

Flexible multibody dynamics: From FE formulations to control and optimization

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Acknowledgement to co-workers:

- Local frame methods: V. Sonneville, A. Cardona, M. Arnold
- Control: A. Lismonde, G. Bastos
- Optimization: E. Tromme



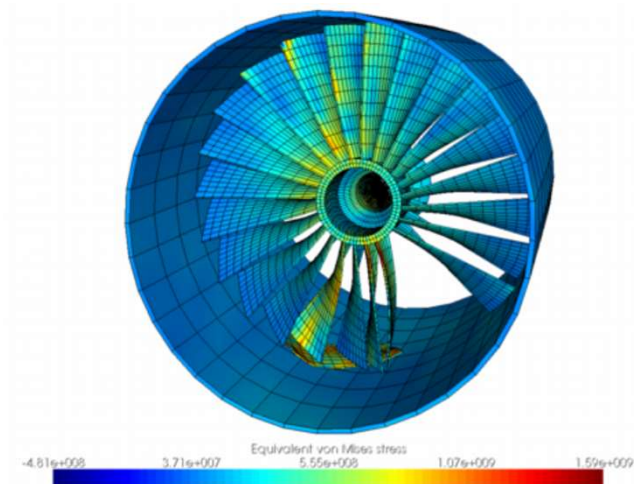
INRIA Rhône-Alpes, Grenoble, July 3, 2017



Dep. of Aerospace & Mechanical Engineering



- 23 research units (~ 130 persons)
- Research fields: aeronautics and space, solid and fluid mechanics, mechanical engineering, materials, energetics and applied maths
- Master degrees: Aerospace, Mechanics and Electromechanics



Dep. of Aerospace & Mechanical Engineering



- 1960s: pioneering development of the FEM in the SAMCEF package (Prof. Fraeijs de Veubeke, Prof. Sander)
- 1989: Extension to flexible multibody systems with MECANO (Prof. Gérardin, Prof. Cardona)
- 1980s: Creation of Samtech (now part of Siemens PLM)
- 2000s: Creation of Open Engineering with the OOFELIE multiphysics package

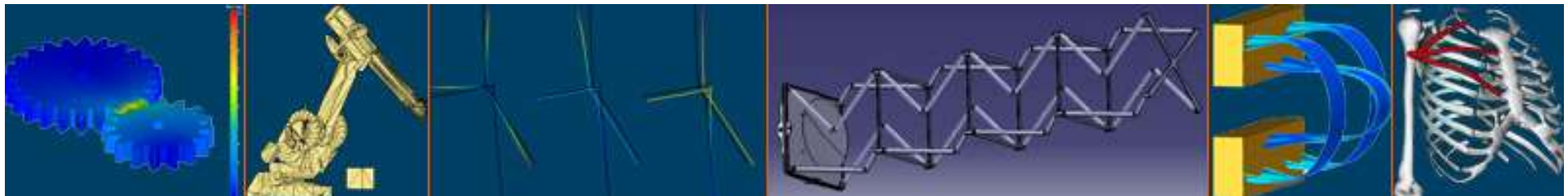
Multibody & Mechatronic Systems Lab

Research interests

- Kinematics, dynamics & control of mechanical systems
- Specific focus on flexibility & vibrations problems
- Numerical (FE) modelling and optimization

Why the nonlinear FE approach?

- Integrated approach to represent flexible bodies with linear or nonlinear behaviour, but also rigid bodies and kinematic joints.
- Differ from the floating frame of reference technique used in standard MBS packages, in which linear elastic models are imported from an external FE software.



Outline

Introduction to our research group

More about MECANO

Local frame approach (rigid systems)

Local frame approach (flexible systems)

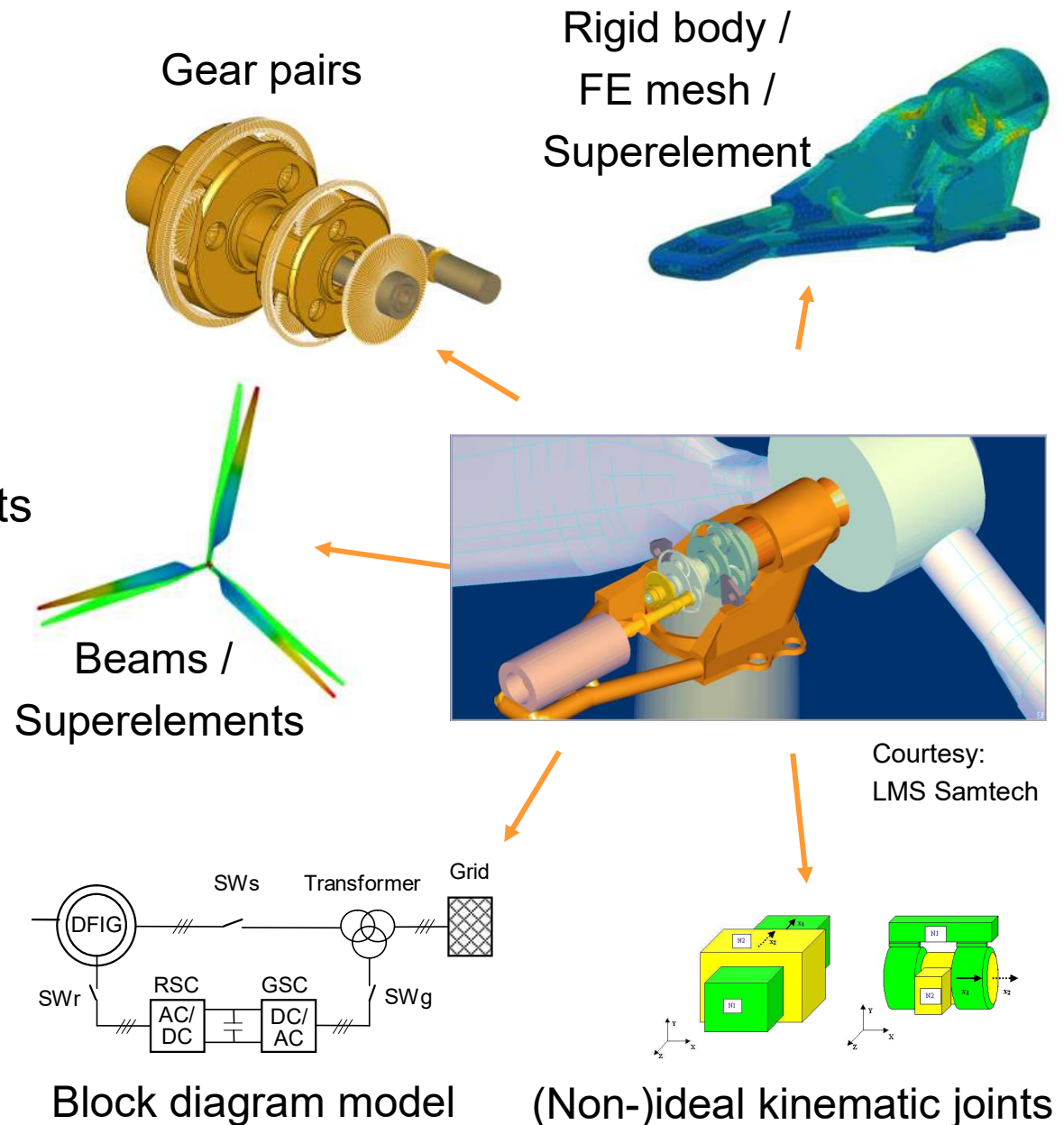
Optimization of MBS components

Control of flexible MBS

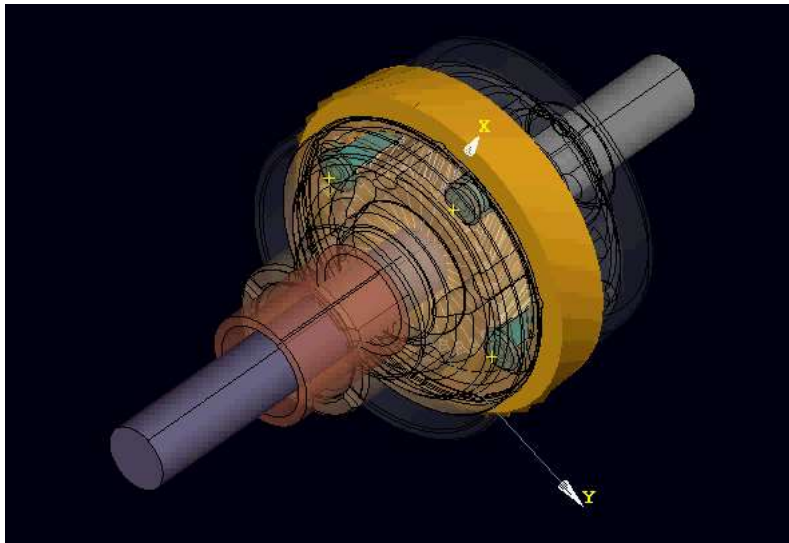
Example 1: wind engineering

Dynamic load prediction in a wind turbine

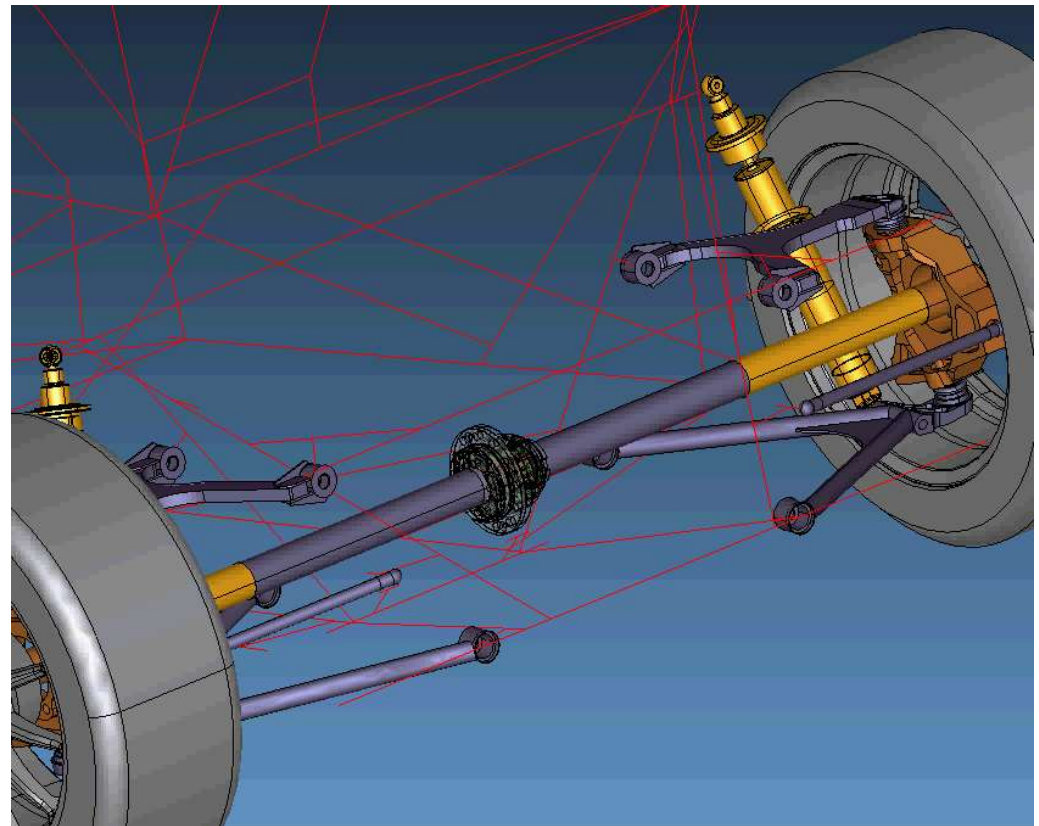
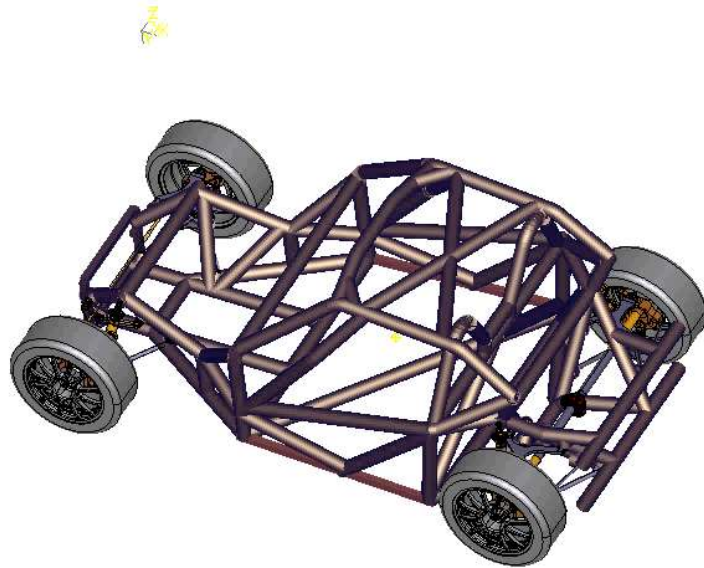
- Importance of flexibility effects
- Contacts and impacts in the drive-train
- Non-mechanical elements



Example 2: differential in a vehicle model



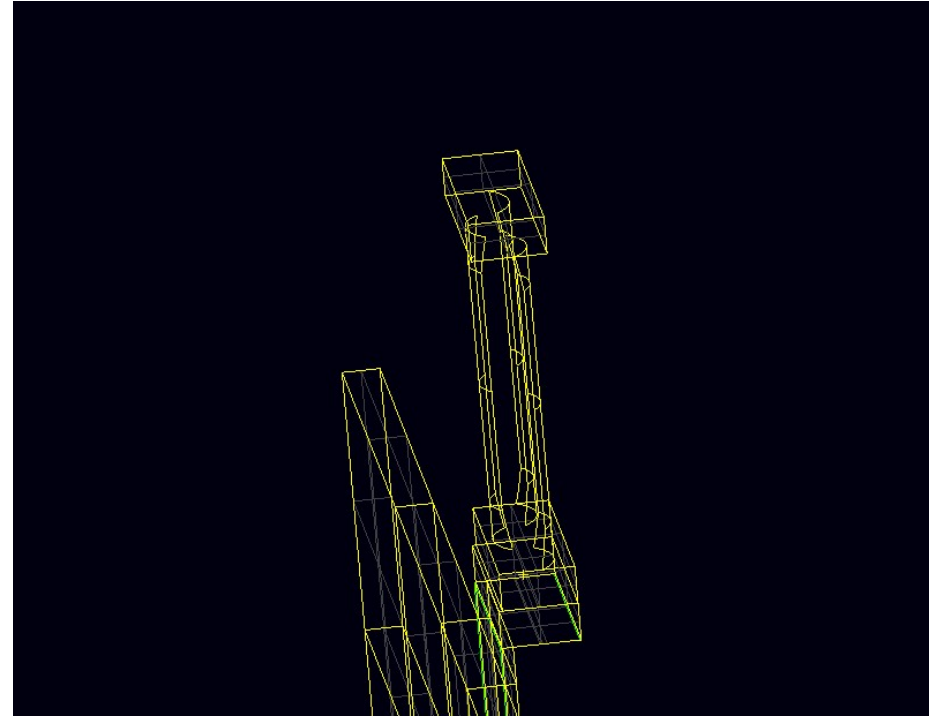
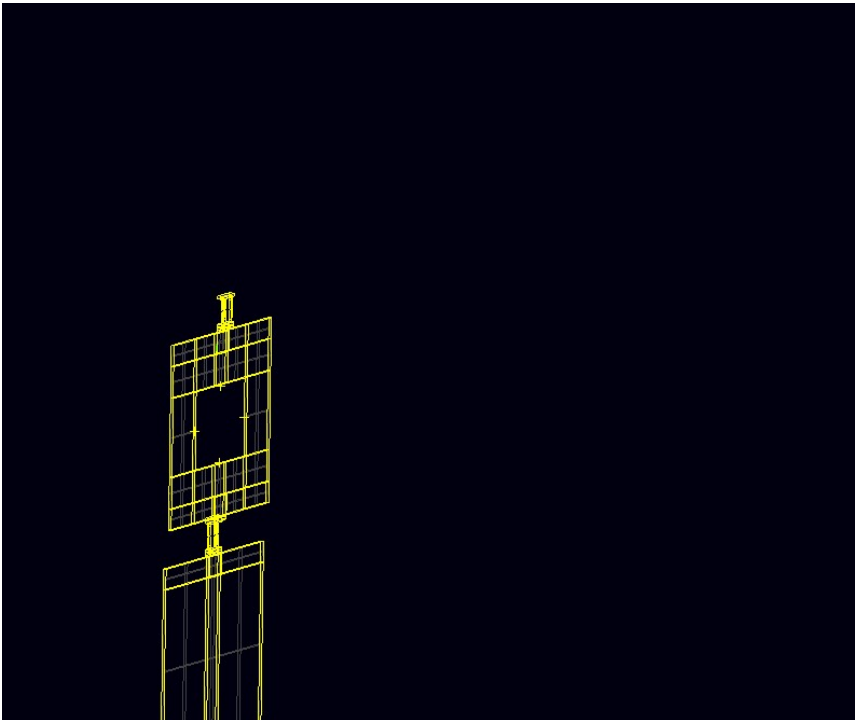
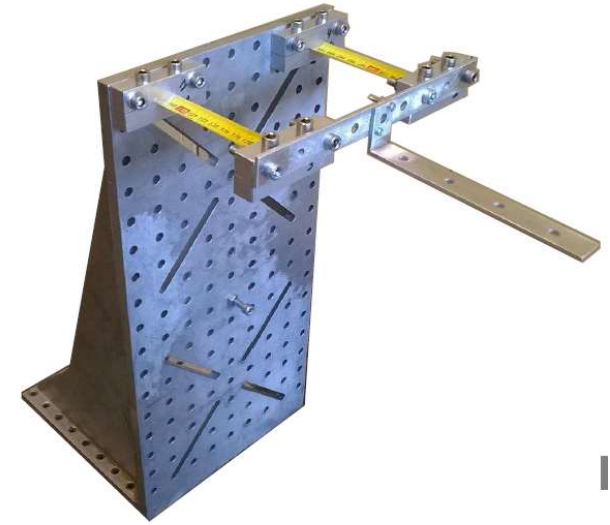
Torsen limited slip differential



