INDEPENDENT COMPONENT ANALYSIS, A SURVEY OF SOME ALGEBRAIC METHODS.

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ABSTRACT

The source separation problem has been addressed in many ways during the last decade, and one of its instances gave birth to Independent Component Analysis (ICA). Iterative methods can be opposed to algebraic ones for the computation of the ICA, and seem to reveal very interesting research tracks. This paper attempts to give an outline of some of the works that have been carried out in the latter area, without pretending to survey exhaustively neither objectively the subject. Bibliographical pointers hopefully compensate for this drawback.

1. SEPARATION OF SOURCES

1.1. Data model

The following linear statistical model (standard in narrow band array processing) is assumed throughout the paper:

$$\mathbf{x} = A\mathbf{s} + \mathbf{b}, \tag{1}$$

where $\mathbf{x}$ and $\mathbf{s}$ are random vectors of size m (the number of sensors), and n (the number of sources), respectively. Vector $\mathbf{b}$ represents the disturbances in the form of an additive noise independent of the source vector, $\mathbf{s}$. The goal is to identify the mixing matrix A, and/or the source vector, $\mathbf{s}$, only from the corresponding output measurements, $\mathbf{x}$. This problem has received the name of Independent Component Analysis (ICA), or (instantaneous) source separation.

For convenience, the j-th column of matrix A is denoted by $\mathbf{a}_j$, $1 \le j \le n$, and the (i,j)-th entry by a_i^j, $1 \le i \le m$. With this notation, eq. 1 also reads

$$x_i = \sum_{j=1}^{n} a_i^j s_j + b_i.$$

We also normalize the problem by taking unit variance sources. This is always possible by an appropriate scaling of the columns of matrix A.

1.2. Some approaches to source separation

Because of space, it is impossible to report on all the works that have been carried out on the subject; in fact, since the first works of Bar-Ness and Jutten, the problem has been widely studied (see [1] and references therein).

Works that have not been reported in [1] and that will not be discussed in the present paper include [2] [3], where the Maximum Likelihood approach is discussed, [4] where comparisons are carried out in real operational situations, and [5] [6] [7] where on-line implementations are proposed.

Other related but *different* problems not subsequently discussed are (i) signals separation (identification of a static model excited by wide-band colored processes) [8] [9], and multivariate blind deconvolution based on (ii) second-order [10] [7] or (iii) high-order statistics [11] [12] [13] [14].

Also note that the identification of multichannel non-monic MA models [15] is linked to problem (iii) and involves the computation of one ICA [16].

2. HIGH-ORDER STATISTICS

2.1. Complex cumulants and their structure

Cumulants are defined as the coefficients of the Taylor expansion of the second characteristic function. As a consequence, like moments, they do not always exist. It is assumed here that components of $\mathbf{x}$ and $\mathbf{s}$ admit finite moments up to order 4. In addition, we focus on cumulants of complex random variables. When all moments up to order r are finite, then so are cumulants up to the same order; in addition, both are related to each other [17]. For instance at order 4 and if $\mathbf{x}$ is jointly circular up to order 4, we have:

$$\mu_{ij} \stackrel{\text{def}}{=} \mathrm{E}\{x_i x_j^*\}\ ; \qquad \mu_{ikjl} \stackrel{\text{def}}{=} \mathrm{E}\{x_i x_j^* x_k x_l^*\} \tag{2}$$

$$\mathrm{Cum}(x_i, x_j^*, x_k, x_l^*) = \mu_{ikjl} - \mu_{ij}\mu_{kl} - \mu_{il}\mu_{kj}. \tag{3}$$

Now, there are two specific properties that cumulants enjoy and not moments: (P1) Any cumulant of order higher than 2 of a Gaussian variable is null, (P2) Any cumulant of a set of variables splitting into two statistically independent subsets is null.

