Safety-Critical Control via Control Barrier Functions: Theory and Applications

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Joint work with

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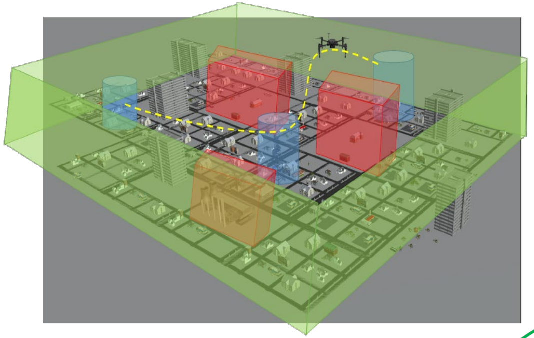
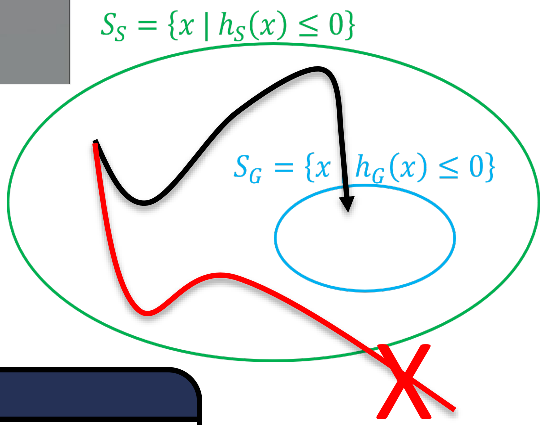
Constrained Safety-Critical Control Synthesis

Let $\dot{x} = f(x) + g(x)u$ where $x \in \mathbb{R}^n, u \in U \subset \mathbb{R}^m$



Assume that:

- There exists a safe set $S_S = \{x \in \mathbb{R}^n \mid h_S(x) \leq 0\}$ where $h_S(x)$ is continuously differentiable
- There exist a goal set $S_G = \{x \in \mathbb{R}^n \mid h_G(x) \leq 0\}$ where $h_G(x)$ are continuously differentiable
- $S_S \cap S_G \neq \emptyset$



Problem statement (Problem 0)

Find a control input $u(t) \in U = \{A_u u \leq b_u\}$ such that for $x(0) \in S_s \setminus S_G$

- $x(t) \in S_S$, $\forall t \geq 0$
- $x(\bar{T}) \in S_G$, for a fixed (given) time $\bar{T}$

Presentation Outline

- Time Constraints
- Input constraints
- Robustness against disturbances
- Adaptation of parameters for optimality and feasibility
- Zero-Order Hold
- Output feedback

Set Invariance (Nagumo's Theorem)

- A set S is invariant under the system dynamics

$$\dot{x} = f(x)$$

if $x(0) \in S$ implies that $x(t) \in S$, for all $t \geq 0$.

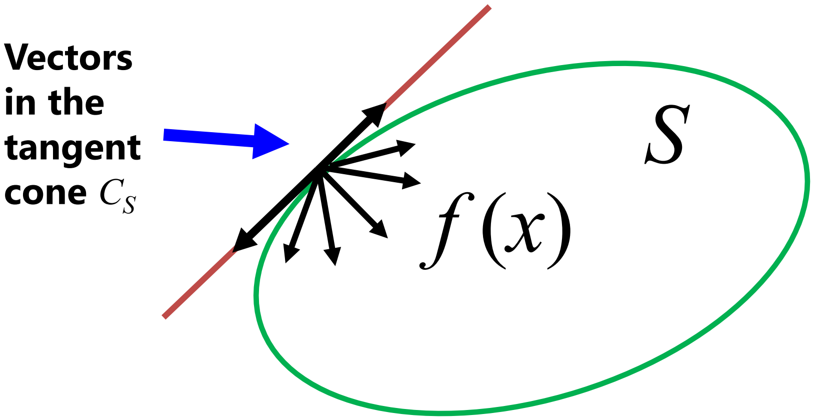
- The necessary and sufficient conditions for set invariance are expressed via Nagumo's theorem.

- Intuitively, Nagumo's Theorem says that the system trajectories $x(t)$ never escape the set S if and only if the vector field of the system on each point of the boundary of the set points inside or tangent to the set

$$f(x) \in C_S(x), \quad \forall x \in \partial S$$

where C_S is the tangent cone of S at x

- For the precise statement, see Blanchini's survey paper "Set Invariance in Control"



$$L_f V(x) = \frac{\partial V(x)}{\partial x} f(x)$$

$$L_g V(x) = \frac{\partial V(x)}{\partial x} g(x)$$

Set Invariance via Zeroing Barrier Functions

- Definition of Zeroing Barrier Function (Ames et al, TAC 2017)

Definition: Let $\dot{x} = f(x)$ where f is locally Lipschitz, and a closet set defined as

$$\mathcal{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}$$
$$\partial\mathcal{C} = \{x \in \mathbb{R}^n : h(x) = 0\}$$
$$\text{Int}(\mathcal{C}) = \{x \in \mathbb{R}^n : h(x) > 0\}$$

where $h : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable.

The function $h : \mathbb{R}^n \to \mathbb{R}$ is called a Zeroing Barrier Function (ZBF) for the set C if there exists an extended class K function α, and a set $\mathcal{D}$ such that $\mathcal{C} \subseteq \mathcal{D} \subset \mathbb{R}^n$ such that for all $x \in \mathcal{D}$

$$L_f h(x) \geq -\alpha(h(x))$$

Proposition: If h is a ZBF on the set D, then the set C is forward invariant.

Remark: The consideration the set D allows for the consideration of disturbances.

Zeroing Control Barrier Functions

- Definition of Zeroing Control Barrier Function (Ames et al, TAC 2017)

Definition: Let $\dot{x} = f(x) + g(x)u$, where $f(x), g(x)$ are locally Lipschitz

$$x \in \mathbb{R}^n, u \in U \subset \mathbb{R}^m$$

A continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$ is called a Zeroing Control Barrier Function (ZCBF) for the set defined as

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if there exists an extended class K function α, and a set $\mathcal{D}$ such that $\mathcal{C} \subseteq \mathcal{D} \subset \mathbb{R}^n$ such that for all $x \in \mathcal{D}$

$$\sup_{u \in U} [L_f h(x) + L_g h(x) u + \alpha(h(x))] \geq 0, \quad \forall x \in \mathcal{D}$$

Performance (CLFs) and Safety (CBFs) via QPs

- Let the following CLF-CBF QP

$$\mathbf{u}^\star(x) = \arg \min_{\mathbf{u}=(u,\delta)\in\mathbb{R}^m\times\mathbb{R}} \frac{1}{2}\mathbf{u}^T H(x)\mathbf{u} + F(x)^T\mathbf{u}$$

s.t.

$$L_f V(x) + L_g V(x) u + c_3 V(x) - \delta \leq 0$$
$$L_f B(x) + L_g B(x) u - \alpha(h(x)) \leq 0$$

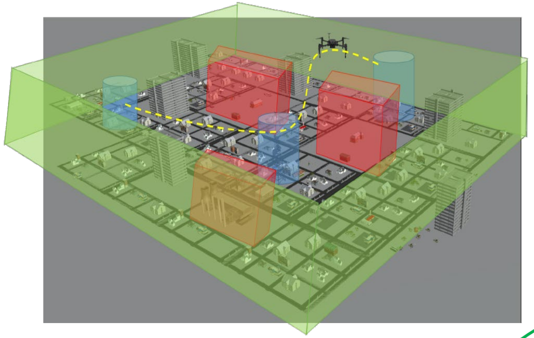
- Theorem 3 [Ames et al, TAC 2017]: Suppose that the following functions are locally Lipschitz:
  - the vector fields f and g in the control system,
  - the gradients of the RCBF $B(x)$ and CLF $V(x)$,
  - the cost function terms $H(x)$ and $F(x)$ in (CLF-CBF QP).

  Suppose furthermore that $L_g B(x) = 0$, for all $x \in \text{Int}(C)$.

  - ❑ Then the solution, $\mathbf{u}^*(x)$, of (CLF-CBF QP) is locally Lipschitz continuous for $x \in \text{Int}(C)$. Moreover, a closed-form expression can be given for $\mathbf{u}^*(x)$.

Input-Constrained Spatiotemporal Control

Let $\dot{x} = f(x) + g(x)u$ where $x \in \mathbb{R}^n, u \in U \subset \mathbb{R}^m$



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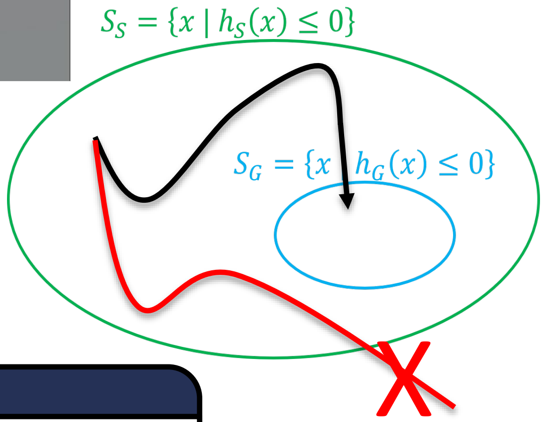

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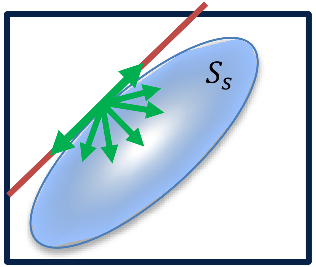
- $x(t) \in S_S$, $\forall t \geq 0$
- $x(\bar{T}) \in S_G$, for a fixed (given) time $\bar{T}$

Set Invariance and Control Barrier Functions

Consider the nonlinear, control-affine dynamical system:



$$\dot{x}(t) = f\big(x(t)\big) + g\big(x(t)\big)u$$

Define $S_s = \{x \in D \mid h_s(x) \le 0\}$, where $h_s : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable

Forward Invariant Set[1]

The set S_s is forward invariant under some $u \in U$ if the following condition holds:

$$\dot{h}_s(x) = L_f h_s(x) + L_g h_s(x)u \le 0, \qquad \text{for all } x \in \partial S_s = \{x \in D \mid h_s(x) = 0\}$$

Definition: Control Barrier Function[2]

h_s is a Control Barrier Function (CBF) for the set S_s if there exists an extended class K_∞ function α such:

$$\inf_{u \in U}\left[L_f h_s(x) + L_g h_s(x)u\right] \le -\alpha\big(h_s(x)\big), \qquad \text{for all } x \in S_s$$

[1] F. Blanchini, "Set invariance in control," Automatica 1999.

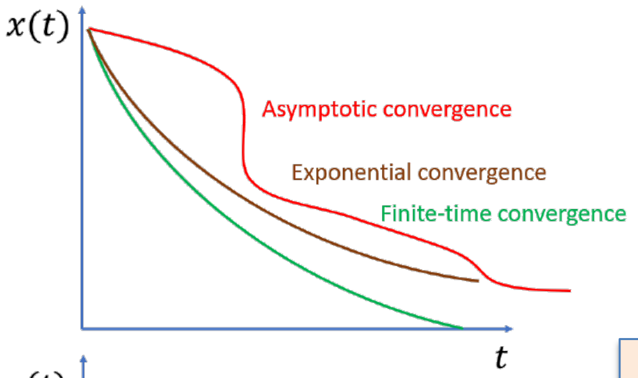
[2] A. D. Ames et al., "Control barrier function based quadratic programs for safety critical systems," IEEE TAC 2017.

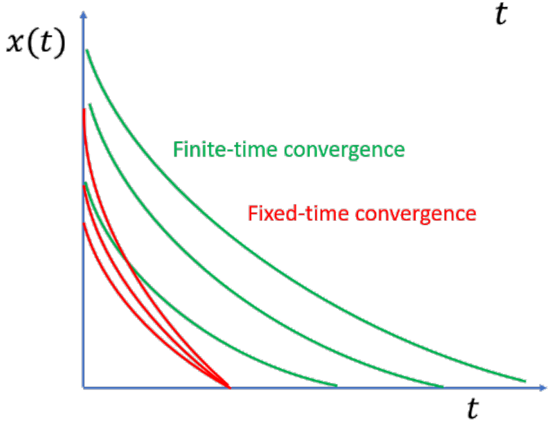
FxT-CLF-CBF QPs

Kunal Garg, Ehsan Arabi (CDC 2019, Automatica 2022)

Finite-Time and Fixed-Time Stability

Let $\dot{x} = f(x)$
where f is continuous, $f(0) = 0$





Finite-time Stability (FTS) (Bhat and Bernstein, 2000)

Theorem 1. *Suppose there exists a positive definite function V for system (1) such that*

$$\dot{V}(x) \le -cV(x)^{\beta},$$

with $c > 0$ and $0 < \beta < 1$. Then, the origin of (1) is FTS with settling time function

$$T(x(0)) \le \frac{V(x(0))^{1-\beta}}{c(1-\beta)}.$$

Fixed-time Stability (FxTS) (Polyakov, 2012)

Theorem 1 ([2]). *Suppose there exists a positive definite function V for system (1) such that*

$$\dot{V}(x) \le -aV(x)^{p} - bV(x)^{q}$$

with $a, b > 0$, $0 < p < 1$ and $q > 1$. Then, the origin of (1) is FxTS with continuous settling time T that satisfies

$$T \le \frac{1}{a(1-p)} + \frac{1}{b(q-1)}.$$

Fixed-Time Control Lyapunov Function

Let $\dot{x} = f(x) + g(x)u$ where $x \in \mathbb{R}^n, u \in U \subset \mathbb{R}^m$

Definition: The continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ is called a **Fixed-Time Control Lyapunov Function** wrt a set S_G (FxT-CLF-S_G) of the system with parameters a_1, a_2, b_1, b_2 if

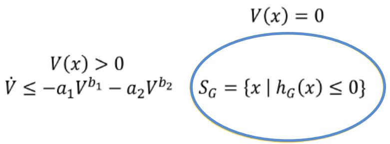
i) It is positive definite wrt a closed set S_G, i.e.,

$$V(x) > 0 \text{ for } x \notin S_G$$
$$V(x) = 0 \text{ for } x \in \partial S_G$$

ii) $\inf_u [L_f V(x) + L_g V(x) u] \leq -a_1 (V(x))^{b_1} - a_2 (V(x))^{b_2}, \ \forall x \notin \text{Int}(S_G)$

where $a_1, a_2 > 0, \ b_1 > 1, \ 0 < b_2 < 1$ satisfy $\frac{1}{a_1(b_1 - 1)} + \frac{1}{a_2(1 - b_2)} \leq \bar{T}$

with $\bar{T}$ being a user-defined time.



$$L_f V(x) = \frac{\partial V(x)}{\partial x} f(x)$$
$$L_g V(x) = \frac{\partial V(x)}{\partial x} g(x)$$

FxT-CLF-CBF QP for Spatiotemporal Control

Consider the following optimization problem:

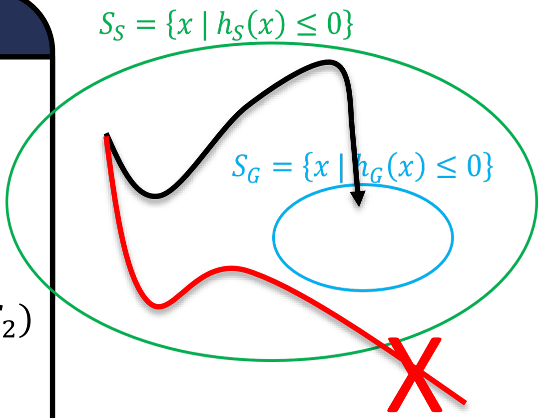
$$\min_{u,\delta_1,\delta_2} \quad \frac{1}{2}\|u\|^2 + p_1{\delta_1}^2 + p_2{\delta_2}^2 + q_1\delta_1$$

$$L_f h_s(x) + L_g h_s(x)u \le -\delta_2 h_s(x) \qquad (C_1)$$

$$L_f h_G(x) + L_g h_G(x)u \le \delta_1 h_G(x) - a_1 \max\{0, h_G(x)\}^{b_1} - a_2 \max\{0, h_G(x)\}^{b_2} \qquad (C_2)$$

$$A_u u \le b_u \qquad (C_3)$$

where $p_1, p_2, q_1 > 0,\ \mu > 1,\ b_1 = 1 + \frac{1}{\mu}, b_2 = 1 - \frac{1}{\mu},\ \ a_1 = a_2 = \frac{\mu\pi}{2\bar{T}}$



- C_1 imposes that: $h_s = 0 \Rightarrow \dot{h}_s \le 0 \Rightarrow$ Control Barrier Function for forward invariance of the set S_s
- C_2 imposes: $\dot{h}_G \le \delta_1 h_G - a_1 h_G^{b_1} - a_2 h_G^{b_2} \Rightarrow$ Relaxed FxT-CLF wrt the set S_G within $\bar{T}$
- C_3 imposes control input constraints

FxT-CLF-CBF QP for Spatiotemporal Control

Consider the following optimization problem:

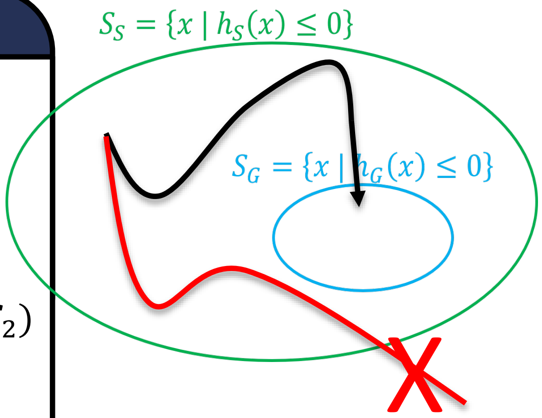
$$\min_{u,\delta_1,\delta_2} \quad \frac{1}{2}\|u\|^2 + p_1{\delta_1}^2 + p_2{\delta_2}^2 + q_1\delta_1$$

$$L_f h_s(x) + L_g h_s(x)u \le -\delta_2 h_s(x) \qquad (C_1)$$

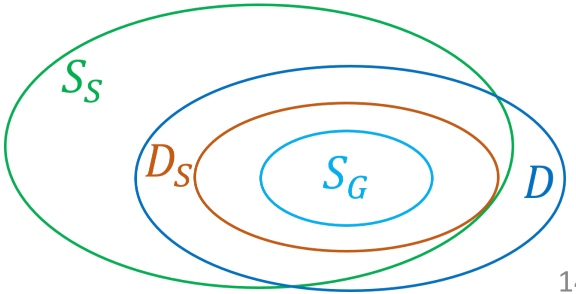
$$L_f h_G(x) + L_g h_G(x)u \le \delta_1 h_G(x) - a_1 \max\{0, h_G(x)\}^{b_1} - a_2 \max\{0, h_G(x)\}^{b_2} \qquad (C_2)$$

$$A_u u \le b_u \qquad (C_3)$$

where $p_1, p_2, q_1 > 0,\ \mu > 1,\ b_1 = 1 + \frac{1}{\mu}, b_2 = 1 - \frac{1}{\mu},\ \ a_1 = a_2 = \frac{\mu\pi}{2\bar{T}}$



