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# Path-Wise Wide-Sense Polynomial Receiver for UMTS communications*

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**Abstract**

In existing DS-CDMA systems, synchronization and channel identification are carried out by cross-correlation with spreading sequences (path-wise correlators). In a second stage, the symbol sequence of the user of interest is obtained by maximum-ratio combining (matched filtering) of the path-wise correlator outputs (Space-Time Rake). Here, we show that this second stage can be improved by using a simple non linear Space-Time receiver, formally polynomial, adapted to the discrete distribution of the symbols transmitted. The gain in performance is most visible when a single antenna element is used for reception.

## 1 Introduction

The context of this work is that of DS-CDMA communications [4]. The work has been carried out with the goal of developing efficient receivers dedicated to UMTS communications, at a reasonable computational cost, but performing better than the standard Space-Time (ST) RAKE receiver.

In standard systems, synchronization and channel parameters are identified by cross-correlation between observed and spreading sequences. Throughout the paper, one assumes that the channel has been identified in a first stage, and our goal is to estimate the symbol sequence of the user of interest. It is assumed that the spreading sequence of this user is known, but those of other users are supposed to be unknown (which is always the case in downlink communications at the mobile receiver, for instance).

The sum of contributions of all interfering users thus appears as a non Gaussian noise of unknown statistics. Yet, most algorithms utilized in the receivers assume that this noise is Gaussian, which is valid only if the number of interfering users is sufficiently large in *all directions*. In practice, there are always some regions of the space in which the number of users is small, so that this simplifying assumption does not hold true.

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## 2 Notation and Modeling

The DS-CDMA signal $x(t)$ transmitted over the channel takes the following form:

$$x(t) = \sum_{n=0}^{N-1} s(n) \sum_{\ell=0}^{G-1} C(\ell)\, g(t - nT_s - \ell T_c) \tag{1}$$

where:

- $g(t)$ denotes the transmit filter (pulse shaping), of finite support, and $t$ the continuous time variable;
- $T_s$ and $T_c$ are the symbol and chip periods, respectively;
- $G$ is the spreading gain, $G = T_s/T_c$;
- $C(\ell)$ denotes the spreading sequence of the user of interest, and $C^u(\ell)$ that of the $u$th interfering user; $\ell$ is thus a discrete variable belonging to $\{0, 1.., G-1\}$;
- $s(n)$ is the transmitted symbol sequence, $1 \leq n \leq N$.

In the present framework, the channel is assumed to be specular, that is, it takes the following form:

$$h_k(t) = \sum_{p=1}^{P} A_{kpn}\, \delta(t - \tau_p) \tag{2}$$

where $h_k(t)$ denotes the channel linking the user of interest and sensor $k$. The signal received on the $k$th sensor of the array, $1 \leq k \leq K$, takes the form:

$$y_k(t) = \sum_m h_k(t-m) x(m) \tag{3}$$

Every path $p$ is delayed by $\tau_p$ and undergoes an attenuation $A_{kpn} = \rho_p \exp\{\jmath \varphi_{pn}\}\, d_{kp}$, where the amplitude $\rho_p$ is constant over at least one symbol period $T_s$, whereas the phase of the reflection coefficient, $\varphi_{pn}$, may change from one symbol to the next one; in addition, the distance between sensors is assumed to be sufficiently small so that there exists only a phase factor, $d_{kp}$, (and no attenuation) depending only on the Direction Of Arrival (DOA), distinguishing between the signals received on different sensors. This phase factor $d_{kp}$ thus naturally depends on the path index $p$ and the sensor index $k$.

Combining (1), (2), and (3), the signal received on the array can be written as:

$$y_k(t) = \sum_{pn\ell} s(n)\, A_{kpn}\, C(\ell)\, g(t - \tau_p - nT_s - \ell T_c) + v_k(t) \tag{4}$$

or, in block form:

$$\boldsymbol{y}(n) = \boldsymbol{H}\, \boldsymbol{s}(n) + \boldsymbol{v}(n) \tag{5}$$

where $\boldsymbol{y}(t)$ denotes the $K-$dimensional vector with components $y_k(t)$, $\boldsymbol{s}(n)$ the $L-$dimensional vector with entries $s(n), s(n-1), \ldots, s(n-L+1)$, where $L\,T_s$ represents the channel length, and $\boldsymbol{H}$ is a $K \times L$ Töplitz matrix built on the channel taps $h_k(n)$.

