ORDER DETECTION AND BLIND IDENTIFICATION OF 2×1 MISO CHANNELS

Carlos Estêvão R. Fernandes,[1] *Pierre Comon and Gérard Favier*

I3S Laboratory–CNRS, University of Nice Sophia Antipolis, France.
{cfernand, favier, pcomon}@i3s.unice.fr

ABSTRACT

In this paper, we investigate the use of output 4th-order cumulants to detect the number of source signals on a multiple-input single-output (MISO) communications channel and blindly identify their respective channel coefficients. More particularly, we are interested in the case of two sources. The proposed cumulant-based order detection principle allows us for recovering the longest channel order and its coefficients. A similar procedure is applied for detecting the presence of a second source as well as estimating its associated channel order and coefficients. When only estimated cumulants are available, we implement two hypothesis tests based on three proposed test-statistics. Computer simulations illustrate the performances obtained with the methods proposed for order detection and channel identification.

***Index Terms*—** Order detection, MISO channels, Blind identification, Higher-order statistics.

1. INTRODUCTION AND PROBLEM DESCRIPTION

We are interested in detecting the number of source signals and identifying channel parameters in the context of a multiple-input single-output (MISO) communication system. The transmitters are supposed to use the same carrier frequency and are located far apart from each other, hence the physical channels are different. The following assumptions hold:

A1 The input signals $s_p[n]$, for every integer $p \in [1, P]$, are mutually independent and temporally i.i.d. (independently and identically distributed).

A2 The random variables s_p, $p \in [1, P]$, have unknown non-Gaussian distributions with non-zero kurtoses.

A3 The channel lengths are bounded by a known value L.

The output signal is the result of a linear combination of the discrete-time input signals, $\mathbf{s}[n] \in \mathbb{C}^P$. After baud rate sampling, the received signal $x[n]$ can be written as follows:

$$x[n] = \sum_{p=1}^{P} \sum_{l=0}^{L_p} h_p[l] s_p[n-l], \qquad (1)$$

where $h_p[l]$, with $l \in [0, L_p]$ and $p \in [1, P]$, are the channel parameters associated with the user p and $h_p[l] = 0, \forall\, l \notin [0, L_p]$, where $L_p < L, \forall\, p \in [1, P]$. Let us define the channel vector $\mathbf{h}_p = [h_p(0), \ldots, h_p(L_p)]^{\mathsf{T}}$ with $p \in [1, P]$. From the observation of $x[n]$ only, our goals are to:

P1: Detect the number of sources P;

P2: Determine the channel lengths L_p for each $p \in [1, P]$;

P3: Identify the vectors $\mathbf{h}_p$ for each $p \in [1, P]$.

The case of overdetermined mixtures (more sensors than sources) has been exhaustively treated in the literature, including static mixtures [1] as well as dynamic ones [2]. On the other hand, underdetermined mixtures have only recently received attention. Our contribution is based on cumulant-matching as in [3]. Due to assumption A1, the 4th-order cumulants of $x[n]$, defined as $C(i,j,k) \triangleq$ *cum*$\{x^*[n], x[n+i], x^*[n+j], x[n+k]\}$, can be expressed in terms of the marginal cumulant contributions of each source as:

$$C(i,j,k) = \sum_{p=1}^{P} C_p(i,j,k), \qquad (2)$$

in which $C_p(i,j,k)$ depends on the unknown channel parameters $h_p[l]$ and can be written as follows [3, 4]:

$$C_p(i,j,k) = \gamma_{s_p} \sum_{l=0}^{L_p} h_p^*[l] h_p[l+i] h_p^*[l+j] h_p[l+k],$$

where $p \in [1, P]$ and γ_{s_p} stands for the *kurtosis* of the source $s_p[n]$. Since $h_p[l] = 0, \forall\, l \notin [0, L_p]$, we have $C_p(i,j,k) = 0$ $\forall\, i,j,k > L_p$. The number P of sources (users) is unknown and assumed to be bounded by two, i.e. $P \leq 2$, so that we consider a MISO channel with two inputs ($P = 2$) and a single output, assuming transmission channels are of different orders, so that $L_1 > L_2$. If a single source is present ($P = 1$) the model still holds with $L_2 = 0$ and $h_2(0) = 0$. Therefore, we get:

$$C(i,j,L_1) = C_1(i,j,L_1) = \gamma_{s_1}\, h_1^*[0]\, h_1[i]\, h_1^*[j]\, h_1[L_1]. \qquad (3)$$