- δ_1, δ_2 ensure feasibility of the QP for all $x \in \mathrm{Int}(S_S) \setminus S_G$
- δ_1 dictates region D of fixed-time convergence (based on Robust FxTS)
- D_S is the set of initial conditions from which safety, time and input constraints are met



K. Garg, E. Arabi and D. Panagou (Automatica, 2022)

Robust Fixed-Time Stability (FxTS)

Theorem on Robust Fixed-Time Stability [K. Garg et al, Automatica 2022]

Let $V:\mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable, positive definite, proper function such that

$$\dot{V} \le -a_1 V^{b_1} - a_2 V^{b_2} + \delta_1 V$$

where $b_1 = 1 + \frac{1}{\mu}$, $b_2 = 1 - \frac{1}{\mu}$, $\mu > 1, a_1 > 0, a_2 > 0, \delta_1 \in R$. Then, for all $x(0) \in D$, where

$$D = \begin{cases} \left\{x \mid V(x) < k^\mu \left(\frac{\delta_1 - \sqrt{\delta_1{}^2 - 4a_1 a_2}}{2a_1}\right)^\mu\right\}, & \text{if } \delta_1 > 2\sqrt{a_1 a_2} \\ \left\{x \mid V(x) < k^\mu \left(\frac{a_2}{a_1}\right)^{\frac{\mu}{2}}\right\}, & \text{if } \delta_1 = 2\sqrt{a_1 a_2} \\ \mathbb{R}^n, & \text{if } \delta_1 < 2\sqrt{a_1 a_2} \end{cases}$$

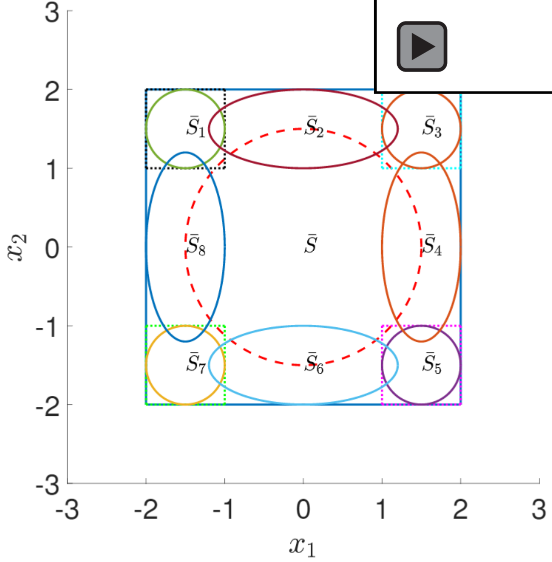
the system trajectories reach the origin within settling time T, where

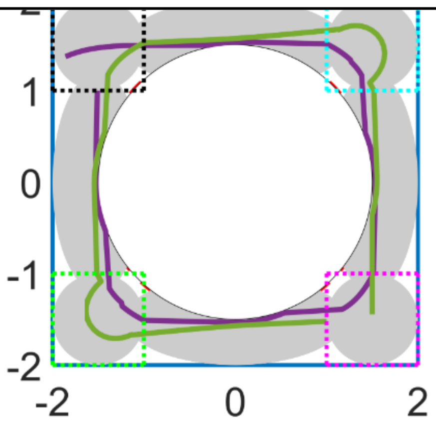
$$T \le \begin{cases} \frac{\mu}{a_1(b-a)}\left(\log\left(\frac{b-ka}{a(1-k)}\right) - \log\left(\frac{b}{a}\right)\right), & \text{if } \delta_1 > 2\sqrt{a_1 a_2} \\ \frac{\mu}{\sqrt{a_1 a_2}}\left(\frac{k}{1-k}\right), & \text{if } \delta_1 = 2\sqrt{a_1 a_2} \\ \frac{\mu}{a_1 k_1}\left(\frac{\pi}{2} - \tan^{-1} k_2\right), & \text{if } 0 < \delta_1 < 2\sqrt{a_1 a_2} \\ \frac{\mu\pi}{2\sqrt{a_1 a_2}}, & \text{if } \delta_1 \le 0 \end{cases}$$

Example: Area patrolling

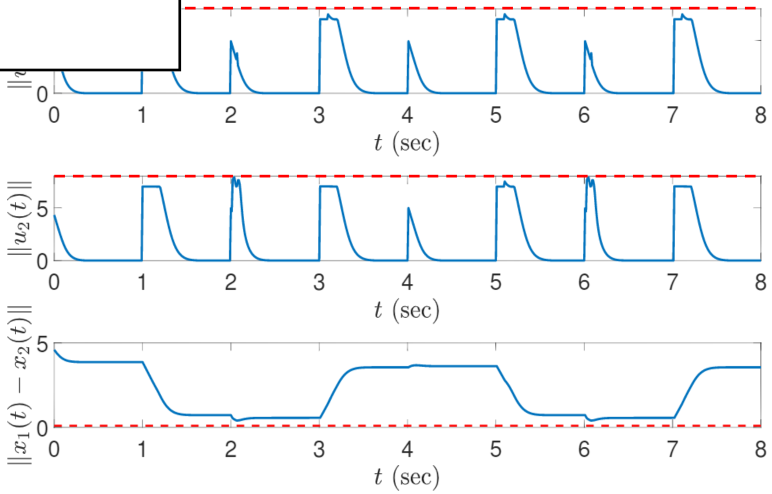
Agent dynamics

$$\dot{x}_i = u_i$$





Closed-loop paths



Control input and inter-agent distance

Robust FxT-CLF-CBF QPs

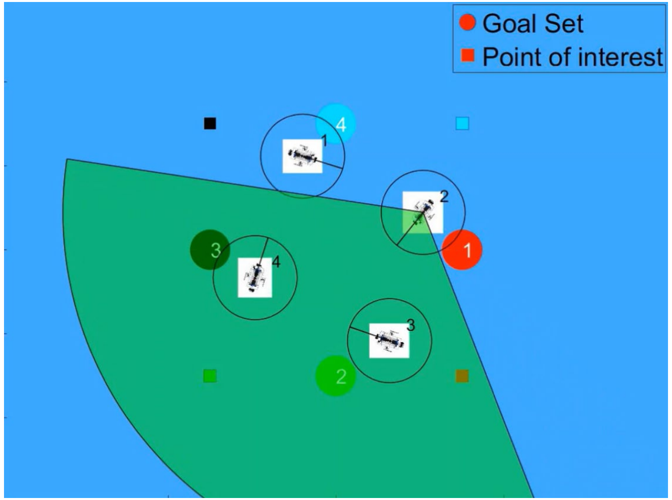
Kunal Garg (ACC 2021)

Constrained Safety Control under Disturbances

Consider the *perturbed* dynamical control system:

$$\dot{x} = f\big(x(t)\big) + g\big(x(t)\big)u + d(t,x), \qquad ||d(t,x)|| \leq \gamma.$$

Estimated state $\hat{x}$ is available with $\qquad ||x - \hat{x}|| \leq \epsilon$



$S_T(t) = \{x \mid h_T(t,x) \leq 0\}$: dynamically-changing safe set

- Moving obstacles or other agents in multi-agent scenario
- $S_T(t) = \left\{x_i(t) \mid h\left(x_i(t), x_j(t)\right) \leq 0, j \neq i\right\}$
- E.g., $h\left(x_i(t), x_j(t)\right) = d_s^2 - ||x_i(t) - x_j(t)||^2$ and $h_T(t, x_i) = \log\left(\sum_j e^{h\left(x_i(t), x_j(t)\right)}\right)$

K. Garg and D. Panagou (ACC 21)

Robust CBF and Robust FxT-CLF

Definition (Robust CBF)

For a set $S_T(t)$: $\{x \mid h_T(t,x) \leq 0\}$, the function $h_T: \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}$ with $||\frac{\partial h_T}{\partial x}|| \leq l_T$ is called robust CBF w.r.t. disturbance $||d|| \leq \gamma$ if there exists $\alpha \in \mathcal{K}$ such that

$$\inf_{u \in \boldsymbol{\mathcal{U}}} \{L_f h_T(t,x) + L_g h_T(t,x)u\} \leq \alpha\big(-h_T(t,x)\big) - l_T\gamma, \qquad \forall x \in S_T(t).$$

Definition (Robust FxT-CLF)

For a set S_G: $\{x \mid h_G(x) \leq 0\}$, the function $h_G: \mathbb{R}^n \to \mathbb{R}$ with $||\frac{\partial h_G}{\partial x}|| \leq l_G$ is called robust fixed-time CLF w.r.t. disturbance $||d|| \leq \gamma$ if

$$\inf_{u \in \boldsymbol{\mathcal{U}}} \{L_f h_G(x) + L_g h_G(x)u\} \leq \delta h_G(x) - \alpha_1 h_G(x)^{\gamma_1} - \alpha_2 h_G(x)^{\gamma_2} - l_G\gamma,$$

for all $x \notin S_G$, with $\delta \in \mathbb{R}, \alpha_1 = \alpha_2 = \frac{\mu\pi}{2T_{ud}}, \gamma_1 = 1 + \frac{1}{\mu}, \gamma_2 = 1 - \frac{1}{\mu}, \mu > 1$.

Robust FxT-CLF-CBF QP

Consider the following quadratic program:

$$\min_{u,\delta_1,\delta_2,\delta_3} \frac{1}{2}u^2 + p_1\delta_1^2 + p_2\delta_2^2 + p_3\delta_3^2 + q_1\delta_1$$

Input constraints: $\text{s.t.} \quad A_u u \leq b_u,$

New FxTS condition for set S_G: $L_f\hat{h}_G(\hat{x}) + L_g\hat{h}_G(\hat{x})u \leq \delta_1\hat{h}_G(\hat{x}) - \alpha_1\hat{h}_G(\hat{x})^{\gamma_1} - \alpha_2\hat{h}_G(\hat{x})^{\gamma_2} - l_G\gamma,$

Forward invariance of set S_S: $L_f\hat{h}_S(\hat{x}) + L_g\hat{h}_S(\hat{x})u \leq -\delta_2\hat{h}_S(\hat{x}) - l_S\gamma,$

For disturbance d

Forward invariance of set $S_T(t)$: $L_f\hat{h}_T(t,\hat{x}) + L_g\hat{h}_T(t,\hat{x})u \leq -\delta_3\hat{h}_T(t,\hat{x}) - \frac{\partial\hat{h}_T}{\partial t}(t,\hat{x}) - l_T\gamma,$

other agents' effect

where l_G, l_S, l_T are Lipschitz constants and

$$\hat{h}_G(\hat{x}) \coloneqq h_G(\hat{x}) + l_G\epsilon, \qquad \hat{h}_S(\hat{x}) \coloneqq h_S(\hat{x}) + l_S\epsilon, \qquad \hat{h}_T(t,\hat{x}) \coloneqq h_T(t,\hat{x}) + l_T\epsilon$$

For state-estimation error

Example: Constrained Multi-Robot Navigation

Consider underactuated underwater autonomous vehicles

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\phi}_i \\ m_{11}\dot{u}_i \\ m_{22}\dot{v}_i \\ m_{33}\dot{r}_i \end{bmatrix} = \begin{bmatrix} u_i \cos\phi_i - v_i \sin\phi_i \\ u_i \sin\phi_i + v_i \cos\phi_i \\ r_i \\ m_{22}v_i r_i + X_u u_i + X_{u|u|}|u_i|u_i \\ -m_{11}u_i r_i + Y_v v_i + Y_{v|v|}|v_i|v_i \\ (m_{11} - m_{22})u_i v_i + N_r r_i + N_{r|r|}|r_i|r_i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tau_{u,i} \\ 0 \\ \tau_{r,i} \end{bmatrix} + \begin{bmatrix} V_w \cos(\theta_w) \\ V_w \sin(\theta_w) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

$z_i = [x_i,\ y_i,\ \phi_i]^T$: configuration vector of the i-th agent

$\tau_{u,i}$: control input along the x-axis ($\|\tau_u\| \leq 10$)

$\tau_{r,i}$: control input along the yaw axis ($\|\tau_r\| \leq 15$)

V_w, θ_w: speed and direction of water current

X_u, Y_v, N_r: linear drag terms, and $X_{u|u|}, Y_{v|v|}, N_{r|r|}$: non-linear drag terms

Example: Constrained Multi-Robot Navigation

Nominal case: $\gamma = \epsilon = 0$

With SEE: $\gamma = 0, \epsilon = 0.5$

With SEE and AD: $\gamma = \epsilon = 0.5$

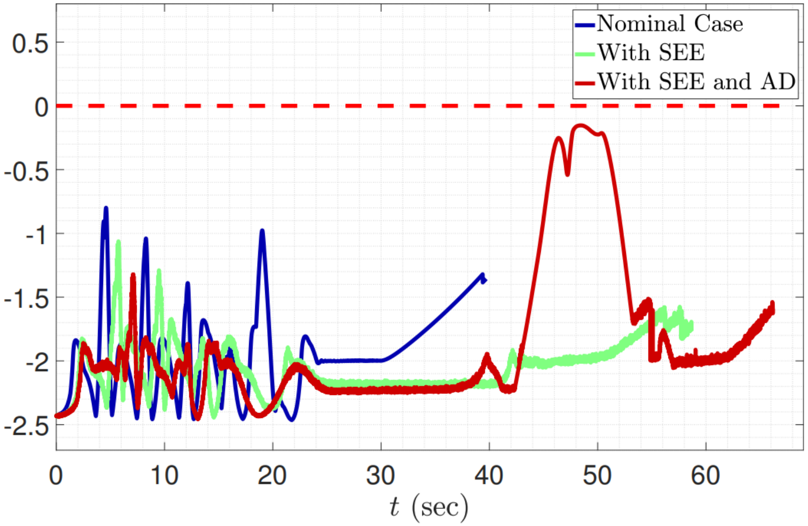
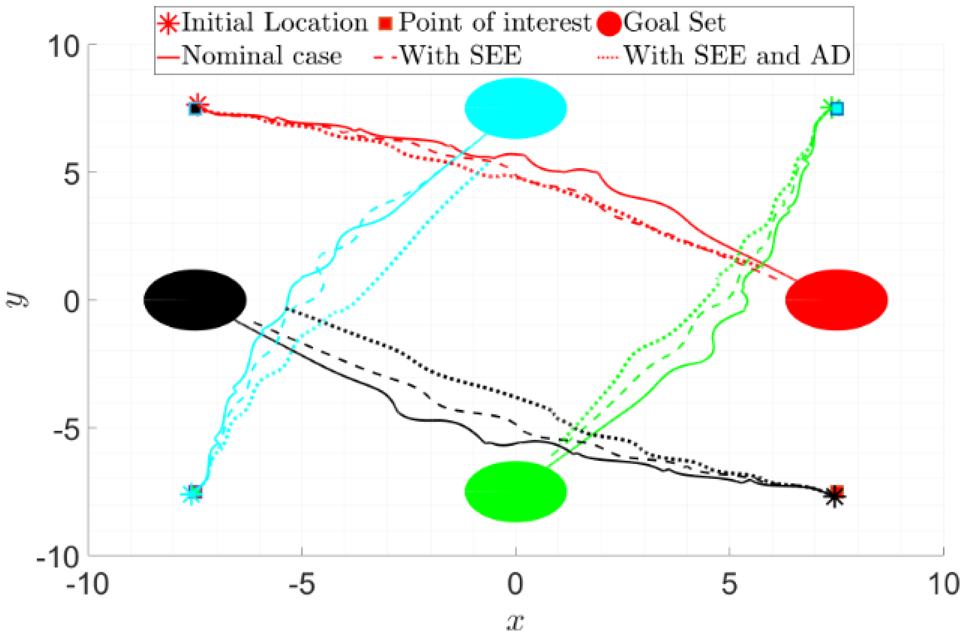
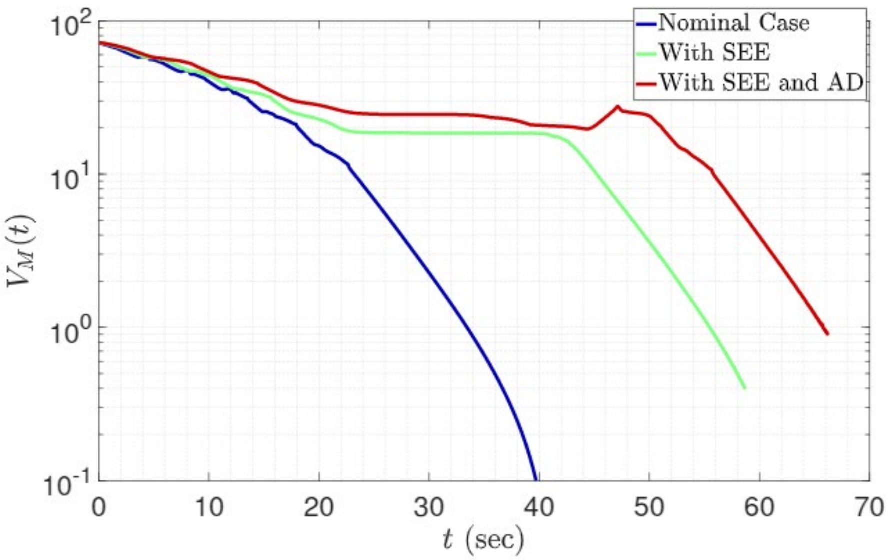


Figure: Pointwise maximum of CBFs $h_M(t) := \max\{h_{ij}, h_R, h_\phi\}$.

Example: Constrained Multi-Robot Navigation



Closed-loop paths traced by the agents.



Pointwise maximum of Lyapunov functions.