Since sources are independent, a first consequence of (P2) is that the source cross-cumulants vanish. The auto-cumulants are denoted

$$\sigma_p \stackrel{\text{def}}{=} \mathrm{Cum}(s_p, s_p^*) = 1 \tag{4}$$

$$\kappa_p \stackrel{\text{def}}{=} \mathrm{Cum}(s_p, s_p^*, s_p, s_p^*). \tag{5}$$

Multi-linearity is also a fundamental property that moments and cumulants share [18]. Using this property together with (P2), then relation (1) yields:

$$\mathrm{Cum}(x_i, x_j^*) = \sum_p \sigma_p\, a_i^p a_j^{p*} + \mathrm{Cum}(b_i, b_j^*) \tag{6}$$

$$\mathrm{Cum}(x_i, x_j^*, x_k, x_l^*) = \sum_p \kappa_p\, a_i^p a_j^{p*} a_k^p a_l^{p*}, \tag{7}$$

wherethere is no noise contribution in eq. (7) since, from property (P1), this term cancels when the noise $\mathbf{b}$ is normally distributed.

2.2. The cumulant tensor and its range space

Denote Q_x the linear map $M \to Q_x(M)$ which associates to any matrix M the matrix $Q_x(M)$ defined component-wise by

$$[Q_x(M)]_i^j \stackrel{\text{def}}{=} \sum_{kl} \mathrm{Cum}(x_i, x_j^*, x_k, x_l^*) M_l^k. \tag{8}$$

Using the above definitions of cumulants, it is readily verified that:

$$Q_x(M) = \mathrm{E}\{(\mathbf{x}^H M \mathbf{x})\ \mathbf{x}\mathbf{x}^H\} - MR_xM - R_x \mathrm{Trace}(MR_x). \quad (9)$$

Linear map Q_x is the 4th-order counterpart of the covariance operator R_x. In the case of source separation, eq. (7) implies that

$$Q_x(M) = A\Lambda_M A^H \qquad \Lambda_M = \mathrm{Diag}(\ldots, \kappa_p \mathbf{a}_p^H M \mathbf{a}_p, \ldots) \quad (10)$$

Thus the range of Q_x is the set of all matrices in the form $A\Lambda A^H$ with Λ diagonal. Equivalently, this is:

$$\mathrm{Range}(Q_x) \stackrel{\mathrm{def}}{=} \mathrm{Span}(\mathbf{a}_1\mathbf{a}_1^H, \ldots, \mathbf{a}_n\mathbf{a}_n^H) \quad (11)$$

$$\mathrm{Range}(Q_x) \stackrel{\mathrm{def}}{=} \{M \mid M = \sum_{p=1,n} \alpha_p \mathbf{a}_p \mathbf{a}_p^H\} \quad (12)$$

$$= \{M \mid M = A\Lambda A^H,\ \Lambda \text{ diagonal}\} \quad (13)$$

Thus, the range of Q_x, termed the '4th-order signal subspace' (FOSS) is a n-dimensional linear matrix space.

An orthonormal basis for the FOSS can be obtained from an eigen-decomposition of Q_x. Actually, it exists a system of m^2 orthonormal 'eigen-matrices' $F_1, \ldots, F_{m^2}$ of size $m \times m$ verifying

$$\mathrm{Trace}(F_i F_j^H) = \delta(i,j) \quad 1 \le i,j \le m^2 \quad (14)$$

$$Q_x(F_i) = \mu_i F_i \quad \mu_i \in R \quad 1 \le i \le m^2 \quad (15)$$

Each F_i is an eigen-matrix and the real scalar μ_i is the corresponding eigen-value. In addition, the eigen-matrices can be chosen to be hermitian. See [19] for a (simple) proof of these facts which hold for any cumulant tensor.

In the case of source separation, where the cumulants have structure (7), we have seen that the range of Q_x has a dimension equal to the number n of sources. Thus there are only n non-zero eigenvalues of Q_x out of m^2. We denote $F_1, \ldots, F_n$ the eigen-matrices associated to these eigenvalues. They form an orthonormal basis of the FOSS, the range space of Q_x :

$$\mathrm{FOSS} = \mathrm{Range}(Q_x) = \mathrm{Span}(F_1, \ldots, F_n). \quad (16)$$

These subspace structures (11) and (16) are exploited below for the blind identification of A.