In the above observation models, the noise $\boldsymbol{v}(n)$ is itself formed of a sum of interferring users of same type, $s^u(n)$; more precisely, we have that:

$$v_k(t) = \sum_{u=1}^{M} \sum_{pn\ell} s^u(n)\, A^u_{kpn}\, C^u(\ell)\, g(t - \tau^u_p - nT_s - \ell T_c) \tag{6}$$

The number of interfering users, $M$, is always assumed to be strictly smaller than the spreading factor: $M \leq G - 1$.

Now, we shall also require subsequently the definitions below:

- For zero-mean processes, stationary at second order, one denotes the inter-correlation function $\mathbf{\Gamma}_{xy}(\tau) = \mathrm{E}\{\boldsymbol{x}(t)\,\boldsymbol{y}(t-\tau)^{\mathrm{H}}\}$. By default, $\mathbf{\Gamma}_{xy}$ denotes the covariance for the zero time lag, $\tau = 0$.
- $\boldsymbol{a} \otimes \boldsymbol{b}$ denotes the Kronecker product between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. It contains in vector form all the cross-products $a_i b_j$, and its dimension is thus equal to the product of the dimensions of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.
- $\boldsymbol{y} \oslash \boldsymbol{y}$ denotes the *symmetric* Kronecker product between $\boldsymbol{y}$ and itself. This vector contains all the *distinct* products $y_i y_j$, $i \geq j$, and is therefore of size $K(K+1)/2$ if $\boldsymbol{y}$ is of size $K$.

# 3 Linear solutions

In linear statistical models such as (5), the optimal estimate of $s(n)$ in the Mean Square Error (MSE) sense is given by [1]:

$$\hat{s}(n) = \mathbf{\Gamma}_{sy}\,\mathbf{\Gamma}_{yy}^{-1}\,\boldsymbol{y} \tag{7}$$

Another writing is useful when the channel $\boldsymbol{H}$ is known:

$$\hat{s}(n) = \mathbf{\Gamma}_{ss}\,\boldsymbol{H}^{\mathrm{H}}\,\mathbf{\Gamma}_{yy}^{-1}\,\boldsymbol{y} \tag{8}$$

This MSE solution is also optimal in the Maximum Likelihood (ML) sense, conditionally to $\boldsymbol{s}(n)$, when the noise $\boldsymbol{v}(n)$ is Gaussian (it is the ML solution when the signal $\boldsymbol{s}(n)$ is deterministic or uniformly distributed). The Space-Time Matched Filter (ST-MF) is a simplification of this solution, that can be obtained by replacing the covariance matrix of $\boldsymbol{y}$ by the identity:

$$\hat{s}_{mf}(n) = \mathbf{\Gamma}_{ss}\,\boldsymbol{H}^{\mathrm{H}}\,\boldsymbol{y} \tag{9}$$

When $\mathbf{\Gamma}_{ss}$ is unknown, it is often replaced by $\boldsymbol{H}^{\mathrm{H}}\,\mathbf{\Gamma}_{yy}^{-1}\,\boldsymbol{H}$. The well known Space-Time RAKE solution actually implements the above ST-MF (see next section), which can be viewed as a ST Zero Forcing solution.

At this point, some conclusions can already be drawn. First, the linear solutions (7) or (8) are generally not optimal in the ML sense when the noise is not Gaussian. However, when the signal $s(n)$ is deterministic or uniformly distributed, the best ML estimate of $s(n)$ may still be linear, even if the noise is not Gaussian, but its expression will take a different form. On the other hand, if the distribution of the signal $s(n)$ is not constant, the optimal solution should be sought in the sense of the Maximum A Posteriori (MAP), and has very little chances to be linear.

These observations argue in favor of a search for non linear solutions, especially when both signal and noise have a discrete distribution, as it is the case in the present context.

Finally, from a general point of view, note that the linear model (5) and the related solutions (8) or (9) are relevant for either despreading or signal estimation. However, a non linear despreading is not envisaged in the present framework because of its excessive computational complexity.

# 4 Path-wise processing in CDMA

As explained above, we shall consider that despreading has been carried out in a linear way. But for various reasons, and particularly because of huge computational reductions, it is convenient to proceed path-wise [2], as elaborated now in the present section.