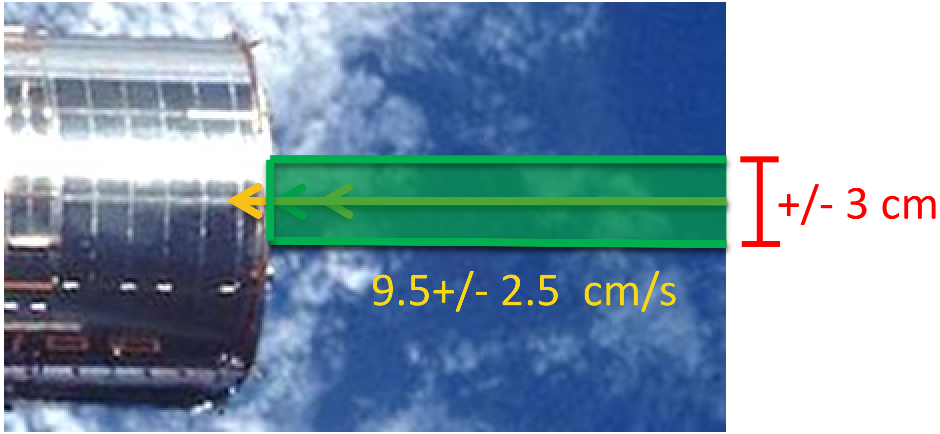
High-Relative Degree CBFs under Input Constraints and Disturbances

Joseph Breeden

(CDC 2021, Automatica 2023)

Safe Spacecraft Docking

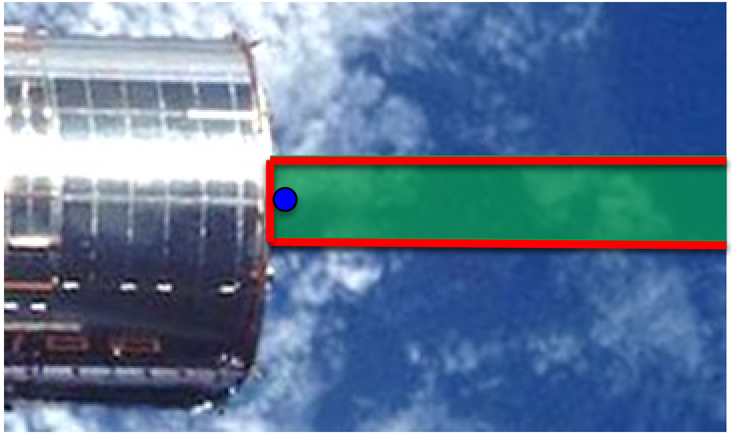
- "Safety" = "meets requirements"
- Spacecraft docking has required tolerances
  - Narrow docking mechanism (cross-track, radial relative position)
  - Docking must occur within specified velocity tolerances (in-track velocity)
- Describe tolerances by a set $S_h \subset \mathbb{R}^n$



J. Breeden, D. Panagou (CDC 21)

Safe Spacecraft Docking

- Spacecraft docking is a "tight tolerance" problem
  1. Safe set is small (in the context of the problem)
  2. Docking target lies close to the boundary of the safe set



Outline and Contributions

1. Achieving provable safety in the presence of input constraints and disturbances (see [6])
2. Extension of safety to allow for tight tolerance objectives
3. Application to spacecraft docking

[6] J. Breeden and D. Panagou, "Robust control barrier functions under high relative degree and input constraints for satellite trajectories," Automatica, 2023, to appear. [Online]. Available: https://arxiv.org/abs/2107.04094

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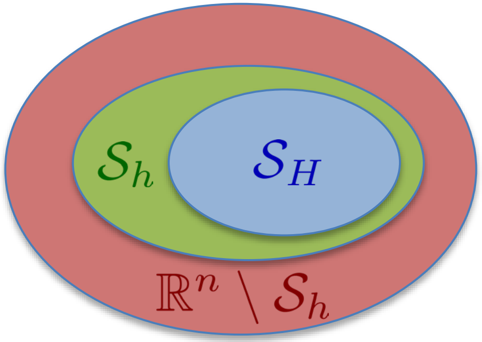
Primary Tool

- Control Barrier Functions (CBFs)
  - A CBF $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ ensures that the system state always lies within a set $\mathcal{S}_h \subset \mathbb{R}^n$

- Our formulation
  - State $x \in \mathbb{R}^n$, control $u \in \mathcal{U} \subset \mathbb{R}^m$, time $t \in \mathcal{T} \subseteq \mathbb{R}$
  - Dynamics $\dot{x} = f(t,x) + g(t,x)(u + w_u) + w_x$ with bounded disturbances $\|w_u\| \leq w_{u,\max}$, $\|w_x\| \leq w_{x,\max}$
  - Safe set: $\mathcal{S}_h(t) = \{x \in \mathbb{R}^n \mid h(t,x) \leq 0\}$ for a given function $h : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ of relative-degree two
  - Design a CBF H such that $\mathcal{S}_H(t) = \{x \in \mathbb{R}^n \mid H(t,x) \leq 0\}$ is a subset of $\mathcal{S}_h(t)$ and then render $\mathcal{S}_H$ forward invariant



Background – Control Barrier Functions

Definition. A $\mathcal{C}^1$ function $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ is a Control Barrier Function (CBF) on a set $\mathcal{X}$ if there exists a locally Lipschitz continuous $\alpha_0 \in \mathcal{K}$ such that $\forall x \in \mathcal{X}(t), t \in \mathcal{T}$,

$$\max_{\substack{\|w_u\| \leq w_{u,\max} \\ \|w_x\| \leq w_{x,\max}}} \inf_{u \in \mathcal{U}} \dot{H}(t, x, u, w_u, w_x) \leq \alpha_0(-H(t, x)) .$$

$$\dot{H}(t,x,u,w_u,w_x) = \underbrace{\partial_t H(t,x) + \nabla H(t,x) f(t,x)}_{\text{known, uncontrolled}} + \underbrace{\nabla H(t,x) g(t,x) u}_{\text{known, controlled}} + \underbrace{\nabla H(t,x) g(t,x) w_u + \nabla H(t,x) w_x}_{\text{unknown, bounded}}$$

(where $\mathcal{K}$ is the set of class-$\mathcal{K}$ functions $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$)

Background – Control Barrier Functions

Definition. A $\mathcal{C}^1$ function $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ is a Control Barrier Function (CBF) on a set $\mathcal{X}$ if there exists a locally Lipschitz continuous $\alpha_0 \in \mathcal{K}$ such that $\forall x \in \mathcal{X}(t), t \in \mathcal{T}$,

$$\inf_{u \in \mathcal{U}} \dot{H}(t, x, u, 0, 0) + W(t, x) \leq \alpha_0(-H(t, x)) \,.$$

- Define $W(t,x) \triangleq \|\nabla H(t,x) g(t,x)\| w_{u,\max} + \|\nabla H(t,x)\| w_{x,\max}$

  which implies $\dot{H}(t, x, u, w_u, w_x)$

$$\in [\dot{H}(t,x,u,0,0) - W(t,x),\ \dot{H}(t,x,u,0,0) + W(t,x)]$$

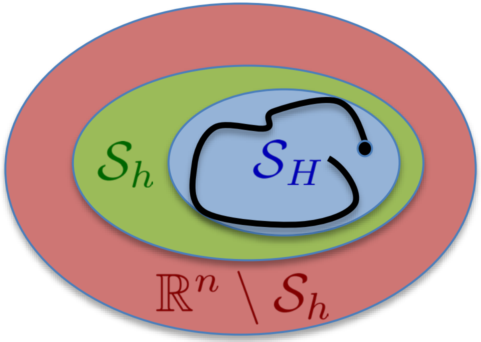
Background – Control Barrier Functions

Lemma ([6, Cor. 17]). Suppose $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ is a CBF on the set $\mathcal{S}_H$. Suppose there exists constants $\eta_1, \eta_2 > 0$ such that W satisfies $W(t,x) \in [\eta_1, \eta_2], \forall x \in \mathcal{S}_H(t), t \in \mathcal{T}$. Let $\alpha_w \in \mathcal{K}$ be locally Lipschitz continuous. Then any control law $u(t,x)$ that is piecewise continuous in t and locally Lipschitz continuous in x, and that satisfies: $\forall x \in \mathcal{S}_H(t), t \in \mathcal{T}$,

$$\dot{H}(t,x,u,0,0) \leq \alpha_w(-H(t,x))W(t,x) - W(t,x) \tag{1}$$

will render the set $\mathcal{S}_H$ forward invariant.

- (1) is called the "CBF condition"
- $\dot{H}(t,x,u,0,0)$ is control-affine
- $\mathcal{S}_H$ is a viability domain



Background – Control Barrier Functions

- CBFs are composable using the CBF condition (1) repeatedly

- Implement controller as an LP or QP satisfying (1) for all i

$$u = \underset{\substack{u \in \mathcal{U} \\ \dot{H}_i \leq \alpha_w(-H_i)W - W,\, \forall i}}{\operatorname{argmin}} \quad u^{\mathrm{T}} J u + F u$$

- LP/QP with dimension m is computationally lightweight and constraints can be easily added/removed

CBFs for Input Constraints and Bounded Disturbances

- Inputs:
  - Safe set function: $h : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$
  - Control input constraints: $\mathcal{U}$
  - Disturbance bounds: $w_{u,\max}, w_{x,\max}$
  - Dynamics: f, g

- Assumptions – see [6]
- Outputs:
  - CBF: $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ such that $\mathcal{S}_H \subseteq \mathcal{S}_h$

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CBFs for Input Constraints and Bounded Disturbances

- Given $h : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ and under certain assumptions in [6, Thm. 9], the following is a CBF for any $\alpha_0 \in \mathcal{K}$

$$H(t,x) \triangleq \Phi^{-1}\left(\Phi(h(t,x)) - \frac{1}{2}\left|\dot{h}_w(t,x)\right|\dot{h}_w(t,x)\right) \qquad (2)$$

where $\dot{h}_w(t,x) \triangleq \max_{\|w_x\| \leq w_{x,\max}} \dot{h}(t,x,w_x)$ is derived from the dynamics f and g, $\Phi : \mathbb{R} \to \mathbb{R}$

input constraints $\mathcal{U}$, and disturbance bounds $w_{u,\max}$ and $w_{x,\max}$

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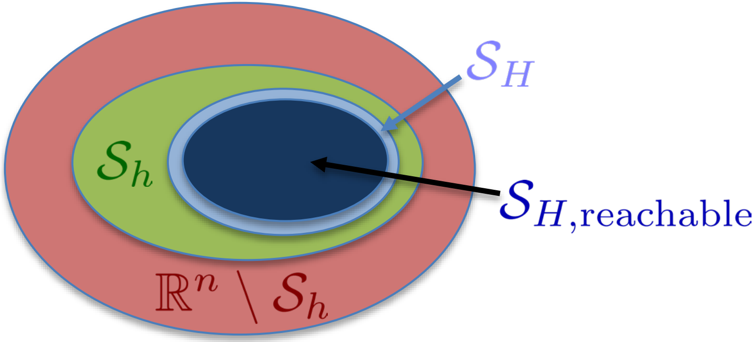
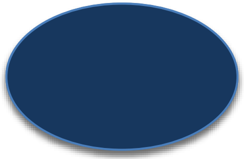
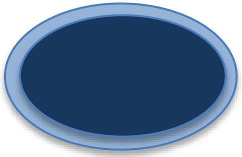
Problem with CBFs

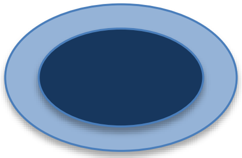
- Robustness to bounded disturbances introduces margins



- The reachable safe set depends on the online disturbances w_u, w_x







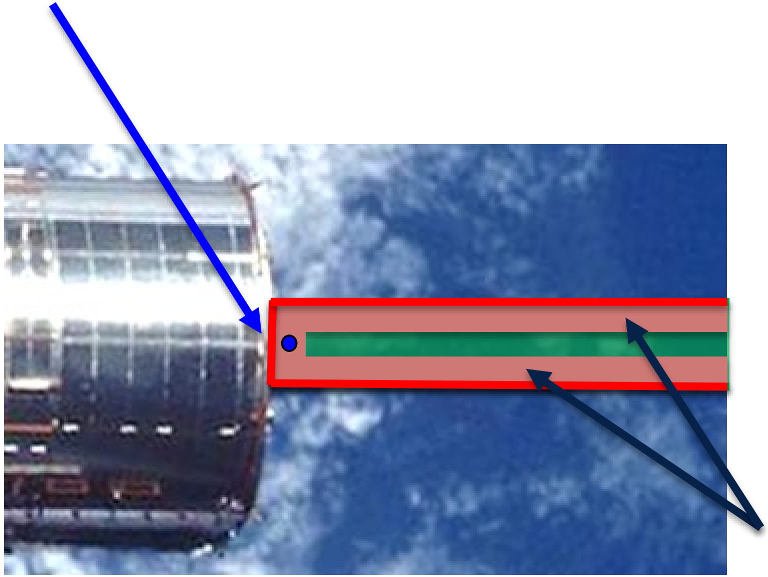
Reachable safe set if
$$\nabla H(t,x)g(t,x)w_u + \nabla H(t,x)w_x = W(t,x)$$

Reachable safe set if
$$\nabla H(t,x)g(t,x)w_u + \nabla H(t,x)w_x = 0$$

Reachable safe set if
$$\nabla H(t,x)g(t,x)w_u + \nabla H(t,x)w_x = -W(t,x)$$

Problem with CBFs

- The conservatism induced by (1) is problematic for tight tolerance objectives because
  1) The reachable safe set may become empty
  2) The target may not be inside the reachable safe set



Margins induced by robustness to worst-case $W(t, x)$

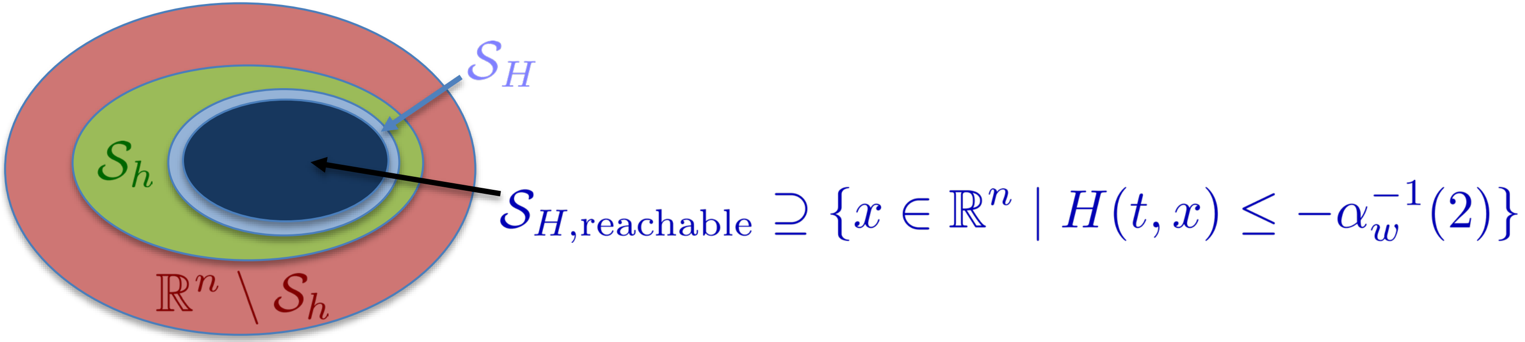
Tuning Robust CBF Margins

$$\dot{H}(t,x,u,0,0) \leq \alpha_w(-H(t,x))W(t,x) - W(t,x) \qquad (1)$$

- With H as in (2), we can choose any α_w

Lemma. If the control input $u(t,x)$ satisfies (1) with equality and $x(t_0) \in \mathcal{S}_H(t_0)$, then $\lim_{t\to\infty} H(t,x) \in [-\alpha_w^{-1}(2), 0]$.

- Choose α_w such that the "effective margin" $\alpha_w^{-1}(2)$ is sufficiently small



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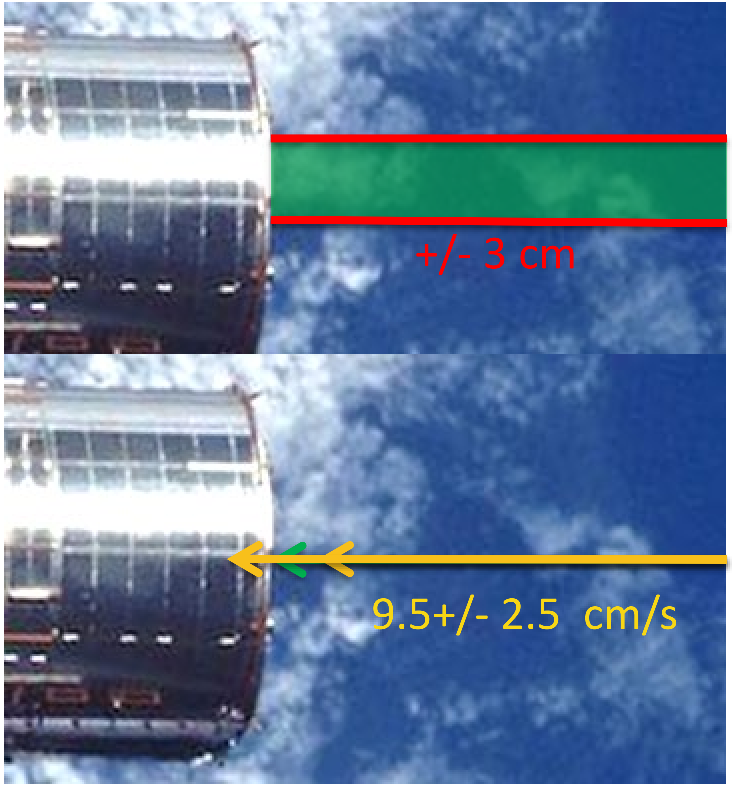
Docking Requirements

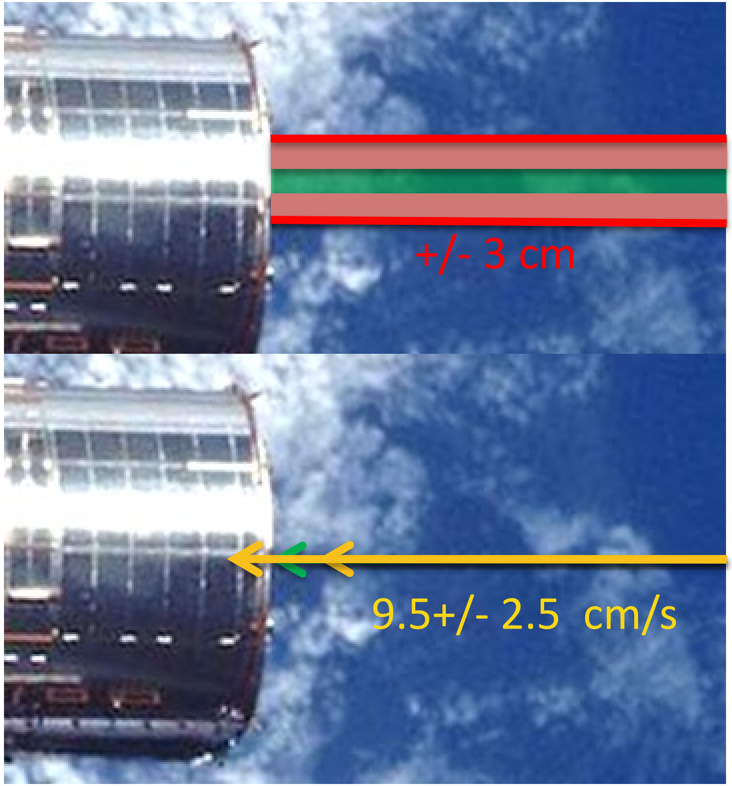
- Given $f, g, \mathcal{U}, w_{u,\max}, w_{x,\max}$