Example 3: compliant structures

MAEVA tape spring hinge

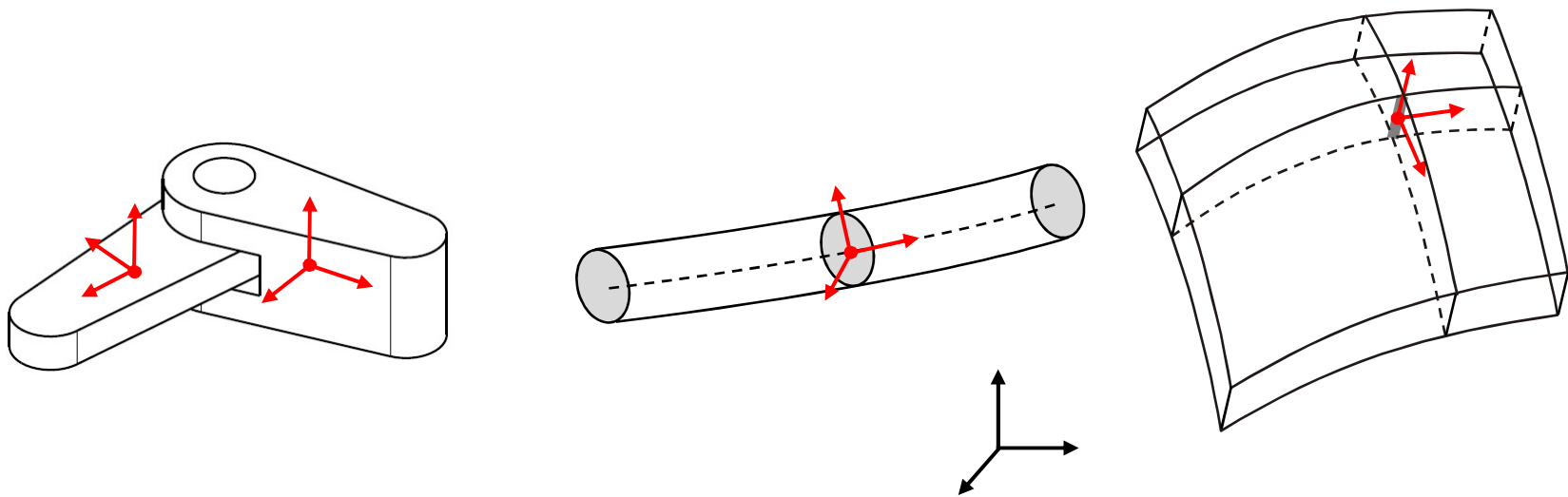
Deployment of solar panels
in a spacecraft



FE approach (Cardona 1989, G eradin & Cardona 2001)

Local frames are used to describe

- The position and orientation of a rigid body
- The position and orientation of the cross-section of a beam as a function of the centerline coordinate
- The position and orientation of the normal director of a shell as a function of the reference surface coordinates



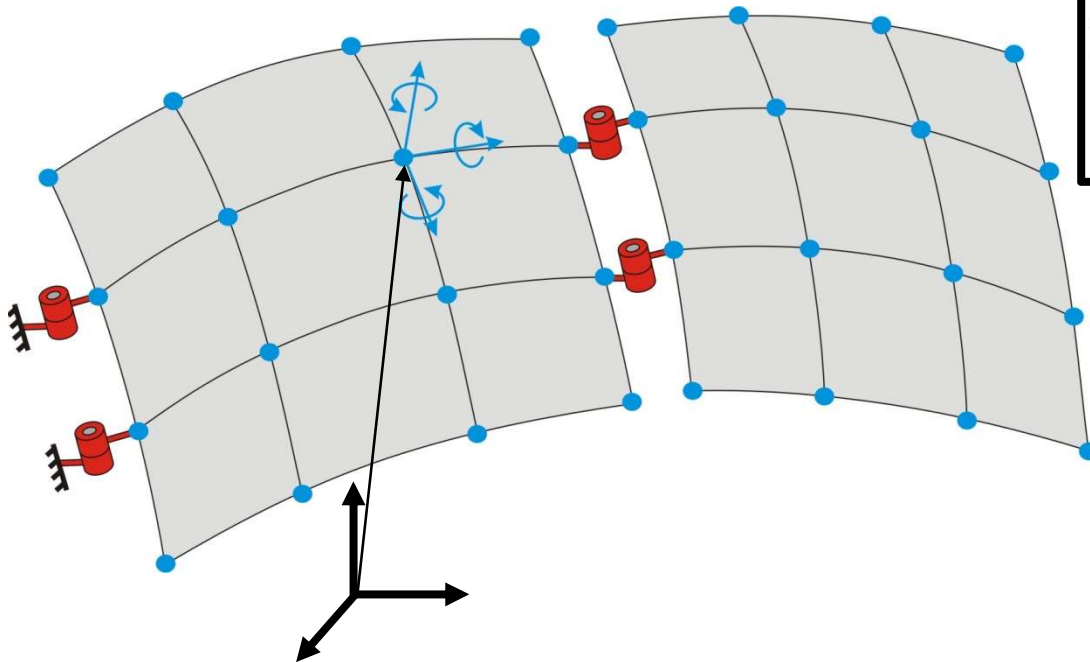
FE approach (Cardona 1989, Géradin & Cardona 2001)

One local frame per node \Rightarrow 6 coordinates per node

- Shape functions for interpolation of translations and rotations
- Kinetic, potential, internal energies written as a function of the coordinates

Kinematic joints & rigidity conditions

- algebraic constraints



$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, t) + \mathbf{B}^T \boldsymbol{\lambda} &= \mathbf{0} \\ \boldsymbol{\Phi}(\mathbf{q}, t) &= \mathbf{0} \end{aligned}$$

Index-3 DAE with
rotation coordinates

Important technical details

- The **rotation parameterization** should be carefully selected as it enters the equations of motion
- The operators behave **nonlinearly** as soon as rotations become large (even though the bodies do not deform much)
- Reduced integration is used to avoid **shear locking** problems in beam and shell formulations
- Incremental rotation representation is used to guarantee **frame invariance** and avoid **singularities**
- Implicit time integration method for the **index-3 DAE**
- Scaling of equations and unknowns is necessary to avoid a **bad numerical conditioning** of the linearized problem
- Numerical damping is needed to **stabilize the constraints**
- Since the index-3 problem is solved directly (constraints at position level), **spurious but transient oscillations** appear in the initial phase

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Motivation: beyond direct analysis

Additional algorithms are needed for control design and optimization

- Optimization algorithms and sensitivity analysis
- BVP solver
- Direct transcription method
- Direct multiple shooting method
- Equivalent static load computation...

Other motivations

- Simulation interactivity (modification of B.C., loadings, etc)
- Robustness of the models w.r.t. loading, trajectory and structural parameters
- Model efficiency (e.g., for real-time control)
- Models with frictional contacts and impacts

Our goal: **simplified** and **efficient** codes which **stick to the physics**
(we should not depend so much on the rotation parameterization)

Local frame approach

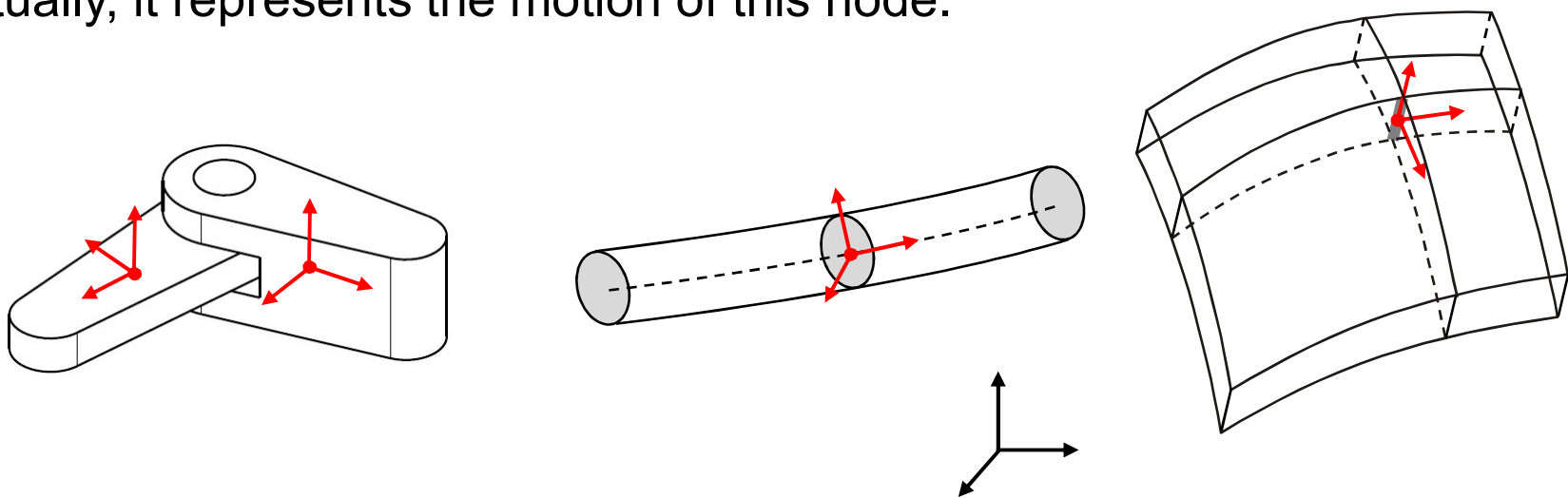
The local frame follows the motion of the body/cross section/director

The local frame is used to represent the equations of motion i.e.

- velocities and acceleration
- deformation gradients (leading to strain measures)
- forces and moments

After FE discretization, a local frame is available for each node.

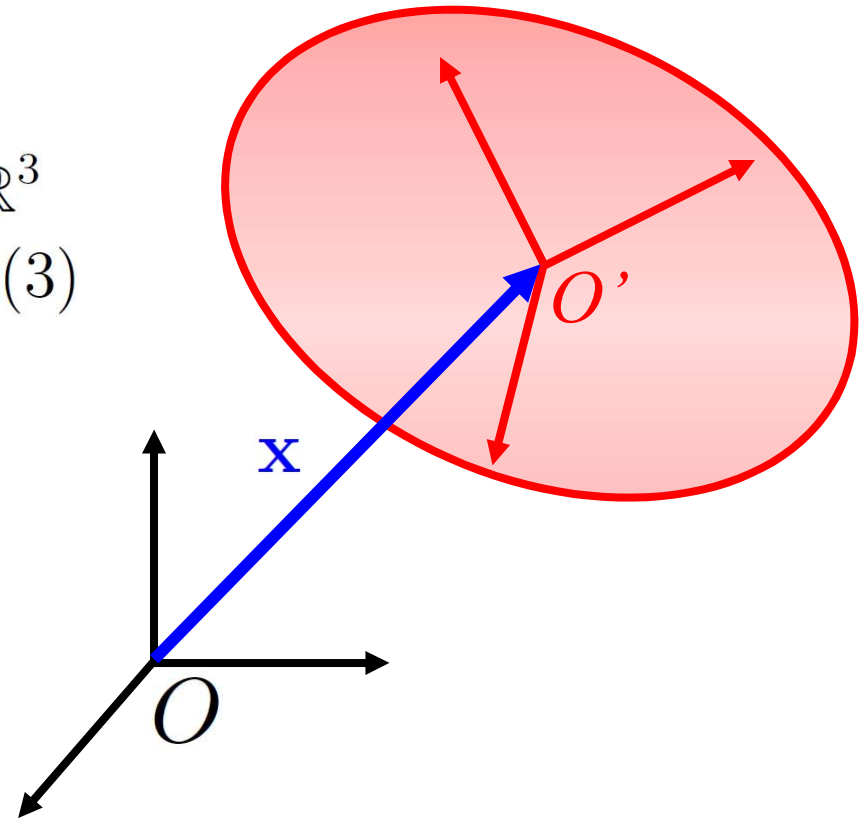
Actually, it represents the motion of this node.



Kinematics of a free rigid body

FE approach \Rightarrow one node at the CM

- One translation vector: $\mathbf{x} \in \mathbb{R}^3$
- One rotation matrix: $\mathbf{R} \in SO(3)$



The special Euclidean group $SE(3)$ is the set of 4 x 4 matrices

$$\mathbf{H} = \begin{bmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \quad \text{with} \quad \mathbf{R} \in SO(3) \quad \text{and} \quad \mathbf{x} \in \mathbb{R}^3$$

Kinematics of a free rigid body

$$q = \mathbf{H} = \begin{bmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

➤ Composition:

$$q_1 q_2 = \begin{bmatrix} \mathbf{R}_1 \mathbf{R}_2 & \mathbf{x}_1 + \mathbf{R}_1 \mathbf{x}_2 \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$

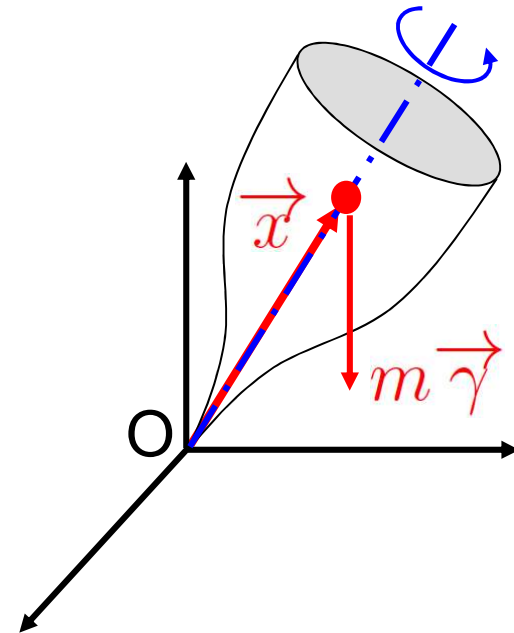
➤ (Lie algebra) representation of velocities: $\dot{q} = q \tilde{\mathbf{v}}$

$$\begin{bmatrix} \dot{\mathbf{R}} & \dot{\mathbf{x}} \\ \mathbf{0}_{1 \times 3} & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{\Omega}} & \mathbf{U} \\ \mathbf{0}_{1 \times 3} & 0 \end{bmatrix}$$

➤ Local frame velocity vector: $\mathbf{v} = \begin{bmatrix} \mathbf{U} \\ \boldsymbol{\Omega} \end{bmatrix}$ with $\begin{cases} \dot{\mathbf{x}} = \mathbf{R}\mathbf{U} \\ \dot{\mathbf{R}} = \mathbf{R}\tilde{\boldsymbol{\Omega}} \end{cases}$

Rotating top example

- One node at the CM undergoes translations and rotations
- The fixed point condition is imposed as a constraint



$$\mathcal{K}(\dot{\mathbf{x}}, \boldsymbol{\Omega}) = \frac{1}{2} m \dot{\mathbf{x}}^T \dot{\mathbf{x}} + \frac{1}{2} \boldsymbol{\Omega}^T \mathbf{J} \boldsymbol{\Omega}$$

$$\mathcal{V}(q) = -\mathbf{x}^T m \boldsymbol{\gamma}$$

$$\boldsymbol{\Phi}(q) = -\mathbf{R}^T \mathbf{x} + \mathbf{X} = \mathbf{0}_{3 \times 1}$$

$$\mathcal{L} = \mathcal{K} - \mathcal{V}$$

Rotating top example

$$\Phi(q) = \mathbf{X} - \mathbf{R}^T \mathbf{x} = \mathbf{0}_{3 \times 1}$$

Using $\dot{\mathbf{x}} = \mathbf{R}\mathbf{U}$, $\dot{\Phi} \Big|_{\Phi=0} = \begin{bmatrix} -\mathbf{I}_3 & -\tilde{\mathbf{X}} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \boldsymbol{\Omega} \end{bmatrix}$

Constant gradient

Local frame velocity

Hamilton principle: $\delta \int_{t_i}^{t_f} (\mathcal{L}(q, \mathbf{v}) - \Phi(q)^T \boldsymbol{\lambda}) dt = 0$

DAE on the special Euclidean group

$$\dot{\mathbf{x}} = \mathbf{R}\mathbf{U}$$

$$\dot{\mathbf{R}} = \mathbf{R}\tilde{\boldsymbol{\Omega}}$$

$$m\dot{\mathbf{U}} + m\boldsymbol{\Omega} \times \mathbf{U} - \boldsymbol{\lambda} = \mathbf{R}^T m\boldsymbol{\gamma}$$

