Stability and output regulation for a cascaded network of 2×2 hyperbolic systems with PI control

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1 Introduction

- PDE hyperbolic systems and cascaded networks
- Boundary control problem
- Output regulation problem
- PI control design

PDE hyperbolic systems and cascaded networks

Engineering applications of PDE hyperbolic systems

- Hydraulic engineering - Saint Venant models
- Road traffic - Burgers equation
- Gas pipeline
- Heat exchanger process
- $\cdots$

Homogeneous first-order hyperbolic systems

Let $\phi \in \mathbb{R}^n$, $A(\phi) \in \mathbb{R}^{n\times n}$, $x \in [0, L]$,$t \in \mathbb{R}_+$,

$$\phi_t + A(\phi)\ \phi_x = 0\ ,\ \phi(0,x) = \phi^0(x)$$

A has n real eigenvalues, i.e $\lambda_i \in \mathbb{R}\ \forall i = 1, 2, ..n$.

If A is independent on ϕ, system is linear. If not, it is quasi-linear.

PDE hyperbolic systems and cascaded networks

Cascaded network

- Popular in practical applications (channels of rivers, gas,$\cdots$)
- n PDE hyperbolic sub-systems
- $n+1$ junctions, 2 free junctions and $n-1$ mixed junctions.

FIGURE: Cascaded network of n systems

A cascaded network can be considered a large PDE hyperbolic system with complex boundary conditions !

Boundary control problem

Boundary conditions

$$f\Big(\phi(0,t),\phi(L,t),U(t)\Big)=0$$

$U(t)$ is control action on the boundary.

- Static control, i.e $U(t)=g(\phi(0,t),\phi(L,t))$.
- Dynamic control, i.e $U(t)=g(\phi(0,t),\phi(L,t))$ + **other dynamic parts**.

Boundary control problem

Find boundary conditions such that :

- The PDE hyperbolic system has a unique solution in the corresponding state space.
- The PDE hyperbolic system is (globally/locally) asymptotically/exponentially stable w.r.t some equilibrium point.

Boundary control problem

Static control laws

- Literatures :
  - (Li Tatsien 1994, Coron et al. 2015) A sufficient boundary condition for the 'zero-point' stability of quasi-linear systems in C^1 norm.
  - (Coron et al. 2008) A sufficient boundary condition for the 'zero-point' stability of quasi-linear systems in H^2 norm.
  - (Hale and Verduyn Lune 1993) A necessary and sufficient boundary condition for the 'zero-point' stability of linear systems in L^2 norm.
- Limits : Not robust with constant perturbations.

Dynamic control laws with integral actions

- Literatures with works of Pohjolainen, Xu, Dos Santos, C. Prieur, D. Georges,...
- Advantages : Robust to constant perturbations.
- Limits : Become a coupling systems of PDE and ODE, difficult to prove stability.

Output regulation problem

Given a system one wants to ensure that outputs $y(t)$ follow references y_r despite disturbances, i.e $y(t) \rightarrow y_r$

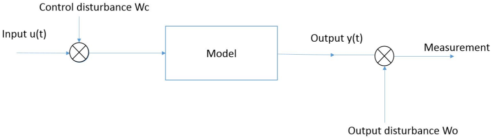


FIGURE: Example of Disturbances

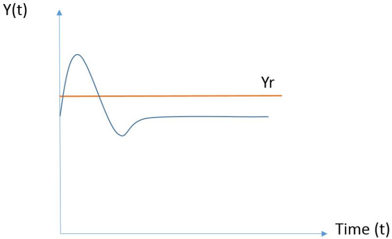


FIGURE: Static error

Disturbances in real model : error of the modelisation, linearisation, sensors, $\cdots$

$\Rightarrow$ Static error between the measurement output and the set-point.

Solution : using the integral action to eliminate the static error.

Output regulation problem

Example : A very trivial system :

$$\dot{\phi} = u + d$$

$$y = \phi$$

State $\phi \in \mathbb{R}$, control $u \in \mathbb{R}$, unknown constant disturbance $d \in \mathbb{R}$, measure $y \in \mathbb{R}$.

Objective : Given a reference y_r in $\mathbb{R}$, design u such that $y \to y_r$.