- Let h_l, h_r describe a docking cylinder
- Require $h_l(t, x(t)) \leq 0$ and $h_r(t, x(t)) \leq 0$ for all t

- Let h be the distance along the docking axis
- Require $h(t_f, x(t_f)) = 0$ and $\dot{h}(t_f, x(t_f)) \in [\gamma_1, \gamma_2]$ for some $t_f < \infty$

Docking Implementation



- Use prior lemma to ensure that $S_{H,\text{reachable}}$ is always nonempty

- Use prior lemma and Theorems 1-3 in paper (which relate H to h) to ensure docking axis requirements are satisfied in finite time

Simulations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & -2n & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} w_{x,1} \\ w_{x,2} \\ w_{u,1} \\ w_{u,2} \end{bmatrix}$$

$h(x) = -x_2 \quad \to H \ (\text{Thm. } 3)$ (in-track distance)

$h_l(x) = x_1 - \Delta \quad \to H_l \ [6, \text{Thm. } 9]$ (left radial constraint)

$h_r(x) = -x_1 - \Delta \quad \to H_r \ [6, \text{Thm. } 9]$ (right radial constraint)

$H_v(x) = \|[\dot{x}_1, \dot{x}_2]\|_\infty - v_{max}$ (velocity constraint)

$$\Delta = 0.03 \text{ m}, \ \ v_{max} = 10 \text{ m/s}, \ \ \mathcal{U} = \{u \in \mathbb{R}^2 \mid \|u\|_\infty \leq 0.082 \text{ m/s}^2\}$$
$$w_{u,\max} = 0.002 \text{ m/s}^2, \ \ w_{x,\max} = 0.001 \text{ m/s}$$

Simulations

$$u(t,x) = \begin{cases} \underset{\substack{u\in\mathcal{U}, \\ u \text{ satisfies (1) for } H, \\ u \text{ satisfies (1) for } H_r \\ u \text{ satisfies (1) for } H_v}}{\operatorname{argmin}} \|u - u_{nom}(t,x)\|^2 & H_l(t,x) > 0 \\ \underset{\substack{u\in\mathcal{U}, \\ u \text{ satisfies (1) for } H, \\ u \text{ satisfies (1) for } H_r, \\ u \text{ satisfies (1) for } H_l \\ u \text{ satisfies (1) for } H_v}}{\operatorname{argmin}} \|u - u_{nom}(t,x)\|^2 & H_l(t,x) \leq 0 \end{cases}$$

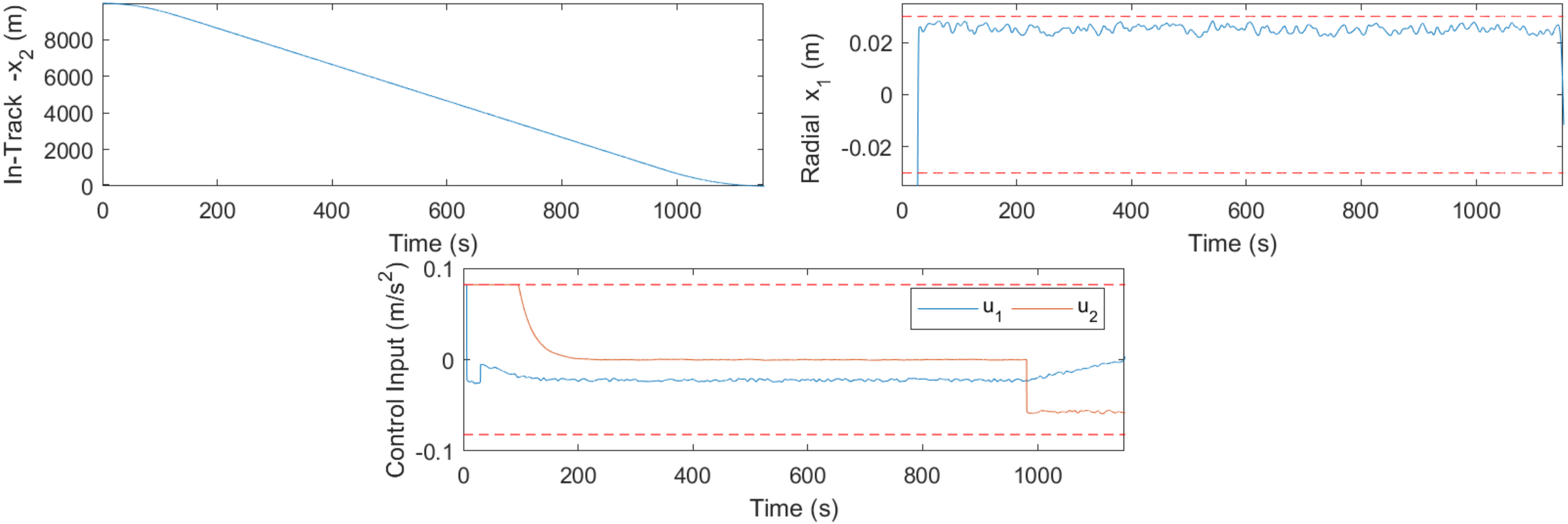
- u_{nom} is an attractive control law (drives x to the origin)
- h_l does not become active until the spacecraft first enters the safe set

Simulation Results

(not to scale)

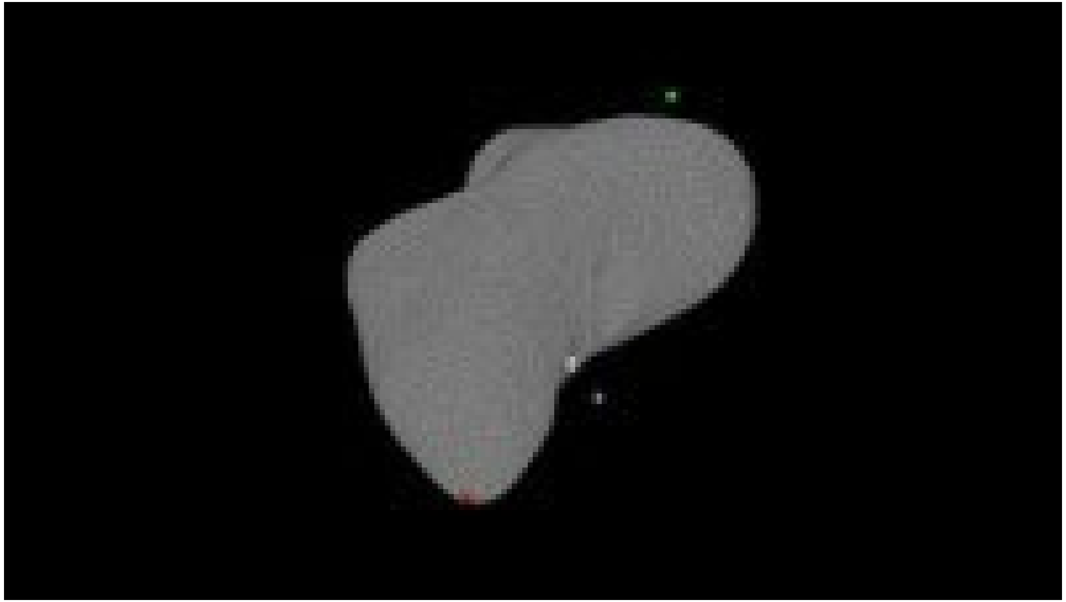
https://youtu.be/RoByiSD__jo

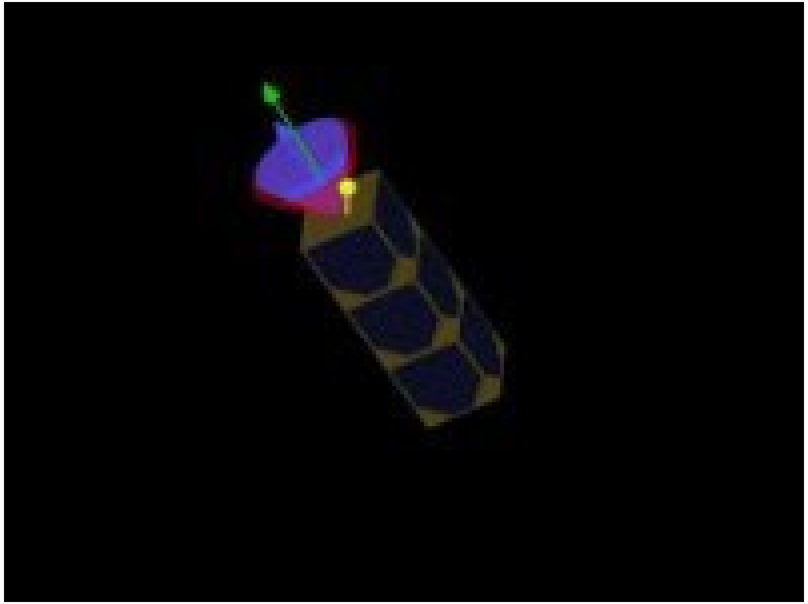
Simulation Results



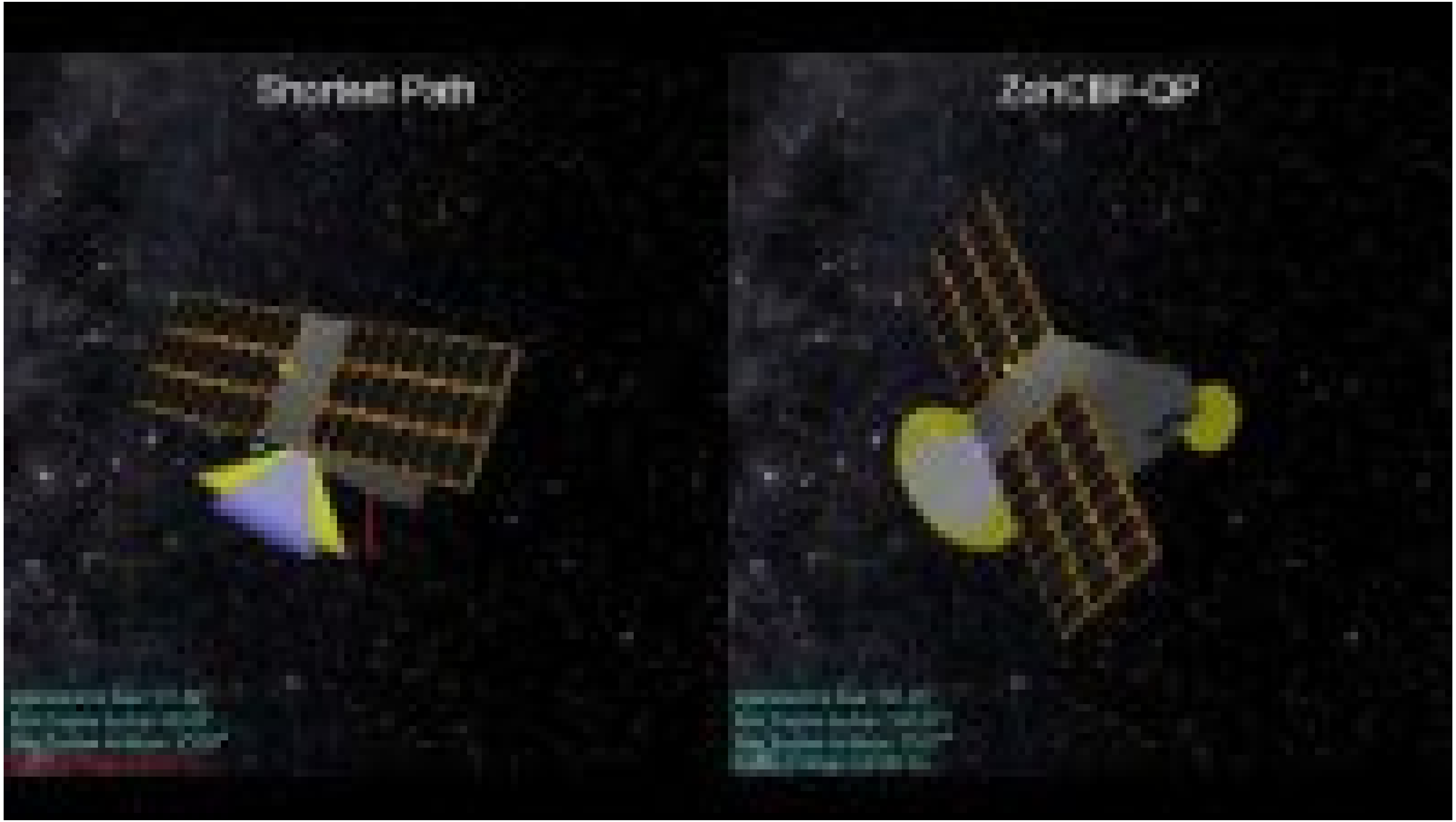
- $\gamma_1 = 0.07 \text{ m/s}, \; \gamma_2 = 0.12 \text{ m/s}$
- Docking velocity of $\dot{h}(t_f, x(t_f)) = 0.11 \text{ m/s}$

More Spacecraft Control Applications





More Spacecraft Control Applications



Adaptation for CBF Validation and Safe Control Synthesis

Mitchell Black
(CDC 2023)

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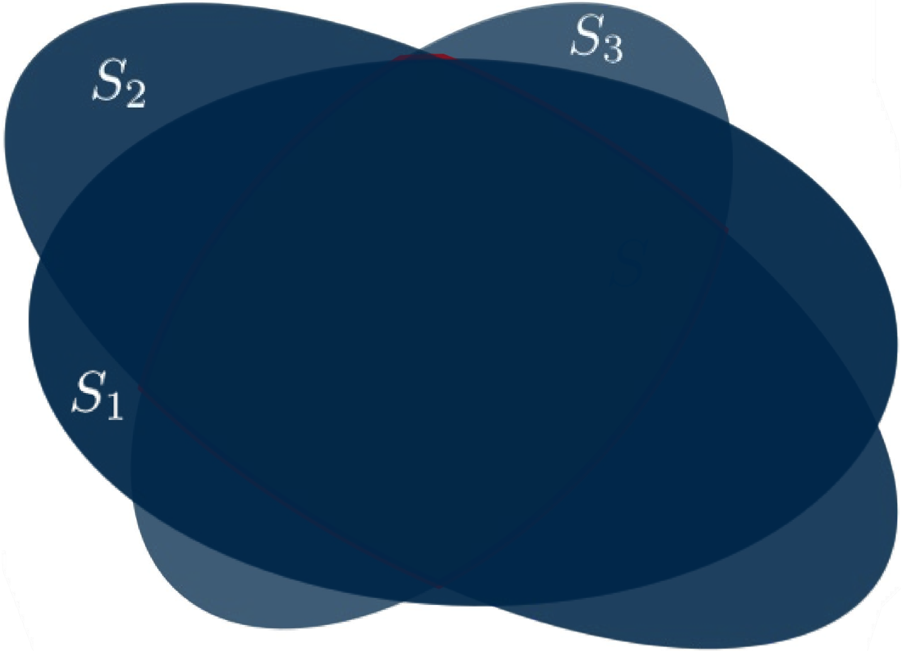
Motivation and Problem

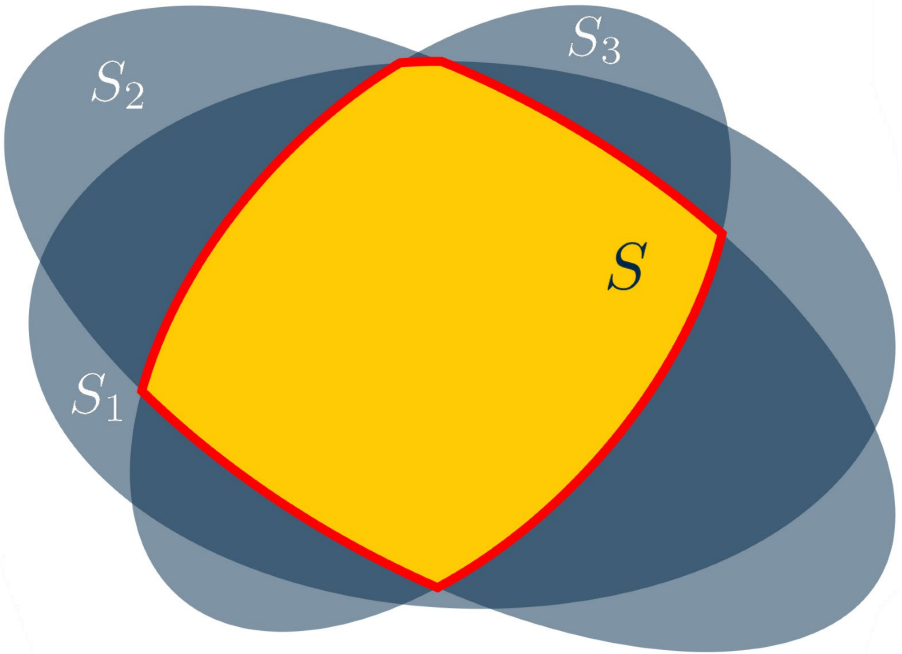


$$S = S_1 \cap S_2 \cap S_3$$

$$S_1 = \{x \in \mathbb{R}^n \mid h_1(x) \geq 0\}$$
$$S_2 = \{x \in \mathbb{R}^n \mid h_2(x) \geq 0\}$$
$$S_3 = \{x \in \mathbb{R}^n \mid h_3(x) \geq 0\}$$

Motivation and Problem

- **Dynamics:** $\dot{x} = f(x) + g(x)u$
- **Problem**: Design a control input u such that the safe set S defined with respect to multiple candidate CBFs h_i is rendered forward-invariant



