Int. Conf. Acoust. Speech and Signal Processing (ICASSP), May 7-11, 2001, Salt Lake City

BLIND MIMO EQUALIZATION AND JOINT-DIAGONALIZATION CRITERIA

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ABSTRACT

We consider the problem of convolutive blind signal separation through the optimization of contrast functions. In this work, we show that some links between contrasts and joint diagonalization criteria can be exhibited in the convolutive case. This allows to devise a constructive algorithm performing MIMO blind equalization, with the help of a joint approximate diagonalization of a set of matrices built from the observations. This analytical algorithm can be run block-wise, which is appropriate in the context of short burst communications.

1. INTRODUCTION

We consider the problem of blind equalization, or *deconvolution*, of Linear Time Invariant (LTI) Multiple-Input Multiple-Output (MIMO) systems. Such a problem is of interest in multi-user wireless communications, where MIMO systems are expected to equalize the observed signals both in space and time. In fact, MIMO equalization aims at eliminating inter-symbol interferences due to possible delays introduced by multi-paths propagation, and co-channel interferences due to the possible presence of simultaneous users in the same band. Examples are found in Space Division Multiple Access (SDMA) or Code Division Multiple Access (CDMA) communications systems.

Other well-known fields of application are those where the generic problem arises; this includes array processing, passive sonar, seismic exploration, speech processing, interception, surveillance...

One can find numerous works on blind equalization of Single-Input Single-Output (SISO) channels using high-order statistics [2] [11], or constant modulus [17] [9] or constant power [8] properties, whereas the multichannel framework, although more recent, already seems to be more powerful; numerous references include [12] [1] [3] [6] [13]. A particular instance of the problem is the static mixture, often referred to as the "source separation" problem, where no inter-symbol interference is considered but only co-channel interference [5] [4]. The latter problem is relevant when the time delays are smaller than the symbol period, for example.

Blind MIMO deconvolution can be carried out with the help of on-line iterative algorithms [15], some of them extracting one source at a time (deflation) [16]. However, block methods become more and more attractive since computer power no longer appears to be an impediment. The use of block (off-line) algorithms presents a number of advantages including: great improvement on the convergence time (both estimation of statistics and optimization), increased facility to handle spurious local extrema, better robustness to loss of synchronization, possibility to work jointly with the reversed-time signal, natural formatting for TDMA transmission, especially short bursts like in the GSM standard [7]. On the other hand, adaptive (on-line) algorithms can also benefit from of block updates, and gain in convergence speed and complexity [10].

In this paper, our goal is to relate blind MIMO deconvolution to joint matrix diagonalization, yielding a constructive block algorithm. We consider the problem of estimating an inverse of the impulse response of a LTI MIMO channel, given only output measurements. Indeed, we estimate an equivalent multivariate Moving Average channel regardless of what the actual channel is. Our approach relies on inverse filter criteria based on high-order statistics.

2. PROBLEM FORMULATION

We consider the *invertible* multichannel LTI system described by

$$\boldsymbol{x}(n) = \sum_{k=-\infty}^{\infty} \boldsymbol{C}(k)\boldsymbol{a}(n-k) \tag{1}$$

where $\boldsymbol{a}(n)$ is the $N-$dimensional vector of sources, $\boldsymbol{x}(n)$ is the $N-$dimensional vector of observations and $\{\boldsymbol{C}\} \stackrel{\text{def}}{=}$