2. CUMULANT-BASED ORDER DETECTION

From equation (3), it is straightforward to obtain:

$$C(i,k,L_1)\, h_1[j] - C(j,k,L_1)\, h_1[i] = 0, \qquad (4)$$

[1] PhD scholarship granted from CAPES agency, Brazilian government.

with $0 \leq i < j \leq L_1$ and $0 \leq k \leq L_1$. From equation (4) we can get a set of $L_1(L_1+1)^2/2$ equations with L_1+1 unknowns, which can be rewritten in a matrix form as follows:

$$\mathbf{C}\mathbf{h}_1 = \mathbf{0}, \tag{5}$$

where $\mathbf{0}$ is the null-vector of dimension $L_1(L_1+1)^2/2$ and $\mathbf{C} \in \mathbb{C}^{(L_1(L_1+1)^2/2)\times(L_1+1)}$. A solution to (5) is obtained by computing the right singular vector of $\mathbf{C}$ associated with the smallest singular value [3]. That solution is optimal in the total least squares (TLS) sense. To avoid the trivial solution we can impose a unit-norm constraint ($\sum_n |h_1[n]|^2 = 1$).

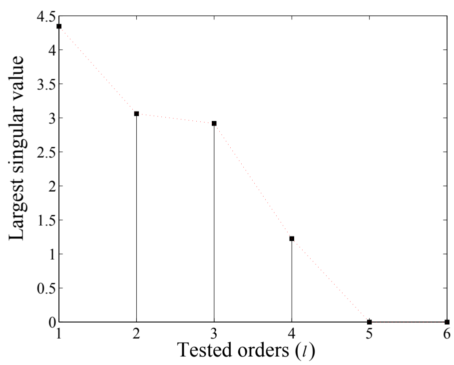

Fig. 1. Profile of the largest SV $\sigma_1(\ell)$ of $\mathbf{C}(\ell)$ for $\ell \in [1, L]$ with $L_1 = 4$, $L_2 = 2$ and $L = 6$.

Let us consider a channel order testing variable $\ell \in [1, L]$, $L > L_1 > L_2$, and build L cumulant matrices $\mathbf{C}(\ell) \in \mathbb{C}^{(\ell(\ell+1)^2/2)\times(\ell+1)}$ satisfying (4) with ℓ in the place of L_1. This yields $\mathbf{C}(\ell)\mathbf{h}_1 = \mathbf{d}$, where $\mathbf{d} = \mathbf{0}$ if $\ell \geq L_1$, as in (5), and $\mathbf{d} \neq \mathbf{0}$ otherwise. That is the main idea behind our channel order detector. Denoting by $\sigma_1(\ell) \geq \ldots \geq \sigma_{\ell+1}(\ell)$ the singular values (SV) of $\mathbf{C}(\ell)$, we note that for $\ell > L_1$, $\mathbf{C}(\ell)$ is the null matrix and clearly we have:

