
<td>4560</td>
<td>2040</td>
</tr>
<tr>
<td>Volume (m³)</td>
<td>19000</td>
<td>6050</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.55</td>
<td>0.1</td>
</tr>
<tr>
<td>RT (s)</td>
<td>1.2</td>
<td>5.0</td>
</tr>
</tbody>
</table>
Figure 4. Normalized sound energy decays along the line such that \( Y = 12 \) m in Fig. 3, i.e. the center line of the main room, for various source-receiver distances \( 5 < r < 25 \) m. Darker decays (below) correspond to small \( r \) while lighter decays (above) correspond to more distant receivers. The dotted line represents the classical model by Cremer et al. [16].

Figure 5. Schematic representation of a room impulse response in single volume.

An illustration of such a signal is shown in Fig. 5. The previously proposed model of sound energy decay can be used to synthesize room impulse responses using various processes. The idea is to apply the given energy decay to pre-filtered noise in order to obtain the reverberation part of impulse response (top of Fig. 6). An inverse Fourier transform of this decaying noise produces the temporal impulse response (bottom of Fig. 6). This sort of process has been used in previous research for perceptual experiments whose purpose was to estimate Just Noticeable Differences (JND) of single and double-slope reverberation from single and coupled spaces, allowing to change decay rates easily while keeping temporal distribution and frequency content unchanged. Frissen et al. [17] applied energy decays to a normally distributed random number sequence and Picard et al. [18] applied energy decays to pink noise.

Refinements can be performed along two different dimensions: the temporal or spectral distribution of energy. Measured room impulse responses show different trends along temporal segments. As can be seen in Fig. 5, the first part of received energy is the direct sound, then the first reflections from the walls and ceiling reach the listener before the density of reflection becomes too high to be considered as discrete, which is called reverberation. Hence simulated room impulse responses should include direct sound and possibly early reflections, which have been proved by Barron [19] to be perceptually influential, in order to sound more realistic. Fig. 7 shows the steps to construct impulse responses with reverberation only, added early reflections, and direct sound. While the room geometry is responsible for intensity and time of arrival of early reflections, intensity of direct sound relative to the rest of impulse response corresponds to the source-receiver distance. Therefore, adding the described steps can be seen as accounting for a type of room and a specific receiver position. Furthermore, the three decay curves presented in Fig. 7, which are backward integrations of the impulse responses as defined by Schroeder [20], are different in the sense that the early decay is steeper with the direct sound and early reflections. Depending on the proportion of change as compared to the case with reverberation only, the modification will be audible, possibly adding clarity to the sound.

Another refinement can be performed, in the frequency domain, consisting in setting different decay rates in the available octave bands. The proposed model of energy decay can be used with various absorption coefficient settings in order to obtain a collection of decay curves, as illustrated in Fig. 8. RT values in uncoupled rooms shown in Table 2 are set depending on the desired absorption in the main room and in the reverberation chamber. These decay curves can be applied successively to noise filtered in frequency bands. The obtained impulse response is closer to reality than before this process because measurements in actual concert halls always present a variation of decay rates, leading to total energy variations in the order
Figure 7. Possible refinements of room impulse response synthesis in temporal domain: addition of early reflections (center) and direct sound (right) to reverberation only (left).

<table>
<thead>
<tr>
<th>Center frequency (Hz)</th>
<th>125</th>
<th>250</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>4000</th>
<th>125</th>
<th>250</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>4000</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT (s)</td>
<td>3.12</td>
<td>2.82</td>
<td>2.11</td>
<td>1.74</td>
<td>1.34</td>
<td>0.96</td>
<td>13.67</td>
<td>10.87</td>
<td>8.46</td>
<td>4.8</td>
<td>3.31</td>
<td>2.63</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.19</td>
<td>0.24</td>
<td>0.32</td>
<td>0.39</td>
<td>0.50</td>
<td>0.61</td>
<td>0.03</td>
<td>0.04</td>
<td>0.06</td>
<td>0.10</td>
<td>0.14</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 2. Reverberation times (RT) and mean absorption coefficients ($\alpha$) per octave bands in uncoupled configuration, from measurements in a scale model of coupled spaces. Energy decay curves presented in Fig. 8 are generated with these values.

Figure 8. Example of double-slope decay curves per octave bands from a coupled volume system, at $r = 20$ m from the source.

of dozens of decibels over frequency bands.

5. CONCLUSION

A model of sound energy decay based on the diffusion equation in coupled spaces is proposed to perform auralization. The process which has been used in previous research consists in applying these sound decays to filtered noise with various possible refinements to produce realistic room impulse responses. Convolving the latter with anechoic sounds allows for hearing sound sources within virtual spaces. This process can be useful in several domains, e.g. virtual reality or architectural acoustic design, where acoustical immersion might be required to experience particular sound environments, among which coupled spaces are often encountered. Further research includes listening tests to estimate the level of sound quality which can be obtained with the proposed energy decay model, as compared to other auralization methods.

6. REFERENCES