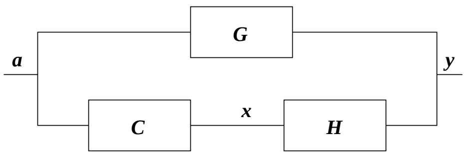
$\{\boldsymbol{C}(n), n \in \mathbb{Z}\}$ is a sequence of $N \times N$ matrices that corresponds to the impulse response of the LTI mixing filter. The multichannel blind deconvolution problem consists of estimating a LTI filter (equalizer) $\{\boldsymbol{H}\} \stackrel{\text{def}}{=} \{\boldsymbol{H}(n), n \in \mathbb{Z}\}$ using only the outputs $\boldsymbol{x}(n)$ of the unknown LTI system $\{\boldsymbol{C}\}$. $\{\boldsymbol{H}\}$ ensures the fact that vector $\boldsymbol{y}(n)$ defined as

$$\boldsymbol{y}(n) = \sum_{k=-\infty}^{\infty} \boldsymbol{H}(k)\boldsymbol{x}(n-k) \qquad (2)$$

restores the N input signals $a_i(n)$, $i \in \{1, \ldots, N\}$. We define the global LTI filter $\{\boldsymbol{G}\} \stackrel{\text{def}}{=} \{\boldsymbol{G}(n), n \in \mathbb{Z}\}$ as follows

$$\boldsymbol{y}(n) = \sum_{k=-\infty}^{\infty} \boldsymbol{G}(k)\boldsymbol{a}(n-k) \stackrel{\text{def}}{=} [\widehat{\boldsymbol{G}}(z)]\boldsymbol{a}(n) \qquad (3)$$

where $\widehat{\boldsymbol{G}}(z) \stackrel{\text{def}}{=} \sum_{k=-\infty}^{\infty} \boldsymbol{G}(k) z^{-k}$ stands for the $N \times N$ transfer matrix and z^{-1} the backward-shift operator.



Processing line: $\boldsymbol{C}$ is the channel, $\boldsymbol{H}$ the equalizer, and $\boldsymbol{G}$ the global system.

Assumptions. The following properties are assumed throughout the paper:

- **A1**. The sources $a_i(n)$, $i \in \{1, \ldots, N\}$, are mutually statistically independent. Each source is a sequence of zero-mean and unit power independent and identically distributed (i.i.d.) real random variables.
- **A2**. $\boldsymbol{a}(n)$ is a random process stationary up to the considered order $r \in \mathbb{N}^*$, *i.e.* $\forall i \in \{1, \ldots, N\}$, the cumulant $\mathsf{Cum}\,[\underbrace{a_i(n), \ldots, a_i(n)}_{r \text{ times}}]$ is independent of n, and can be denoted compactly $\mathsf{C}_r[a_i]$.
- **A3**. At most one of the considered source cumulants is zero.
- **A4**. The transfer matrix $\widehat{\boldsymbol{G}}(z)$ satisfies $\widehat{\boldsymbol{G}}(z)\widehat{\boldsymbol{G}}^{\mathrm{H}}(1/z^*) = \boldsymbol{I}$ where $\boldsymbol{I}$ is the $N \times N$ identity matrix. In other words, the matrix $\widehat{\boldsymbol{G}}(z)$ is para-unitary.

The assumption **A4**, although stronger than that of [6] for $p = 2$, can be satisfied after a second order prewhitening of the observations, followed by a spatial standardization. Those two operations can be carried out with the help of classical algorithms, such as spectral minimum phase factorization and Principal Component Analysis.

Indeterminations. Since sources are assumed to be unobservable, some inherent indeterminations in their restitution subsist: in most cases, the order as well as the power and the time delay cannot be identified. Actually, these limitations combine the inherent indeterminations of the source separation problem together with those of the classical blind scalar deconvolution problem. Hence, signals are said to be separated if and only if (iff) the global LTI system $\widehat{\boldsymbol{G}}(z)$ reads

$$\widehat{\boldsymbol{G}}(z) = \boldsymbol{D}(z)\boldsymbol{\Lambda}\boldsymbol{P} \qquad (4)$$

where $\boldsymbol{D}(z) = \mathrm{Diag}(z^{-k_1}, \ldots, z^{-k_N})$ with $k_j \in \mathbb{Z}^N$, $1 \leq j \leq N$, $\boldsymbol{\Lambda}$ is an invertible constant diagonal matrix and $\boldsymbol{P}$ a permutation matrix. Note that, because $\widehat{\boldsymbol{G}}(z)$ satisfies (4) and **A4**, the entries of $\boldsymbol{\Lambda}$ are of unit modulus.