- If $u = -(y - y_r) \Rightarrow$ equilibrium is stable but $y \nrightarrow y_r$.
- If $u = -(y - y_r) - z$, where $\dot{z} = y - y_r \Rightarrow$ equilibrium is stable and $y \to y_r$.

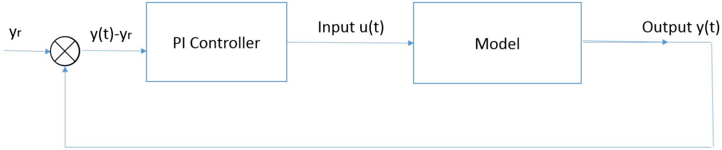
Conclusion : The integral term added rejects the constant disturbance.

PI control design

- PI controller is a type of dynamic boundary control law :

$$u(t) = K_P(y(t) - y_r) + K_I z(t) \ , \ \dot{z} = y(t) - y_r$$

  - Measured output on the boundary $y(t) = g(\phi(0,t), \phi(L,t))$
  - Input $u(t)$, reference y_r
  - Gain parameter matrices K_p, K_I.
- Schema of closed-loop system :



- Objective : Design PI controller (determine K_P and K_I) such that :
  - **Stability** of closed-loop system
  - **Output regulation** : $y(t) \to y_r$

Plan

Network model

- n PDE hyperbolic systems

$$\begin{cases} \partial_t \phi_{i1}(x,t) + \lambda_{i1}\ \partial_x \phi_{i1}(x,t) &= 0 \\ \partial_t \phi_{i2}(x,t) - \lambda_{i2}\ \partial_x \phi_{i2}(x,t) &= 0 \end{cases} \quad , \ x \in [0,L],\ \ t \in [0,\infty),\ \ i = \overline{1,n}$$

  where two states $\phi_{i1},\ \phi_{i2} :\ [0,L] \times [0,\infty) \to \mathbb{R}$ and $\lambda_{i1} > 0,\ \lambda_{i2} > 0.$

- Boundary conditions defined at junctions

$$\begin{cases} \phi_{i2}(L,t) &= R_{i2}\phi_{i1}(L,t)\ + u_i(t) \\ \phi_{i1}(0,t) &= R_{i1}\phi_{i2}(0,t) + \alpha_i \phi_{(i-1)1}(L,t) + \delta_i \phi_{(i-1)2}(L,t)\ , \end{cases} \quad , \ i = \overline{1,n}$$

  where $\phi_{01} = \phi_{02} = 0.$

- n measured outputs

$$y_i(t) = a_i \phi_{i1}(L,t) + b_i \phi_{i2}(L,t) + y_{ir}$$

PI structure and state space

- Design n PI controllers at each juctions

$$u_i(t) = K_{iP}(y_i(t) - y_{ir}) + K_{iI} z_i(t) \;,\; \dot{z}_i = y_i(t) - y_{ir}$$

$K_{iP} \in \mathbb{R}$ and $K_{iI} \in \mathbb{R}$ to be designed.

- Consider the state space of closed-loop network :

$$\mathbb{E} = \left((L^2(0,L))^2 \times \mathbb{R}\right)^n$$

with the norm associated

$$||Y||^2_{\mathbb{E}} = \sum_{i=1}^{n} \left(||\phi_{i1}(.,t)||^2_{L^2(0,L)} + ||\phi_{i2}(.,t)||^2_{L^2(0,L)} + z_i^2(t) \right)$$

where $Y = (\phi_{11}, \phi_{12}, z_1, \cdots, \phi_{n1}, \phi_{n2}, z_n) \in \mathbb{E}$

Main result

Two hypothesises

- $H_1 : a_i \neq 0 \ \forall i = \overline{1,n}$
- $H_2 : a_i + b_i R_{i2} \neq 0 \ \forall i = \overline{1,n}$

Theorem (Trinh-Andrieu-Xu 2017)

There exists $\mu^* > 0$ such that, if two hypothesises H_1 and H_2 are satisfied, for each $\mu \in (0, \mu^*)$ and

$$K_{iP} = \frac{-R_{i2}}{a_i} \ , \ K_{iI} = -\mu \, \frac{(b_i + a_i R_{i1} e^{\mu L})(a_i + b_i R_{i2})}{a_i} \quad , \ \forall i = \overline{1,n}$$