In order to focus our attention, consider a fixed sampling instant $nT_s + \ell T_c$. Because we have assumed that the channel (2) has been identified in a preliminary stage, it is possible to compute, for every path $p$, and every sensor $k$, the output of the matched filter associated with the shaping filter $g(t - nT_s - \ell T_c)$:

$$\bar{y}_{kp\ell}(n) \stackrel{\text{def}}{=} \int_t y_k(t + \tau_p)\, g(t - nT_s - \ell T_c)^* dt \tag{10}$$

Replacing $y_k$ by its expression as a function of channel parameters yields:

$$\bar{y}_{kp\ell}(n) = A_{kpn}\, C(\ell)\, s(n) + w_{kp\ell}(n) + v_{kp\ell}(n) \tag{11}$$

where we find two types of noises. The first one, $w_{kp\ell}(n)$, contains inter-chip interferences from the user of interest. The second one, $v_{kp\ell}(n)$, stems from interfering users, and will be partially eliminated by projection onto the spreading sequence $C(\ell)$.

As already mentioned, in view of expression (11), it appears that we have a linear model, and that we could despread non linearly. But we shall proceed again linearly via a standard matched filter scheme. We get easily, assuming that the spreading sequences are all orthonormal:

$$\bar{y}_{kp}(n) \stackrel{\text{def}}{=} \sum_{\ell=1}^{G} C(\ell)^* \int_t y_k(t + \tau_p)\, g(t - nT_s - \ell T_c)^* dt \tag{12}$$

or

$$\bar{y}_{kp}(n) \stackrel{\text{def}}{=} A_{kpn} s(n) + w_{kp}(n) + v_{kp}(n).$$

Now, this equation defines a stochastic process at the symbol rate $T_s$.

Next, express the components of the inter-user noise $v_{kp}(n)$. By using (12) and (6), we get:

$$v_{kp}(n) = \sum_{\ell} C(\ell) \int_t g(t - \ell T_c)^* \sum_{uq\ell' n'} s^u(n')\, A^u_{kqn}\, C^u(\ell')\, g(t + \tau_p - \tau^u_q - n'T_s - \ell' T_c)$$

or, after integration over the real line:

$$v_{kp}(n) = \sum_{u} \sum_{qn'\ell\ell'} s^u(n') A^u_{qkn'} C(\ell') C^u(\ell) \mathbf{\Gamma}_g(\tau_p - \tau^u_q + (n - n')T_s + (\ell - \ell')T_c)$$

where, if $g(t)$ is a raised cosine,

$$\mathbf{\Gamma}_g(\tau) = \operatorname{sinc}(\pi\, \tau / T_c)\, \frac{\cos(\pi \beta\, \tau / T_c)}{1 - 4\beta^2\, \tau^2 / T_c^2}.$$

We can proceed similarly for Inter-Chip (IC) and Inter-Symbol (IS) interferences. More precisely, the noise $w_{kp}(n)$ can be split into $w^{IC}_{kp}(n) + w^{IS}_{kp}(n)$; firstly:

$$w^{IC}_{kp}(n) = s(n) \sum_{q \neq p} \sum_{\ell\ell'} A_{kqn} C(\ell') C(\ell)\, \mathbf{\Gamma}_g(\tau_p - \tau_q + (\ell - \ell')T_c) \; - s(n) A_{kpn} \tag{13}$$

because $\mathbf{\Gamma}_g(0) = 1$ and $\sum_\ell C(\ell)\, C(\ell + \Delta) = \delta(\Delta)$. Note that this noise is proportional to $s(n)$. Secondly:

$$\begin{aligned} w^{IS}_{kp}(n) &= \sum_q A_{kq} \sum_{\ell\ell'} C(\ell)^* C(\ell')\ [s(n-1)\mathbf{\Gamma}_g(\tau_p - \tau_q + (\ell - \ell')T_c + T_s) \\ &\quad + s(n+1)\mathbf{\Gamma}_g(\tau_p - \tau_q + (\ell - \ell')T_c - T_s)] \end{aligned}$$

The noise data matrix, of entries $w_{kp}(n) = w^{IC}_{kp}(n) + w^{IS}_{kp}(n)$, is then of rank 3, since it is of the form $\boldsymbol{w}(n) = \boldsymbol{w}^{ic} s(n) + \boldsymbol{w}^{is+}\, s(n+1) + \boldsymbol{w}^{is-}\, s(n-1)$, $1 \leq n \leq N$, and since vectors $\boldsymbol{w}^{ic}$, $\boldsymbol{w}^{is+}$, and $\boldsymbol{w}^{is-}$ are generally linearly independent. In practice, $\boldsymbol{w}^{ic}$ is dominant, but $\boldsymbol{w}^{is+}$ and $\boldsymbol{w}^{is-}$ are not negligible.