Notations. Let us define some useful notations. The set of matrix sequences satisfying assumption **A4** is denoted by $\mathcal{U}$. The set of source random vectors satisfying assumptions **A1** to **A4** is represented by $\mathcal{A}$. The subset of $\mathcal{U}$ of matrix sequences such that (4) is satisfied, often referred to as *trivial filters*, is denoted by $\mathcal{P}$. Finally, the set of random vectors $\boldsymbol{y}(n)$ satisfying (3) where $\boldsymbol{a} \in \mathcal{A}$ and $\{\boldsymbol{G}\} \in \mathcal{U}$ is denoted by $\mathcal{Y}$.

3. CONTRAST FUNCTIONS

3.1. New Contrast functions

First we generalize some contrasts available in the instantaneous case to the convolutive one. With this goal, let us introduce the following notation

$$\begin{aligned}&\mathsf{C}^{\boldsymbol{y}}_{p,r}[i, \boldsymbol{j}, \boldsymbol{\ell}] = \\ &\mathsf{Cum}[\underbrace{y_i(n), \ldots, y_i(n)}_{p \text{ terms}}, \underbrace{y_{j_1}(n-\ell_1), \ldots, y_{j_q}(n-\ell_q)}_{q=r-p \text{ terms}}]\end{aligned} \qquad (5)$$

where $r = p + q$ stands for the cumulant order and

$$\begin{aligned}\boldsymbol{j} &= (j_1, \ldots, j_q)\\ \boldsymbol{\ell} &= (\ell_1, \ldots, \ell_q)\ .\end{aligned}$$

Let us also consider the following set of indices: $\mathsf{J} = \{1, \ldots, N\}^q$ and the set of delays $\mathsf{L} = \mathbb{Z}^q$. We have the following first result:

Proposition 1 *Let r, p and q be three integers such that $r \geq 3$, $2 \leq p \leq r$ and $q = r - p$, the function*

$$\mathcal{J}_{p,r}(\boldsymbol{y}) = \sum_{i=1}^{N} \sum_{\boldsymbol{j} \in \mathsf{J}} \sum_{\boldsymbol{\ell} \in \mathsf{L}} \left|\mathsf{C}^{\boldsymbol{y}}_{p,r}[i, \boldsymbol{j}, \boldsymbol{\ell}]\right|^2 \qquad (6)$$

is a contrast for 2nd order standardized white observations $\boldsymbol{y}(n) \in \mathcal{Y}$.

Proof. It is easily seen that for a given order r of cumulants, we have $\mathcal{J}_{p,r}(\boldsymbol{y}) \leq \mathcal{J}_{2,r}(\boldsymbol{y})$. Now recalling that $\mathcal{J}_{2,r}(\boldsymbol{y})$ is a contrast [14], then $\mathcal{J}_{2,r}(\boldsymbol{y}) \leq \mathcal{J}_{2,r}(\boldsymbol{a})$. Because the sources are i.i.d. statistically independent signals, all their cross-cumulants are zero and $\mathcal{J}_{p,r}(\boldsymbol{a}) = \mathcal{J}_{2,r}(\boldsymbol{a})$. Then considering the above results altogether, $\mathcal{J}_{p,r}(\boldsymbol{y}) \leq \mathcal{J}_{p,r}(\boldsymbol{a})$. Finally, because $\mathcal{J}_{2,r}(\boldsymbol{y})$ is a contrast, it is not difficult to see that the equality $\mathcal{J}_{p,r}(\boldsymbol{y}) = \mathcal{J}_{p,r}(\boldsymbol{a})$ holds only for separating states. Thus $\mathcal{J}_{p,r}(\boldsymbol{y})$ is a contrast. $\diamondsuit$