Then, we have :

- *Existence and uniqueness of solutions in $\mathbb{E}$*
- *The exponential stability of 'zero' point in $\mathbb{E}$.*
- *With initial conditions in $\left((H^1(0,L))^2 \times \mathbb{R}\right)^n$, Output regulation, i.e*

$$\lim_{t\to\infty} |y_i(t) - y_{ir}| = 0 \ , \forall i = \overline{1,n}.$$

About the theorem

$$y_i(t) = a_i\phi_{i1}(L,t) + b_i\phi_{i2}(L,t) + y_{ir}$$

$$K_{iP} = \frac{-R_{i2}}{a_i}\ , \ K_{iI} = -\mu\ \frac{(b_i + a_i R_{i1} e^{\mu L})(a_i + b_i R_{i2})}{a_i}\quad , \ \forall i = \overline{1,n}$$

Two output conditions (two hypothesises) for our PI control design :

- H_1 for existence of our PI controller.

$$a_i \neq 0\ \forall i = \overline{1,n}$$

- H_2 for having dynamic feedback (by integral action) , i.e $K_{iI} \neq 0$.

$$a_i + b_i R_{i2} \neq 0\ \forall i = \overline{1,n}$$

Lyapunov candidate functional

Use Lyapunov techniques $\Leftrightarrow$ construct a candidate Lyapunov function.

$$\mathbb{V}(\phi_{11}, \phi_{12}, z_1, \cdots, \phi_{n1}, \phi_{n2}, z_n) = \sum_{i=1}^{n} p_i V_i$$

where

$$V_i(\phi_{i1}, \phi_{i2}, z_i) = \int_0^L \begin{pmatrix} \phi_{i1} e^{-\frac{\mu x}{2}} \\ \phi_{i2} e^{\frac{\mu x}{2}} \\ z_i \end{pmatrix}^T P_i \begin{pmatrix} \phi_{i1} e^{-\frac{\mu x}{2}} \\ \phi_{i2} e^{\frac{\mu x}{2}} \\ z_i \end{pmatrix} dx$$

with

$$P_i = \begin{pmatrix} 1 & 0 & q_{i3} \\ 0 & q_{i1} & q_{i4} \\ q_{i3} & q_{i4} & q_{i2} \end{pmatrix}$$

Here $p_i > 0$ and $q_{i1}, q_{i2}, q_{i3}, q_{i4}$ need to be designed.

Lyapunov candidate functional

$$\mathbb{V} = \sum_{i=1}^{n} p_i \int_0^L \begin{pmatrix} \phi_{i1} e^{-\frac{\mu x}{2}} \\ \phi_{i2} e^{\frac{\mu x}{2}} \\ z_i \end{pmatrix}^T \begin{pmatrix} 1 & 0 & q_{i3} \\ 0 & q_{i1} & q_{i4} \\ q_{i3} & q_{i4} & q_{i2} \end{pmatrix} \begin{pmatrix} \phi_{i1} e^{-\frac{\mu x}{2}} \\ \phi_{i2} e^{\frac{\mu x}{2}} \\ z_i \end{pmatrix} dx$$

- If $q_{i2} = q_{i3} = q_{i4} = 0$, this is the Lyapunov functionnal of Bastin, Coron and Andréa Novel 2009 for a cascaded network.
- If $n = 1$ and $q_{i3} = q_{i4} = 0$, this is the Lyapunov functionnal of Bastin and Coron 2016 for a single system.
- By adding the new terms ($q_{i3}, q_{i4} \neq 0$) and n positive parameters p_i, it allows to deal with dynamic feedback of cascaded network of n systems.