Our main point is that, after path-wise matched filter and linear despreading, we can obtain a linear model with a simpler *static form*:

$$\bar{\boldsymbol{y}}(n) \stackrel{\text{def}}{=} \boldsymbol{a}\, s(n) + \boldsymbol{\nu}(n), \quad \boldsymbol{\nu}(n) \stackrel{\text{def}}{=} \sum_u a^u s^u(n) \tag{14}$$

where

$$a_{kp} \stackrel{\text{def}}{=} \sum_q A_{kpn} \mathbf{\Gamma}_g(\tau_p - \tau_q)$$

does not depend on $n$ if $A_{kpn}$ does not over a frame, as usually admitted. In this model, vector $\boldsymbol{y}(n)$ is of size $KP$ and not of size $K$ anymore: the diversity has increased. On the other hand, $s(n)$ is now scalar. Lastly, $s(n)$ and $\boldsymbol{\nu}(n)$ are statistically independent if the so-called "noise" term $w^{IC}_{kp}(n)$, which is proportional to $s(n)$, is pulled into the signal part, whereas the other one, $w^{IS}_{kp}$ is added to the noise part, $\boldsymbol{\nu}(n)$. Under these conditions, independence is insured because $s(n)$ is an iid sequence.

With these notations, the ST-RAKE takes the simple form:

$$\hat{s}(n) = \boldsymbol{a}^{\mathrm{H}} \bar{\boldsymbol{y}}(n) / ||\boldsymbol{a}||^2 \tag{15}$$

Lastly, signal and noises have a discrete distribution, and we shall assume in this paper that for all users, the symbol sequences belong to the same QPSK alphabet. In other words, we shall assume that

$$s(n)^4 = 1, \forall n. \tag{16}$$

# 5 Wide-Sense Polynomial receivers

With the goal of limiting the computational complexity, the only non linearities that will be considered are formal polynomials in the variables $\boldsymbol{y}(n)$ and $\boldsymbol{y}^*(n)$, hence the name of Wide-Sense Polynomials (WSP). The idea is to augment the actual observation $\boldsymbol{y}(n)$ by a WSP virtual observation, $\boldsymbol{z}(n)$, of the form (limiting to the degree 3):

$$\boldsymbol{z}(n) = \begin{bmatrix} y^{\oslash 3*}(n) \\ y^{\oslash 2}(n) \otimes y^*(n) \\ y^{\oslash 2}(n) \\ y^{\oslash 2*}(n) \\ y(n) \otimes y^*(n) \\ y^*(n) \\ y(n) \otimes y^{\oslash 2*}(n) \\ y^{\oslash 3}(n) \end{bmatrix} \tag{17}$$

| | |
|---|---|
| $E\{s(n)s^*(n)\} = 1$ | $E\{\nu(n)\nu^*(n)\} = \Gamma_\nu$ |
| $E\{s^2(n)\} = 0$ | $E\{\nu^2(n)\} = 0$ |
| $E\{s^3(n)\} = 0$ | $E\{\nu^3(n)\} = 0$ |
| $E\{s^2(n)s^*(n)\} = 0$ | $E\{\nu^2(n)\nu^*(n)\} = 0$ |
| $E\{s^4(n)\} = 1$ | $E\{\nu^4(n)\} = M_\nu$ |
| $E\{s^3(n)s^*(n)\} = 0$ | $E\{\nu^3(n)\nu^*(n)\} = 0$ |
| $E\{s^2(n)s^{2*}(n)\} = 1$ | $E\{\nu^2(n)\nu^{2*}(n)\} = K_\nu$ |
| $E\{s^5(n)\} = 0$ | $E\{\nu^5(n)\} = 0$ |
| $E\{s^4(n)s^*(n)\} = 0$ | $E\{\nu^4(n)\nu^*(n)\} = 0$ |
| $E\{s^3(n)s^{2*}(n)\} = 0$ | $E\{\nu^3(n)\nu^{2*}(n)\} = 0$ |
| $E\{s^6(n)\} = 0$ | $E\{\nu^6(n)\} = 0$ |
| $E\{s^5(n)s^*(n)\} = 1$ | $E\{\nu^5(n)\nu^*(n)\} = Q_\nu$ |
| $E\{s^4(n)s^{2*}(n)\} = 0$ | $E\{\nu^4(n)\nu^{2*}(n)\} = 0$ |
| $E\{s^3(n)s^{3*}(n)\} = 1$ | $E\{\nu^3(n)\nu^{3*}(n)\} = N_\nu$ |

Table 1: Properties of signal $s(n)$ and noise $\nu(n)$ when users are QPSK.