Now one can even notice that the function $\mathcal{J}_{p,r}(\boldsymbol{y})$ can always be written as

$$\mathcal{J}_{p,r}(\boldsymbol{y}) = \mathcal{I}_r(\boldsymbol{y}) + \mathcal{C}_{p,r}(\boldsymbol{y}) \tag{7}$$

where

$$\mathcal{I}_r(\boldsymbol{y}) = \sum_{i=1}^{N} \left|\mathsf{C}_r[y_i]\right|^2 \tag{8}$$

and

$$\mathcal{C}_{p,r}(\boldsymbol{y}) \stackrel{\text{def}}{=} \sum_{i=1}^{N} \sum_{\boldsymbol{j}\in\mathsf{J}_c} \sum_{\boldsymbol{\ell}\in\mathsf{L}^*} \left|\mathsf{C}^{\boldsymbol{y}}_{p,r}[i,\boldsymbol{j},\boldsymbol{\ell}]\right|^2 . \tag{9}$$

In the above definition, the sets J_c and L^* are defined respectively as $\mathsf{J}_c = \mathsf{J} \setminus \mathsf{J}_d$ with $\mathsf{J}_d = \{(1,\ldots,1),\ldots,(N,\ldots,N)\}$ and $\mathsf{L}^* = \mathbb{Z}^q \setminus \{(0,\ldots,0)\}$. Let us remark that the function $\mathcal{I}_r(\boldsymbol{y})$ involves only auto-cumulants and was proved to be a contrast in [6]. On the contrary, the function $\mathcal{C}_{p,r}(\boldsymbol{y})$ involves only cross-cumulants and thus is zero when considering the source vector, *i.e.* $\mathcal{C}_{p,r}(\boldsymbol{a}) = 0$. Based on the above remark we can propose the following result:

Proposition 2 *Let r, p and q be three integers such that $r \geq 3$, $2 \leq p \leq r$ and $q = r - p$, with $\lambda \leq 1$ the function*

$$\mathcal{J}_{p,r,\lambda}(\boldsymbol{y}) = \mathcal{I}_r(\boldsymbol{y}) + \lambda\mathcal{C}_{p,r}(\boldsymbol{y}) \tag{10}$$

is a contrast for 2nd order standardized white observations $\boldsymbol{y}(n) \in \mathcal{Y}$.

Proof. The proof follows the same lines as the one for proposition 1. Briefly, because $\lambda \leq 1$ then $\mathcal{J}_{p,r,\lambda}(\boldsymbol{y}) \leq \mathcal{J}_{p,r}(\boldsymbol{y})$. Now according to proposition 1 $\mathcal{J}_{p,r}(\boldsymbol{y}) \leq \mathcal{J}_{p,r}(\boldsymbol{a})$. Moreover $\mathcal{J}_{p,r,\lambda}(\boldsymbol{a}) = \mathcal{J}_{p,r}(\boldsymbol{a}) = \mathcal{I}_r(\boldsymbol{a})$. Then considering the above results altogether, $\mathcal{J}_{p,r,\lambda}(\boldsymbol{y}) \leq \mathcal{J}_{p,r,\lambda}(\boldsymbol{a})$. Finally, because $\mathcal{J}_{p,r}(\boldsymbol{y})$ is a contrast, it is not difficult to see that the equality $\mathcal{J}_{p,r,\lambda}(\boldsymbol{y}) = \mathcal{J}_{p,r,\lambda}(\boldsymbol{a})$ holds only for separating states. Thus $\mathcal{J}_{p,r,\lambda}(\boldsymbol{y})$ is a contrast. $\diamondsuit$

3.2. Link with a joint diagonalization criterion

From now on, we take $p = 2$; this will be justified by the proposition below. Assume that the MIMO equalizer $\{\boldsymbol{H}\}$ is of finite length L. Then, for all i, $1 \leq i \leq N$:

$$y_i(n) = \sum_{\alpha=0}^{L-1} \sum_{a=1}^{N} H_{ia}(\alpha)\, x_a(n-\alpha) \tag{11}$$