Design of Lyapunov functional

$$V_i(\phi_{i1},\phi_{i2},z_i)=\int_0^L \begin{pmatrix}\phi_{i1}e^{-\frac{\mu x}{2}}\\ \phi_{i2}e^{\frac{\mu x}{2}}\\ z_i\end{pmatrix}^T \begin{pmatrix}1 & 0 & q_{i3}\\ 0 & q_{i1} & q_{i4}\\ q_{i3} & q_{i4} & q_{i2}\end{pmatrix}\begin{pmatrix}\phi_{i1}e^{-\frac{\mu x}{2}}\\ \phi_{i2}e^{\frac{\mu x}{2}}\\ z_i\end{pmatrix} dx$$

Lemma (For sub-functional V_i)

Let $q_{i1}, q_{i2}, q_{i3}, q_{i4}$ be defined as follows :

$$q_{i1} > \frac{3\lambda_{i1}R_{i1}^2}{\lambda_{i2}},\quad q_{i2}=\mu e^{\mu L}\lambda_{i2}q_{i1},\quad q_{i3}=\mu e^{\frac{3\mu L}{2}}\frac{a_i\lambda_{i2}q_{i1}}{\lambda_{i1}},\quad q_{i4}=\mu e^{\frac{3\mu L}{2}}a_iR_{i1}q_{i1}.$$

Then there exists $\mu^*>0$, $M_i>0$ and $\gamma_i>0$ such that for all $\mu\in(0,\mu^*)$

1. $\frac{1}{M_i}V_i(\phi_{i1},\phi_{i2},z_i)\leqslant ||\phi_{i1}(.,t)||^2_{L^2(0,L)}+||\phi_{i2}(.,t)||^2_{L^2(0,L)}+z_i^2(t)\leqslant M_iV_i(\phi_{i1},\phi_{i2},z_i)$
2. $\dot V_i(t)\leqslant -\gamma_iV_i(t)-F_i(t)+G_{i-1},$

$$\text{where}\quad F_i(t)=\frac14 z_i^2(t)\,k_i^2\lambda_{i2}q_{i1}e^{\mu L}+\phi_{i1}^2(L,t)\frac{\lambda_{i1}e^{-\mu L}}{2},$$

$$G_{i-1}=\phi_{(i-1)1}^2(L,t)\lambda_{i1}\alpha_i^2\left(3+\frac{4\lambda_{i1}^2q_{i3}^2e^{-\mu L}}{k_i^2\lambda_{i2}q_{i1}}\right)+z_{i-1}^2(t)\lambda_{i1}\beta_i^2\left(3+\frac{4\lambda_{i1}^2q_{i3}^2e^{-\mu L}}{k_i^2\lambda_{i2}q_{i1}}\right)$$

Sketch of proof

V_i is definite positive

$$V_i(\phi_{i1}, \phi_{i2}, z_i) = \int_0^L \begin{pmatrix} \phi_{i1} e^{-\frac{\mu x}{2}} \\ \phi_{i2} e^{\frac{\mu x}{2}} \\ z_i \end{pmatrix}^T P_i \begin{pmatrix} \phi_{i1} e^{-\frac{\mu x}{2}} \\ \phi_{i2} e^{\frac{\mu x}{2}} \\ z_i \end{pmatrix} dx$$

With μ small enough, prove that P_i is symmetric positive definite (SDP)

Consider $\dot{V}_i$

$$\dot{V}_i = -\int_0^L \begin{pmatrix} \phi_{i1}(x,t) e^{-\frac{\mu x}{2}} \\ \phi_{i2}(x,t) e^{\frac{\mu x}{2}} \\ z_i(t) \\ \phi_{i1}(L,t) \end{pmatrix}^T Q_i \begin{pmatrix} \phi_{i1}(x,t) e^{-\frac{\mu x}{2}} \\ \phi_{i2}(x,t) e^{\frac{\mu x}{2}} \\ z_i(t) \\ \phi_{i1}(L,t) \end{pmatrix} dx - F_i(t) + G_{i-1}$$

With μ small enough, prove that $Q_i \in \mathbb{R}^{4\times 4}$ is SDP

$$\Rightarrow \forall t \in \mathbb{R}_+, \ \exists \gamma_i > 0, \ \dot{V}_i(t) \leqslant -\gamma_i V_i(t) - F_i(t) + G_{i-1}.$$

Design of Lyapunov functional

$$\mathbb{V}(\phi_{11}, \phi_{12}, z_1, \cdots, \phi_{n1}, \phi_{n2}, z_n) = \sum_{i=1}^{n} p_i V_i$$

Lemma (For global functional $\mathbb{V}$)

