Towards More Efficient Transportation: Modeling and Control of Dynamical Flow Networks

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My background and research interests

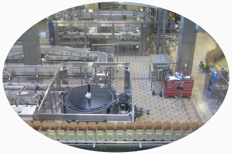
Background:

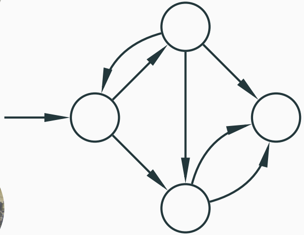
- Ph.D. from Lund University in 2019
- Post-doc positions at Georgia Tech, EPFL, and University of Trento

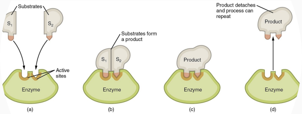
Research interests:

- Non-linear flow dynamics in network
- Compartmental models for macroscopic traffic modeling:
  Space allocation and utilization of dedicated bus lanes, macroscopic modeling of ride-hailing services
- Game theory:
  Stackelberg games for charging of electric ride-hailing services, resource allocation games

Networks with non-linear dynamics and (some) mass conservation





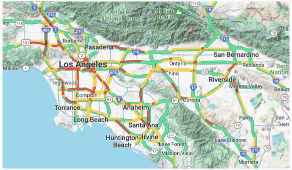


Non-linear dynamics:

- flow/reaction dynamics (e.g., limited flow capacity on the links)
- storage capacity constraints (e.g., limited road space)
- simultaneous service constraints (e.g., traffic signals)

The need for control in transportation networks

With growing sensing and communication abilities in transportation systems, physical resources are often the limiting factors.



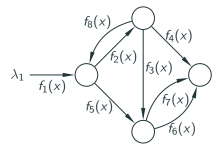
Can we design feedback solutions to ensure efficient and resilient use of the limited physical resources?

Granularity Levels in Transportation Modeling

Vehicle

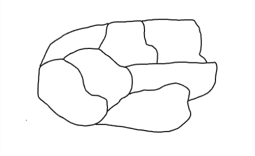
Discrete quantity on road/link level

Network



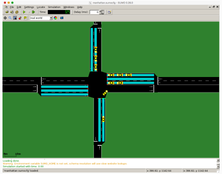
Continuous quantity on road/link level

Macroscopic



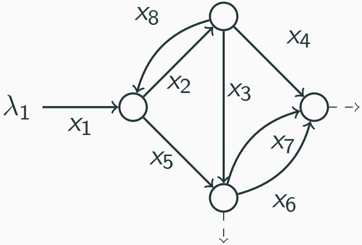
Continuous quantity in regional compartments

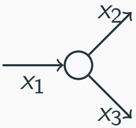
Simulator



Dynamical flow network model

- Directed multigraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
  - $\mathcal{V}$ – set of nodes
  - $\mathcal{E}$ – set of links (edges)
- State space – mass on each link $x \in \mathcal{X} \subset \mathbb{R}^{\mathcal{E}}_{\geq 0}$
- Exogenous inflow – $\lambda(t) \in \mathbb{R}^{\mathcal{E}}_{\geq 0}$
- Routing matrix $0 \leq R_{ij}(x, t) \leq 1$
  - the fraction of outflow from link i to link j
  - $\sum_j R_{ij}(x, t) \leq 1$
  - $1 - \sum_j R_{ij}(x, t)$ fraction of flow that leaves the network
- Outflow function – $f_i(x, t)$





30 % turning left

70 % turning right

$R_{1,2} = 0.3,\ R_{1,3} = 0.7$

DFN – Where are the actuators?