Next, denote the $r-$th order cumulant multi-correlation function of $\boldsymbol{x}$ as

$$\begin{aligned}\boldsymbol{T}_{\boldsymbol{a},\boldsymbol{b}}(\boldsymbol{\alpha},\boldsymbol{\beta}) = \mathsf{Cum}[&x_{a_1}(n-\alpha_1),\\ x_{a_2}(n-\alpha_2),\, x_{b_1}(n-\beta_1),\ldots,&x_{b_q}(n-\beta_q)]\end{aligned} \tag{12}$$

where $\boldsymbol{a}$ and $\boldsymbol{\alpha}$ are vectors of size $p = 2$, and $\boldsymbol{b}$ and $\boldsymbol{\beta}$ are vectors of size $q = r - 2$. For every fixed value of a_2, $\boldsymbol{b}$, α_2, and $\boldsymbol{\beta}$, the cumulant tensor $\boldsymbol{T}_{\boldsymbol{a},\boldsymbol{b}}(\boldsymbol{\alpha},\boldsymbol{\beta})$ is a $N \times L$ matrix with indices a_1 and α_1, and can thus be stored in a vector of size NL. The same remark holds for indices a_2 and α_2 when other indices are fixed. As a consequence, when the vector indices $\boldsymbol{b}$ and $\boldsymbol{\beta}$ are fixed, the cumulant tensor $\boldsymbol{T}_{\boldsymbol{a},\boldsymbol{b}}(\boldsymbol{\alpha},\boldsymbol{\beta})$ can be stored in a $NL \times NL$ matrix that can be denoted $\boldsymbol{N}(\boldsymbol{b},\boldsymbol{\beta})$. Then, with these notations, we have the following result:

Proposition 3 *The contrast $\mathcal{J}_{2,r}(\boldsymbol{y})$ can be written as a criterion of joint diagonalization of a set of $N^q L^q$ matrices:*

$$\mathcal{J}_{2,r}(\boldsymbol{y}) = \sum_{\boldsymbol{b}} \sum_{\boldsymbol{\gamma}} ||\mathbf{Diag}\{\mathcal{H}^{\mathrm{T}}\, \boldsymbol{N}(\boldsymbol{b},\boldsymbol{\gamma})\, \mathcal{H}\}||^2 \tag{13}$$

where $\boldsymbol{b}$ and $\boldsymbol{\gamma}$ are vectors containing $q = r - 2$ indices, varying in $\{1,\ldots,N\}$ and $\{0,\ldots,L-1\}$, respectively, and $\mathcal{H}$ is semi-unitary.

Proof. First, replace in the definition (6) of $\mathcal{J}_{2,r}(\boldsymbol{y})$ the cumulants of $\boldsymbol{y}$ by their expression as a function of those of $\boldsymbol{x}$:

$$\begin{aligned}\mathsf{C}^{\boldsymbol{y}}_{p,q}[i,\boldsymbol{j},\boldsymbol{\ell}] = \sum_{\boldsymbol{a},\boldsymbol{b}} \sum_{\boldsymbol{\alpha},\boldsymbol{\beta}} H_{ia_1}(\alpha_1) H_{ia_2}(\alpha_2) H_{j_1b_1}(\beta_1)\ldots\\ H_{j_qb_q}(\beta_q)\, T_{\boldsymbol{a},\,\boldsymbol{b}}(\boldsymbol{\alpha},\boldsymbol{\beta}+\boldsymbol{\ell})\end{aligned} \tag{14}$$

Yet, the para-unitary condition **A4** on $\widehat{\boldsymbol{G}}(z)$ implies that $\widehat{\boldsymbol{H}}(z)$ is also para-unitary, which itself yields

$$\sum_{j\ell} H_{jr}(\tau+\ell)\, H_{jr'}(\tau'+\ell) = \delta_{rr'}\,\delta_{\tau\tau'}$$