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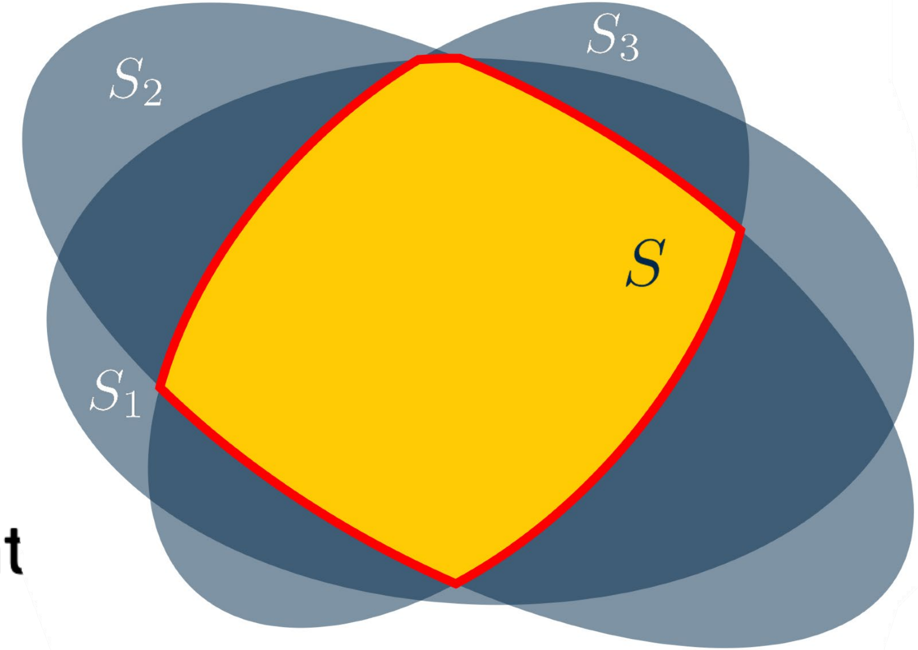
- **Solution**: Synthesize one Consolidated Control Barrier Function from the constituent candidate CBFs



$$S = S_1 \cap S_2 \cap S_3$$

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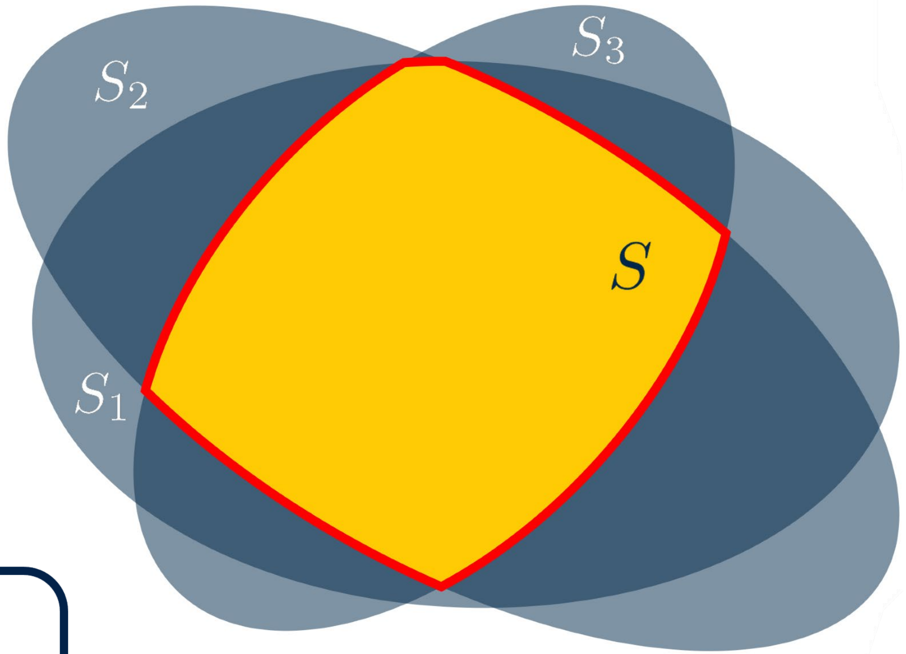
$$H(\boldsymbol{x}, \boldsymbol{k}) = 1 - \sum_{s=1}^{c} \phi\Big(h_s(\boldsymbol{x}), k_s\Big),$$



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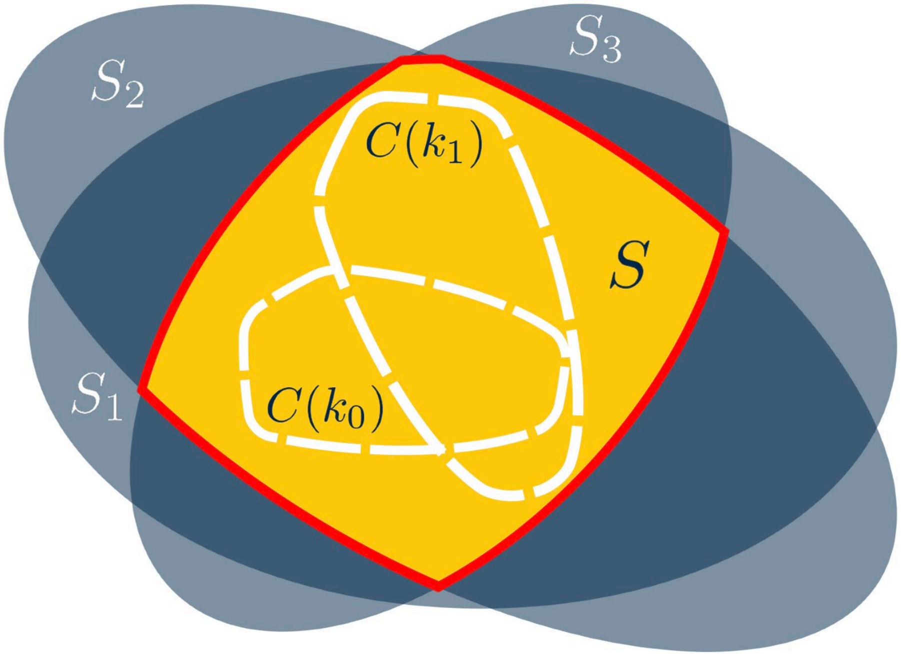
$\phi : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_+$, continuously differentiable
$\phi(h_s, 0) = \phi(0, k_s) = \phi(0, 0) = 1$. $\phi \in \mathcal{LL}$
e.g. $e^{-h_s k_s}$ satisfies conditions for ϕ



$$S = S_1 \cap S_2 \cap S_3$$

$$C(k) = \{x \in \mathbb{R}^n \mid H(x, k) \geq 0\}$$

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Motivation and Problem

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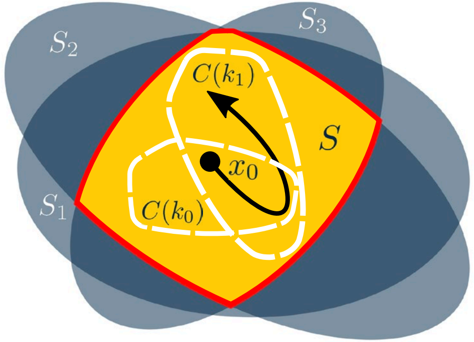
e.g. $e^{-h_s k_s}$ satisfies conditions for ϕ

- **Problem:**

$$\dot{H} = L_f H + L_g H u \geq -\alpha(H)$$

if $L_g H = 0$ when $L_f H + \alpha(H) < 0$

- **Solution:** Adapt the weights k such that $L_g H \neq 0$



$S = S_1 \cap S_2 \cap S_3$
$C(k) = \{x \in \mathbb{R}^n \mid H(x, k) \geq 0\}$

$$\dot{\boldsymbol{k}} = \underset{\boldsymbol{\mu} \in \mathbb{R}^c}{\arg\min} \ \frac{1}{2}(\boldsymbol{\mu} - \boldsymbol{\mu}_0)^T \boldsymbol{P} (\boldsymbol{\mu} - \boldsymbol{\mu}_0)$$

$$\text{s.t.}$$

$$\boldsymbol{\mu} + \alpha_k(\boldsymbol{k} - \boldsymbol{k}_{min}) \geq 0, \quad (k_{min} > 0)$$

$$\boldsymbol{p}^T \boldsymbol{Q} \dot{\boldsymbol{p}} + \boldsymbol{p}^T \dot{\boldsymbol{Q}} \boldsymbol{p} + \alpha_p(h_p) \geq 0, \quad (L_g H \neq \boldsymbol{0}_{1 \times M})$$

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Numerical Case Studies

- Multi-Robot Goal-Reaching in Constrained Warehouse Environment

Dynamics: Bicycle

$$
\begin{aligned}
\dot{x}_i &= v_i \left(\cos\psi_i - \sin\psi_i \tan\beta_i\right) \\
\dot{y}_i &= v_i \left(\sin\psi_i + \cos\psi_i \tan\beta_i\right) \\
\dot{\psi}_i &= \frac{v_i}{l_r} \tan\beta_i \\
\dot{\beta}_i &= \omega_i \\
\dot{v}_i &= a_i,
\end{aligned}
$$

Controller: Decentralized C-CBF-QP

$$
\boldsymbol{u}_i^* = \underset{\boldsymbol{u}_i \in \mathcal{U}_i}{\arg\min} \frac{1}{2} \|\boldsymbol{u}_i - \boldsymbol{u}_i^0\|^2
$$

s.t.

$$
a + \boldsymbol{b}_i \boldsymbol{u}_i \geq d, \quad \text{(C-CBF Condition)}
$$

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Numerical Case Studies

- Decentralized Swarm Reorganization

Dynamics: Bicycle

$$
\begin{aligned}
\dot{x}_i &= v_i \left(\cos\psi_i - \sin\psi_i \tan\beta_i\right) \\
\dot{y}_i &= v_i \left(\sin\psi_i + \cos\psi_i \tan\beta_i\right) \\
\dot{\psi}_i &= \frac{v_i}{l_r} \tan\beta_i \\
\dot{\beta}_i &= \omega_i \\
\dot{v}_i &= a_i,
\end{aligned}
$$

Controller: Decentralized C-CBF-QP

$$
\boldsymbol{u}_i^* = \underset{\boldsymbol{u}_i \in \mathcal{U}_i}{\arg\min}\ \frac{1}{2}\|\boldsymbol{u}_i - \boldsymbol{u}_i^0\|^2
$$

s.t.

$$
a + \boldsymbol{b}_i \boldsymbol{u}_i \geq d, \quad \text{(C-CBF Condition)}
$$

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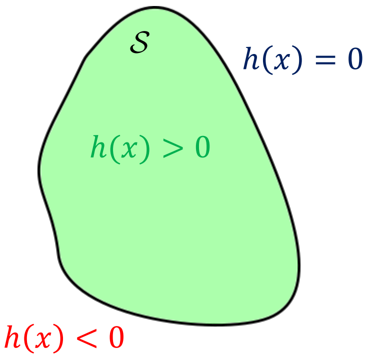
Rate-Tunable CBFs for Feasibility and Optimality

Hardik Parwana
(CDC 2022)

Review of CBFs and Effect of their Parameters



Let the dynamical system

$$\dot{x} = f(x) + g(x)u$$

Given a constrained (safe) set

$$\mathcal{S} \triangleq \{x \in \mathcal{X} : h(x) \geq 0\},$$
$$\partial\mathcal{S} \triangleq \{x \in \mathcal{X} : h(x) = 0\},$$
$$\mathrm{Int}(\mathcal{S}) \triangleq \{x \in \mathcal{X} : h(x) > 0\}$$

Sufficient condition on set invariance

The set S is rendered invariant if h is a CBF, that is, there exists an extended classK function α such that

$$\sup_{u \in \mathcal{U}} \left[\dot{h}(x, u)\right] \geq -\alpha(h(x)), \quad \forall x \in \mathcal{S}$$

On the boundary: $\alpha(0) = 0 \implies \dot{h}|_{h=0} \geq 0$

If the constraints are time-varying

$$\sup_{u \in \mathcal{U}} \left[\dot{h}(t, x, u)\right] \geq -\alpha(h(t, x)), \quad \forall x \in \mathcal{D} \subset \mathcal{S}$$

$$\min \quad ||u - u_{desired}||^2$$
$$\dot{h} \geq -h$$

$$\min \quad ||u - u_{desired}||^2$$
$$\dot{h} \geq -2h$$

$$\min \quad ||u - u_{desired}||^2$$
$$\dot{h}_1 \geq -1h_1$$
$$\dot{h}_2 \geq -6h_2$$

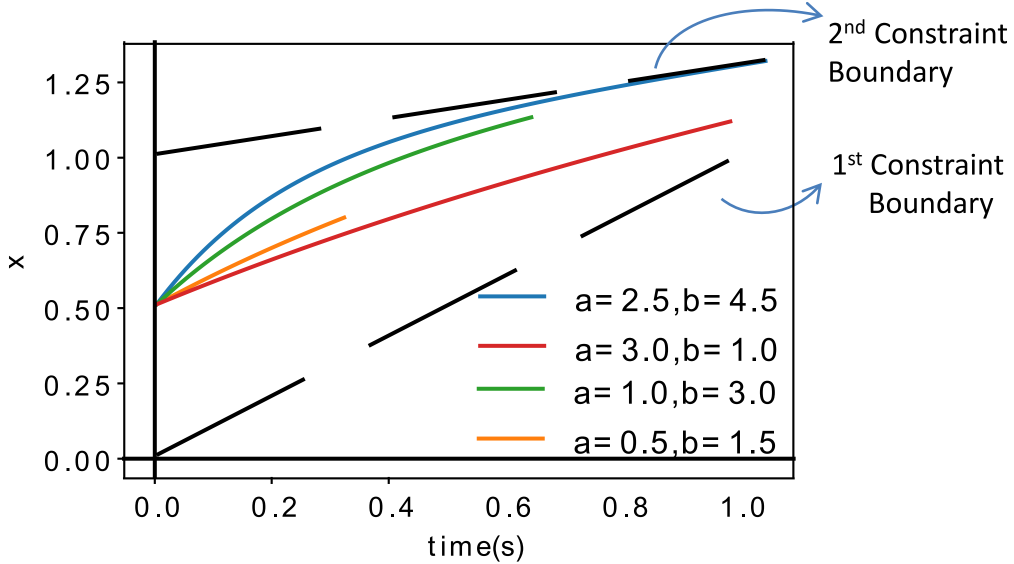
Review of CBFs and Effect of their Parameters

Revisiting the Adaptive Cruise Control Example

$$\begin{aligned} \max \quad & u \\ \text{s.t.} \quad & \dot{h}_1(t,x,u) \geq -a h_1(t,x) \\ & \dot{h}_2(t,x,u) \geq -b h_2(t,x) \end{aligned}$$

CBF for vehicle in front

CBF for vehicle at back



Design adaptation schemes for a, b to address
- Optimality of CBFs
- Feasibility of CBFs

Most previous approaches either
- Relax the constraints [1]
- Do offline sampling in parameter space [2,3]

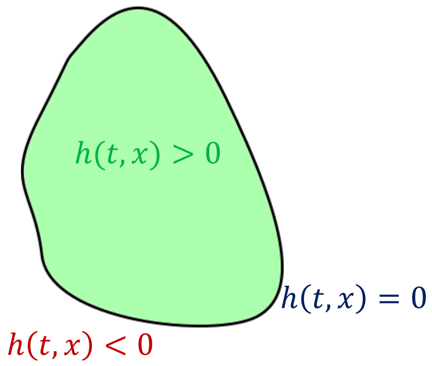
[1] J. Zeng, B. Zhang, Z. Li, and K. Sreenath, “Safety-critical control using optimal-decay control barrier function with guaranteed point-wise feasibility,” in 2021 American Control Conference (ACC). IEEE, 2021, pp. 3856–3863
[2]W. Xiao, C. A. Belta, and C. G. Cassandras, “Feasibility-guided learning for constrained optimal control problems,” in 2020 59th IEEE Conference on Decision and Control (CDC). IEEE, 2020, pp. 1896–1901
[3] Tan, Xiao, and Dimos V. Dimarogonas. "Compatibility checking of multiple control barrier functions for input constrained systems." *2022 IEEE 61st Conference on Decision and Control (CDC)*. IEEE, 2022.

High-Relative Degree CBF Conditions

Given $\dot{x} = f(x) + g(x)u$

And a safe set

$$\mathcal{S}(t) \triangleq \{x \in \mathcal{X} : h(t,x) \geq 0\},$$
$$\partial\mathcal{S}(t) \triangleq \{x \in \mathcal{X} : h(t,x) = 0\},$$
$$\text{Int}(\mathcal{S}(t)) \triangleq \{x \in \mathcal{X} : h(t,x) > 0\}$$



CBF condition for safety (forward invariance)

$\exists\, u$ s.t. $\dot{h}(t,x,u) \geq -\alpha(h(t,x))$

Higher Order CBF

$$\psi^0(t,x) = h(t,x),$$
$$\psi^k(t,x) = \dot{\psi}^{k-1}(t,x) + \boxed{\alpha^k(\psi^{k-1}(t,x))}$$
$$k \in \{1,2,..,r\}$$

We consider parametric classK functions

$\alpha_i^k(\theta_{\alpha_i^k}, h)$ with parameter $\theta_{\alpha_i^k}$

For example

$$\alpha^k(h) = \nu^k h,\ \nu \in \mathbb{R}^+$$

$\theta_{\alpha^k} = \nu^k$ is the parameter

CBF – QP controller

$$u(x,\theta_\alpha) = \arg\min_u \ ||u - u_d||^2$$
$$\text{s.t.} \quad \psi^r(t,x,u,\theta_\alpha) \geq 0$$

Rate Tunable Control Barrier Functions

Each individual agent is subject to time-varying environment → Cannot assume fixed backup safe set (Robust CBFs conservative)

Offline search for a CBF: find a barrier function h and corresponding classK functions α^k
- Too expensive to verify CBF condition over the whole state space (consider all possibilities with every other agent in the environment)
- Too conservative to have the same CBF over the whole state space
- Heterogeneous agents

→ Need online adaptation methods

Rate Tunable CBFs

Consider the augmented system (1)

$$\begin{bmatrix} \dot{x} \\ \dot{\theta}_\alpha \end{bmatrix} = \begin{bmatrix} f(x) + g(x)u \\ f_\alpha(x, \theta_\alpha) \end{bmatrix} \qquad (1)$$

with C^{r+1} barrier function h with relative degree r and corresponding derived higher order barrier functions given in (2).

$$\begin{aligned} \psi^0(t,x) &= h(t,x), \\ \psi^k(t,x) &= \dot{\psi}^{k-1}(t,x) + \alpha^k(\psi^{k-1}(t,x)) \\ & k \in \{1,2,..,r\} \end{aligned} \qquad (2)$$

Then h is a RT-CBF for (1) starting at initial state $x_0 \in S$ if

$$\sup_{u \in \mathbb{R}^m} \left[\psi^r(x, u, \dot{\theta}_\alpha) \right] \geq 0 \; \forall t > 0$$

Example

$$\alpha^k(h) = \nu^k h, \; \nu^k \in \mathbb{R}^+$$

RT-CBF allows designing $\dot{\nu}^k$ and changing ν^k

↑ the faster the trajectory approaches the boundary

↓ the slower the trajectory approaches the boundary

Problem Setting

Consider N **agents** $i = 1,2, \ldots, N$ with dynamics

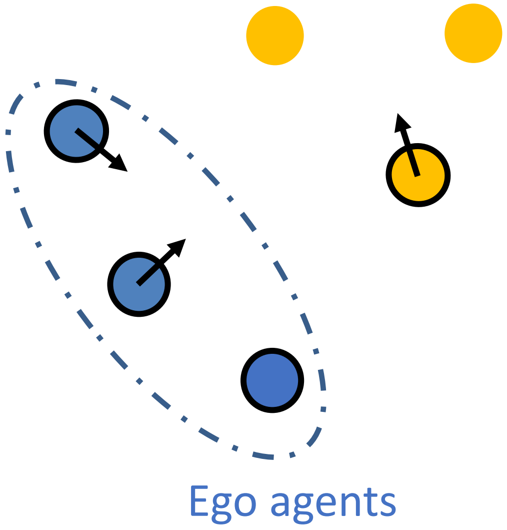
$$\dot{x}_i = f(x_i) + g(x_i)u_i$$

- Agent *i* measures the state x_j of its neighbor agents *j*
- Agent *i* measures the state derivative $\dot{x}_j$ of its neighbor agents OR has a bounded estimate of closed-loop dynamics of its neighbor agents (with a known Lipschitz bound on F(x))

$$\dot{x}_j \in F(x)$$

- designs its own control input of form $\min_{u_i} \quad J(u_i, u_{d_i})$

$$\text{s.t.} \quad h_{ij}(t) \geq 0 \; \forall \, j$$



Objective

Given a group of agents V, comprising of **cooperative**, **uncooperative**, and **adversarial** agents whose identities are unknown, design a controller to be used by ego agent i so that

- Safety is preserved for all time: $h_{ij}(t) \geq 0 \; \forall \, j \in Ni, \forall t > 0$
- Deviation between actual and nominal control input is minimized.