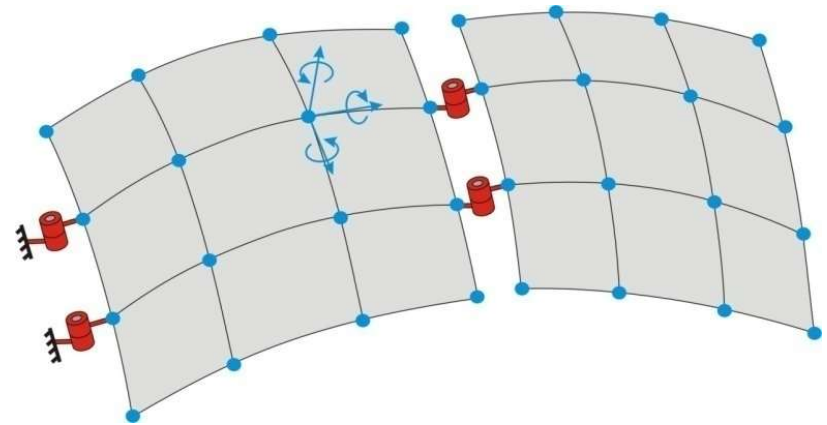
$$\mathbf{J}\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times \mathbf{J}\boldsymbol{\Omega} + \mathbf{X} \times \boldsymbol{\lambda} = \mathbf{0}_{3 \times 1}$$

$$\mathbf{X} - \mathbf{R}^T \mathbf{x} = \mathbf{0}_{3 \times 1}$$

- Coordinate free
- Quadratic compatibility eq.
- Linear reaction forces
- Constant mass matrix
- Quadratic (but coupled) inertia forces
- Orientation-dependent gravity forces

Configuration of a multibody system

N nodal variables
 M kinematic joints



The configuration is represented by a matrix

$$q = \text{diag}(\mathbf{H}_1, \dots, \mathbf{H}_N, p_1, \dots, p_M)$$

which belongs to the k -dimensional Lie group

$$G = SE(3) \times \dots \times SE(3) \times G_1 \times \dots \times G_M$$

Since q needs to satisfy m kinematic constraints $\Phi(q)$,
the configuration space is a submanifold of dimension $k-m$

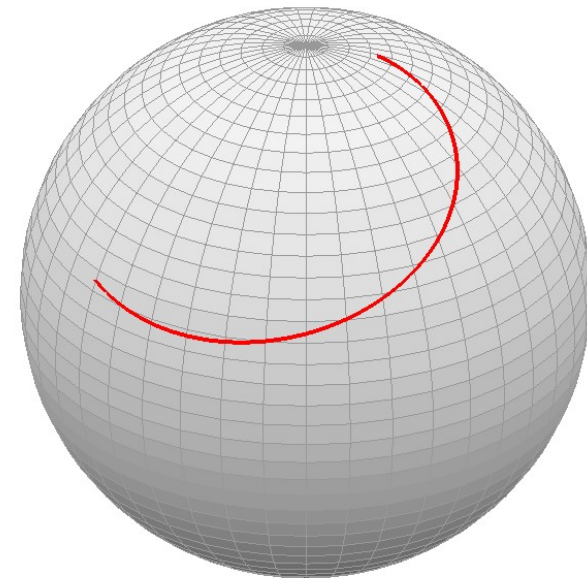
$$N = \{q \in G : \Phi(q) = \mathbf{0}\}$$

Equations of motion in the local frame

Index-3 DAE on a Lie group (no parameterization):

$$\begin{aligned}\dot{q} &= q\tilde{\mathbf{v}} \\ \mathbf{M}\dot{\mathbf{v}} - \widehat{\mathbf{v}}^T \mathbf{M}\mathbf{v} &= -\mathbf{g}(\mathbf{q}, t) - \mathbf{B}^T(q)\boldsymbol{\lambda} \\ \Phi(q) &= \mathbf{0}_{m \times 1}\end{aligned}$$

- The configuration is described by the matrix q
- The velocity is described by a vector \mathbf{v} and the matrix $\tilde{\mathbf{v}}$
- If the initial conditions are on the group, the solution of the DAE will stay on the group for $t \geq 0$



Equations of motion in the local frame

Index-3 DAE on a Lie group (no parameterization):

$$\begin{aligned}\dot{q} &= q\tilde{\mathbf{v}} \\ \mathbf{M}\dot{\mathbf{v}} - \widehat{\mathbf{v}}^T \mathbf{M}\mathbf{v} &= -\mathbf{g}(\mathbf{q}, t) - \mathbf{B}^T(q)\boldsymbol{\lambda} \\ \boldsymbol{\Phi}(q) &= \mathbf{0}_{m \times 1}\end{aligned}$$

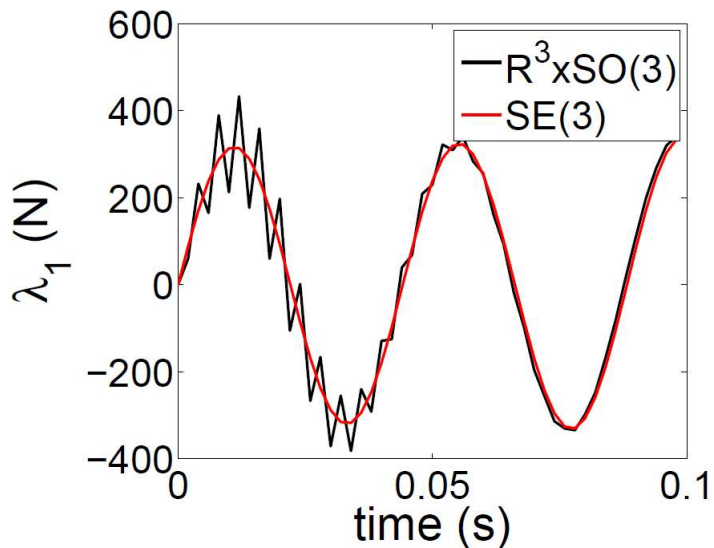
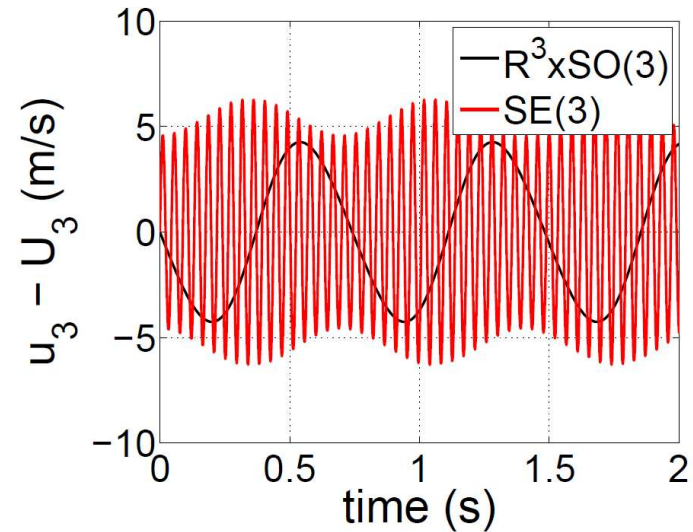
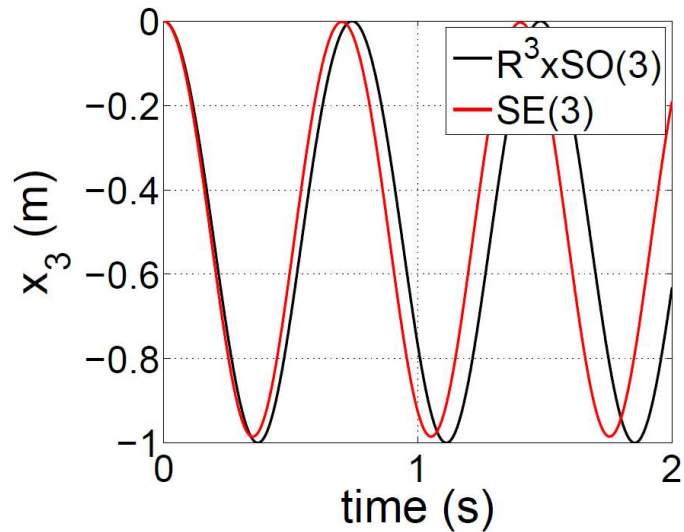
- The configuration is described by the matrix q
- The velocity is described by a vector \mathbf{v} and the matrix $\tilde{\mathbf{v}}$

Time integration on a Lie group

- Euler implicit
$$\begin{aligned}q_{n+1} &= q_n \exp(h \tilde{\mathbf{v}}_{n+1}) \\ \mathbf{v}_{n+1} &= \mathbf{v}_n + h \dot{\mathbf{v}}_{n+1}\end{aligned}$$
- Lie group generalized- α method (B. and Cardona 2010, B., Cardona and Arnold 2012)

Rotating top example

Generalized- α method, $h = 0.002$ s, $\rho = 0.8$



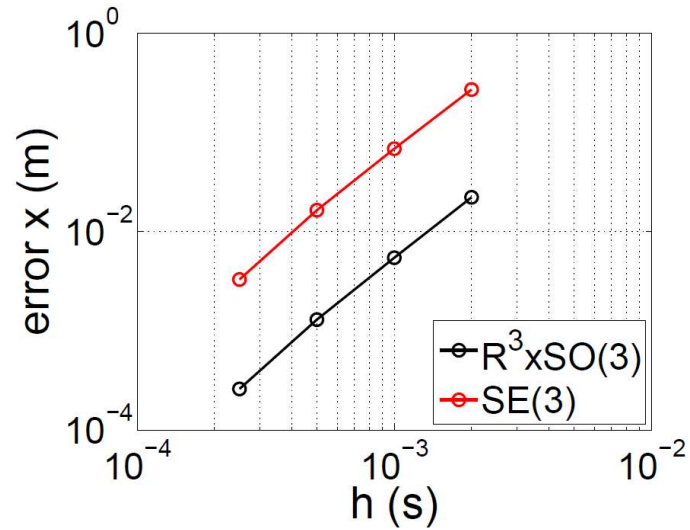
Mean number of Newton iterations

	Updated St	Frozen St
$R^3 \times SO(3)$	2.69	/
$SE(3)$	2	2.96

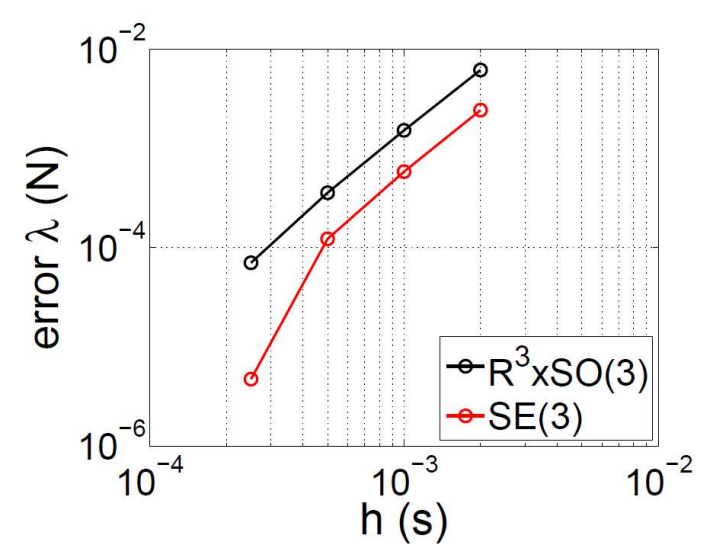
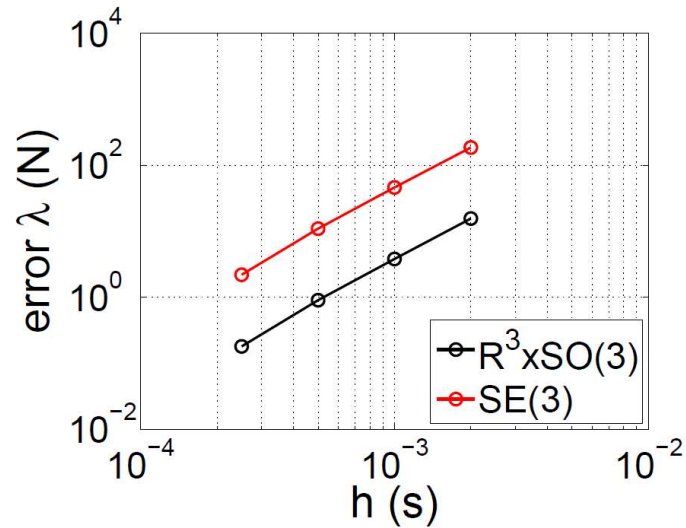
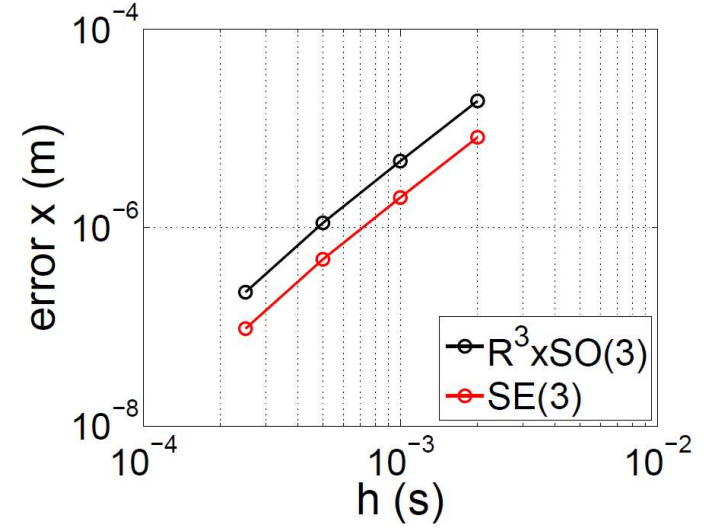
Hidden constraints are automatically satisfied by the $SE(3)$ solution

Rotating top example

High initial velocity



Low initial velocity



Intermediate summary 1

Local frame approach (rigid systems)

- Rotations and translations are treated as a whole
- Velocities, accelerations & forces are defined in the local frame
- Rigid body constraints block the relative motion in the local frame
⇒ "linear" behaviour
- Joint formulations only involve the relative motion
- Nonlinearities are reduced
- DAEs on a Lie group can be solved numerically

Outline

Introduction to our research group

More about MECANO

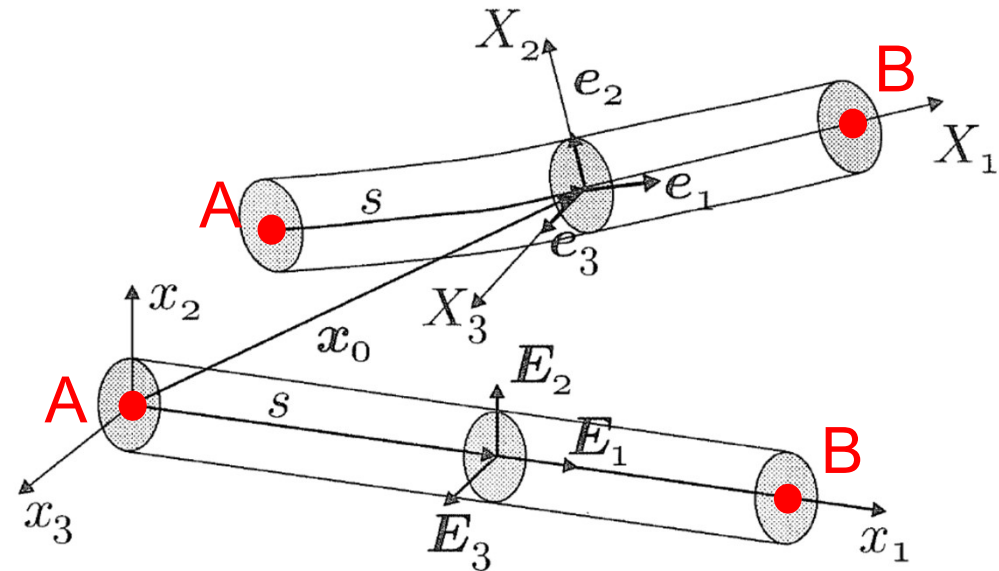
Local frame approach (rigid systems)

Local frame approach (flexible systems)

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Control of flexible MBS

Flexible beam formulation



- Timoshenko-type geometrically exact model
(cross sections do not deform)
- Translation and rotation fields $\mathbf{x}(s), \mathbf{R}(s)$
- Interpolation from nodal values $(\mathbf{x}_A, \mathbf{x}_B)$ and $(\mathbf{R}_A, \mathbf{R}_B)$
- Strain energy : bending, torsion, traction and shear

Beam finite element formulation

Rotational & translational dofs in geometrically exact beam formulations

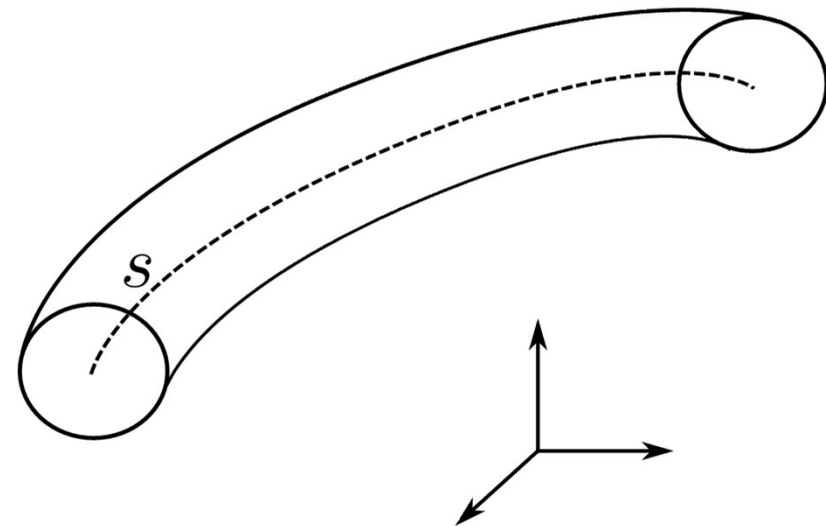
- Independent interpolation of rotation and translation (Simo 1985)
- Coupled interpolation using an helicoidal approximation (Borri & Bottasso, 1994)