Let $q_{i1}, q_{i2}, q_{i3}, q_{i4}$ be defined in Lemma of sub functional V_i, and p_i be defined as follows

$$p_1 > 0 \ , \ p_{i+1} = \epsilon p_i$$

Then there exists $\epsilon > 0$ and $\mu^* > 0$ such that for every $\mu \in (0, \mu^*)$, we have :

1. *There exists $M > 0$ such that*

$$\frac{1}{M}\mathbb{V}(\phi_{11}, \phi_{12}, z_1, \cdots, \phi_{n1}, \phi_{n2}, z_n) \leqslant ||(\phi_{11}, \phi_{12}, z_1, \cdots, \phi_{n1}, \phi_{n2}, z_n)||^2_{\mathbb{E}}$$
$$\leqslant M\ \mathbb{V}(\phi_{11}, \phi_{12}, z_1, \cdots, \phi_{n1}, \phi_{n2}, z_n)\ .$$

2. *There exists $\gamma > 0$ such that*

$$\dot{\mathbb{V}}(t) \leqslant -\gamma \mathbb{V}(t).$$

Sketch of proof

$$\mathbb{V}(\phi_{11}, \phi_{12}, z_1, \cdots, \phi_{n1}, \phi_{n2}, z_n) = \sum_{i=1}^{n} p_i V_i$$

$\mathbb{V}$ is definite positive

$$\sum_{i=1}^{n} \frac{p_i}{M_i} V_i \leqslant ||(\phi_{11}, \phi_{12}, z_1, \cdots, \phi_{n1}, \phi_{n2}, z_n)||_{\mathbb{E}}^2 \leqslant \sum_{i=1}^{n} p_i M_i V_i$$

Employing the definite positive property of Lemma for sub functional V_i, one finds the proof.

$\dot{\mathbb{V}}$ is definite negative

$$\dot{\mathbb{V}}(t) \leqslant -\sum_{i=1}^{n} p_i \gamma_i V_i(t) - \sum_{i=1}^{n} z_i^2(t)\,(p_i\ A_i - p_{i+1} B_i) - \sum_{i=1}^{n} \phi_{i1}^2(L,t)\,(p_i\ C_i - p_{i+1} D_i)$$

With $p_{i+1} = \epsilon p_i$, choosing ϵ enough small, we have

$$\forall t \in \mathbb{R}_+,\ \exists \gamma > 0,\ \dot{\mathbb{V}}(t) \leqslant -\gamma \mathbb{V}(t).$$

Proof of Theorem

Unique solution and 'zero' stability

- (Existence and uniqueness of solutions)

  Choosing initial condition

  $$(\phi_{11}^0(x), \phi_{12}^0(x), z_1^0, \cdots, \phi_{n1}^0(x), \phi_{n2}^0(x), z_n^0) \in \mathbb{E}, \quad \forall x \in [0, L]$$

  $\Rightarrow$ Closed-loop system with PI controller has a unique solution in $\mathbb{E}$ (using idea in [Coron and Bastin 2008]).

- (Exponential stability of 'zero' point in $\mathbb{E}$)

  Directly deduced from the Lemma for global functional $\mathbb{V}$.

Proof of Theorem

Output regulation

- With initial condition

$$\left(\phi_{11}^0(x), \phi_{12}^0(x), z_1^0, \cdots, \phi_{n1}^0(x), \phi_{n2}^0(x), z_n^0\right) \in \left((H^1(0,L))^2 \times \mathbb{R}\right)^n, \quad \forall x \in [0,L]$$

  and the exponential stability of 'zero' in $\mathbb{E}$

$$\rightarrow \text{'zero' stability in } \left((H^1(0,L))^2 \times \mathbb{R}\right)^n \textbf{ by closed graph theorem}$$

- With

$$\lim_{t\to\infty} ||\phi_{i1}||_{H^1(0,L)} = 0 \ , \ \lim_{t\to\infty} ||\phi_{i2}||_{H^1(0,L)} = 0$$

  and **Sobolev embedding theorem**, we have

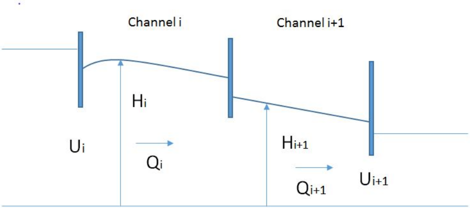
$$\lim_{t\to\infty} \phi_{i1}(x,t) = 0 \ , \ \lim_{t\to\infty} \phi_{i2}(x,t) = 0 \ \forall x \in [0,L]$$