### 5.1 Diversity 1 case

To simplify the notation, let us explain the principles in the case of a diversity 1 in a first stage. Because all users are QPSK distributed, they satisfy relation (16), and consequently all relations summarized in table 1, where $\Gamma_v$ and $K_v$ are real numbers, and $M_v$ and $Q_v$ complex numbers, all depending on the noise statistics.

Now denote the augmented observation as

$$\boldsymbol{Y}(n) = \begin{bmatrix} y(n) \\ \boldsymbol{z}(n) \end{bmatrix}$$

where $\boldsymbol{z}(n)$ is defined in (17). Then, the ST MF WSP and ST MSE WSP equalizers are defined as follows:

$$\begin{aligned} \hat{s}^{wsp}_{mf}(n) &= \boldsymbol{\Gamma}_{sY}\boldsymbol{Y}(n) && (18) \\ \hat{s}^{wsp}_{mse}(n) &= \boldsymbol{\Gamma}_{sY}\boldsymbol{\Gamma}_Y^{-1}\boldsymbol{Y}(n) && (19) \end{aligned}$$

Yet, from table 1, it appears that $\boldsymbol{\Gamma}_Y$ is of the form:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{12}^* & A_{22} & A_{32}^* & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{13} & A_{32} & A_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{13} & A_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{12}^* & A_{13} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{11} & A_{13} & A_{12}^* \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{13} & A_{22} & A_{32}^* \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{12} & A_{32} & A_{22} \end{bmatrix}$$

and that

$$\boldsymbol{\Gamma}_{sY} = \Big[a,\ a^{3*},\ a^2a^*,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\Big]$$

where: $A_{11} = aa^* + \Gamma_v$, $A_{12} = a^4 + M_v$, $A_{13} = a^2a^{2*} + K_v$, $A_{22} = a^3a^{3*} + N_v$ and $A_{32} = a^*a^5 + Q_v$. It is thus clear that: (i) covariance $\boldsymbol{\Gamma}_Y$ contains four distinct blocks,

| $p$ | $\tau_p$(ns) | $\rho_p$(dB) | $\rho_p$(dB) | DOA |
|---|---|---|---|---|
| 1 | | | $-7$ | $\Delta/2$ |
| 2 | 0 | 0 | $-2$ | 0 |
| 3 | | | $-7$ | $-\Delta/2$ |
| 4 | 310 | $-1$ | $-4$ | $\Delta/2-\delta$ |
| 5 | | | $-4$ | $-\Delta/2+\delta$ |
| 6 | 710 | $-9$ | $-9$ | $2\Delta$ |
| 7 | 1090 | $-10$ | $-10$ | $-2\Delta$ |
| 8 | 1730 | $-15$ | $-15$ | $3\Delta$ |
| 9 | 2510 | $-20$ | $-20$ | $4\Delta$ |

Table 2: Amplitude of reflection coefficients and DOAs for every path of the channel proposed by France Telecom R&D (FTRD) in 1998 [3]. The chip period is $T_c = 244$ns, $\Delta = 25^o$, as suggested by FTRD. We added parameter $\delta$, set here to $\delta = 5^o$.

so that equations decouple in the linear system to be solved for $\boldsymbol{F}$, $\boldsymbol{\Gamma}_Y\boldsymbol{F} = \boldsymbol{\Gamma}_{Ys}$, (ii) the right hand side is non zero only for the 3 equations of the first block.

As a consequence, the only augmented observations that are useful for the WSP equalizer are $y^{3*}(n)$ and $y^2(n)y^*(n)$.

### 5.2 General case: diversity $KP$

In a similar manner, though with more cumbersome notations, it can be proved that the only useful augmented observations are $\boldsymbol{y}^{\oslash 3*}(n)$ and $\boldsymbol{y}^{\oslash 2}(n) \otimes \boldsymbol{y}^*(n)$. From now on, we shall assume that $\boldsymbol{z}(n)$ contains only these two WSP terms. Vector $\boldsymbol{z}(n)$ is thus of size $PK(PK-1)(PK-2)/6 + P^2K^2(PK-1)/2$.

## 6 Simulation results

### 6.1 Channel generation

**Uplink.** For the reverse link, up to 15 interfering users have been considered, each with a spreading factor of $G = 16$. Each user, including the user of interest, propagates through a channel impinge on the array with a total power of 1. The transmit filter has a rolloff factor of $\beta = 0.22$.

For the user of interest, the channel is composed of $P = 9$ paths and 6 distinct delays. According to a model proposed by France Telecom R&D, the DOAs, the delays, and the amplitude of reflection coefficients, are given by table 2.