Originality:

Formulation in the local frame

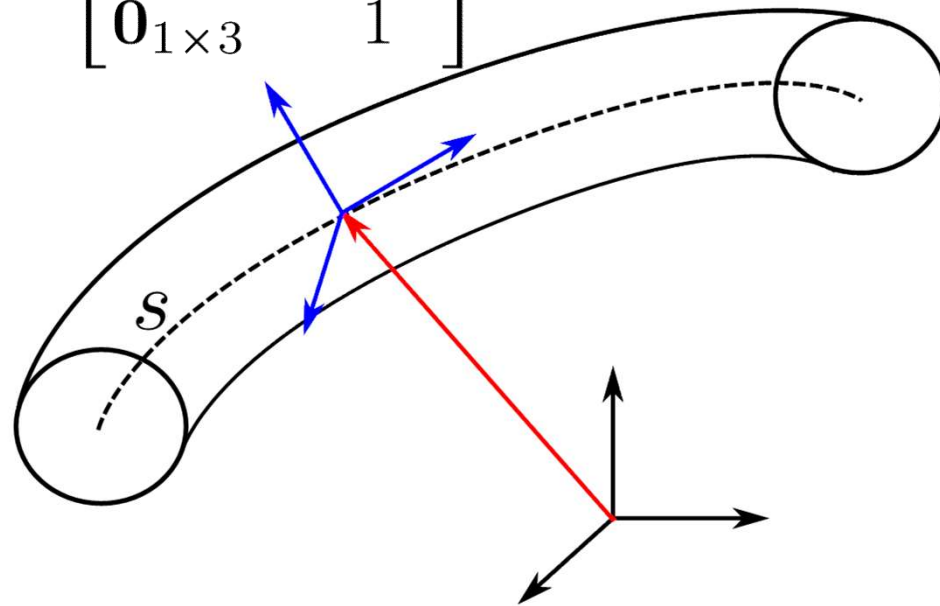
Assumption in this talk:

undeformed configuration is straight



Kinematics of the beam on SE(3)

$$\mathbf{H}(s) = \begin{bmatrix} \mathbf{R}(s) & \mathbf{x}(s) \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix}$$



Local frame representation of strains

"Pose gradient" in the local frame

$$\begin{aligned}\frac{d(\mathbf{H})}{ds} &= \mathbf{H}\tilde{\mathbf{f}} \\ &= \mathbf{H}(s)(\tilde{\mathbf{f}}^0 + \tilde{\boldsymbol{\epsilon}})\end{aligned}$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} \boldsymbol{\gamma} \\ \boldsymbol{\kappa} \end{bmatrix} \begin{array}{l} \nearrow \boldsymbol{\gamma} = \mathbf{R}^T(s) \frac{d\mathbf{x}(s)}{ds} - \mathbf{f}_U^0 \\ \searrow \boldsymbol{\kappa} = \text{vect}(\mathbf{R}^T d\mathbf{R}/ds) \end{array}$$

Intrinsic beam formulation

$$\frac{d\mathbf{H}}{ds} = \mathbf{H}(\tilde{\mathbf{f}}^0 + \tilde{\boldsymbol{\epsilon}}) \quad \frac{d(\mathbf{H})}{dt} = \mathbf{H}\tilde{\mathbf{v}}$$
$$\delta(\mathcal{W}_{int}) = \int_0^L \delta(\boldsymbol{\epsilon})^T \mathbf{K} \boldsymbol{\epsilon} ds \quad \delta(\mathcal{K}) = \int_0^L \delta(\mathbf{v})^T \mathbf{M}_C \mathbf{v} ds$$

Local form of the dynamic equilibrium (12-dimensional PDE)

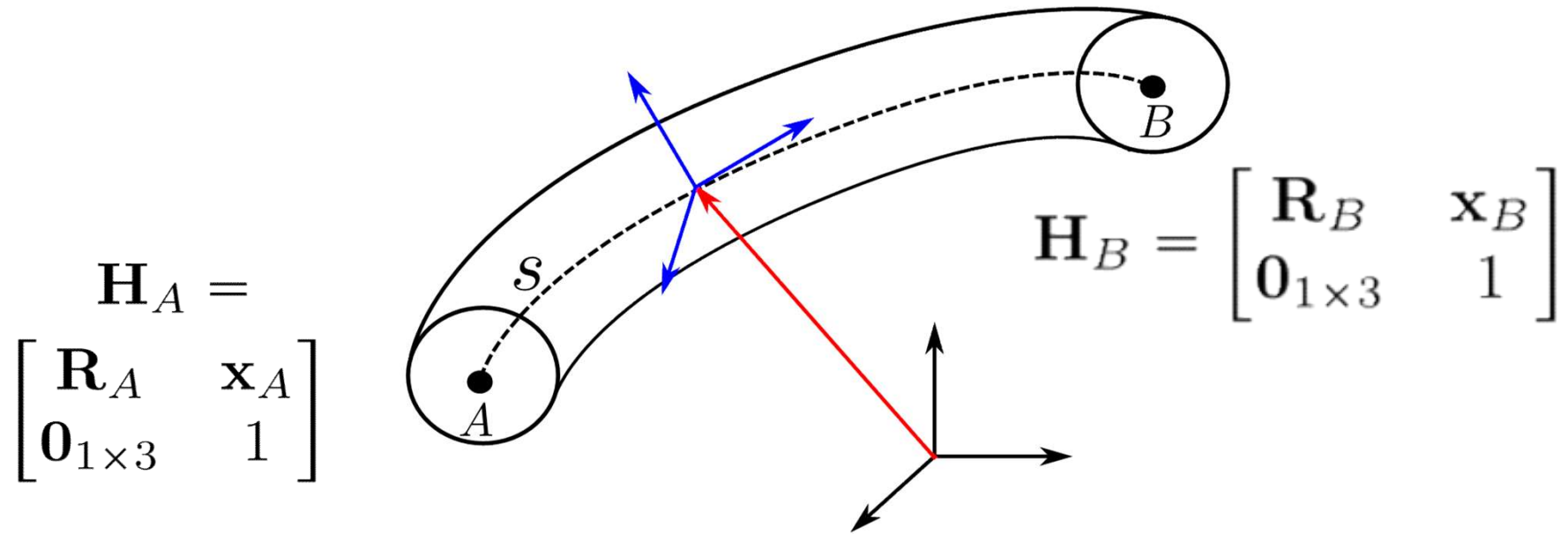
$$\frac{d\mathbf{v}}{ds} = \frac{d\boldsymbol{\epsilon}}{dt} + \hat{\mathbf{v}}\boldsymbol{\epsilon}$$
$$\mathbf{M}_C \dot{\mathbf{v}} - \hat{\mathbf{v}}^T \mathbf{M}_C \mathbf{v} + \mathbf{K} \frac{d}{ds}(\boldsymbol{\epsilon}) - \hat{\mathbf{f}}^T \mathbf{K} \boldsymbol{\epsilon} + \mathbf{g}_{ext} = 0$$

Dynamic equilibrium in terms of \mathbf{f} and \mathbf{v} only



No need to know actual position and orientation

FE interpolation field



Interpolation on the special Euclidean group:

$$\mathbf{H}(s) = \mathbf{H}_A \exp(s \tilde{\mathbf{f}}^*) \quad \text{with} \quad \tilde{\mathbf{f}}^* = \frac{\log(\mathbf{H}_A^{-1} \mathbf{H}_B)}{L}$$

Discretized strains

Simple analytical expression of the interpolated strains

$$\tilde{\boldsymbol{\epsilon}} = \log(\mathbf{H}_A^{-1} \mathbf{H}_B) / L - \tilde{\mathbf{f}}^0$$

- They depend on the **relative configuration between node A and B**, i.e., they are invariant under rigid body motion.
- They do not depend on the coordinate along the beam.

The shape functions can thus **represent exactly a constant strain field** in the element.

The same observations hold for the internal forces and the tangent stiffness matrix.

No shear locking in pure bending

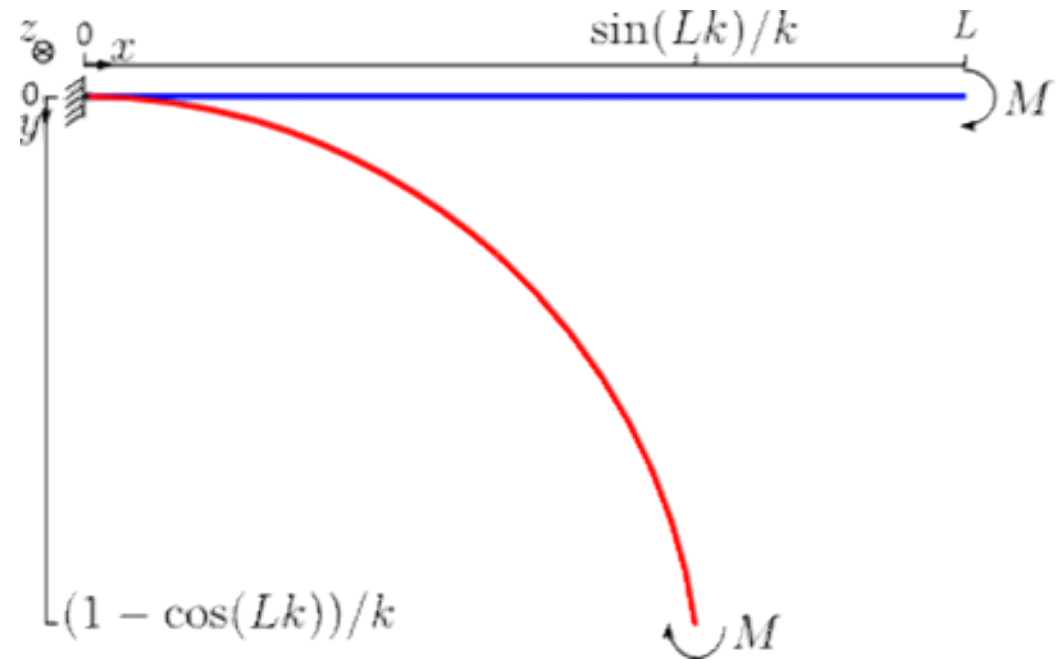
The interpolation field can represent exactly any helicoidal curve (constant curvature and torsion)

$$k = \frac{M}{EI}$$

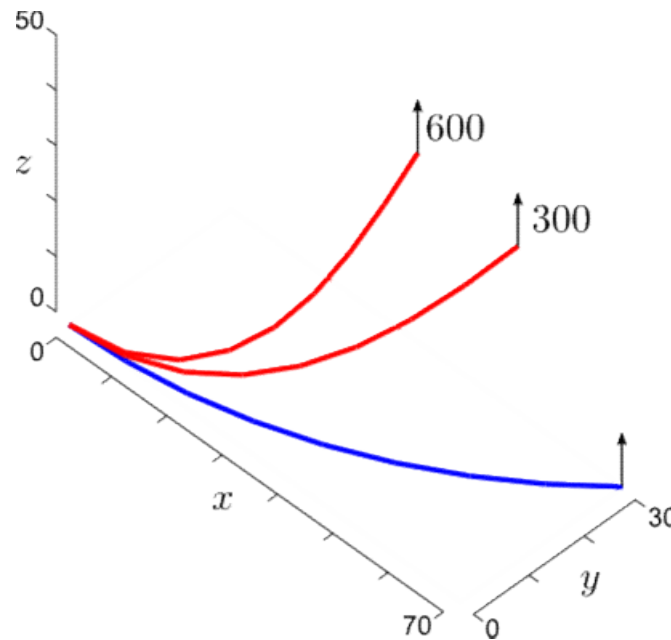
$$x(s) = \frac{1}{k} \sin(sk)$$

$$y(s) = \frac{1}{k} (1 - \cos(sk))$$

$$z(s) = 0$$

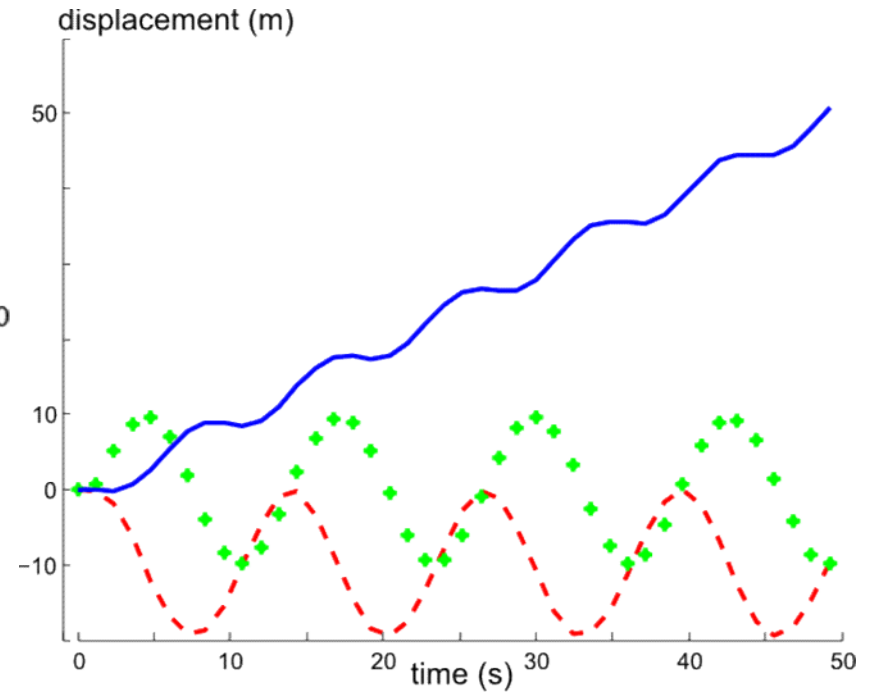
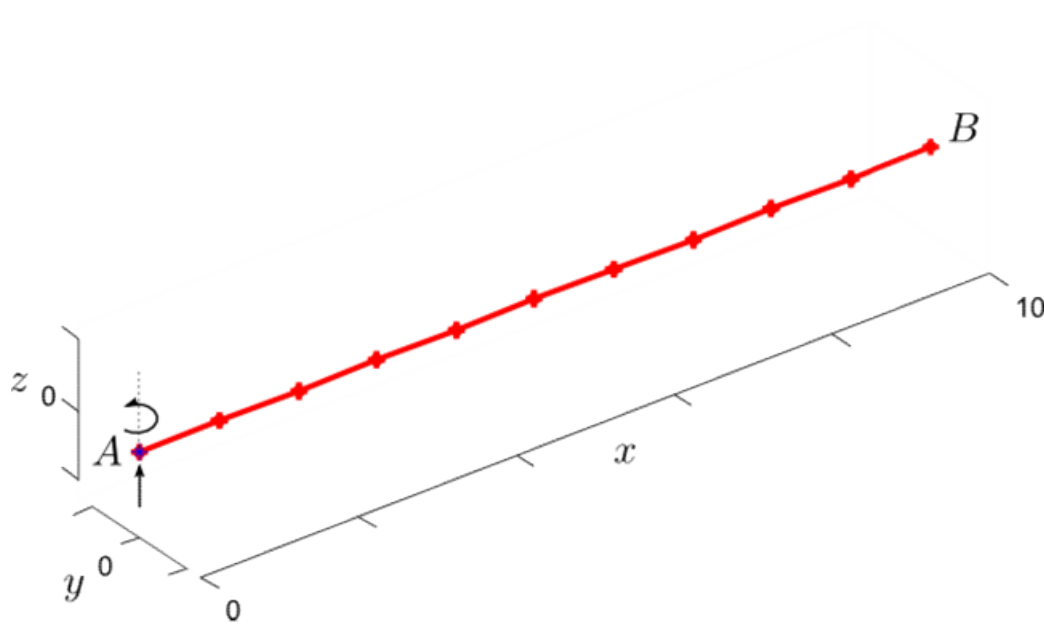


Static example



Load	$SE(3)$ formulation				Simo and Vu-Quoc			
	Nit	x	y	z	Nit	x	y	z
300	9	58.84	22.30	40.03	13	58.84	22.33	40.08
600	14	47.23	15.76	53.28	27	47.23	15.79	53.37

Dynamic example

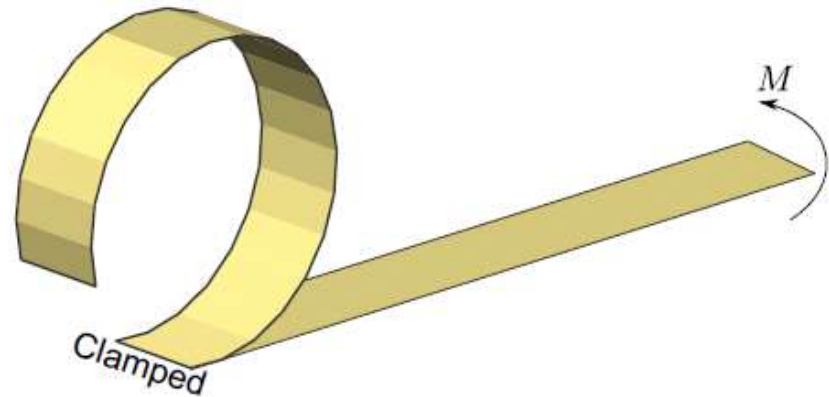


This problem was solved without updating the iteration matrix in the Newton iterations

Nonlinear shell: Example 1

360° roll-up of a clamped beam [Simo & Fox 1989]

- Poisson ratio: $\nu = 0$
- Pure bending situation
- Static solution: constant curvature



Numerical model 1

- 2 quadrangular elements
- \mathbf{K}_t is updated at each Newton iteration
- Exact solution in 1 load step