RT-CBFs for Heterogeneous Multi-Agent Coordination

Types of neighbor agent behavior:

Cooperative: other agent shares responsibility for safety

Uncooperative: other agent does not share responsibility for safety. Safe as long as it does not intercept other agent's path.

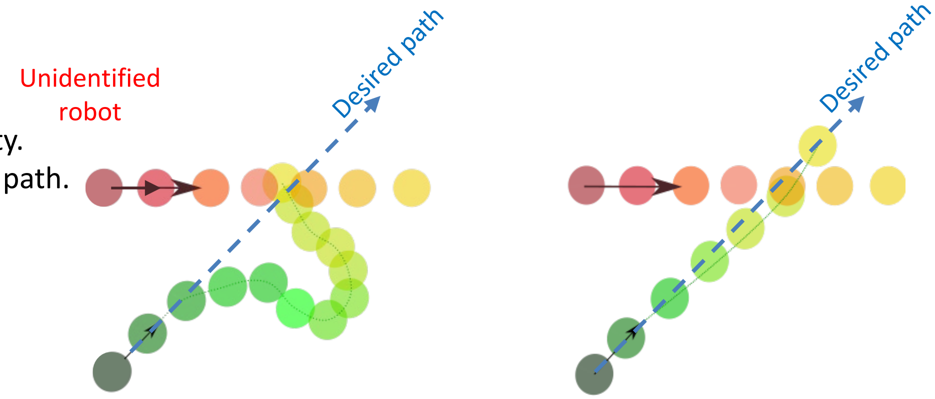
Adversarial: other agent actively seeks collision.

Previous Works

- either assume to know identities beforehand[1] or
- tend to treat all agents in the same fashion[2][3].

Problem:

Treating all agents **with same controller** is too conservative.



(a) (Identity unaware) (b) (Identity aware)

Idea: Develop a continuous ***trust factor ρ*** based on observations

Heuristic:

- If other agent *contributes* positively to *safety*, trust it more and relax the constraint
- If other agent *contributes* negatively to *safety*, trust it less and tighten the constraint

Safety Constraints — Using Control Barrier Functions(CBF)

Contribution to Safety — Ease of satisfaction of CBF conditions

[1] Borrmann, U., Wang, L., Ames, A. D., & Egerstedt, M. (2015). Control barrier certificates for safe swarm behavior. *IFAC-PapersOnLine*, *48*(27), 68-73.
[2] Usevitch, J., & Panagou, D. (2021, May). Adversarial resilience for sampled-data systems using control barrier function methods. In *2021 American Control Conference (ACC)* (pp. 758-763). IEEE.
[3] Multi-Robot Adversarial Resilience using Control Barrier Functions

RT-CBFs Results: Relative Degree 1

Consider first-order barrier functions

Consider ego agent i and any other agent j

$$\dot{h}_{ij} = \frac{\partial h_{ij}}{\partial x_i}\dot{x}_i + \frac{\partial h_{ij}}{\partial x_j}\dot{x}_j \geq -\nu_{ij} h_{ij}$$

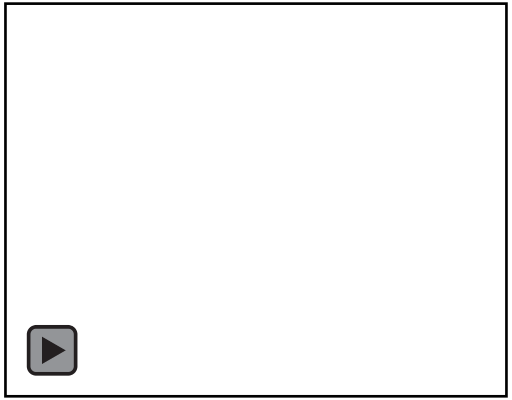
Best case ease of satisfaction

$$\frac{\partial h_{ij}}{\partial x_j}\dot{x}_j \geq -\nu_{ij} h_{ij} - \max\left\{\frac{\partial h_{ij}}{\partial x_i}\dot{x}_i\right\}$$

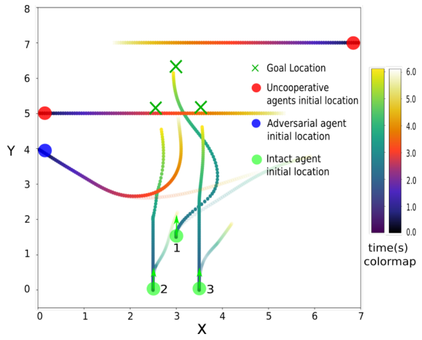
A linear program! (max computed while respecting other CBF constraints)

Of the form $A_{ij} v \geq b_{ij}$

Constraint margin $m_{ij} = A_{ij}\dot{x}_j + b_{ij}$



Constant



Trust factor computation

Distance based

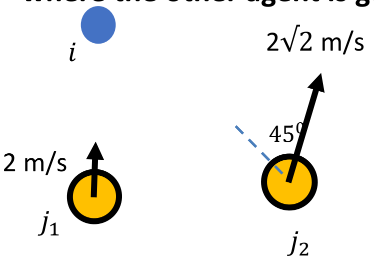
$$\rho_{m_{ij}} = f_d(m_{ij})$$

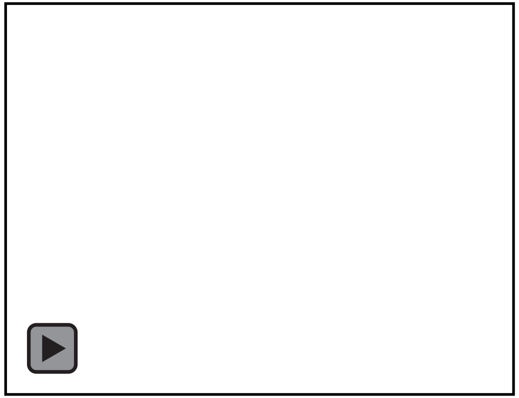
$$f_d : \mathbb{R}^+ \to [0, 1]$$

Example: $f_d(d) = \tanh(\beta d)$

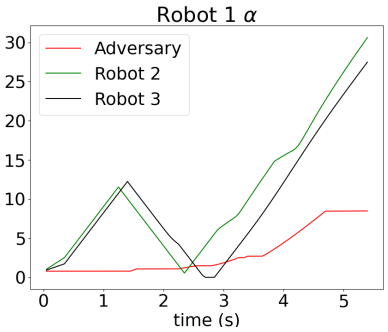
$$\dot{\nu}_{ij} = f_{ij}(\rho_{ij})$$

Direction based - Belief of where the other agent is going

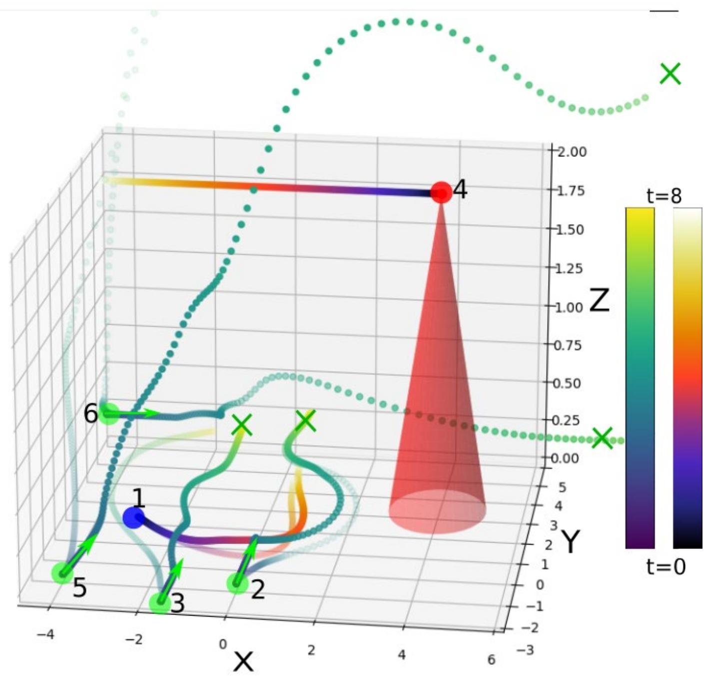




Trust-based Adaptation



RT-CBFs Results: High Relative Degree



2,3: unicycles
5,6 double integrators
1: adversary
4 uncooperative surveillance agent

arameter

e parameter

Nominal Motion

Fixed Parameter

Trust-based Adaptation

Top View

Top View

Predictive CBFs

Hardik Parwana (ICRA 2022)
Joseph Breeden (CDC 2022)
Mitchell Black (ACC 2023)

Predictive Adaptation for Policy Optimization (Ongoing)

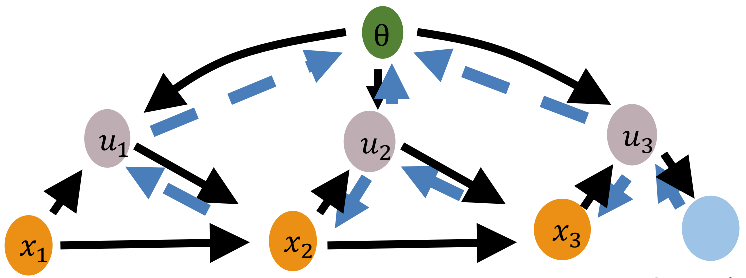
H. Parwana and D. Panagou "Recursive Feasibility Guided Optimal Parameter Adaptation of Differential Convex Optimization Policies for Safety-Critical Systems" (ICRA 2022)

Model Predictive Adaptation

$$\min_{u_t,\delta} \quad J(u) = (u_t - u_{d_t})^T P (u_t - u_{d_t}) + Q\delta^2$$

$$\begin{aligned} \text{s.t.} \quad & V(t+1, x_{t+1}) \leq (1 - \alpha_0)V(t,x) + \delta \\ & h_1(t+1, x_{t+1}) \geq (1 - \alpha_1)h_1(t, x_t) \\ & h_2(t+1, x_{t+1}) \geq (1 - \alpha_2)h_2(t, x_t) \\ & \quad . \\ & h_N(t+1, x_{t+1}) \geq (1 - \alpha_N)h_N(t, x_t) \end{aligned}$$

$$\theta = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ . \\ . \\ \alpha_N \end{bmatrix} \in \mathbb{R}^{N+1}$$



FEASIBILITY

Multiple hard constraints ⇒ Does a solution exist **for all states x ?**

- No, depends on the parameter θ

PERFORMANCE

How do resulting trajectories vary as we change parameters?

Predict future states and rewards over policy u_t -> Evaluate Performance -> Update control parameters θ with constrained gradient descent to preserve feasibility and improve performance

- Note that each controller u_t is a CBF-CLF program
- Hence the method performs online tuning of controller

Results: Target Tracking (Ideal Case)

- Follower
- Leader
- Desired Location

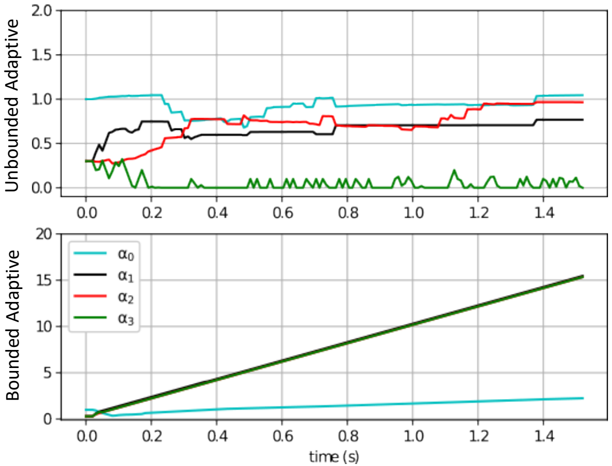
Follower is given the model of the leader's motion
Leader: single integrator
Follower: unicycle

Follower's Objective: Achieve the desired location of leader in the FoV for maximum reward

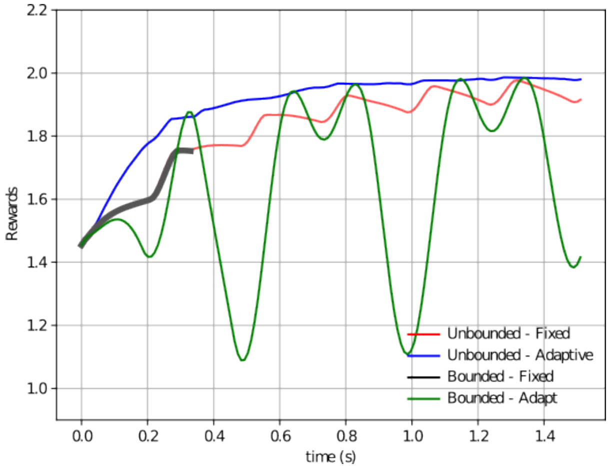
Follower's Policy: CBF-QP
CBFs: 3 (min dist, max dist, FoV angle)

	No adaptation	With adaptation
Without input bounds		
With input bounds		

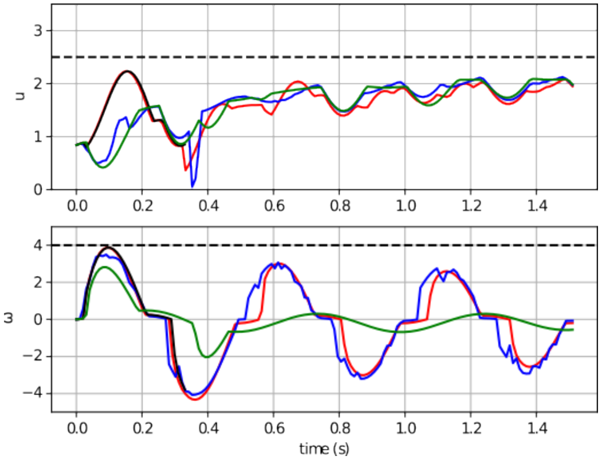
Results: Target Tracking (Ideal Case)



CLF and CBF parameters



Moving average Reward



Control Inputs

Encoding the Predictive CBF

- Given a time $\tau_i \in \boldsymbol{M}(t,x)$ and a nondecreasing function $m_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, define the "Predictive CBFs":

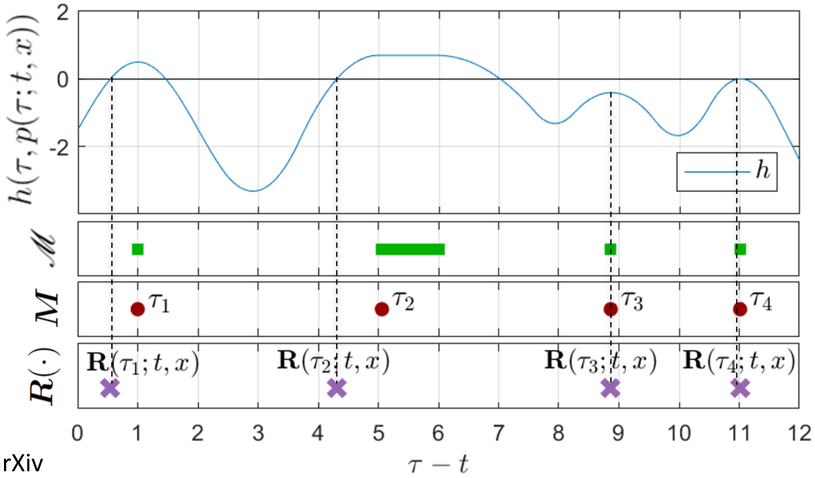
$$H_i(t,x) \triangleq h(\tau_i, p(\tau_i; t, x)) - m_i(\boldsymbol{R}(\tau_i; t, x) - t)$$

Amount by which safety is violated, or amount of margin

Time to make correction*

- Choose m_i so that $H_i(t_0, x_0) \leq 0$

J. Breeden and D. Panagou "Predictive Control Barrier Functions for Online Safety Critical Control" (CDC 2022, Outstanding Student Paper Award)



*See also Black et al., "Future-Focused Control Barrier Functions for Autonomous Vehicle Control", arXiv

Future-Focused Control Barrier Function (FF-CBF)

Future-Focused Collision Avoidance.

$$h_{\tau,ij}(\boldsymbol{z}_i, \boldsymbol{z}_j) = \min_{\tau \in [0,\bar{\tau}]} \|\hat{\boldsymbol{\xi}}_{ij}(t+\tau)\|^2 - (2R)^2$$

$$\tau^* = \arg\min_{\tau \in \mathbb{R}} \|\hat{\boldsymbol{\xi}}_{ij}(t+\tau)\|^2 - (2R)^2$$

$$\hat{\tau}^* = -\frac{\xi_x \nu_x + \xi_y \nu_y}{\nu_x^2 + \nu_y^2 + \varepsilon}$$

← (boundedness)

$(\hat{\tau} \in [0, \bar{\tau}])$

$$\hat{\tau} = \hat{\tau}^* K_0(\hat{\tau}^*) + (\bar{\tau} - \hat{\tau}^*) K_{\bar{\tau}}(\hat{\tau}^*)$$