The phase of reflection coefficients have a random phase uniformly drawn in $[0,\ \pi[$. The channel of each interfering user is composed of 1 to 9 paths (this number is randomly drawn in every realization), their amplitude is also randomly drawn and then normalized (in order for the total power to be unity), the DOAs are randomly drawn within $[\pi/6,\ 5\pi/6]$, and the phase of the reflection coefficients are drawn randomly in $[0,\ \pi[$. Spreading sequences are kept fixed (Hadamard), and form an orthonormal set (if not shifted in time).

**Downlink.** For the forward link, only the channel of the user of interest is generated, the procedure being the same as for the reverse link, as described above.

## 6.2 Performances

Several equalizers are considered in this section:

- The ST-RAKE given by (15), labeled "Linear Rake Hs" in figure 1 (left),
- The ST-RAKE given by $\hat{s}_{mf}(n) = \boldsymbol{\Gamma}_{sy}\boldsymbol{y}(n)$, labeled "Linear Rake S",
- The linear MSE equalizer (7), labeled "Linear MSE S",
- The RAKE (18), with $\boldsymbol{z}(n) = \boldsymbol{y}^{\oslash 2}(n) \otimes \boldsymbol{y}^{*}(n)$, labeled "cubic12 Rake S",
- The MSE (19), with $\boldsymbol{z}(n) = \boldsymbol{y}^{\oslash 2}(n) \otimes \boldsymbol{y}^{*}(n)$, labeled "cubic12 MSE".

Performances of the "cubic30" RAKE and MSE equalizers, involving the virtual WSP observation $\boldsymbol{z}(n) = \boldsymbol{y}^{\oslash 3*}(n)$, are not reported here, because they turned out to be slightly less attractive than those of the "cubic12". In addition, the joint use of $\boldsymbol{y}^{\oslash 2}(n) \otimes \boldsymbol{y}^{*}(n)$ and $\boldsymbol{y}^{\oslash 3*}(n)$ was not significantly better than that of $\boldsymbol{y}^{\oslash 2}(n) \otimes \boldsymbol{y}^{*}(n)$ alone, and its complexity was considerably larger.

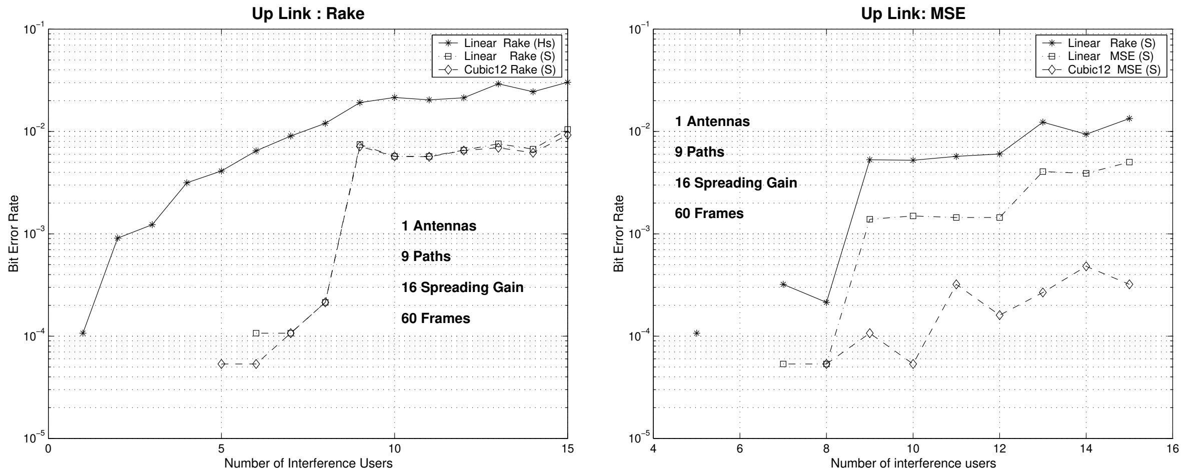


Figure 1: Bit Error Rate for RAKE (left) and MSE (right) receivers.

The Bit Error Rates (BER) reported in figure 1 (left) shows that the cubic "Rake S" does not perform significantly better than its linear version. On the other hand, from figure 1 (right), it is clear that the cubic equalizer is much more attractive in its MSE configuration rather than in its RAKE form. It is also obvious that WSP terms bring a very significant improvement compared to the linear MSE receiver.