3. ORTHOGONAL DECOMPOSITIONS

In this section, we consider blind identification of the mixture based on 'orthogonal structures'. A *whitening matrix* W is an $n \times m$ matrix such that WA is $n \times n$ unitary. The whitened observations are obtained by forming $\mathbf{z} = W\mathbf{x} = WA\mathbf{s} + W\mathbf{b}$. Since WA is unitary, the whitened observations $\mathbf{z}$ form a unitary mixture corrupted by the additive noise vector $W\mathbf{x}$. For a given whitening matrix W, we denote U the unitary matrix $U = WA$ and we have

$$\mathbf{z} = W\mathbf{x} = U\mathbf{s} + W\mathbf{b} \qquad UU^H = I \quad (17)$$

Whitening reduces the problem to inverting a unitary mixture which is definitely a simpler (and better conditioned) problem.

Second-order information (*i.e.* the covariance matrix of $\mathbf{x}$) can be used to whiten the observations. A standard procedure to this end may be summarized by the following steps: i) Estimate R_x the covariance matrix of the observations. ii) Estimate the signal part of R_x *i.e.* the matrix $R_x - R_b = AA^H$. iii) Find W such that $W(R_x - R_b)W^H = I$. For step ii) to be feasible, some a priori information must be available about the noise distribution. A simple instance is when $R_b = \sigma^2 I$: the noise amplitude σ can be consistently estimated if $m > n$. An even simpler case is when the noise covariance matrix is known beforehand or negligible.

With a random vector $\mathbf{z}$ having the structure (17), let us evaluate $Q_z(M)$ for some $n \times n$ matrix M. According to (10), one has

$$Q_z(M) = U\Lambda_M U^H \qquad \Lambda_M = \mathrm{Diag}(\ldots, \kappa_p \mathbf{u}_p^H M \mathbf{u}_p, \ldots) \quad (18)$$

where $\mathbf{u}_p$ denote the p-th column of matrix U. Thus matrix U diagonalizes $Q_z(M)$ and the columns of the unknown U can be identified to the eigen-vectors of $Q_z(M)$ for any matrix M. Actually, for the identification to be possible, the eigen-values of $Q_z(M)$ must be distinct. The simplest instance of this idea is to take $M = I$. In this case, relation (9) reduces to

$$Q_z(I) = \mathrm{E}\{|\mathbf{z}|^2 \mathbf{z}\mathbf{z}^H\} - R_z^2 - R_z \mathrm{Trace}(R_z)$$

and identifiability is granted if the sources have different 4th-order cumulants. These simple algebraic techniques to identify the unitary matrix U have been investigated for instance in [20, 21].

3.1. Joint diagonalization and contrast functions

In this section, we describe how identification of the unitary matrix U by diagonalization can be made more efficient. The idea is to *jointly and approximately* diagonalize a set of matrices in the form $Q_z(M)$ for different values of M [22]. The joint approximate diagonalization by a unitary matrix V of an arbitrary set of s matrices $N_1, \ldots, N_s$ may be defined as the minimization of the joint-diagonality criterion $c(V)$:

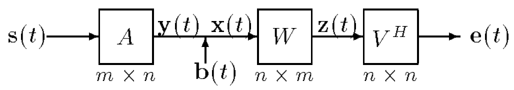
$$C(V) \stackrel{\mathrm{def}}{=} \sum_{r=1,s} |\mathrm{Diag}(V^H N_r V)|^2 \quad (19)$$

where $|\mathrm{diag}(.)|$ is the norm of the vector build from the diagonal of the matrix argument. By involving several matrices $Q_z(M)$ into a unique criterion of joint diagonality, the statistical performance is expected to improve.

By jointly diagonalizing the n eigen-matrices $F_1, \ldots, F_n$ of Q_z, identifiability is granted regardless of the kurtosis of the sources (but at most one source with zero kurtosis is allowed). Further, the joint diagonality criterion (19) may be shown to be equal to :

$$C(V) = \sum_{i,k,l=1,n} |\mathrm{Cum}(e_i, e_i^*, e_k, e_l^*)|^2 \quad (20)$$

where vector $\mathbf{e}$ is the 'output' of the unitary matrix V^H *i.e.* $\mathbf{e} = V^H \mathbf{z}$. When $V = U$, the output $\mathbf{e}$ is a copy of the source signals.