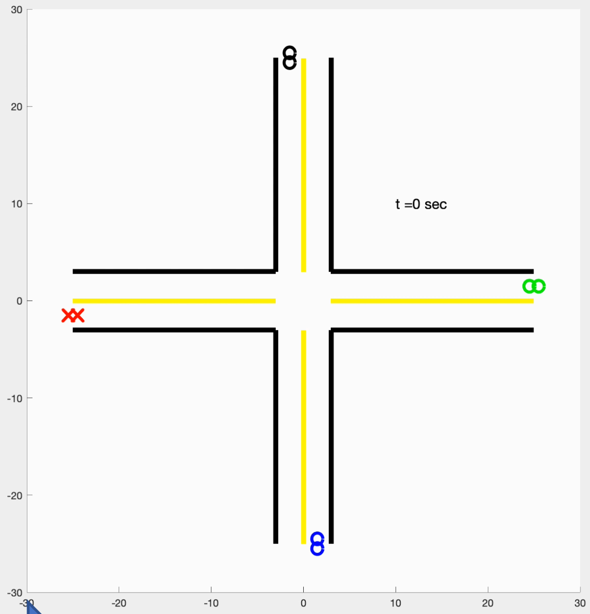
$$K_\delta(s) = \tfrac{1}{2} + \tfrac{1}{2}\tanh\left(k(s-\delta)\right),\ k > 0$$

FF-CBF.

$$h_{\hat{\tau},ij}(\boldsymbol{z}_i, \boldsymbol{z}_j) = \|\hat{\boldsymbol{\xi}}_{ij}(t+\hat{\tau})\|^2 - (2R)^2$$

→

M. Black, M. Jankovic, A. Sharma and D. Panagou "Future-Focused Control Barrier Functions for Autonomous Vehicle Control" (ACC 2023)



FF-CBF-QP control law

Overview of Predictive CBF

- We have presented a new framework for constructing CBFs for generic safety functions h using future trajectory predictions
- The Predictive CBF H_1 takes into account the future trajectories the system is expected to follow and modifies these trajectories before reaching unsafe states
- Compared to MPC, the Predictive CBF
  - followed similar trajectories in simulation
  - yields a pointwise control-affine safety constraint
    - Results in a convex QP control law even for nonlinear dynamics and constraints
    - QP is m-dimensional (where $u \in \mathbb{R}^m$) instead of mN-dimensional as in MPC
  - evaluates safety over a continuous predicted trajectory without fixed sampling (important for satellite simulations or other rapidly evolving systems)

Ongoing Work

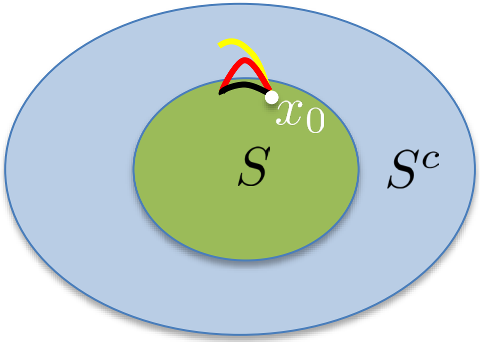
- Provably guaranteed input constraint satisfaction
  - Currently, input constraint satisfaction is achieved via tuning
- Distributed Systems
- Predicti
- Improvi

CBFs under ZOH Control

Joseph Breeden, Kunal Garg
(LCSS 2021)

Safety under ZOH Control

- Trajectories that are safe in continuous time may not be safe under digital controllers, such as zero-order-hold
  - Trajectories may:
    - Exit the safe set between time steps and return before the next control cycle
    - Exit the safe set and not return

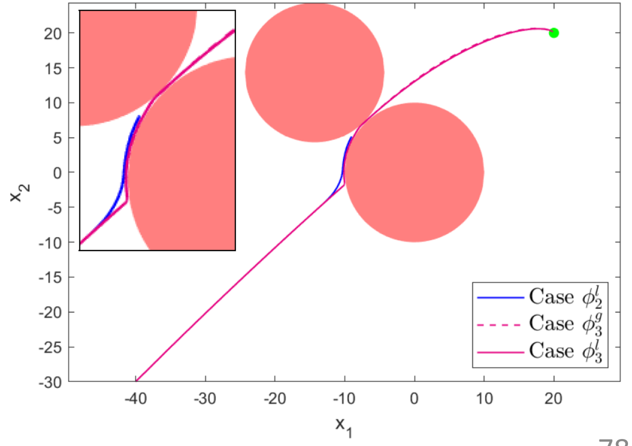


$$l_1(x) = l_{L_f h}(x) + l_{L_g h}(x) u_{\max} + l_{\alpha(h)}$$
$$\Delta(x) = \sup_{z \in \mathcal{R}(x,T), u \in U} \|f(z) + g(z)u\|$$

Theorem 1. *Let α be a locally Lipschitz class-$\mathcal{K}$ function. Then a control input $u(t) = u_k, \forall t \in [kT, (k+1)T)$, where u_k satisfies*

$$L_f h(x_k) + L_g h(x_k) u_k \leq \alpha(-h(x_k)) - l_1(x_k) T \Delta(x_k)$$

at the sample times $x_k = x(kT)$, renders the set S forward invariant.



J. Breeden, K. Garg, D. Panagou (LCSS 21)

Safety under ZOH for Continuous-Time CBFs

Theorem 1. *Let α be a locally Lipschitz class-$\mathcal{K}$ function. Then a control input $u(t) = u_k, \forall t \in [kT, (k+1)T)$, where u_k satisfies*

$$L_f h(x_k) + L_g h(x_k) u_k \leq \alpha(-h(x_k)) - l_1(x_k) T \Delta(x_k)$$

at the sample times $x_k = x(kT)$, renders the set S forward invariant.

- Compare to the condition in Cortez et al. (TCST 2019)

$$L_f h(x_k) + L_g h(x_k) u_k \leq \alpha(-h(x_k)) - \frac{l_1 \Delta}{l_2} \left(e^{l_2 T} - 1\right)$$

where $l_2(x) = L_f h(x) + L_g h(x) u_{\max}$

Safety under ZOH for Continuous-Time CBFs

$$v(x, z, u) = L_f h(z) - L_f h(x) + (L_g h(z) - L_g h(x))u - \alpha(-h(z)) + \alpha(-h(x))$$

Theorem 2. *Let α be a class-$\mathcal{K}$ function. Then a control input $u(t) = u_k, \forall t \in [kT, (k+1)T)$, where u_k satisfies*

$$L_f h(x_k) + L_g h(x_k) u_k \leq \alpha(-h(x_k)) - \sup_{z \in \mathcal{R}(x_k, T), u \in \mathcal{U}} v(x_k, z, u)$$

at the sample times $x_k = x(kT)$, renders the set S forward invariant.

- Same idea as Theorem 1, now using computed differences instead of Lipschitz constants, so both margins are smaller than in Theorem 1

Safety under ZOH for Discrete-Time CBFs

- Introduce $\eta(T,x) = \max\left\{ \sup_{z\in\mathcal{R}(x,T),u\in U} \nabla_z[\dot{h}(z,u)](f(z)+g(z)u), 0 \right\}$
  - This is a bound on the second derivative when u is constant

Theorem 3. *Let $\gamma \in (0,1]$. Then a control input $u(t) = u_k, \forall t \in [kT,(k+1)T)$, where u_k satisfies*

$$L_f h(x_k) + L_g h(x_k) u_k \leq -\frac{\gamma}{T} - \frac{1}{2}\eta(T,x_k)$$

at the sample times $x_k = x(kT)$, renders the set S forward invariant.

Safety under ZOH for Continuous-Time CBFs

- System: $\dot{x} = f(x) + g(x)u$
- Safe set: $S = \{x \in \mathbb{R}^n \mid h(x) \leq 0\}$
- Introduce:
  - $\mathcal{R}(x, T)$: Set of states reachable from x in times $t \in [0, T]$
  - $l_{L_f h}(x), l_{L_g h}(x), l_{\alpha(h)}(x)$: Lipschitz constants of $L_f h$, $L_g h$, $\alpha(-h)$, respectively, on the set $\mathcal{R}(x, T)$, where α is class-$\mathcal{K}$
  - $u_{\max} = \max_{u \in U} \|u\|$
  - $l_1(x) = l_{L_f h}(x) + l_{L_g h}(x) u_{\max} + l_{\alpha(h)}$
  - $\Delta(x) = \sup_{z \in \mathcal{R}(x,T), u \in U} \|f(z) + g(z)u\|$
- A method is "global" when $l_1 = \sup_{x \in \mathcal{S}} l_1(x)$, $\Delta = \sup_{x \in \mathcal{S}} \Delta(x)$, etc.

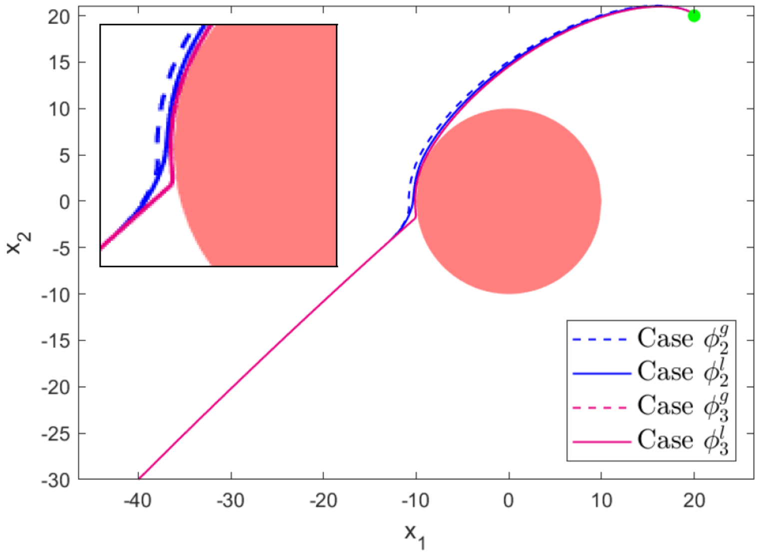
Comparing Methods

- Controller Margin
  - Quantifies the additional control authority required for provable safety under a ZOH controller
    - Prior work: $\nu_0(T) = \frac{l_1 \Delta}{l_2}\left(e^{l_2 T} - 1\right)$
    - Theorem 1: $\nu_1(T, x_k) = l_1(x_k) T \Delta(x_k) \implies \nu_1(T, x_k) \leq \nu_0(T), \ \forall x_k \in S$
- Physical Margin
  - Quantifies the effective shrinkage of the safe set due to the controller margin
- We want both margins to be as small as possible while keeping safety

Example

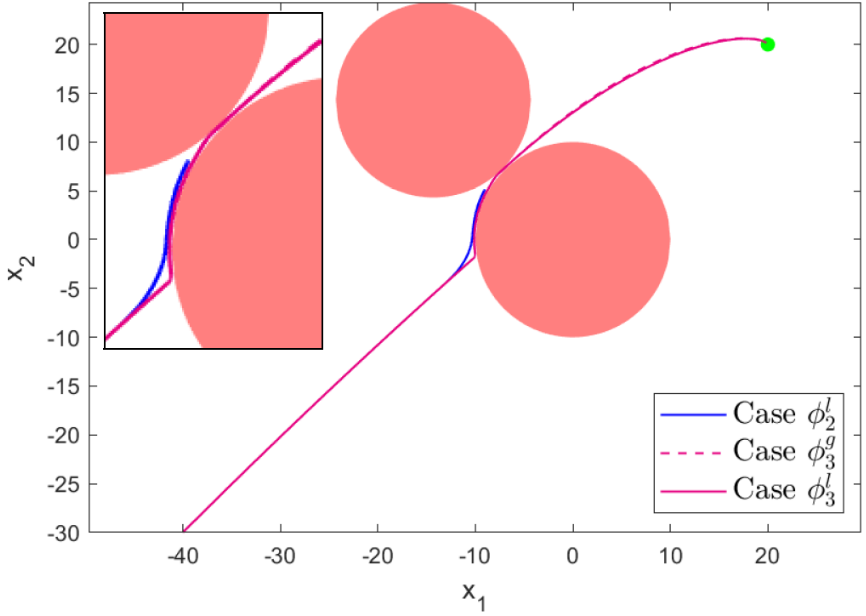
- A unicycle moving around an obstacle
- ϕ_i^l: Safety using Theorem *i* in a local sense
  ϕ_i^g: Safety using Theorem *i* in a global sense (maximized over all safe x)

- The values of l_1 and Δ are so high that the agent turns away from the target when using the safety method in Theorem 1.
- Using local values improves performance but at greater computational cost



Example

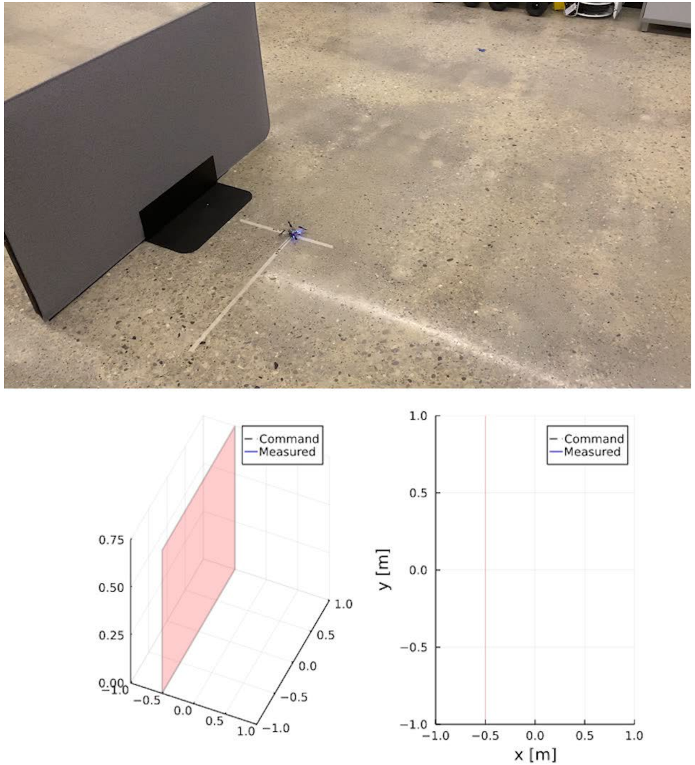
- When there are two obstacles, Theorem 2 may fail as well
- The agent remains safe, but becomes stuck between the obstacles due to physical margin
- Theorem 3 will also fail for obstacles sufficiently close together (see δ_3 in the paper)



CBFs under Output Feedback
or
How to interface Observers and Controllers for Safety

Devansh Agrawal

(LCSS 2022)



Baseline CBF Controller

Control Barrier Functions with Observers

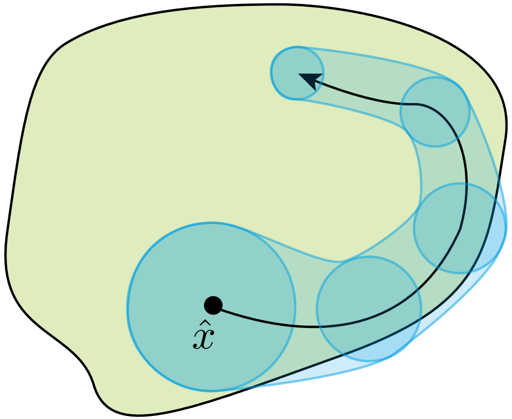
$$\dot{x} = f(x) + g(x)u + g_d(x)d(t)$$
$$y = c(x) + c_d(x)v(t)$$

measurements

bounded disturbances $\|d(t)\| \leq \bar{d}$

bounded disturbances $\|v(t)\| \leq \bar{v}$

$$\mathcal{S} = \{x \in \mathcal{X} : h(x) \geq 0\}$$



Thm: If h is a robust CBF, and $\mathcal{U} = \mathbb{R}^m$, a safe controller is

$$\pi(x) = \underset{u}{\operatorname{argmin}} \ \|u - \pi_d(x)\|^2$$
$$st. \ L_f h(x) + L_g h(x)u - \|g_d(x)\| \bar{d} \geq -\alpha(h(x))$$

where π_d is a desired control input.

stable observer + stable controller $\nRightarrow$ stable observer-controller
similarly,
stable observer + safe controller $\nRightarrow$ safe observer-controller

Measurement-Robust CBF [1]

vision-based state estimation
assumes c(x) is invertible
noiseless sensors
SOCP-controller

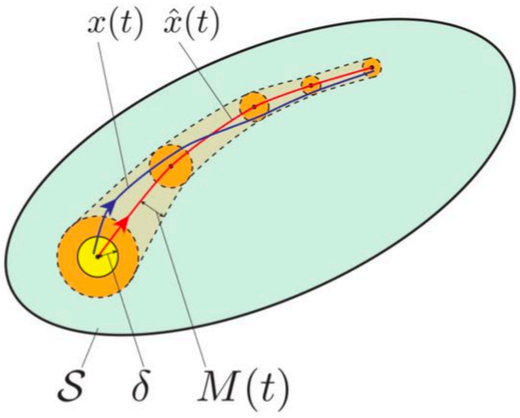
Stochastic CBFs [2, 3]

stochastic system
applicable only to EKF observer
probabilistic safety guarantee

[1] Dean, et. al CoRL, 2021
[2] Clark, ACC 2019
[3] Jahanshahi, IFAC, 2020

Two solution approaches

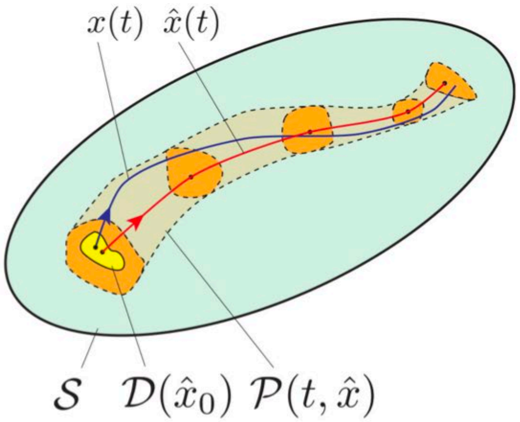
1) Input-to-State Stable (ISS) observers



estimation error is bounded
bound is non-increasing
[ex. Luenberger Observers]

$$\|x_0 - \hat{x}_0\| \leq \boxed{\delta} \implies \|x(t) - \hat{x}(t)\| \leq \boxed{M(t)} \quad \forall t$$

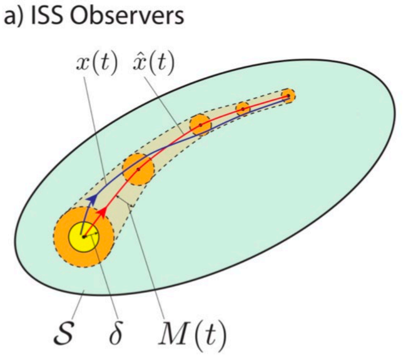
2) Bounded-Error (BE) observers

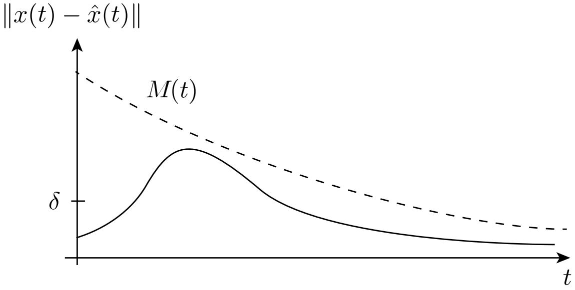


estimation error lies in known set
size and shape change over time
[ex. Deterministic Extended Kalman Filters]

$$x(0) \in \boxed{\mathcal{D}(\hat{x}_0)} \implies x(t) \in \boxed{\mathcal{P}(t, \hat{x})} \quad \forall t$$

Approach 1: Input-to-State-Stable (ISS) Observers





$\dot{x} = f(x) + g(x)u + g_d(x)d(t)$

$\dot{\hat{x}} = p(\hat{x}, y) + q(\hat{x}, y)u$

Lipschitz constant of h

Notice: $h(\hat{x}) \geq \boxed{\gamma_h} M(t) \implies h(x) \geq 0$

Def: A function h is an Observer-Robust CBF if

$$\sup_{u \in \mathcal{U}} L_p h(\hat{x}, y) + L_q h(\hat{x}, y) u \geq -\alpha(h(\hat{x}) - \gamma_h M(0))$$

for all $\hat{x} \in \mathcal{S}$ and possible outputs y.