# 7 MSE Reduction

In this section, we investigate the theoretical decrease in MSE yielded by appending a virtual WSP observation, $\boldsymbol{z}(n)$, to the actual observation, $\boldsymbol{y}(n)$.

Denote the linear MSE estimation given by (7) as

$$\hat{s}_1(n) = \boldsymbol{R}_{sy}\boldsymbol{R}_y^{-1}\boldsymbol{y}(n) \tag{20}$$

and the other stacked WSP MSE estimation as

$$\hat{s}_2(n) = [\boldsymbol{R}_{sy}\boldsymbol{R}_{sz}]\boldsymbol{\Gamma}_{yz}^{-1}[\boldsymbol{y}^{\mathrm{T}}(n)\boldsymbol{z}^{\mathrm{T}}(n)]^{\mathrm{T}} \tag{21}$$

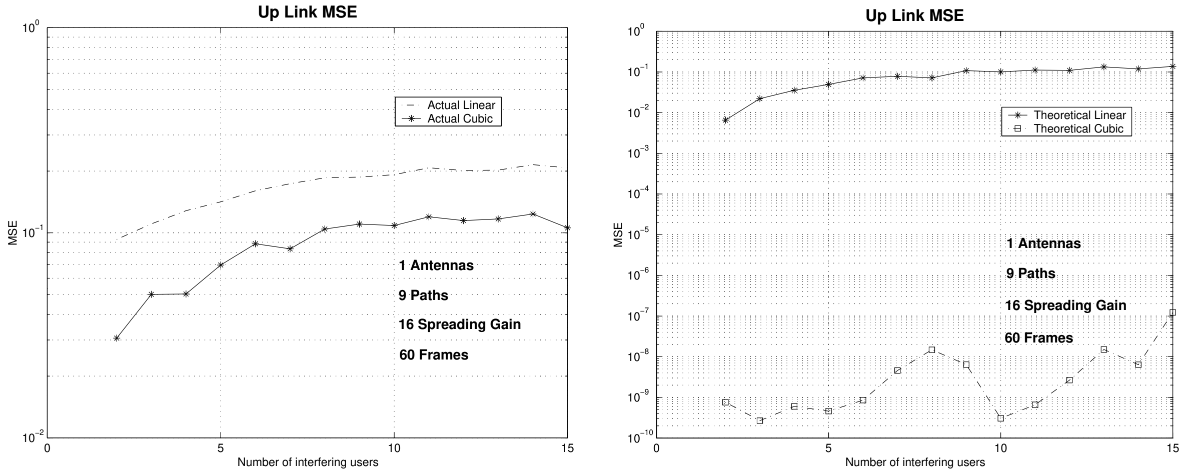


Figure 2: Mean Square Error for the MSE receivers in the reverse link. Left: actual; Right: theoretical.

where $\boldsymbol{\Gamma}_{yz} = \begin{bmatrix} \boldsymbol{R}_y & \boldsymbol{R}_{zy} \\ \boldsymbol{R}_{yz} & \boldsymbol{R}_z \end{bmatrix}$, $\boldsymbol{R}_{sy} = \boldsymbol{R}_{ys}^{\mathrm{H}}$ , $\boldsymbol{R}_{zy} = \boldsymbol{R}_{yz}^{\mathrm{H}}$ and $\boldsymbol{z}(n)$ is either one of the two first terms in (17), or both. Their MSEs are

$$\begin{aligned} \varepsilon_1 &= \boldsymbol{R}_s - \boldsymbol{R}_{sy}\boldsymbol{R}_y^{-1}\boldsymbol{R}_{ys} \\ \varepsilon_2 &= \boldsymbol{R}_s - [\boldsymbol{R}_{sy}\boldsymbol{R}_{sz}]\boldsymbol{\Gamma}_{yz}^{-1}[\boldsymbol{R}_{ys}{}^{\mathrm{T}}\boldsymbol{R}_{zs}{}^{\mathrm{T}}]^{\mathrm{T}} \end{aligned}$$

Now the improvement brought by using WSP observations can be defined as the decrease in MSE, namely $\Delta\varepsilon = \varepsilon_1 - \varepsilon_2$. It is always positive or zero and takes the following general form:

$$\Delta\varepsilon = \boldsymbol{\Gamma}_{zys}^{\mathrm{H}}\boldsymbol{\Gamma}_{zyz}^{-1}\boldsymbol{\Gamma}_{zys} \tag{22}$$

where

$$\boldsymbol{\Gamma}_{zys} \stackrel{\text{def}}{=} [\boldsymbol{R}_{zs} + \boldsymbol{R}_{zy}\boldsymbol{R}_y^{-1}\boldsymbol{R}_{ys}] \quad \text{and} \quad \boldsymbol{\Gamma}_{zyz} \stackrel{\text{def}}{=} [\boldsymbol{R}_z - \boldsymbol{R}_{zy}\boldsymbol{R}_y^{-1}\boldsymbol{R}_{yz}]$$