$$\sigma_1(\ell) = \ldots = \sigma_{\ell+1}(\ell) = 0. \tag{6}$$

On the other hand, if we take $\ell = L_1$ we get $\sigma_i(L_1) \neq 0$, $i = 1, \ldots, L_1$, with $\sigma_1(L_1) \geq \ldots \geq \sigma_{L_1}(L_1)$ and $\sigma_{L_1+1}(L_1) = 0$. Therefore, the ℓ largest SV of $\mathbf{C}(\ell)$ only take non-zero values for $\ell \leq L_1$ and we can detect the order L_1 of the longest channel by looking for the non-zero values of $\sigma_i(\ell)$, $i \in [1, \ell]$, for tested orders $\ell \in [1, L]$. To illustrate this reasoning, fig. 1 plots the largest SV, $\sigma_1(\ell)$, of the cumulant matrices $\mathbf{C}(\ell)$ for values of ℓ in $[1, L]$, with $L = 6$, $L_1 = 4$ and $L_2 = 2$. Perfect knowledge of the cumulant values was assumed. The figure clearly shows that $\sigma_1(\ell) = 0$, $\forall\, \ell \geq 5$ and the first non-zero value is reached for $\ell = L_1 = 4$.

3. LONGEST CHANNEL IDENTIFICATION

In practice, cumulants are affected by estimation errors so that (6) is no longer valid for $\ell > L_1$. Our problem is then to search a rupture point in the profile of a given SV of $\mathbf{C}(\ell)$, e.g. $\sigma_i(\ell)$ for a fixed $i \in [1, \ell]$. Actually, it would be interesting to include not only one SV $\sigma_i(\ell)$ but as many as possible. Ideally, for each $\ell \in [1, L]$, the ℓ largest singular values $\sigma_1(\ell), \ldots, \sigma_{\ell+1}(\ell)$ of matrix $\mathbf{C}(\ell)$ should be taken into account for the computation of a single test variable $\eta(\ell)$ containing information about the whole dynamic of $\mathbf{C}(\ell)$; a decision threshold η_{thr} must be established so that $\eta(\ell) > \eta_{thr}$ only if $\ell \leq L_1$.

3.1. Test variables

Three test variables $\eta(\ell)$ are proposed in Table 1. Note that $\eta_A(\ell)$ and $\eta_B(\ell)$ consist of geometrical means involving the SV of $\mathbf{C}(\ell)$. On the other hand, $\eta_C(\ell)$ can be directly derived from $\mathbf{C}(\ell)$ without singular value decomposition (SVD) computations, since $\eta_C(\ell) = \|\mathbf{C}(\ell)\|_F$; this fact represents a great advantage of $\eta_C(\ell)$ over $\eta_A(\ell)$ and $\eta_B(\ell)$.

Table 1. Computation of test variables.

$\eta_A(\ell) = \left(\prod_{i=1}^{\ell} \sigma_i(\ell) - \sigma_{\ell+1}(\ell)\right)^{1/\ell}$
$\eta_B(\ell) = \dfrac{\left(\prod_{i=1}^{\ell} \sigma_i(\ell)\right)^{1/\ell}}{\sigma_{\ell+1}(\ell)}$
$\eta_C(\ell) = \sqrt{\sum_{i=1}^{\ell+1} \lvert\sigma_i(\ell)\rvert^2}$

In order to detect L_1, we search a rupture in the profile of the test variables, as follows:

1. Initialize the length variable as $\ell = L$;
2. Estimate the output cumulants $C(i, k, \ell)$ for $0 \leq i \leq \ell$ and $0 \leq k \leq \ell$ and build matrix $\mathbf{C}(\ell)$ (as explained in section 2);
3. Compute the SVD of $\mathbf{C}(\ell)$, and denote $\sigma_1(\ell), \ldots, \sigma_{\ell+1}(\ell)$ its singular values;
4. Denote by $\mathbf{v}_\ell \in \mathbb{C}^{\ell+1}$ the singular vector of $\mathbf{C}(\ell)$ associated with $\sigma_{\ell+1}(\ell)$. This is the optimal solution of (5) in the TLS sense [3];
5. Determine the test variable $\eta_A(\ell)$, $\eta_B(\ell)$ or $\eta_C(\ell)$ defined in Table 1;
6. Decrease ℓ by 1 unit and repeat steps 2 to 5 until $\ell = 1$.