Routing matrix $R(x,t) = R(x,t,u(x,t))$:

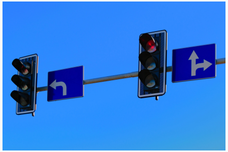
- Route guiding apps (Dynamic traffic assignments)
- Congestion-avoiding human behavior

Outflow control $f(x,t) = f(x,t,u(x,t))$:

- Variable speed limits
- Traffic signal control

Inlet control (adding additional links):

- Ramp metering



Dynamical Flow Networks

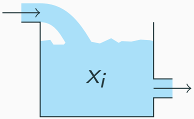
Dynamics follows from the conservation of mass

$$\dot{x}_i = \lambda_i(t) + \sum_{j \in \mathcal{E}} R_{ji} f_j(x) - f_i(x)\,, \quad \forall i \in \mathcal{E}$$

which can be equivalently written as

$$\dot{x} = \lambda(t) - (I - R^T) f(x)$$

$$\lambda_i + \sum_j R_{ji} f_j(x)$$



$f_i(x)$

Assumption: For the rest of this talk, the routing matrix is supposed to be static and outflow connected (i.e., there is always a path to leave the network)

Decentralized traffic signal control

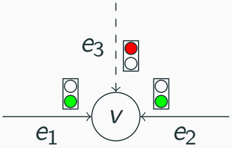
Problem: Design a decentralized control strategy that limits the outflows f at every node so vehicles do not collide

G. Nilsson and G. Como. "Generalized Proportional Allocation Policies for Robust Control of Dynamical Flow Networks", IEEE TAC 2022

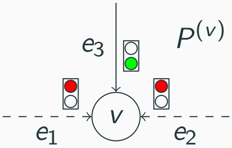
DFN – Traffic signal control problem

- Assume point queue dynamics with outflow capacity $c_i > 0$ so $f_i(x,t) = c_i u_i(x)$
- The traffic signal controller's $u(x)$ task is to decide the fraction of time each phase should be activated
- The phases can for every node be described by a binary phase matrix $P^{(v)}$

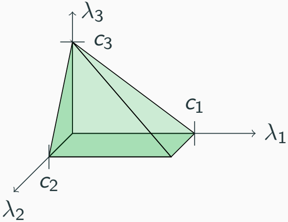
Phase 1



Phase 2



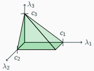
$$P^{(v)} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Generalized Proportional Allocation

Compute the control action locally for every junction by solving a convex optimization problem ($\kappa_v > 0$ introduced to model clearance time):

$$u^{(v)}(x) \in \operatorname{argmax}_{\mu \in \mathcal{U}_v} \sum_{i \in \mathcal{E}_v} x_i \log((P^{(v)}\mu)_i) + \kappa_v \log(1 - \mathbf{1}^T \mu)$$

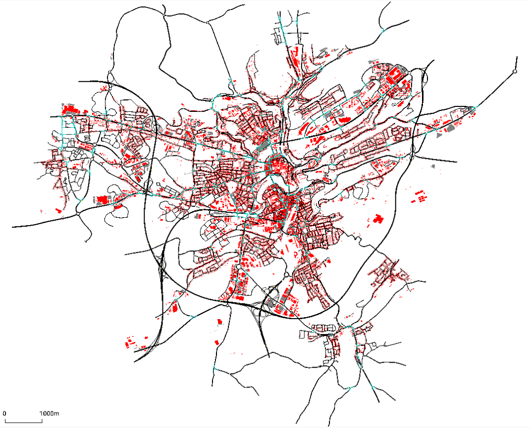
Theorem
If $a = (I - R^T)^{-1}\lambda \in int(Z)$ where $\mathcal{Z}$ is the set of all feasible control actions (cf. 

), the GPA controller stabilizes (bounded states) the dynamical flow network.

N.B. 1) $a = (I - R^T)^{-1}\lambda \in \mathcal{Z}$ is a necessary condition for any control strategy
2) The control action is local in every node
3) The controller requires no demand or network topology information

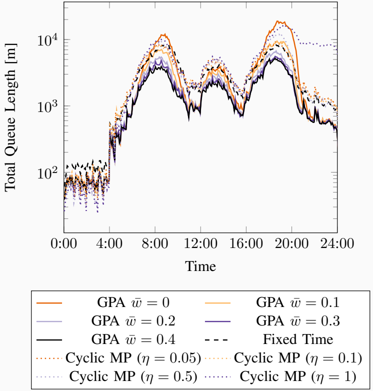
Validation in a microscopic traffic simulator

- Simulates all traffic in Luxembourg during a full day in SUMO
- About 200 signalized junctions
- We added sensors to measure queues (50 m)
- Control action solved in real-time by CVXPY
- Results compared to the standard fixed-time plan that comes with the scenario