An interesting point is the similarity between the joint diagonality criterion (20) and the ICA criterion developped in [1]:

$$C'(V) \stackrel{\mathrm{def}}{=} \sum_{i=1,n} |\mathrm{Cum}(e_i, e_i^*, e_i, e_i^*)|^2. \quad (21)$$

4. TETRADIC STRUCTURES

Another interesting line of approach for blind identification of matrix A is to solve for it in eq. (7) : the point here is to use only 4th-order statistics and to give up the idea pre-whitening. There are two benefits to this approach.

First, 4th-order cumulants are "insensitive" to Gaussian noise while 2nd-order cumulants are not. Thus the "4th-order only" approach does not require to model the noise to design consistent estimates as long as the noise is normally distributed.

Second, the columns of A cannot be orthogonalized if there are more sources than sensors. Thus, giving up the idea of pre-whitening opens the possibility of identifying more sources than sensors. This possibility is explored in this section : we focus on the algebraic equation (7) where the 4th-order cumulant appear as a sum of n tetrads.

Note that the tetradic structure represented by eq. (7) is not difficult to solve when the equation holds exactly (it never does in practice due to modeling errors and finite sample effects) and when $n \leq m$ (no more source than sensors). A possible solution is along the following lines.

Let N_1 and N_2 be two elements of the FOSS *i.e.* two matrices in the range of Q_x (for instance, N_1 and N_2 may be obtained as $N_1 = Q_x(M_1)$ and $N_2 = Q_x(M_2)$ where M_1 and M_2 are 'randomly selected' matrices). Then according to (10), one has $N_1 = A\Lambda_1 A^H$ and $N_2 = A\Lambda_2 A^H$ for some diagonal matrices Λ_1 and Λ_2. Matrix A is then easily computed from the generalized eigen-values of the pencil $(A\Lambda_1 A^H,\ A\Lambda_2 A^H)$.

The problem with this kind of approach is twofold. First, the behavior of the estimates is unpredictible when the cumulants are not exact but are estimated from a finite data set. Second, only a limited part of the 4th-order information is involved in the computation of the estimates. Indeed the previous example shows that Q_x largely over-determines A since two arbitrary elements in the range space of Q_z are enough to determine matrix A. Thus it seems advisable to base esimation of A on the whole (overdetermined) set of equations (7).

4.1. Tensor diagonalization

In this section, discussion is limited to the real case. The problem can actually be addressed in a more general framework. Indeed, given a real super-symmetric[1] fourth-order tensor T, one would like to compute the following decomposition into "rank-one" tensors:

$$T_{ijkl} = \sum_{p=1}^{r} \lambda_p a_i^p a_j^p a_k^p a_l^p \tag{22}$$

forgetting about complex conjugation. The minimal value of integer r can be considered as the "rank" of T, and denoted $\omega(T)$. Let

$$r_1 = \left(\begin{array}{c} n+3 \\ 4 \end{array} \right); \quad r_2 = \left(\begin{array}{c} n+2 \\ 3 \end{array} \right)$$

It turns out that the space of fourth-order symmetric tensors is of dimension r_1, but that the maximal reachable rank [23] is only r_2; moreover, it can be proved that the rank is in general even much smaller [23]. For instance, with 5 sensors, the generic rank is 15, and with 7 sensors, it is 30. Clearly, the number of identifiable sources is (generically) bounded by these numbers.

One could imagine to call vectors a^p the eigen-vectors of T, and the scalars λ_p its eigen-values. Unfortunately, the decomposition is in general not unique, and except for $m = 7$, there are in general infinitely many solutions.

[1] A 4th-order tensor is said to be *super-symmetric* if it remains invariant under all the possible permutations of its indices.

4.2. Subspace based identification

In this section, we consider identification of (the columns of) A based on the tetradic structure and the notion of 4th-order signal subspace introduced in section (2.2.). A key point is that the FOSS itself determines the columns of A. Actually, a technique similar in spirit to MUSIC can be devised for blind identification. Define the "rank-one manifold" (ROM) as the set of all rank-one matrices. Matrices $\mathbf{a}_1\mathbf{a}_1^H, \ldots, \mathbf{a}_n\mathbf{a}_n^H$ belong to the ROM by definition and they also span the FOSS.