In step 4, we construct a collection of vectors, one of which corresponds to the actual longest channel vector $\mathbf{h}_1 \in \mathbb{C}^{L_1+1}$. Decision about which is the good one should be taken based on the profile of the test variables $\eta_A(\ell)$, $\eta_B(\ell)$ or $\eta_C(\ell)$. A decision threshold η_{thr} must be established so that $\eta(\ell) > \eta_{thr}$ if and only if $\ell \leq L_1$. A reasonable choice for the decision threshold is the geometrical mean of the test variable for $\ell \in [1, L]$, so that:

$$\eta_{thr} = \left(\prod_{\ell=1}^{L} \eta(\ell) \right)^{1/L}, \tag{7}$$

where $\eta(\ell)$ takes the values of either $\eta_A(\ell)$, $\eta_B(\ell)$ or $\eta_C(\ell)$.

3.2. H_1/H_2 hypothesis test

Based on the variable $\eta(\ell)$, the proposed hypothesis test considers the following hypotheses:

- Hypothesis H_1: $\ell > L_1$
- Hypothesis H_2: $\ell = L_1$

We assume that H_1 holds true for $\ell = L$ and test $\eta(\ell)$ with respect to a threshold η_{thr} for $\ell = L, \ldots, 1$. We accept H_1 if $\eta(\ell) < \eta_{thr}$, otherwise we reject it and accept H_2, thus determining the order $\ell_o = L_1$ of the longest channel. After that, step 4 of the previously described procedure yields:

$$\hat{\mathbf{h}}_1 = \mathbf{v}_{\ell_o}, \quad \ell_o = L_1. \tag{8}$$

In the next section, we propose a method to estimate the order L_2 of the shortest channel and its coefficient vector $\mathbf{h}_2$, if it exists. We also discuss the case of channels of same order.

4. SHORTEST CHANNEL IDENTIFICATION

Recalling from section (1) that $C_p(i, j, k) = 0, \forall\, k > L_p$, we note that $C_2(i, j, k) = 0$ for $k > L_2$ and hence $C(i, j, k) = C_1(i, j, k), \forall\, k > L_2$. For $k = L_2$, we get:

$$C_2(i, j, L_2) = \gamma_{s_2} h_2^*[0] h_2[i] h_2^*[j] h_2[L_2]. \tag{9}$$

From definition (2) with $P = 2$, we can estimate the marginal cumulant contribution of the source $s_2[n]$ as $\hat{C}_2(i, j, k) = C(i, j, k) - \hat{C}_1(i, j, k)$, where $\hat{C}_1(i, j, k)$ is obtained as follows from the channel parameters $\hat{h}_1[l]$ estimated in (8):

$$\hat{C}_1(i, j, k) = \gamma_{s_1} \sum_{l=0}^{L_1} \hat{h}_1^*[l] \hat{h}_1[l+i] \hat{h}_1^*[l+j] \hat{h}_1[l+k].$$

Therefore, the marginal cumulant $C_2(i, j, k)$ appears to be a good metric to detect the presence (or absence) of a second source with shorter channel. If such a source is present, $C_2(i, j, k)$ will be non-zero for some values of $k \in [1, L_1]$. Precisely, it equals zero for $k > L_2$ and it is non-zero for $k \leq L_2$. Hence, a new test variable $\rho(\kappa)$ can be set up to detect the presence of the second source so that:

$$\begin{cases} \rho(\kappa) = 0, & \kappa > L_2 \\ \rho(\kappa) \neq 0, & \kappa \leq L_2. \end{cases} \tag{10}$$

We will define the test variable as $\rho(\kappa) = \|\mathbf{C}_2(\kappa)\|_F^2$, i.e. the squared Frobenius norm of matrix $\mathbf{C}_2(\kappa) \in \mathbb{C}^{(\kappa+1)\times(\kappa+1)}$ in which the element in position (i, j) equals $C_2(i-1, j-1, \kappa)$, $i, j \in [1, \kappa+1]$.