  Therefore,

$$\lim_{t\to\infty} |y_i(t) - y_{ir}| = 0$$

Plan

Cascade network of n Saint-Venant hydraulic systems



- Cascade network

$$\begin{cases} \frac{\partial}{\partial t}\begin{pmatrix} H_i \\ Q_i \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{B_i} \\ \frac{Q_i^2}{B_i H_i} + gB_i & \frac{2Q_i}{B_i H_i} \end{pmatrix} \frac{\partial}{\partial x}\begin{pmatrix} H_i \\ Q_i \end{pmatrix} = 0 \ , \\ y_i(t) = H_i(L,t) \ (output \ \ measurement) \end{cases}$$

- Boundary conditions :

$$Q_i^2(L,t) = \alpha_i\Big(H_i(L,t) - U_i(t)\Big) \ \forall i = \overline{1,n} \ and \ Q_1(0,t) = Q_0(constant)$$

- Conservation law of discharges : $Q_j(L,t) = Q_{j+1}(0,t) \ , \ j = \overline{1, n-1}$

Linearized model

- Linearized network with $h_i = H_i - H_i^*$, $q_i = Q_i - Q_0$

$$\begin{cases} \dfrac{\partial}{\partial t}\begin{pmatrix} h_i \\ q_i \end{pmatrix} + \begin{pmatrix} 0 & \dfrac{1}{B_i} \\ \dfrac{-Q_0^2}{B_i(H_i^*)^2} + gB_iH_i^* & \dfrac{2Q_0}{B_iH_i^*} \end{pmatrix} \dfrac{\partial}{\partial x}\begin{pmatrix} h_i \\ q_i \end{pmatrix} = 0 \\ y_i(t) = h_i(L,t) + H_i^* \end{cases}$$

- Boundary conditions :

$$2Q_0 q_i(L,t) = \alpha_i\Big(h_i(L,t) - u_i(t)\Big) \ \forall i = \overline{1,n}$$

$$q_1(0,t) = 0$$

- Conservation law of discharges :

$$q_j(L,t) = q_{j+1}(0,t)\ ,\ j = \overline{1,n-1}$$

Network in characteristic form

Using the change of coordinates

$$h_i = \phi_{i1} + \phi_{i2},\ \ q_i = (B_i\sqrt{gH_i^*} + \frac{Q_0}{H_i^*})\phi_{i1} - (B_i\sqrt{gH_i^*} - \frac{Q_0}{H_i^*})\phi_{i2}$$

- Network in new coordinates

$$\begin{cases} \partial_t \phi_{i1}(x,t) + \lambda_{i1}\ \partial_x \phi_{i1}(x,t) & = 0 \\ \partial_t \phi_{i2}(x,t) - \lambda_{i2}\ \partial_x \phi_{i2}(x,t) & = 0 \\ y_i(t) = \phi_{i1}(L,t) + \phi_{i2}(L,t) + H_i^*\ , \end{cases}$$

where $\lambda_{i1} = \sqrt{gH_i^*} + \dfrac{Q_0}{B_i H_i^*} > 0\ , \lambda_{i2} = \sqrt{gH_i^*} - \dfrac{Q_0}{B_i H_i^*} > 0.$

- Boundary conditions at junctions

$$\begin{aligned} \phi_{i2}(L,t) &= R_{i2}\phi_{i1}(L,t)\ + u_i(t) \\ \phi_{i1}(0,t) &= R_{i1}\phi_{i2}(0,t) + \alpha_i \phi_{(i-1)1}(L,t) + \delta_i \phi_{(i-1)2}(L,t)\ , \end{aligned}$$