Thus each of these matrices belong to the intersection FOSS $\cap$ ROM. Actually, it can be shown under mild conditions that any matrix belonging to the intersection FOSS $\cap$ ROM necessarily is in the form $\alpha \mathbf{a}_p \mathbf{a}_p^H$ for some scalar α and some index p in $1, \ldots, n$.

It follows that matrices $\mathbf{a}_1, \ldots, \mathbf{a}_n$ can be identified by searching for the (possibly approximate) intersetion between FOSS and ROM. This is indeed very reminiscent of the MUSIC algorithm for bearing estimation where one looks for the intersections between the (second-order) signal subspace and the array manifold.

In practice, only an estimate ofthe FOSS is available : the FOSS is typically represented by the set $\hat{F}_1, \ldots, \hat{F}_n$ of the n most significant eigen-matrices of the (estimated) linear map $\hat{Q}_x$, which is itself determined by the estimated 4th-order cumulants of $\mathbf{x}$.

In [24], two iterative algorithms are described that search for the rank one matrices that are as close as possible to the "empirical FOSS". *i.e.* the matrix space linearly spanned by matrices $\hat{F}_1, \ldots, \hat{F}_n$.

Another line of investigation is to develop algebraic methods that solve directly eq. (7) for all the colums of A simultaneously. Such an algorithm is described in [25] based on the whole set of cumulants. This algorithm assumes that the cumulant tensor exactly is a sum of n tetrads. The next subsection explains a possible way to deal with sample cumulants.

4.3. Structure forcing

The problem of feeding sample statistics (estimated cumulants) to algorithms designed to process "true statistics" may be alleviated to some extent by the device of "signal enhancement". In the case of source separation, it turns out that the tetradic structure can be forced upon the sample cumulants by using only two simple properties: rank and symmetry. This is described next.

Cumulants tensors are super-symmetric, indeed, and so is any tetrad $a_i a_i^* a_k a_l^*$ or any sum of tetrads. Thus the 4th-order cumulant tensor of $\mathbf{x}$ is both supersymmetric and rank-n. The sample cumulant tensor is super-symmetric in general (for any reasonable estimate of the cumulants) but is not exactly rank-n because the finite sample effects do not allow all the cross cumulants to go to zero as they do when the data size tends to infinity.

Henceforth, in the context of source separation, "cumulant enhancement" consists in computing the rank-n supersymmetric tensor which is as close as possible to the sample cumulants. This operation can be implemented by alternate projections of the original (estimated) cumulant tensor onto the space of rank-n tensors (by truncating the SVD) and onto the space of super-symmetric tensors (by a simple symmetrization). The process of iterating alternate projections ultimately results in a tensor which is both rank-n and super-symmetric. The key point here is that, in the vicinity of tetradic tensors, rank-n super-symmmetric tensors necessarily are a sum of n tetrads. In other words, cumulant enhancement forces the tetradic structure. Consequently, all algebraic algorithms applied to enhanced cumulants will produce the same estimate.

In some sense, cumulant enhancement does the statistical part of the job. The remaining algebraic part of the processing is irrelevant to the quality of estimation. More details (and a more rigorous exposition) can be found in [26].

4.4. Other approaches

To our knowledge, there are very few works in the open literature that have addressed the ICA problem with more sources than sensors. Beside the references previously mentioned in section 4., other works include only [27] [28].

5. CONCLUSION

One can distinguish between two families of approaches, depending on whether they require spatial second-order prewhitening or not. The use of prewhitening makes the task easier in several respects, but has the inconvenience to limit the number of identifiable sources to the number of sensors. By avoiding the use of second-order statistics, one allows to identify much more sources than sensors. However, if algorithms resorting to prewhitening have reached a quite acceptable reliability level, those based only on high-order statistics still belong to the prospective domain and need to be further improved. This statement shows that there are many open problems that deserve to be addressed in the near future.

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