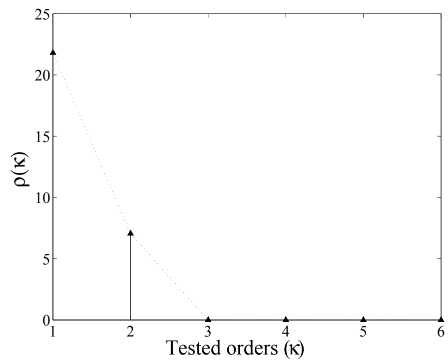

Fig. 2. Test variable $\rho(\kappa)$ for $\kappa \in [1, L]$ with $L_1 = 4$, $L_2 = 2$ and $L = 6$.

Figure 2 illustrates the detection of L_2 using $\rho(\kappa)$ for $\kappa \in [1, L]$. The channels considered here are the same as those of section 2 ($L_1 = 4$, $L_2 = 2$ and $L = 6$). Perfect knowledge of the cumulant values was assumed. The figure clearly shows that the first non-zero value of $\rho(\kappa)$ is reached at $\kappa = L_2 = 2$.

4.1. H_3/H_4 hypothesis test

Let us now consider the more realistic case where the output cumulants are not known and need to be estimated. In that case, the estimation errors corrupting $C(i, j, k)$ imply inaccuracies in the estimation of the longest channel $\hat{\mathbf{h}}_1$, which leads to a bad reconstruction of $C_1(i, j, k)$. This means that cumulative errors are involved in the calculation of $\hat{C}_2(i, j, k)$. Hence, (10) is no longer valid and another hypothesis test must be performed in order to detect the length L_2 of the shortest channel. For this purpose, a decision threshold ρ_{thr} must be defined so that $\rho(\kappa) > \rho_{thr}$ only if $\kappa \leq L_2$. Here again, the geometrical mean seems to be a good choice:

$$\rho_{thr} = \left(\prod_{k=1}^{L} \rho(k) \right)^{1/L}. \tag{11}$$

and the hypothesis test is defined so that:

- Hypothesis H_3: $\kappa > L_2$
- Hypothesis H_4: $\kappa = L_2$

Starting with $\kappa = L_1$ and successively decreasing κ, we accept H_3 when $\rho(\kappa) < \rho_{thr}$, otherwise we accept H_4 and get an estimate of L_2. If the test variable never reaches the threshold, then we conclude that a single source is present.

Once we have determined L_2, we can use $\hat{C}_2(i, j, L_2)$ to estimate the parameter vector $\mathbf{h}_2$ of the shortest channel by solving a linear system similar to (5) in the TLS sense.

4.2. Channels with identical lengths

In the case where $L_1 = L_2$, our approach is no longer valid. Equation (3) becomes $C(i, j, L_1) = C_1(i, j, L_1) +$

$C_2(i, j, L_1)$, so that the output cumulant contains information on both channels. Hence, the estimation method described in section 3 will not result in a fair estimate of $\mathbf{h}_1$ and (8) does not hold true. Nevertheless, the hypothesis test H_1/H_2 can still be used to determine L_1. In addition, we can detect the presence of the second source and determine whether the channels have the same or different orders. Proceeding to the estimation of the longest channel as described in section 3, we can use $\hat{\mathbf{h}}_1$ to compute $\rho(\kappa)$ as described earlier in this section. Then we can implement the test on the shortest channel, which will fail for hypothesis H_3 at $\kappa = L_1$, indicating that both channels are of same order. Therefore, the proposed tests H_1/H_2 and H_3/H_4 can also be used for the case of channels of same length. However, we cannot identify the channel parameters in this case. In fact, there is no means to see the sum of two linear processes as a scalar linear process [5].

5. COMPUTER SIMULATIONS

The results shown in this section were obtained from computer experiments simulating the MISO system (1) with the two following channels: $\mathbf{h}_1 = [1.0,\ 0.61 - 1.18i,\ -0.59 - 0.53i,\ -0.82 + 0.21i]^{\mathsf{T}}$ and $\mathbf{h}_2 = [1.0,\ -0.29 - 0.33i,\ -0.62 - 0.73i]^{\mathsf{T}}$, so that $L_1 = 3$ and $L_2 = 2$. We assumed $L = 5$ and the 4th-order cumulants were estimated from an output sequence of 50000 data samples. No additive noise is present.