Here R_{i1}, R_{i2},α_i, δ_j are constants

Application of PI control design

PI controller design

$$u_i(t) = K_{iP}(y_i(t) - H_i^*) + K_{iI}\int_0^t (y_i(s) - H_i^*)ds$$

where

$$\left.\begin{aligned} K_{iP} &= \frac{-2Q_0(B_i\sqrt{gH_i^*} + \dfrac{Q_0}{H_i^*}) + \alpha_i}{2Q_0(B_i\sqrt{gH_i^*} - \dfrac{Q_0}{H_i^*}) + \alpha_i} \\ K_{iI} &= -\mu\ (1 + e^{\mu L}\frac{\sqrt{gH_i^*} + \dfrac{Q_0}{B_iH_i^*}}{\sqrt{gH_i^*} - \dfrac{Q_0}{B_iH_i^*}})\frac{4Q_0(B_i\sqrt{gH_i^*}}{2Q_0(B_i\sqrt{gH_i^*} - \dfrac{Q_0}{H_i^*}) + \alpha_i} \end{aligned}\right. \quad , \ \forall i = \overline{1, n}$$

μ is tuning parameter chosen small enough.

Numerical simulations

- Numerical application for 3 channels (n=3),
  - Length $L = 100\ m$, base width $B = 4\ m$.
  - Set-points $H_1^* = 10\ m$, $H_2^* = 8\ m$, $H_3^* = 6.5\ m$, constant discharge $Q_0 = 7\ m^3/s$.
  - Output disturbances $w_{1o} = 0.1$, $w_{2o} = 0.2$, $w_{2o} = 0.15$; and control disturbances $w_{1c} = 0.02$, $w_{2c} = 0.03$, $w_{2c} = 0.01$.
- Simulations for the output regulation

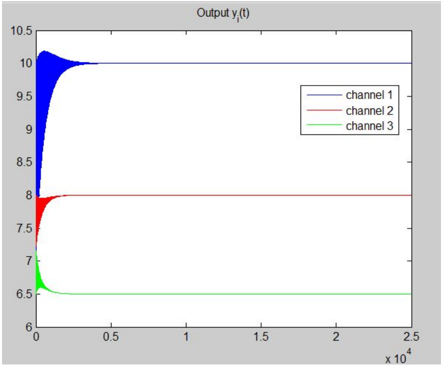


FIGURE: Output measurements $y_i(t)$

Numerical simulations

- Simulations for the stability

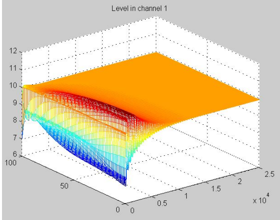


FIGURE: $H_1(x, t)$

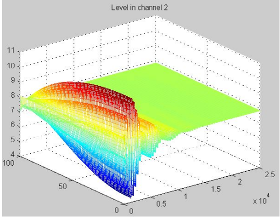


FIGURE: $H_2(x, t)$

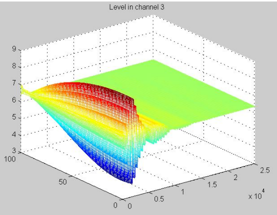


FIGURE: $H_3(x, t)$

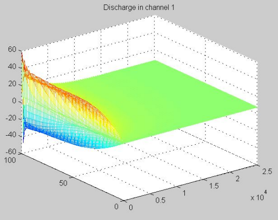


FIGURE: $Q_1(x, t)$

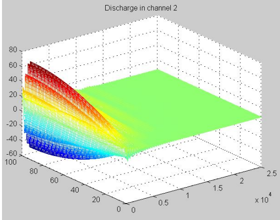


FIGURE: $Q_2(x, t)$

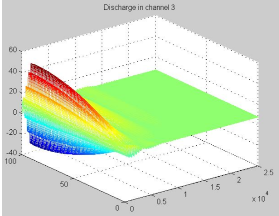


FIGURE: $Q_3(x, t)$

Plan

Conclusions

Obtained results

- Study a network class of n linear 2×2 hyperbolic systems.
- Design n boundary PI controllers at each junction.
- Prove the stability of the closed-loop system in L^2 norm and output regulation based on Lyapunov direct method.
- Apply the control design for a practical network of n fluid flow Saint Venant systems.

Perspectives

- Extend the PI control design for networks of 2×2 nonlinear hyperbolic PDE systems.
- Study the problem of optimal PI controllers (eg. the optimal value of μ).

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THANK YOU FOR YOUR ATTENTION !