Numerical model 2

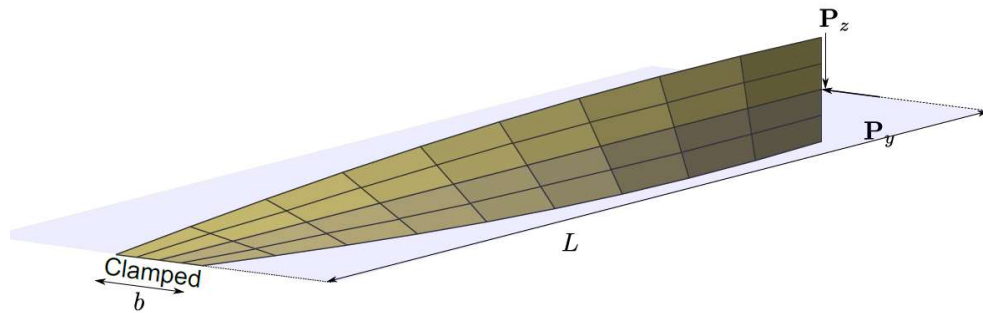
- 2 quadrangular elements
- \mathbf{K}_t is **not updated**
- Exact solution in 2 load steps

Numerical model 3

- 4 quadrangular elements
- \mathbf{K}_t is **not updated**
- Exact solution in 1 load step

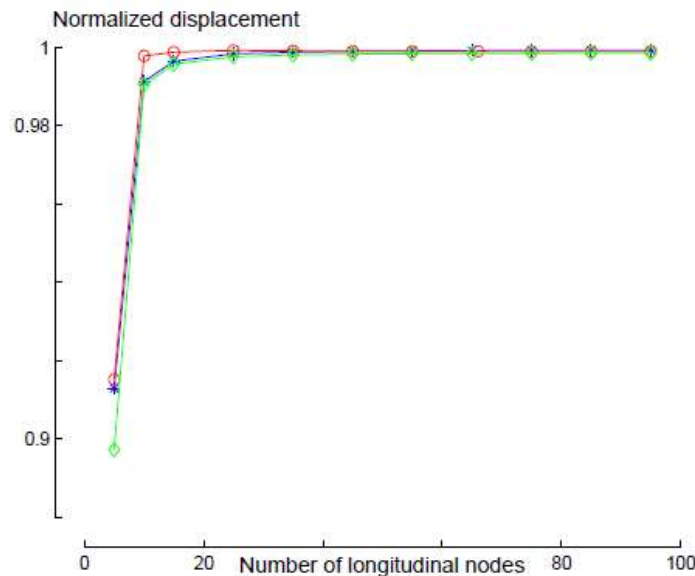
No shear locking (without any numerical trick)

Nonlinear shell: example 2

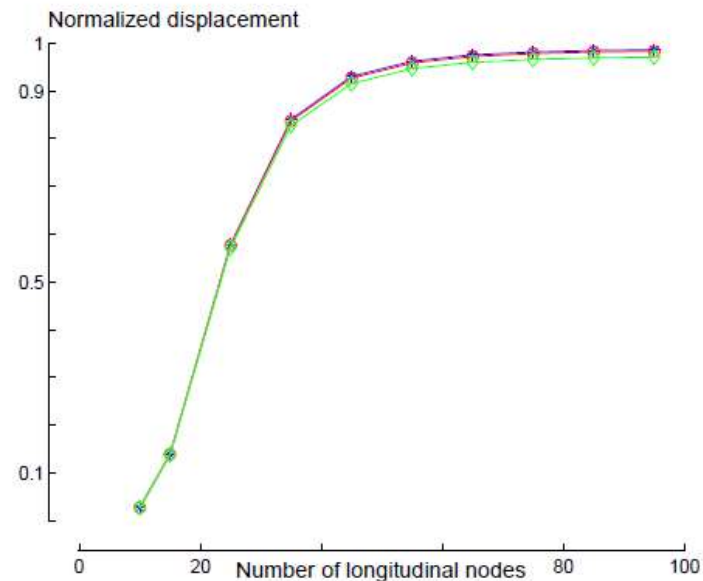


Pre-twisted beam

- Linear static deformation
- 2 load cases



(a) Thick case: 0.32 m



(b) Thin case: 0.0032 m

No shear/membrane locking (without any numerical trick)

Local frame vs. corotational frame

Local frame formulation:

- The local frame is defined from the kinematic assumption (rigid body, beam, shell).
- Several local frames may coexist in a single finite element (e.g., a beam with two nodes).
- The local frame is a nodal quantity, which is shared by all elements connected to the node (\Rightarrow FE assembly).
- The equations of motion are written in the local frame

Corotational frame formulation:

- An additional definition is needed for the corotational frame
- The corotational frame is unique for each element
- The corotational frame is internal to the element (it is not assembled)
- The equations are finally written in the inertial frame (the corotational frame is only used at an intermediate step)

Intermediate summary 2

Local frame approach (flexible systems)

- FE formulation in the **local frame**
- No parameterization of the equations of motion
- For beam and shells, \mathbf{g}_{int} and \mathbf{K}_t are insensitive to rigid body motions
- Fluctuations of $\mathbf{K}_t \rightarrow 0$ when the mesh is refined
- Finite motion problems are solved successfully **without updating the tangent stiffness matrix**, if the mesh is « sufficiently fine »
- No locking problem is observed (helical interpolation)
- More detail: PhD thesis by Valentin Sonneville and related papers

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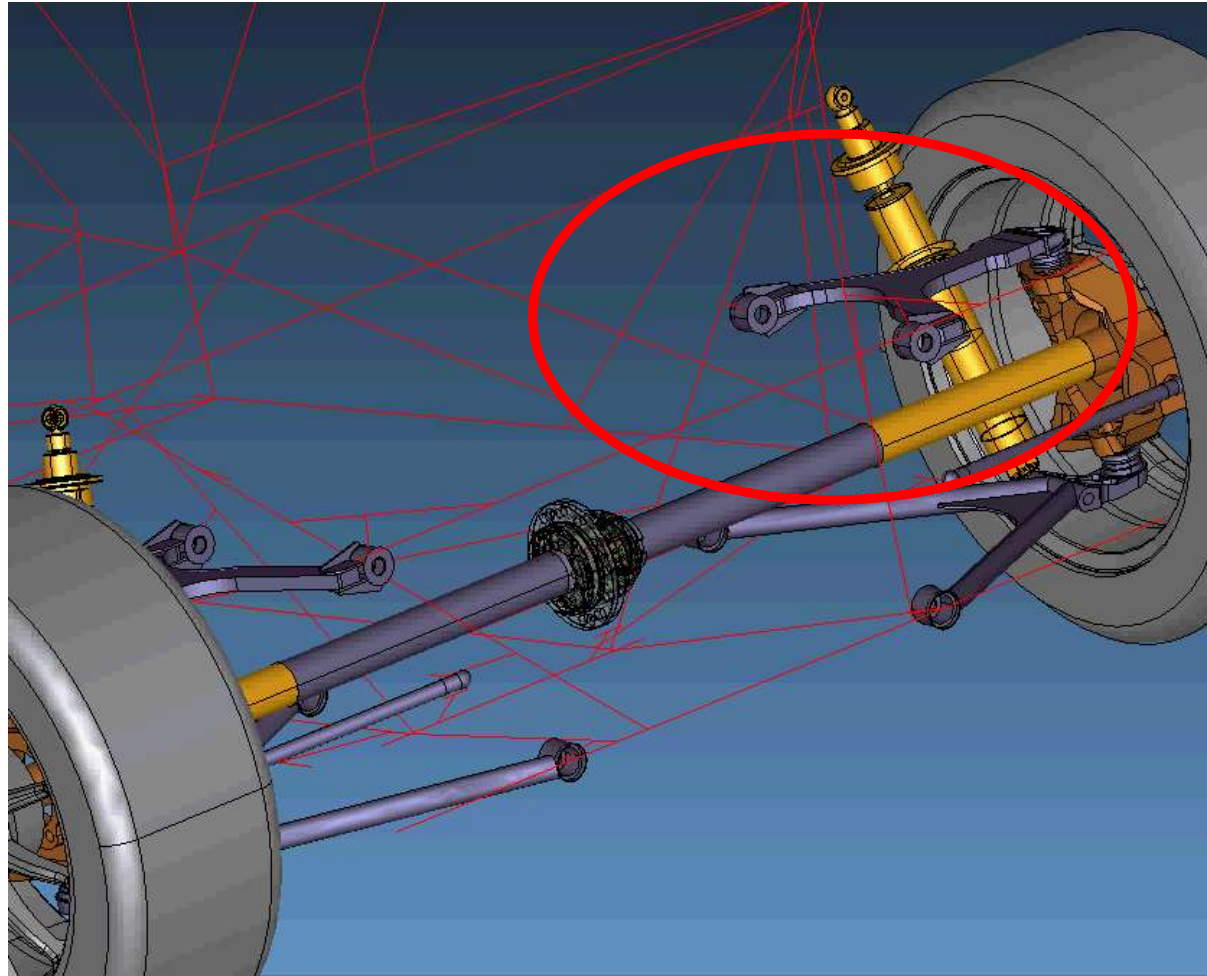
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Control of flexible MBS

Optimization of MBS components

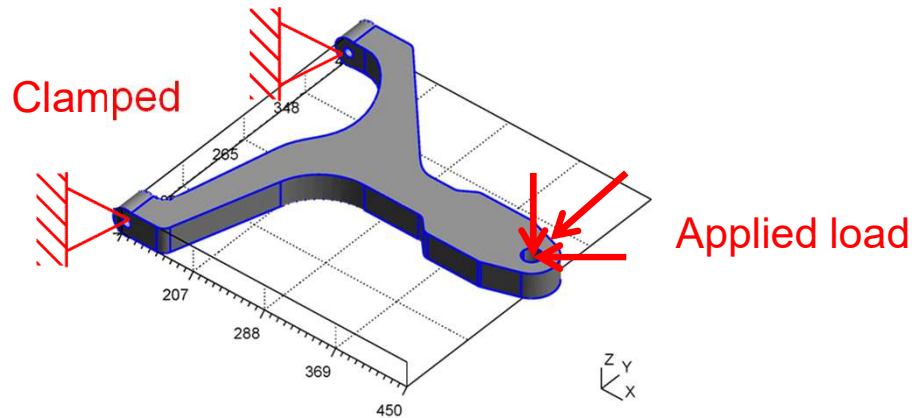
Lane-change maneuver



(Virlez, 2014)

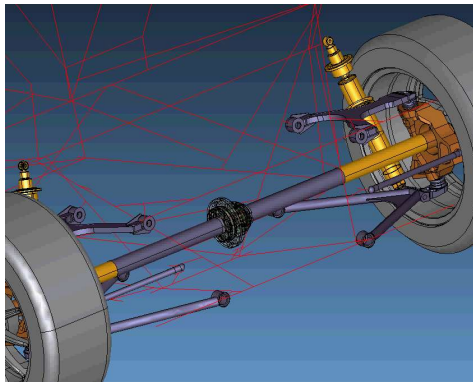
Optimization of mechanical systems

❑ Component-based approach

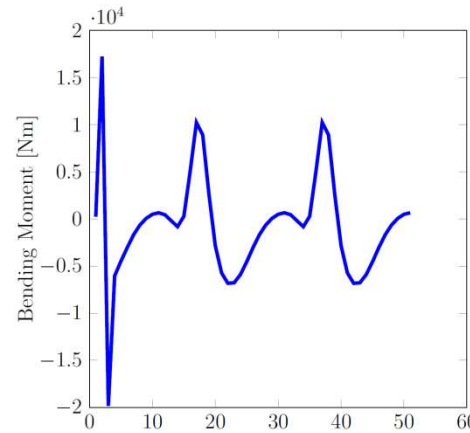


- Experience - Empirical load case - Standard
- Dynamic amplification
- ➔ Not optimal wrt to the real loading
- ➔ Iteration with the MBS team, slow and inefficient

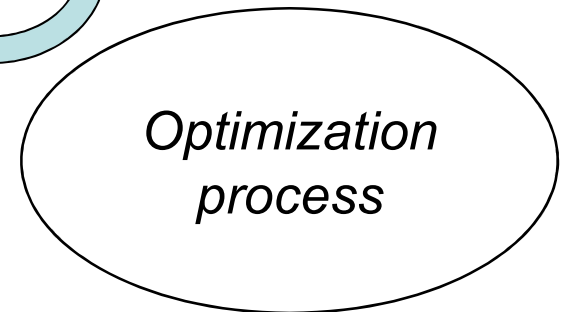
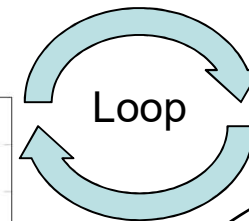
❑ System-based approach



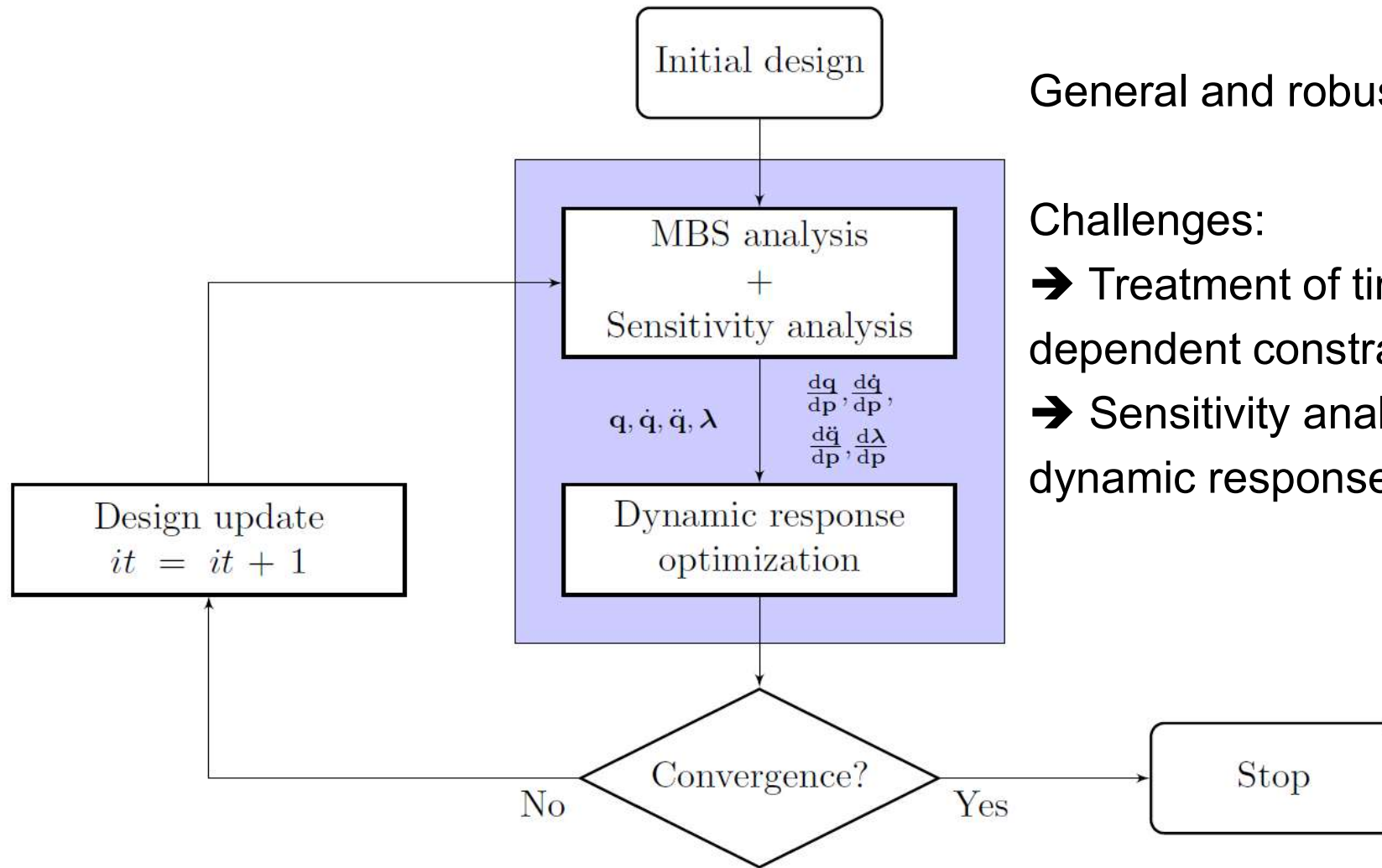
MBS Simulation



Time response



Fully coupled method



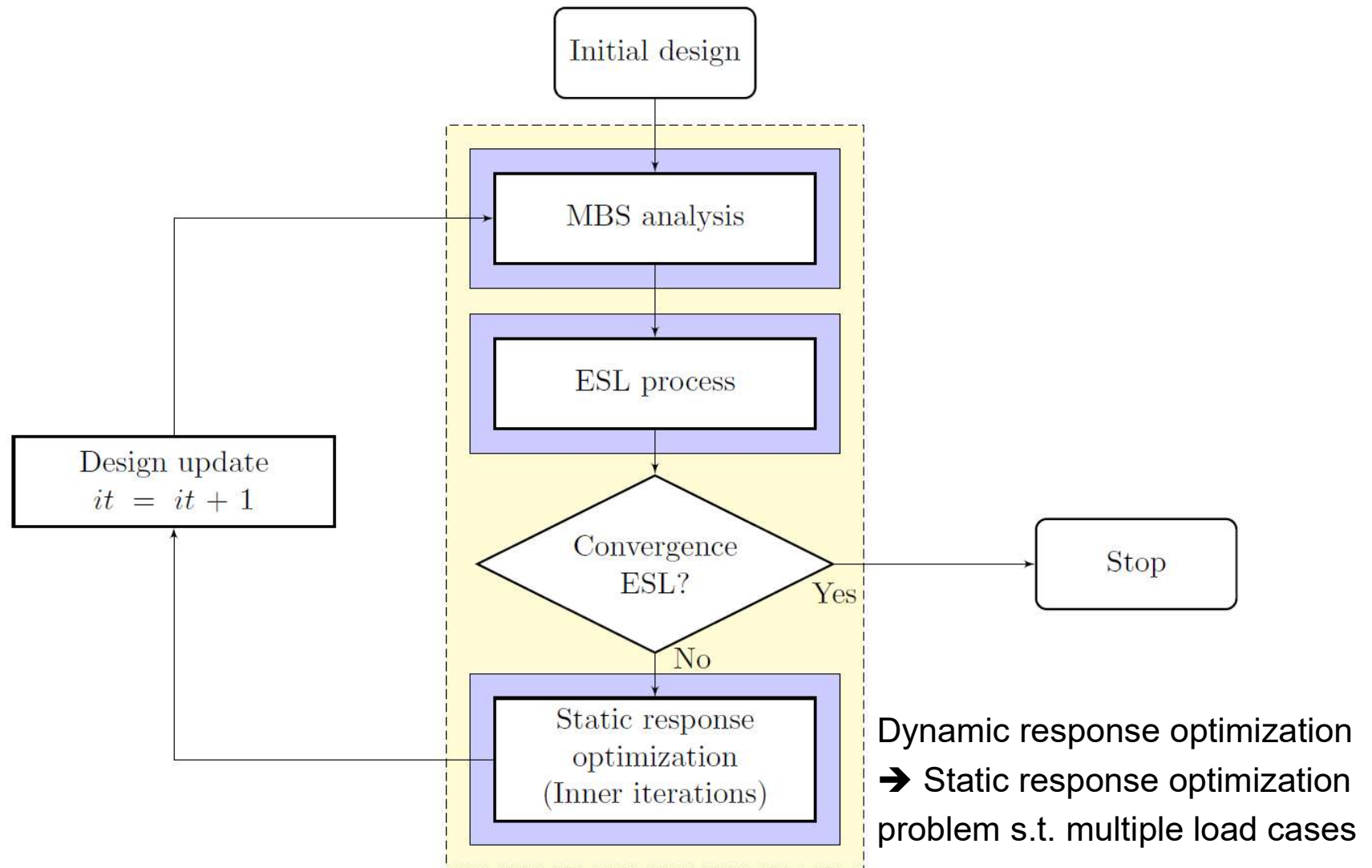
General and robust method

Challenges:

➔ Treatment of time-dependent constraints

➔ Sensitivity analysis of the dynamic response is costly

Weakly coupled method



Equivalent Static Load

- ❑ Aims at mimicking the dynamic loading

- ❑ ESL definition for MBS component optimization (Kang et al., 2005):

*“When a dynamic load is applied to a MBS, the equivalent static load for an **isolated body** is defined as the static load that produces the same relative displacement field as the one created by the dynamic load at an arbitrary time in a **body-attached frame**.”*

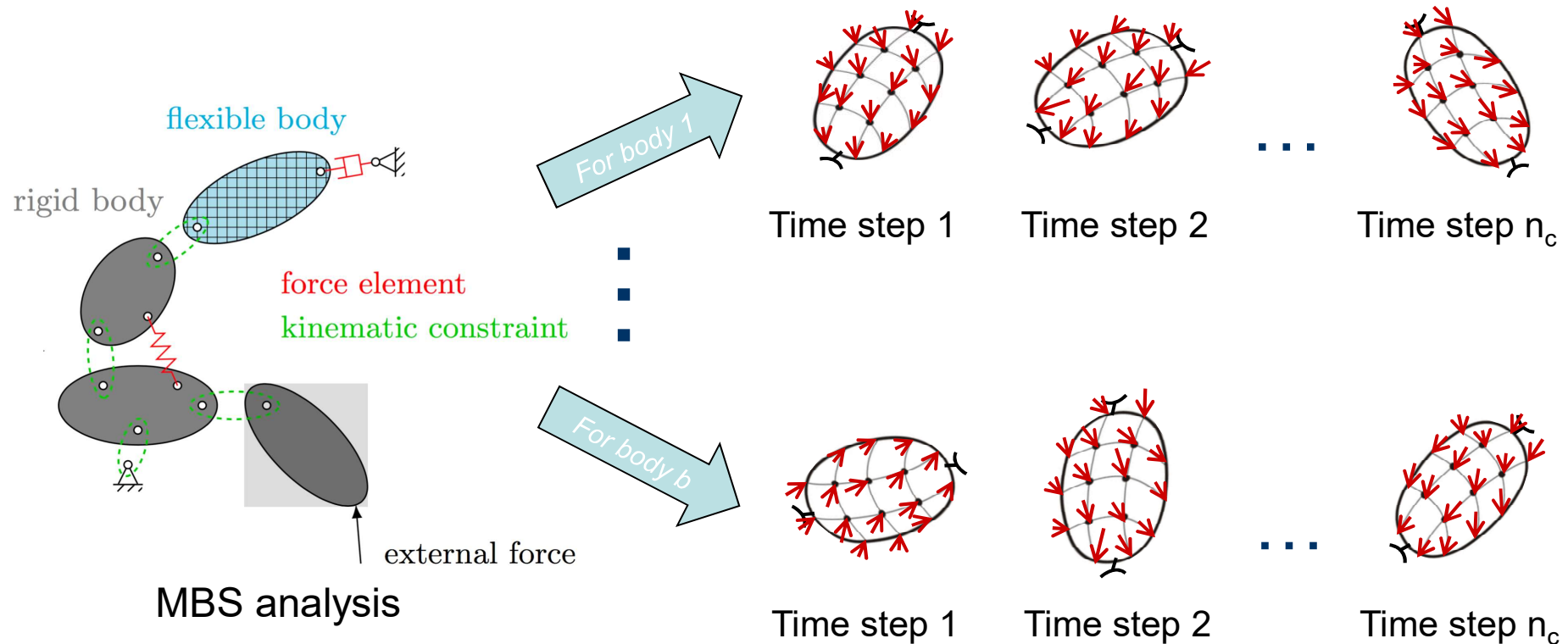
- ❑ General mathematical concept

At the component level, define $\mathbf{g}_{eq}^b(t)$

Such that $\mathbf{K}^b(\mathbf{p}) \mathbf{q}_{st}^b = \mathbf{g}_{eq}^b(t)$ gives $\mathbf{q}_{st}^b = \mathbf{q}^b(t)$.

Number of ESLs = number of integration time steps X number of components

Weakly coupled method



- Optimization of **isolated** components under “**system**” load cases
- One static response optimization problem under multiple load cases
- Efficient method

Generalization of the ESL method

- Definition of the ESL at the **system** level

*“When a dynamic load is applied to a MBS, the generalized equivalent static load is defined as the static load at the **system level**, that produces the same deformed configuration of the mechanism as the one created by the dynamic load at an arbitrary time”*

- Equations of motion

$$\begin{aligned}\dot{q} &= q\tilde{\mathbf{v}} \\ \mathbf{M}\dot{\mathbf{v}} - \hat{\mathbf{v}}^T \mathbf{M}\mathbf{v} &= -\mathbf{g}(\mathbf{q}, t) - \mathbf{B}^T(q)\boldsymbol{\lambda} \\ \Phi(q) &= \mathbf{0}_{m \times 1}\end{aligned}$$

- System-level static load case (rigid modes are fixed)

$$\begin{aligned}\mathbf{g}_{int} + \mathbf{B}^T \boldsymbol{\lambda} &= \mathbf{g}_{eq}(t) \\ \Phi^*(q_{st}) &= \mathbf{0}\end{aligned}$$

General form of the optimization problem

- Mathematical programming approach

$$\underset{\mathbf{p}}{\text{minimize}} \quad f_0(\mathbf{p}, q_{st}(t), \boldsymbol{\lambda}(t))$$

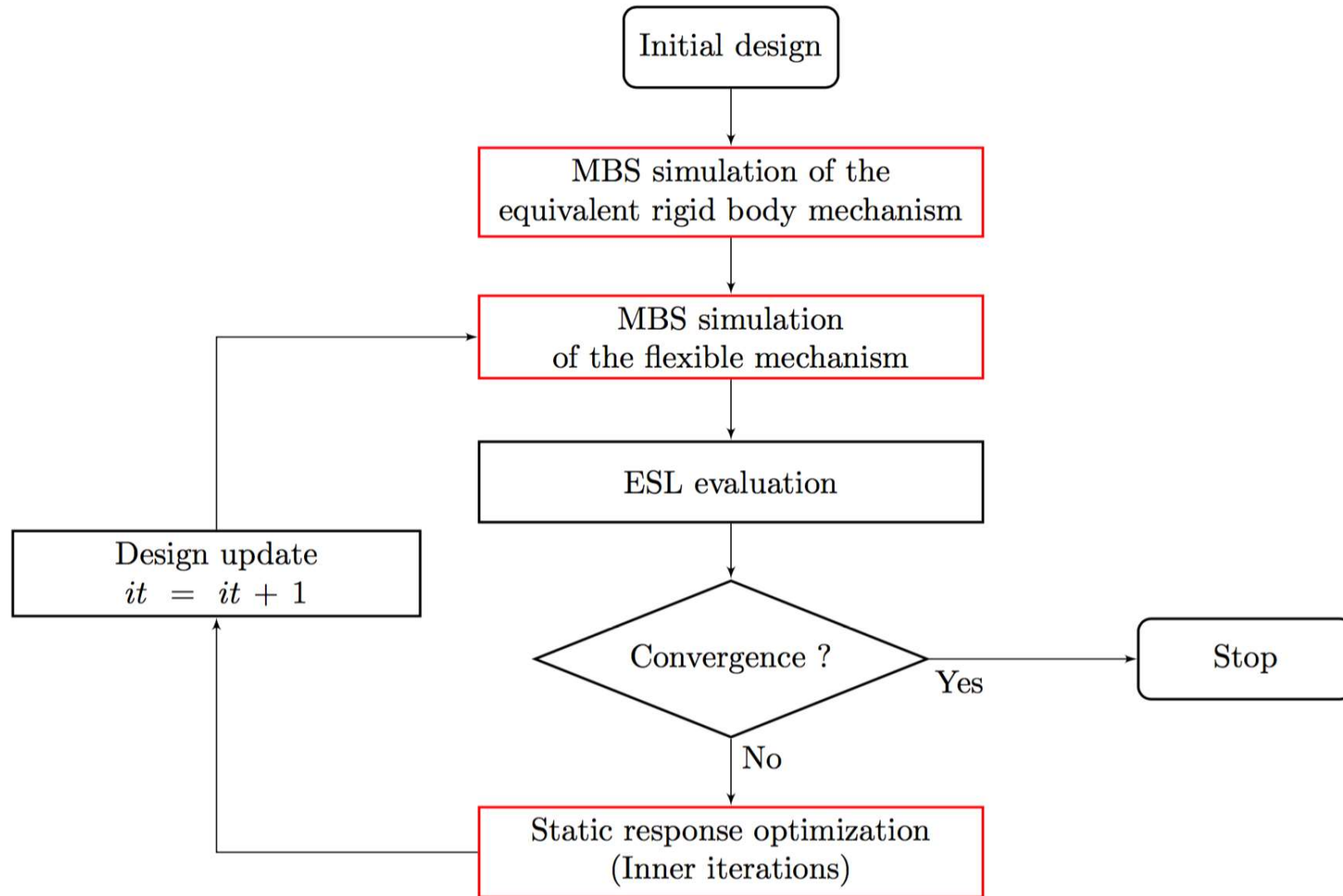
$$\text{subject to} \quad \mathbf{g}_{int} + \mathbf{B}^T \boldsymbol{\lambda} = \mathbf{g}_{eq}(t)$$
$$\boldsymbol{\Phi}^*(q_{st}) = \mathbf{0}$$

$$f_j(\mathbf{p}, q_{st}(t), \boldsymbol{\lambda}(t)) \leq \bar{f}_j(t), \quad j = 1, \dots, n_c,$$

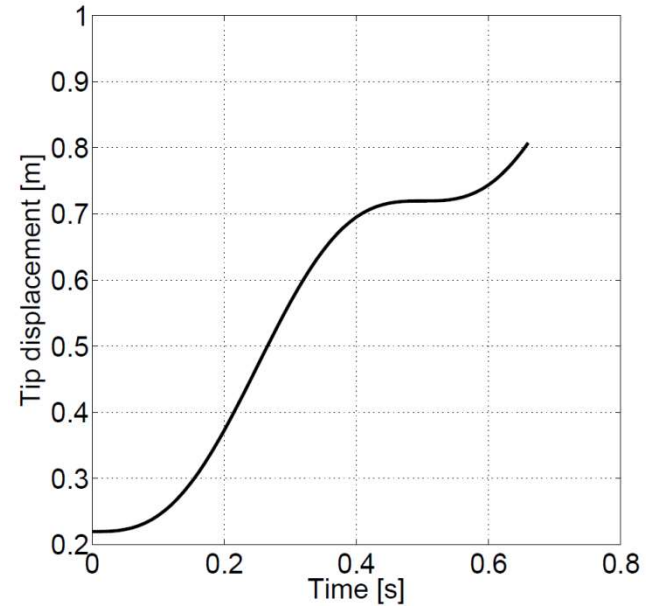
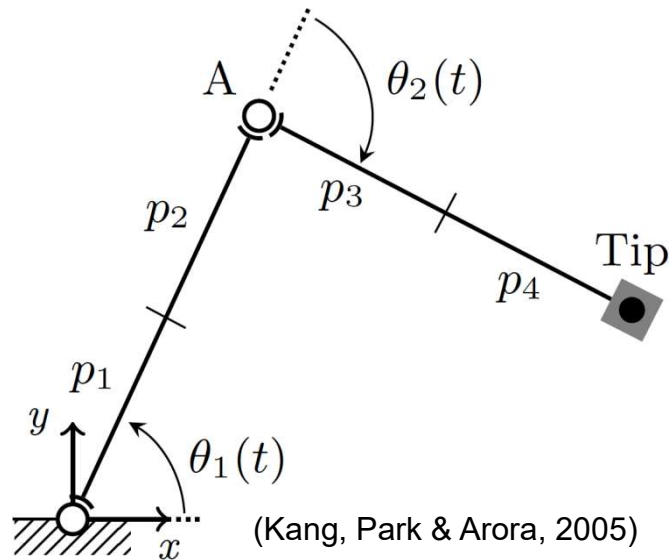
$$\underline{p}_i \leq p_i \leq \bar{p}_i, \quad i = 1, \dots, n_v.$$

- Optimizer: ConLin, MMA, GCM, IpOpt...
- Large displacements, material nonlinearities...
- Velocities and accelerations are not available
- Local frame formalism: constant tangent stiffness matrix!

Iterative scheme



Two dofs robot



minimize $W(\mathbf{p})$

subject to $Tip\ Trajectory\ Error \leq 0.001\ mm,$

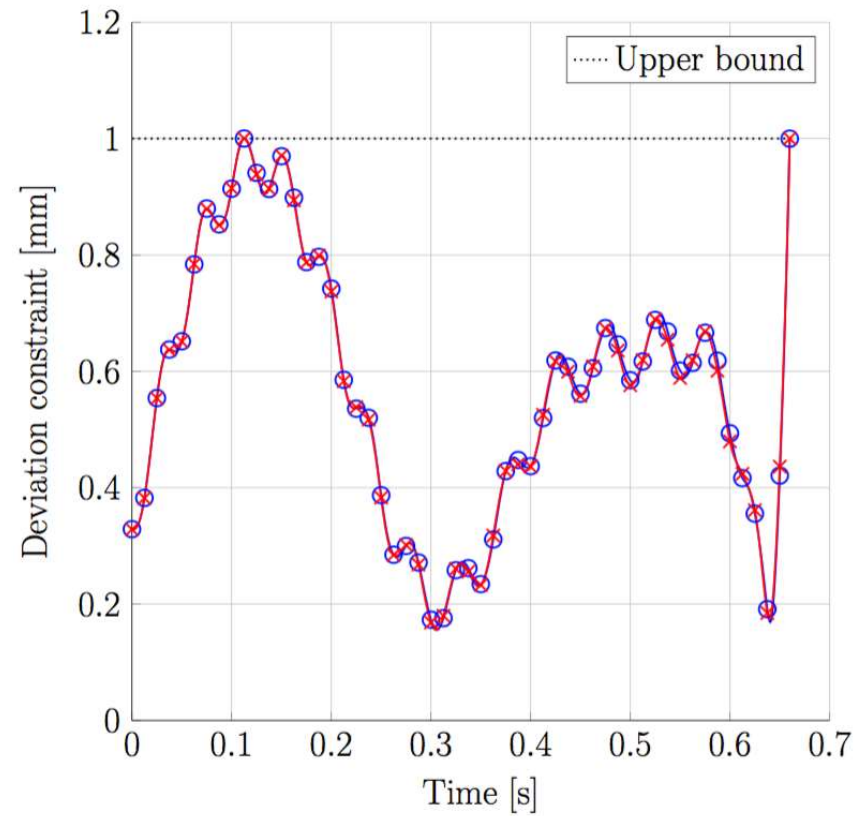
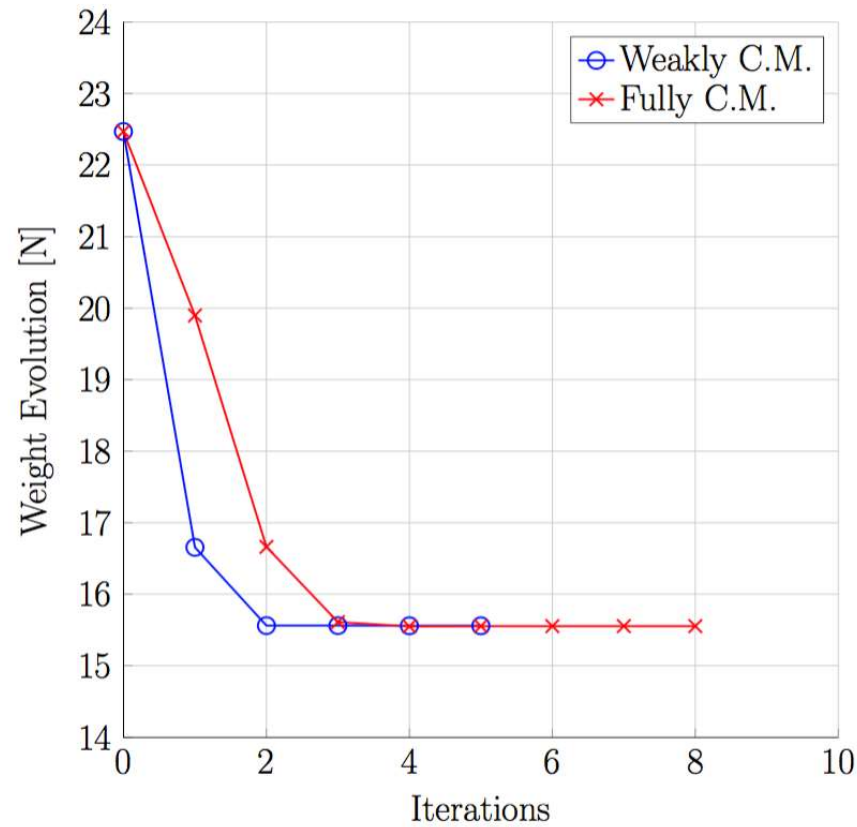
$$[-75 \ -75]^T \leq \sigma_{j,i} \leq [75 \ 75]^T \text{ MPa}, \quad j = 1, \dots, n_c,$$