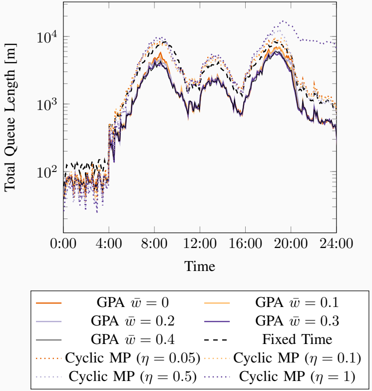


Luxembourg scenario - Results

$\kappa = 5$



$\kappa = 10$



G. Nilsson and G. Como, “A Micro-Simulation Study of the Generalized Proportional Allocation Traffic Signal Control”. IEEE Transactions on Intelligent Transportation Systems. 2020

Input-to-State Stability of Dynamical Flow Networks

G. Nilsson and S. Coogan. "The Strong Integral Input-to-State Stability Property in Dynamical Flow Networks", IEEE TAC 2024

Dynamical Flow Networks – Desired stability properties

Dynamics follows from the conservation of mass

$$\dot{x}_i = \lambda_i(t) + \sum_{j \in \mathcal{E}} R_{ji} f_j(x) - f_i(x) , \quad \forall i \in \mathcal{E} .$$

which can be equivalently written as

$$\dot{x} = \lambda(t) - (I - R^T) f(x)$$

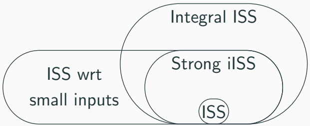
Desired stability properties:

- If $\lambda = 0$, the state should converge to zero
- If the flow functions are bounded from above, there should exist a bounded set $\mathcal{D}$ s.t. for $\lambda \in \mathcal{D}$ the state stays bounded (region of stability)
- (The state grows with the inflows if $\lambda \notin \mathcal{D}$)

Strong integral Input-to-State Stability (Strong iISS)

Definition
[Chaillet, Angeli, Ito 2014] A system $\dot{x} = f(x, u)$ is Strong iISS if there exist $\Lambda > 0$, $\beta(x, t) \in \mathcal{KL}$ (strictly increasing in x, decreasing in t) and $\mu_1(x), \mu_2(x), \mu(x) \in \mathcal{K}_\infty$ (strictly increasing in x) such that for all u, all x_0 and all $t \geq 0$ it holds that

$$\|x(t)\| \leq \beta(\|x_0\|, t) + \mu_1\left(\int_0^t \mu_2(\|u(s)\|)ds\right) \longleftarrow \text{Integral ISS}$$



$$\|u\| < \Lambda \Rightarrow \|x(t)\| \leq \beta(\|x_0\|, t) + \mu\left(\operatorname*{ess\,sup}_{t'>0} \left\|u(t')\right\|\right) \longleftarrow \text{ISS}$$

In our setting: u is the exogenous inflow, i.e., $u(t) = \lambda(t)$

The Strong iISS property in dynamical flow networks

Assumptions on the flow functions:

- $f_i(x) \geq 0$ and $f_i(x) = 0$ if and only if $x_i = 0$
- $\liminf_{\|x\| \to +\infty, x \in \mathcal{X}} \sum_{i \in \mathcal{E}} f_i(x) > 0$

Theorem

A dynamical flow network satisfying the stated assumptions about the routing matrix and flow functions is Strong iISS. In particular, it is ISS with respect to inputs satisfying

$$\operatorname*{ess\,sup}_{t \geq 0} \sum_{i \in \mathcal{E}} a_i(t) < \liminf_{\|x\| \to +\infty, x \in \mathcal{X}} \sum_{i \in \mathcal{E}} f_i(x)\,,$$

where $a(t) = (I - R^T)^{-1} \lambda(t)$.

N.B.: Results extend to multi-commodity settings.

Summary and Outlook

- A model for Dynamical Flow Networks (DFNs)
- Throughput-optimality for a specific type of outflow controllers (GPA)
- General framework for ISS analysis of DFNs

Outlook

- Input condition for ISS conservative $\leftrightarrow$ GPA non-conservative - Close the gap?
- State and time-dependent routing matrices?
- Control design for the transient behavior?

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