Thus, taking the square of (14), making the change of variables $\gamma_k = \beta_k + \ell_k$, and eliminating the unuseful indices leads to

$$\begin{aligned}\mathcal{J}_{2,r}(\boldsymbol{y}) = \sum_{i\boldsymbol{a}\boldsymbol{a}'\boldsymbol{\alpha}\boldsymbol{\alpha}'\boldsymbol{b}\boldsymbol{b}'\boldsymbol{\gamma}\boldsymbol{\gamma}'} H_{ia_1}(\alpha_1)H_{ia_2}(\alpha_2)H_{ia'_1}(\alpha'_1)H_{ia'_2}(\alpha'_2)\\ \cdot T_{\boldsymbol{a},\boldsymbol{b}}(\boldsymbol{\alpha},\boldsymbol{\gamma}) \cdot T_{\boldsymbol{a}',\boldsymbol{b}'}(\boldsymbol{\alpha}',\boldsymbol{\gamma}')\,,\end{aligned}$$

which can be rearranged into

$$\mathcal{J}_{2,r}(\boldsymbol{y}) = \sum_{i\boldsymbol{b}\boldsymbol{\gamma}} \Big| \sum_{\boldsymbol{a}\boldsymbol{\alpha}} H_{ia_1}(\alpha_1) H_{ia_2}(\alpha_2) \cdot T_{\boldsymbol{a},\boldsymbol{b}}(\boldsymbol{\alpha},\boldsymbol{\gamma}) \Big|^2 .$$

Lastly, grouping indices a_j and α_j together in a single index $\boldsymbol{p}_j$, one can remark that the L matrices $\boldsymbol{H}(\alpha)$ can be stored in a $NL \times N$ matrix, $\mathcal{H}$, so that eventually $\mathcal{J}_{2,r}(\boldsymbol{y}) = \sum_{i\boldsymbol{b}\boldsymbol{\gamma}} |\sum_{\boldsymbol{p}_1\boldsymbol{p}_2} \mathcal{H}_{\boldsymbol{p}_1 i}\mathcal{H}_{\boldsymbol{p}_2 i} N_{\boldsymbol{p}_1\boldsymbol{p}_2}(\boldsymbol{b},\boldsymbol{\gamma})|^2$. Here the para-unitary property of $\boldsymbol{H}(\tau)$ implies that $\mathcal{H}^{\mathrm{T}}\mathcal{H} = \boldsymbol{I}_N$. $\diamondsuit$

3.3. Results in the complex case

For the sake of simplicity, our derivations have been carried out in the case of real input and channel. However, it can be seen that the above three propositions can be generalized to the complex case of sources and mixtures. More precisely, the output cumulant in (5) can be defined with any number of complex conjugates, which imposes the use of two additional indices and complicates the notation, hence our presentation in the real case. The only restriction is then that at most one such source cumulant may be null. Next, for $p = 2$, look at the first two terms in the cumulant (5), *i.e.*, the two terms with the same index. If only one of them is conjugated, then one has to replace the transposition in (13) by the Hermitian transposition. Otherwise, if both terms are (or are not) conjugated, the transposition stays as is in (13).

4. CONCLUSION

The framework of contrast functions is now well established and of recognized interest. Contrasts are widely utilized in static Blind Source Separation, but much less in Dynamic BSS. On the other hand, in TDMA communications (as GSM), it is necessary to equalize the channel from a single burst, that is, less than 200 symbols. Block methods, long discarded because of their high computational complexity, now hold appeal, and show a better efficiency on short data lengths.

These statements argue in favor of the proposed algorithm, which solves the MIMO equalization problem with the help of Joint Approximate Diagonalization (JAD) of several matrices. Issues currently under investigation include the robustness with respect to Gaussian noise and to channel order misdetection, and the behavior with random specular channels drawn according to the Clarke model.

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