$$0.02\ m \leq p_i \leq 0.06\ m, \quad i = 1, \dots, 4.$$

Multi-component constraint

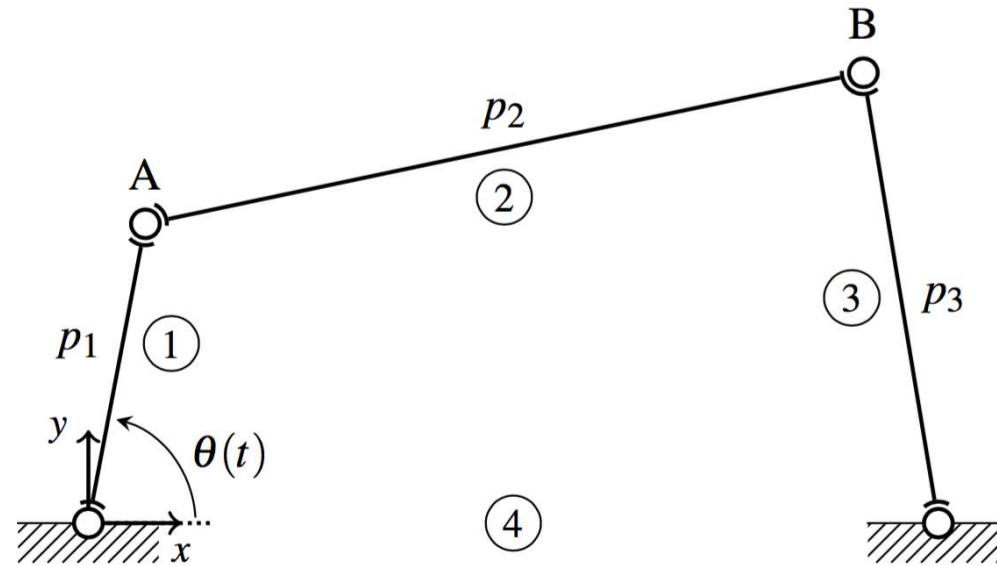
where n_c equals the number of integration time steps.

Two dofs robot



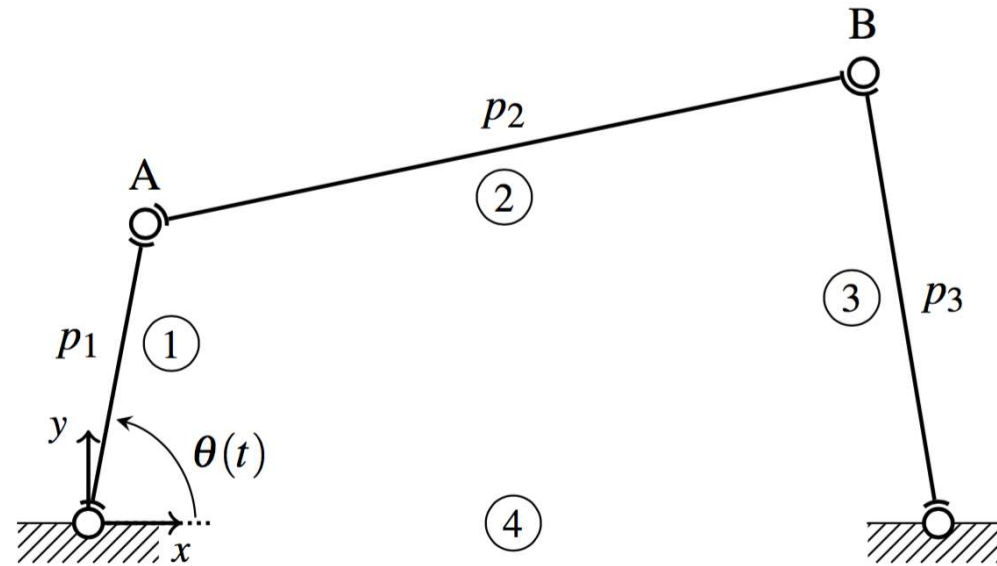
	p_1, mm	p_2, mm	p_3, mm	p_4, mm	W, N	MBS analysis	Inner iteration
Proposed method	56.18	44.55	35.02	23.60	15.56	6	11
F. C. method	55.66	45.32	34.93	23.47	15.56	9	/

Four bar mechanism



- Impose motion at the revolute joint
- 6 beam elements per arm
- 3 design variables → beam diameter

Four bar mechanism

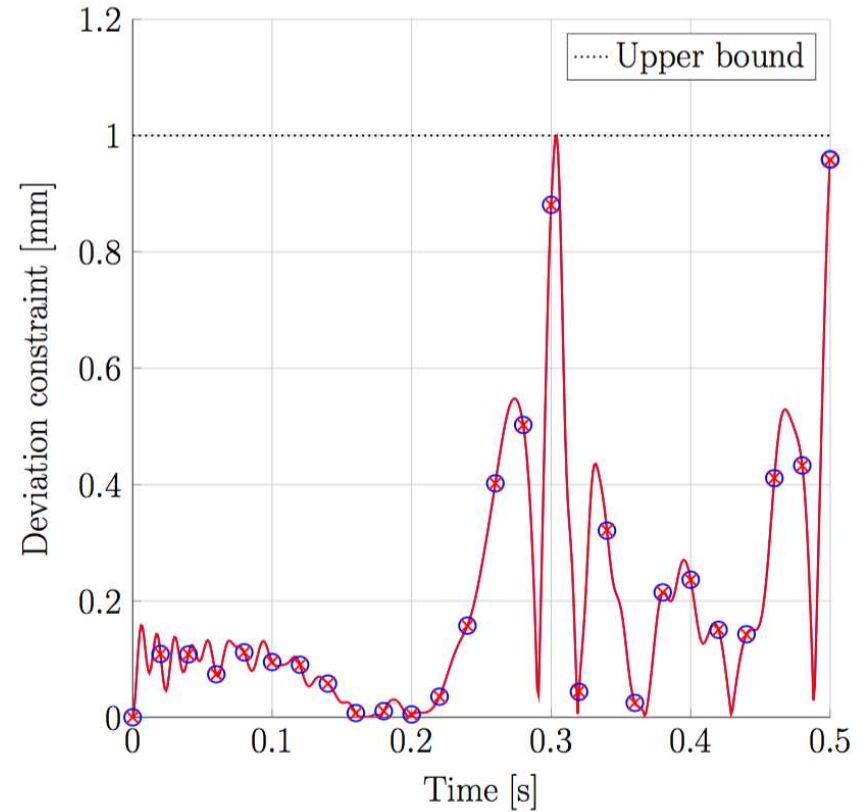
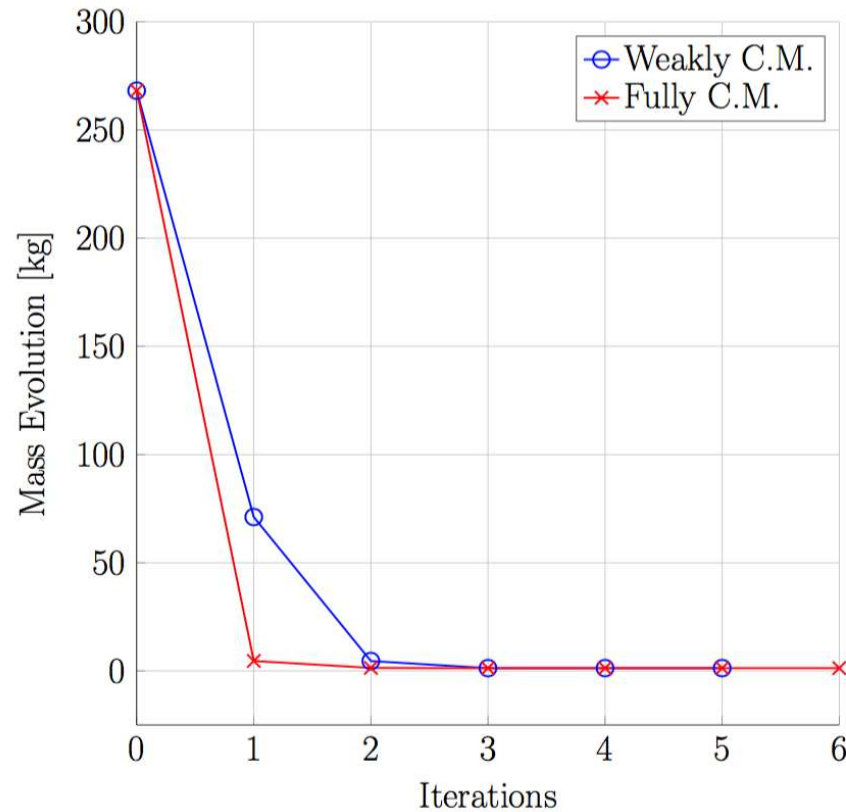


$$\underset{\mathbf{p}}{\text{minimize}} \quad m(\mathbf{p})$$

$$\text{subject to} \quad \sqrt{\Delta x_A^2(t_j) + \Delta y_A^2(t_j)} \leq 0.001 \text{ m}, \quad j = 1, \dots, 501,$$
$$0.015 \text{ m} \leq p_i \leq 0.5 \text{ m}, \quad i = 1, \dots, 3,$$

The system-based ESL naturally accounts for the closed-loop conditions.

Four bar mechanism



	p_1, mm	p_2, mm	p_3, mm	m, kg	MBS analysis	Inner iteration
Proposed method	25.717	15.000	15.000	1.253	6	19
F. C. method	25.720	15.000	15.000	1.253	7	/

Intermediate summary 3

- Optimization of mechanical systems using a system-based approach
- Equivalent static loads are defined at system level
- Local frame formalism simplifies the formulation of the equivalent static problem
- Good convergence of the weakly coupled optimization was observed in the examples

Future work:

Comparison of the efficiency between the weakly coupled method and the fully coupled method

Outline

Introduction to our research group

More about MECANO

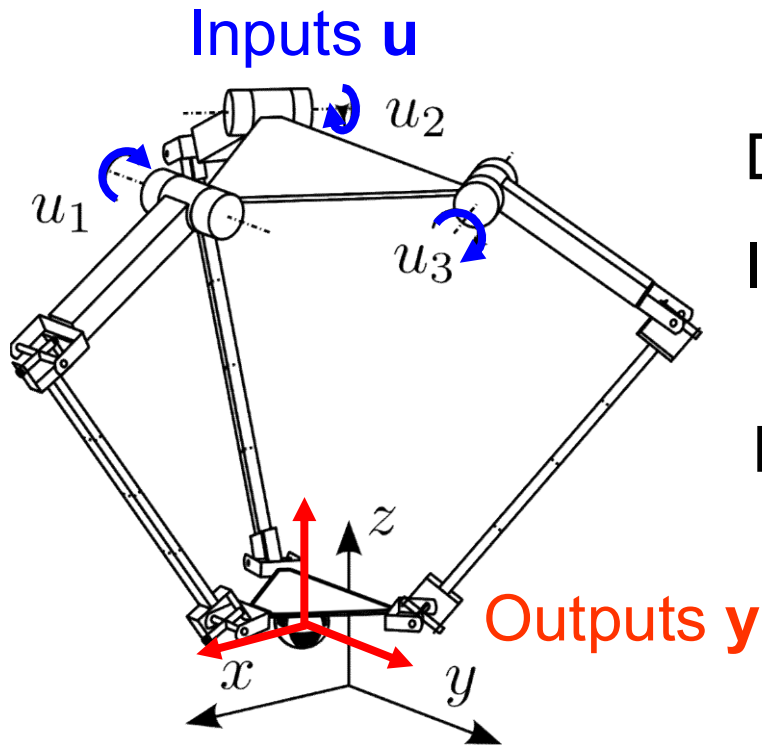
Local frame approach (rigid systems)

Local frame approach (flexible systems)

Optimization of MBS components

Control of flexible MBS

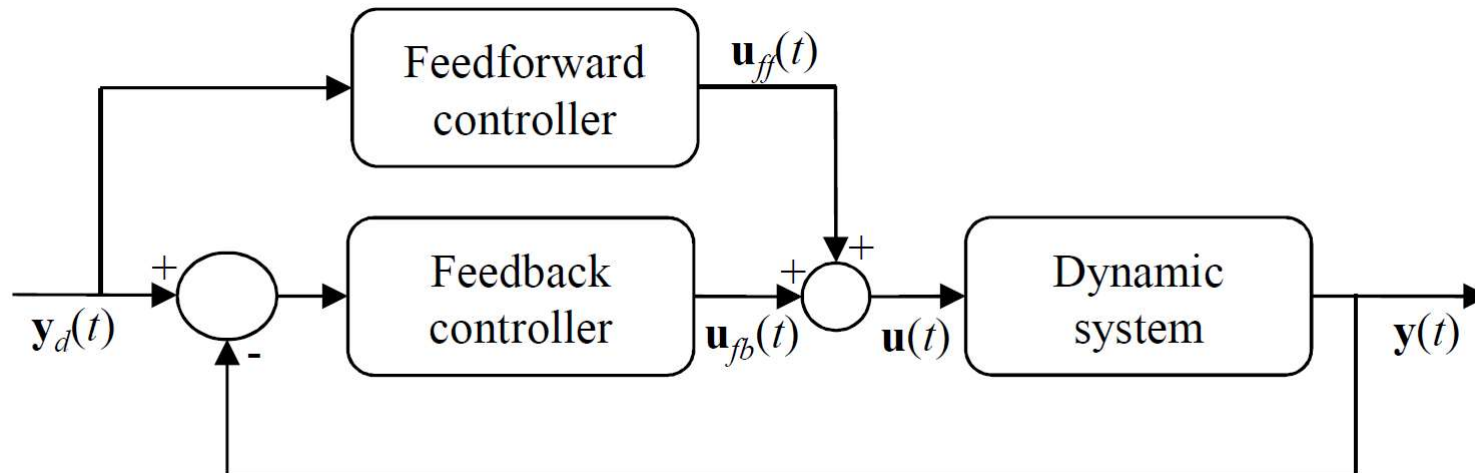
Inverse dynamics: motivation



Direct dynamics: $u(t) \longrightarrow y(t) = ?$

Inverse dynamics: $y(t) \longrightarrow u(t) = ?$

Flexible systems are underactuated



Finite element approach

\mathbf{q} = configuration variable

- collects the 3D translations & rotations of the FE nodes
- evolves a nonlinear space with a Lie group structure
- treated as a n-dimensional vector in a first step

Equations of motion - Differential Algebraic Equations (DAE)

$$\begin{aligned}\dot{\mathbf{q}} &= \mathbf{q} \tilde{\mathbf{v}} \\ \mathbf{M}(q) \dot{\mathbf{v}} + \mathbf{f}(q, \mathbf{v}) + \mathbf{B}^T \boldsymbol{\lambda} &= \mathbf{A}(q) \mathbf{u}(t) \\ \boldsymbol{\Phi}(q) &= \mathbf{0} \\ \mathbf{y} &= \mathbf{h}(q)\end{aligned}$$

Classically, it is used for simulation (forward time integration)

Here: [extension to inverse dynamics computation](#)

Inverse dynamics: formulation

For a given $\mathbf{y}_d(t)$, find $\mathbf{u}(t)$ such that

$$\begin{aligned}\dot{q} &= q \tilde{\mathbf{v}} \\ \mathbf{M}(q) \dot{\mathbf{v}} + \mathbf{f}(q, \mathbf{v}) + \mathbf{B}^T \boldsymbol{\lambda} &= \mathbf{A}(q) \mathbf{u} \\ \Phi(q) &= \mathbf{0} \\ \mathbf{h}(q) &= \mathbf{y}_d(t)\end{aligned}$$