Thm 1: If h is observer-robust CBF, and the initial conditions satisfy

$$\hat{x} \in \hat{\mathcal{X}}_0 = \{\hat{x} : h(\hat{x}) > \gamma_h M(0)\}$$ (state estimate starts sufficiently inside safe set)

$$x \in \mathcal{X}_0 = \{x : \|x - \hat{x}\| \leq \delta\}$$ (true state starts close to estimate)

then the set of safe control inputs is u st.

$$\boxed{L_p h(\hat{x}, y) + L_q h(\hat{x}, y) u} \geq -\alpha(h(\hat{x}) \boxed{- \gamma_h M(t)}) \boxed{+ \gamma_h \dot{M}(t)}$$

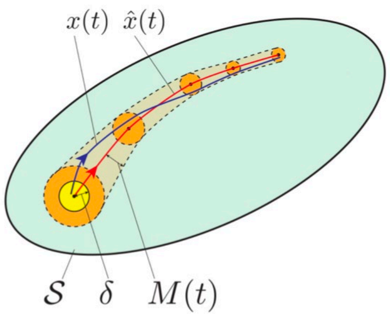
$\dot{h}(\hat{x}, u)$

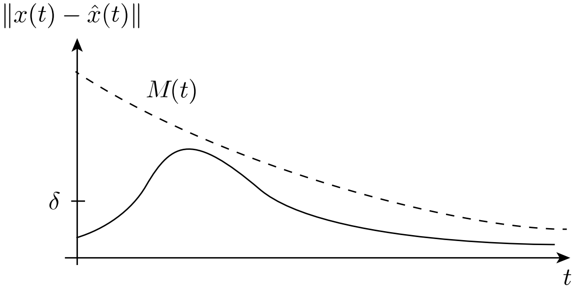
more conservative due to current est. error

less conservative due to decreasing est. error

Approach 1: Input-to-State-Stable (ISS) Observers

a) ISS Observers





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then the set of safe control inputs is u st.

$$L_p h(\hat{x}, y) + L_q h(\hat{x}, y)u \geq -\alpha(h(\hat{x}) - \gamma_h M(t)) + \gamma_h \dot{M}(t)$$

Interesting case: Suppose $\alpha(r) = \alpha r$, and $\dot{M} + \alpha M \leq 0$
Sufficient to choose u s.t.

linear class-K

exponential observer convergence

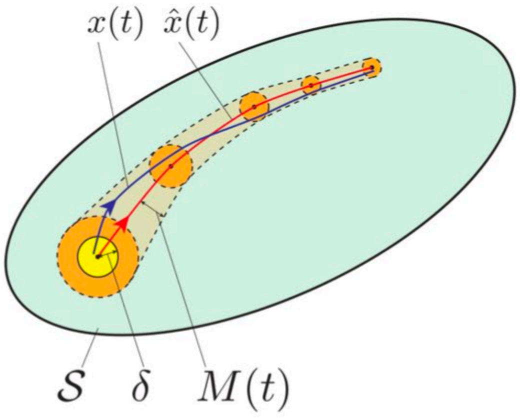
$$L_p h(\hat{x}, y) + L_q h(\hat{x}, y)u \geq -\alpha h(\hat{x}) + \alpha \gamma_h M + \gamma_h \dot{M}$$
$$= -\alpha h(\hat{x}) + \gamma_h (\underbrace{\dot{M} + \alpha M}_{\text{negative}})$$
$$L_p h(\hat{x}, y) + L_q h(\hat{x}, y)u \geq -\alpha h(\hat{x})$$

which doesn't depend on γ_h, M!

∴ *agrees with general principle:*
design observers to converge faster than controllers for good performance

Two solution approaches

1) Input-to-State Stable (ISS) observers



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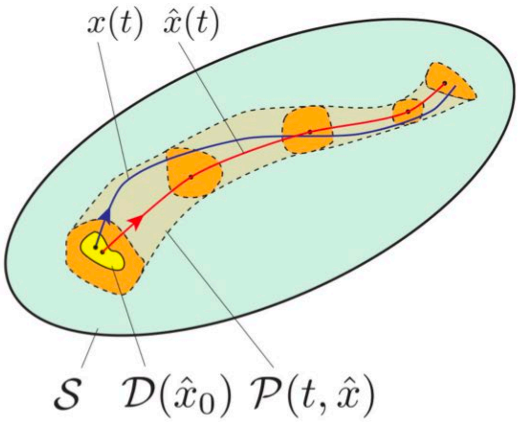

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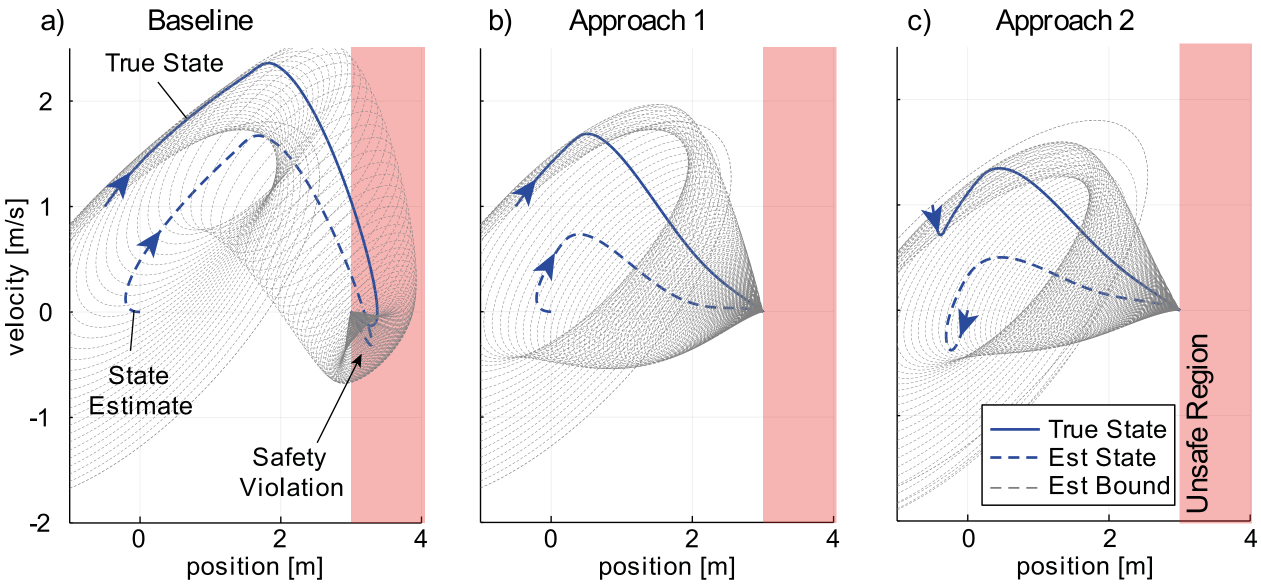
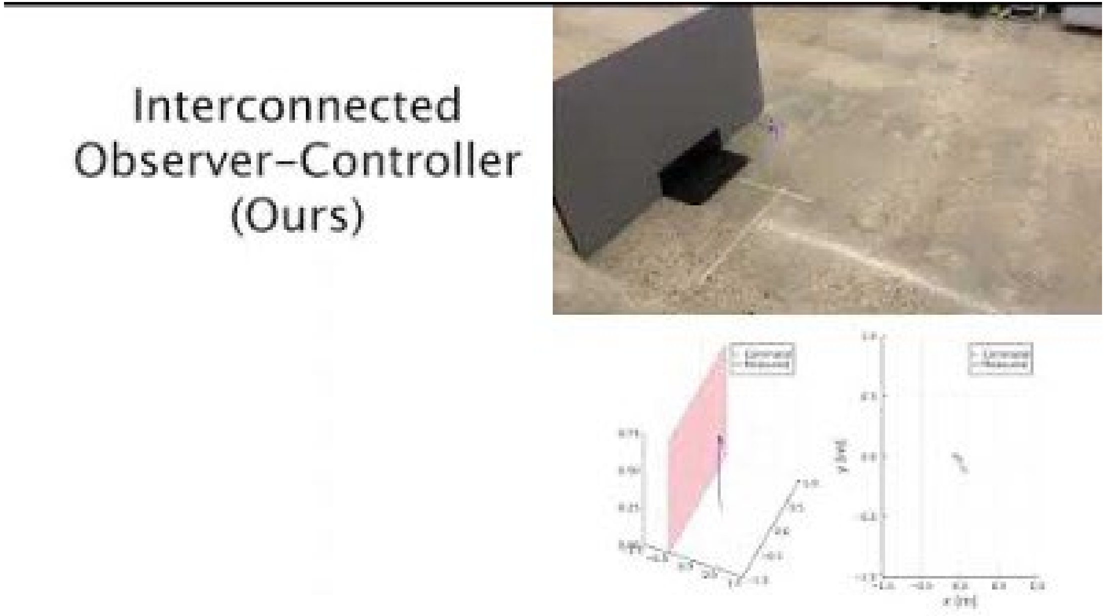
Simulations – Double Integrator



Even without disturbances, considering state estimation is necessary to ensure safety
Both proposed approaches ensure safety
Not clear which approach is better



Summary

considering observer errors in designing safety critical controller is important

Next steps

How to reduce conservativeness due to disturbances or worst-case assumptions?

How to represent safe set efficiently?

How to certify correctness of perception? [1]

[1] Rosen et al. IJRR 2018

“Safe and Robust Observer-Controller Synthesis using Control Barrier Functions”
Devansh R Agrawal and Dimitra Panagou, L-CSS/CDC 2022

Some Conclusions from this Presentation…

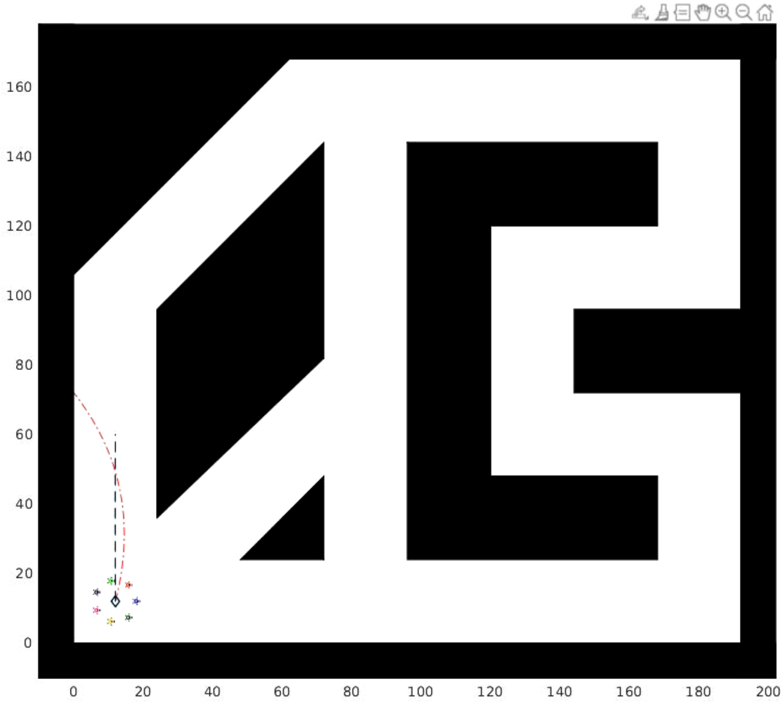
- CBFs are an effective methodology to enforce safety and other specifications
- Have been studied extensively in recent years under various assumptions and settings
- Robustness under disturbances/measurement errors and adaptation of CBF/model parameters
- Future work:
  - Relax assumptions and make things less myopic
    - Deal with Input Constraints and Prediction **Online, and for Multi-Agent Systems**
  - How can CBF theory
    - Learning uncertainty with safety
    - **Intention models of other agents/obstacles**
    - Certify the full-stack autonomy : perception, reasoning, planning and control

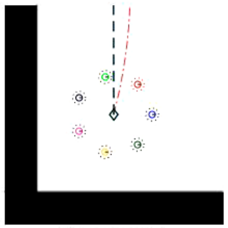
And Some More Work!

Adversarially-Robust CBFs

James Usevitch
(TAC 2022, TRO 2022)

Adversarially-Robust Multi-Agent Systems





Close-up View

Challenge:

Adversaries may compromise information and safety in manned/unmanned teams

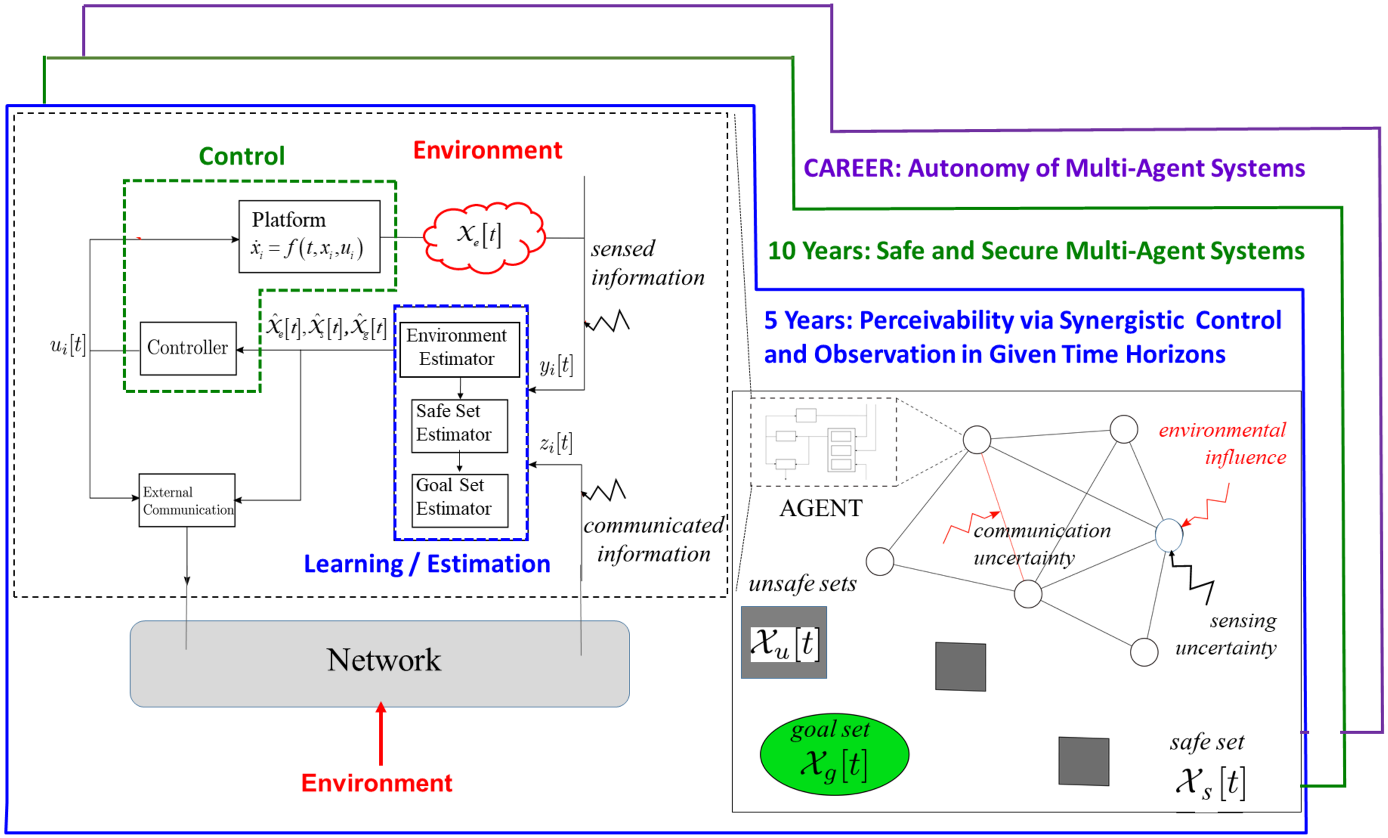
J. Usevitch and D. Panagou, "Resilient Trajectory Propagation in Multi-Robot Networks," *IEEE Transactions on Robotics*, 2022.

ARC U.S.ARMY DEVCOM GROUND VEHICLE SYSTEMS CENTER

Simulations: 2D Unicycles

- Adversarial agents are red circles, normal agents are blue circles
- Unicycle dynamics; CBF controls computed via input-output linearized controller
- Adversaries have lower maximum angular / linear speed constraints than normal agents
- Adversarial agents apply maximum control effort to pursue closest normal agent
- QP controller takes into account adversarial behavior and collision avoidance simultaneously.
- Safety maintained by normally-behaving agents

J. Usevitch and D. Panagou, "Adversarial Resilience for Sampled-Data Systems under High-Relative-Degree Safety Constraints," ***IEEE Transactions on Automatic Control*****, 2022.**


CAREER: Autonomy of Multi-Agent Systems
10 Years: Safe and Secure Multi-Agent Systems
5 Years: Perceivability via Synergistic Control and Observation in Given Time Horizons
Control
Environment
Platform
$\dot{x}_i = f(t, x_i, u_i)$
$\mathcal{X}_e[t]$
sensed information
Controller
$\hat{\mathcal{X}}_e[t], \hat{\mathcal{X}}_s[t], \hat{\mathcal{X}}_g[t]$
$u_i[t]$
Environment Estimator
$y_i[t]$
Safe Set Estimator
$z_i[t]$
External Communication
Goal Set Estimator
communicated information
Learning / Estimation
Network
Environment
AGENT
environmental influence
communication uncertainty
unsafe sets
$\mathcal{X}_u[t]$
sensing uncertainty
goal set
$\mathcal{X}_g[t]$
safe set
$\mathcal{X}_s[t]$

Sponsors

Thank you!

Questions?