The proof is not reproduced for reasons of space. This result extends that of [5]. We can observe that the improvement is strictly positive if: (i) $\boldsymbol{z}$ is not too much correlated with $\boldsymbol{y}$ (otherwise the MSE solution is ill-conditioned), and (ii) $\boldsymbol{z}$ is correlated enough with either $s$ or $\boldsymbol{y}$.

**Actual versus theoretical performances.** Actual performances are obtained by generating the whole processing line ($x(t)$, $h(t)$, and $y(t)$, matched filter and then despreading). On the other hand, the theoretical performances (in terms of MSE) are obtained by generating directly $\boldsymbol{a}$ in (14), and by computing errors $\varepsilon_i$ with the help of relations of the form:

$$\varepsilon_1 = 1 - \boldsymbol{a}^{\mathrm{H}}(\boldsymbol{a}\boldsymbol{a}^{\mathrm{H}} + \boldsymbol{R}_v)\boldsymbol{a}, \quad \boldsymbol{R}_v = \sum_u \boldsymbol{a}^u\boldsymbol{a}^{u\mathrm{H}}$$

The difference between theoretical and actual performances, visible in figure 2 for the uplink, are essentially due to the loss in orthogonality between spreading sequences, when shifted. However, performances are significantly improved by a WSP receiver, even in the "actual" implementation. In the downlink case, the improvement is less striking, but still significant, as shown in figure 3.

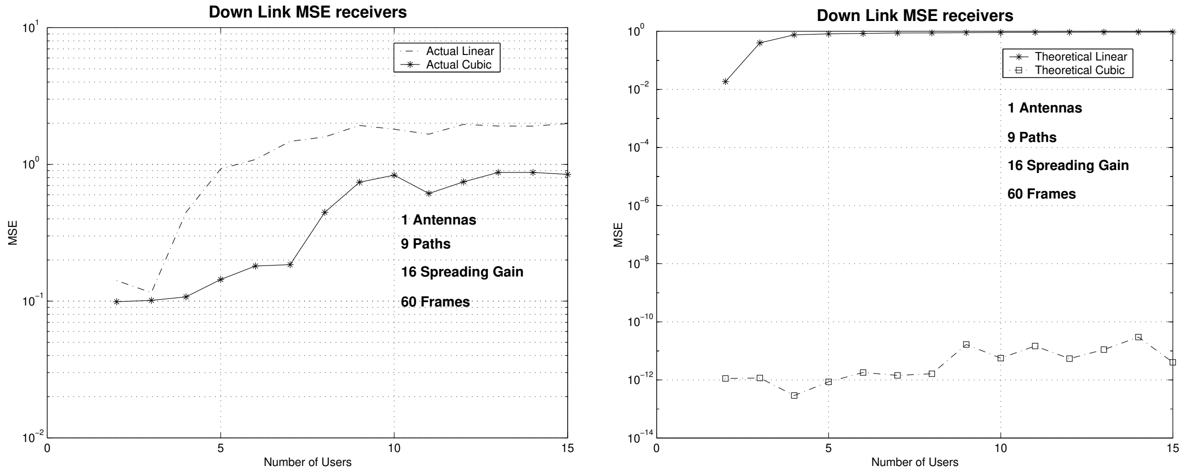


Figure 3: Mean Square Error for the MSE receivers in the forward link. Left: actual; Right: theoretical.

# 8 Concluding remarks

Wide-Sense Polynomial (WSP) non linearities have been demonstrated to bring significant improvement in equalizing a known channel. One of the strongest advantage of this particular non linearity lies in its simplicity of implementation. It has been shown that, for QPSK modulated users, only one family of WSP terms of degree 3 is really useful.

Because of numerical complexity, despreading is done linearly in a standard manner (in Matched Filter form), but one could theoretically envisage to build a joint despreading-equalization along the same lines.

The gain of the WSP processing is most visible for $K = 1$ array element, and is not yet demonstrated for a larger number of antennas, at least according to the results we have obtained so far.

Lastly, computer simulations have also been run in the downlink context, and similar results have been obtained from a qualitative point of view. So, the above conclusions are the same.

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