Table 2. Longest channel detection test: $P(\eta < \eta_{thr})$.

	$\eta = \eta_A$	$\eta = \eta_B$	$\eta = \eta_C$
$\ell = 5$	100%	100%	100%
$\ell = 4$	100%	100%	100%
$\ell = 3$	2.7%	0.88%	6.26%
Success rate	97.3%	99.1%	93.7%

Figure 3 shows the histogram plots for the test variable η_A, built from the sample output cumulants over 5000 Monte Carlo runs. Similar plots for η_B and η_C were also obtained. Table 2 shows the probability of $\eta(l) < \eta_{thr}$ highlighting the success rate of the L_1 detection. For $\ell \leq 2$ all the probabilities were zero. Note that η_B gives the best probability of good decisions on the channel order L_1 with only 0.88% probability of false alarm. On the other hand, although η_C yields the poorest performance among the three considered criteria, it could be worthwhile considering to use it since it does not require any SVD computation and can be derived directly from $||\mathbf{C}(\ell)||_F$. In this case, the threshold level could be revised in order to adjust the sensitivity of the detector (e.g., reducing η_{thr} of an amount of 10% increases the success rate to 98.2%.).

6. CONCLUSION

In this paper, a method using the output 4th-order cumulants has been proposed to detect the number of sources and identify their respective transmission channels for a 2×1 communication system. The same principle applies for any $P \times 1$ MISO channel. The proposed order detection principle is based on the search for the highest value of a test-variable so that the cumulant matrix is non-zero. After extracting the contributions of that channel from the output cumulants, a similar procedure is applied for detecting the presence of a second source as well as its associated channel order and coefficients. Because there is no means to see the sum of two linear processes as a scalar linear process, we cannot estimate parameters of channels of same order. However, it would also be possible to detect the number of sources via a linearity test [5]. Considering estimated cumulants, two hypothesis tests based on three proposed test-statistics have been implemented. Some perspectives include a tensor approach based on a test of the tensor rank. The asymptotic distribution of the proposed test variables also needs to be investigated.

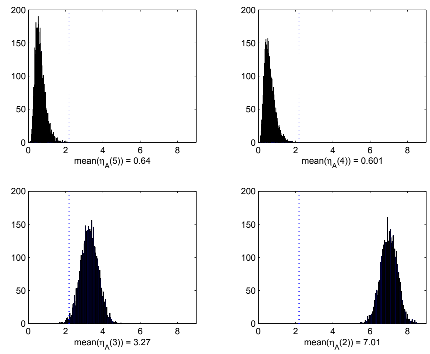

Fig. 3. Histogram plots for η_A (mean value of η_{thr} is 2.19).

Acknowledgement

The authors would like to thank the anonymous reviewers for their relevant and useful remarks.

7. REFERENCES

[1] P. Comon, "Independent Component Analysis, a new concept ?," *Signal Processing, Elsevier*, vol. 36, no. 3, pp. 287–314, apr. 1994, Special issue on Higher-Order Statistics.

[2] E. Moulines, P. Duhamel, J. F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Sig. Proc.*, vol. 43, no. 2, pp. 516–525, feb. 1995.

[3] P. Comon, "MA identification using fourth order cumulants," *Signal Processing, Elsevier*, vol. 26, no. 3, pp. 381–388, 1992.

[4] D. R. Brillinger, and M. Rosenblatt, "Computation and interpretation of kth-order spectra," *Spectral Analysis of Time Series*, Wiley, B. Harris (ed.), New York, USA, pp. 189–232, 1967.

[5] M. Hinich, "Testing for Gaussianity and linearity of a stationary time series," *Journal of Time Series Analysis*, vol. 3, no. 3, pp. 169–176, 1982.