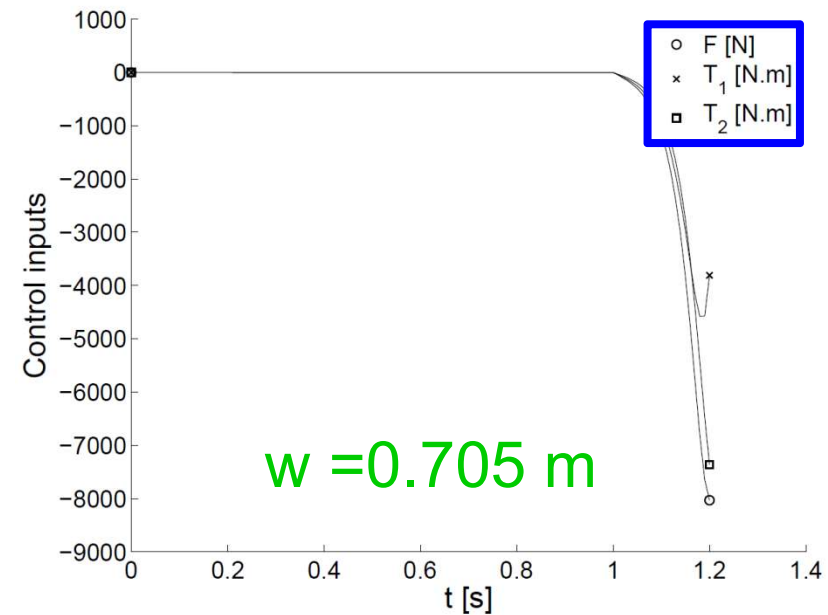
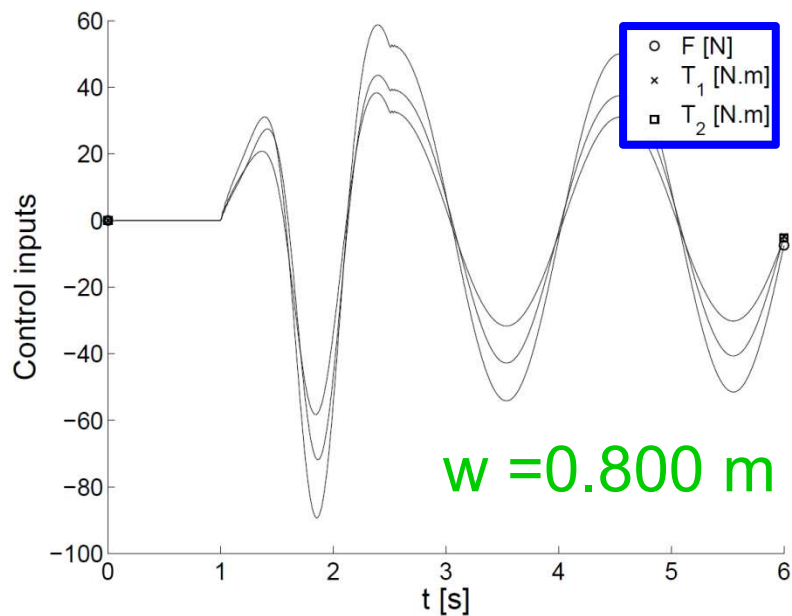
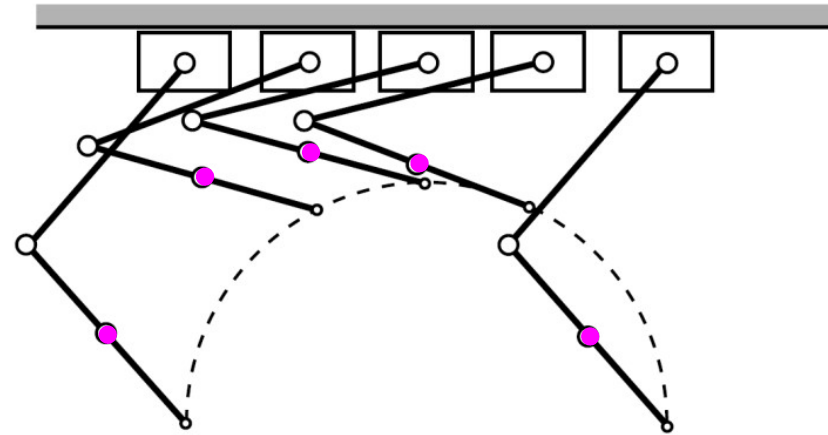
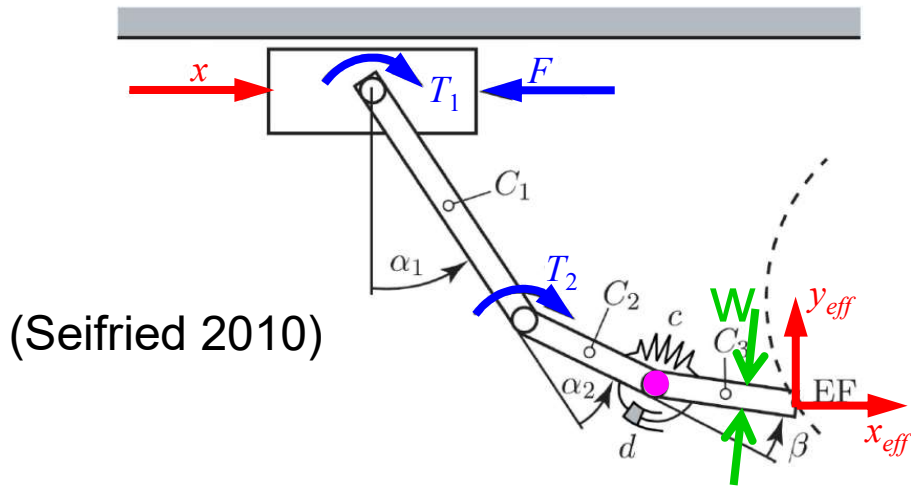
Servo constraint

(Blajer & Kolodziejczyk 2004)

This DAE can have 0, one, several or an infinite number of solutions, which are not necessarily causal.

If $\dim(\mathbf{u}) = \dim(\mathbf{y})$, a meaningful solution can be obtained by forward time integration of the DAE only if the internal dynamics is stable.

Forward DAE integration: example



DAE stable inversion

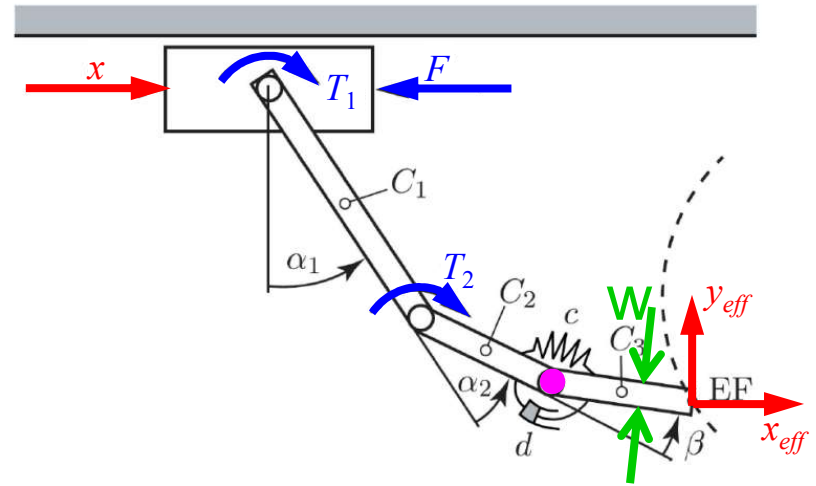
The index 3 DAE is an **implicit representation of the internal dynamics**

$$\begin{aligned}\dot{q} &= q \tilde{\mathbf{v}} \\ \mathbf{M}(q) \dot{\mathbf{v}} + \mathbf{f}(q, \mathbf{v}) + \mathbf{B}^T \boldsymbol{\lambda} &= \mathbf{A}(q) \mathbf{u} \\ \boldsymbol{\Phi}(q) &= \mathbf{0} \\ \mathbf{h}(q) &= \mathbf{y}_d(t)\end{aligned}$$

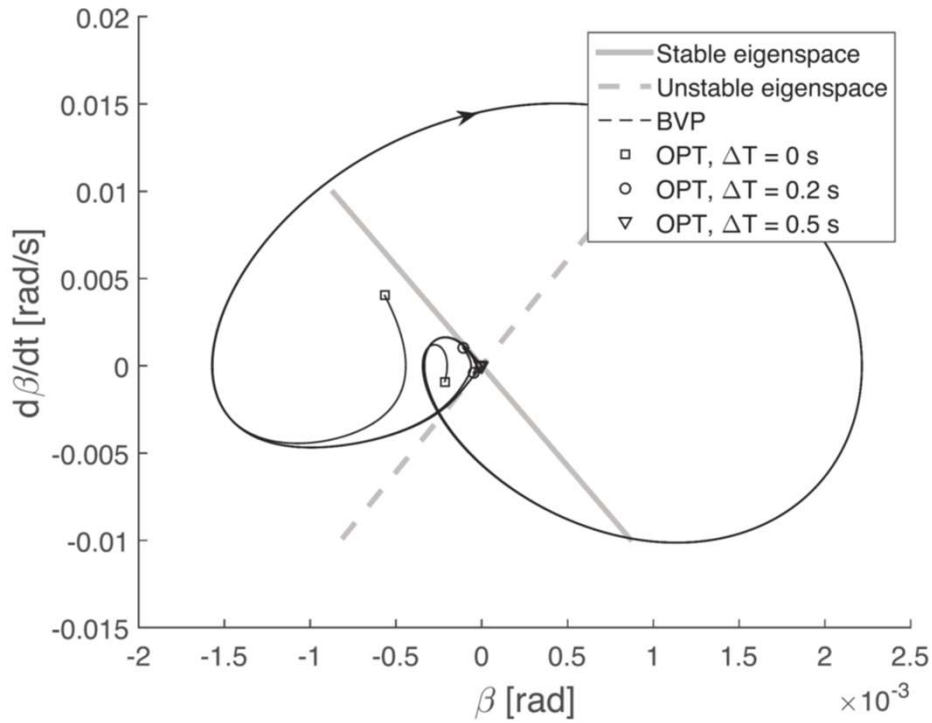
- Initial conditions are relaxed \Rightarrow Stable inversion methods:
 - Boundary Value Problem (Devasia, Chen & Paden 1996)
 - Optimization (Bastos, Seifried & B. 2013)

- Numerical solution based on **DAE methods**
 - direct collocation (Bastos, Seifried & B. 2013)
 - multiple shooting (B., Bastos & Seifried 2014)
 - in both cases, generalized- α time discretization
 - extension to Lie group systems (Lismonde, Sonnevillle & B. 2016)

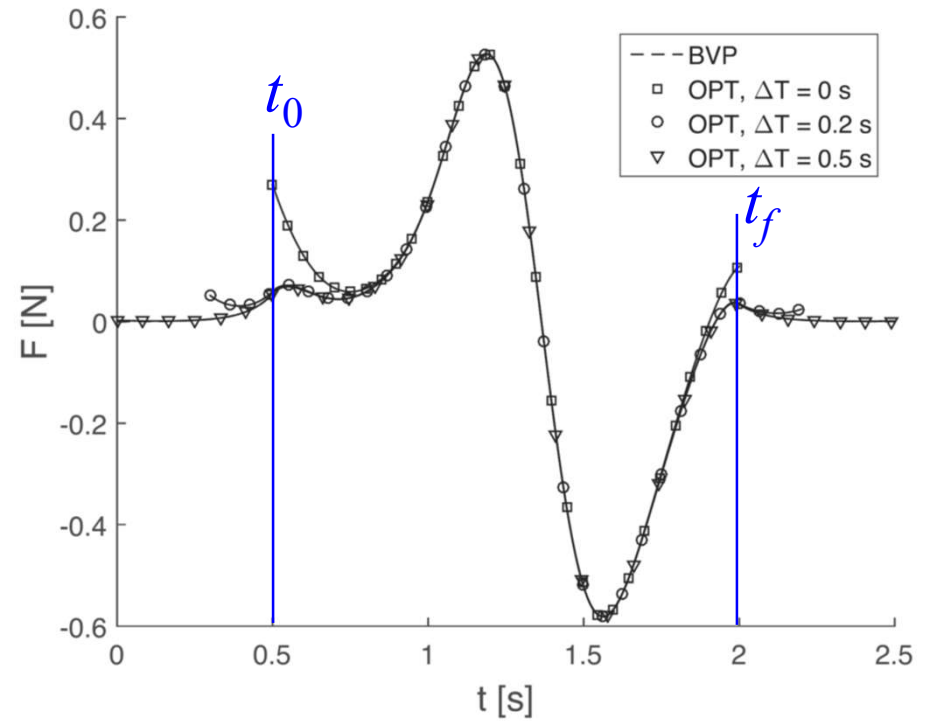
Example



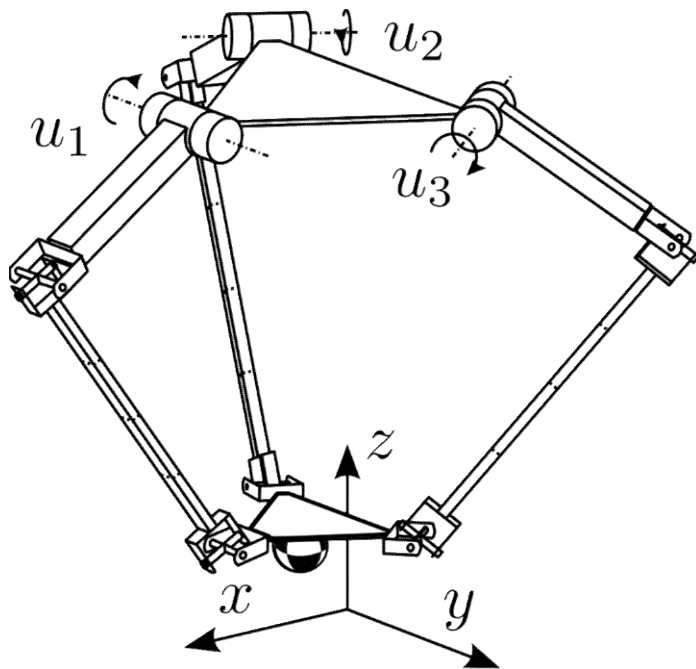
Internal dynamics trajectory



Control force F

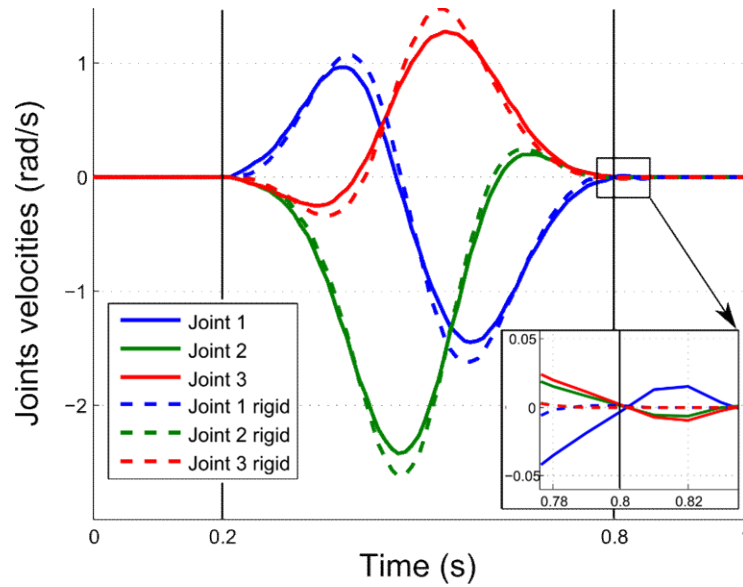


Parallel robot

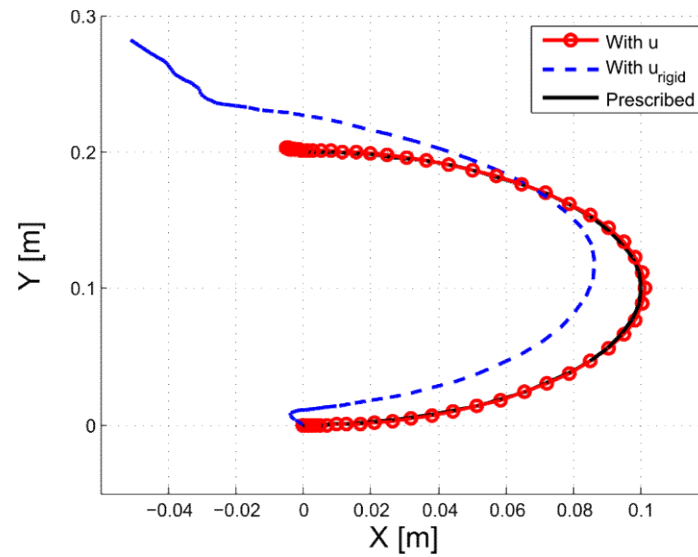


- Parallel robot with **3 rigid dof**.
- Made up of 2 tubular links (1/10 thick):
 1. **Rigid links** (3):
Alu, 0.25 x 0.05 x 0.05 m.
 2. **Flexible links** (3 x 4 beams):
Alu, 0.51 x 0.075 x 0.0075 m.
- **Point mass** at the end-effector (0.1 kg).
- Trajectory: **half-circle** with 0.1 m radius in the **xy plane**, to be completed in **0.6 s**.
- Analysis: **1st unstable pole at 24 Hz**.

Parallel robot



Velocity of joints before and after optimization



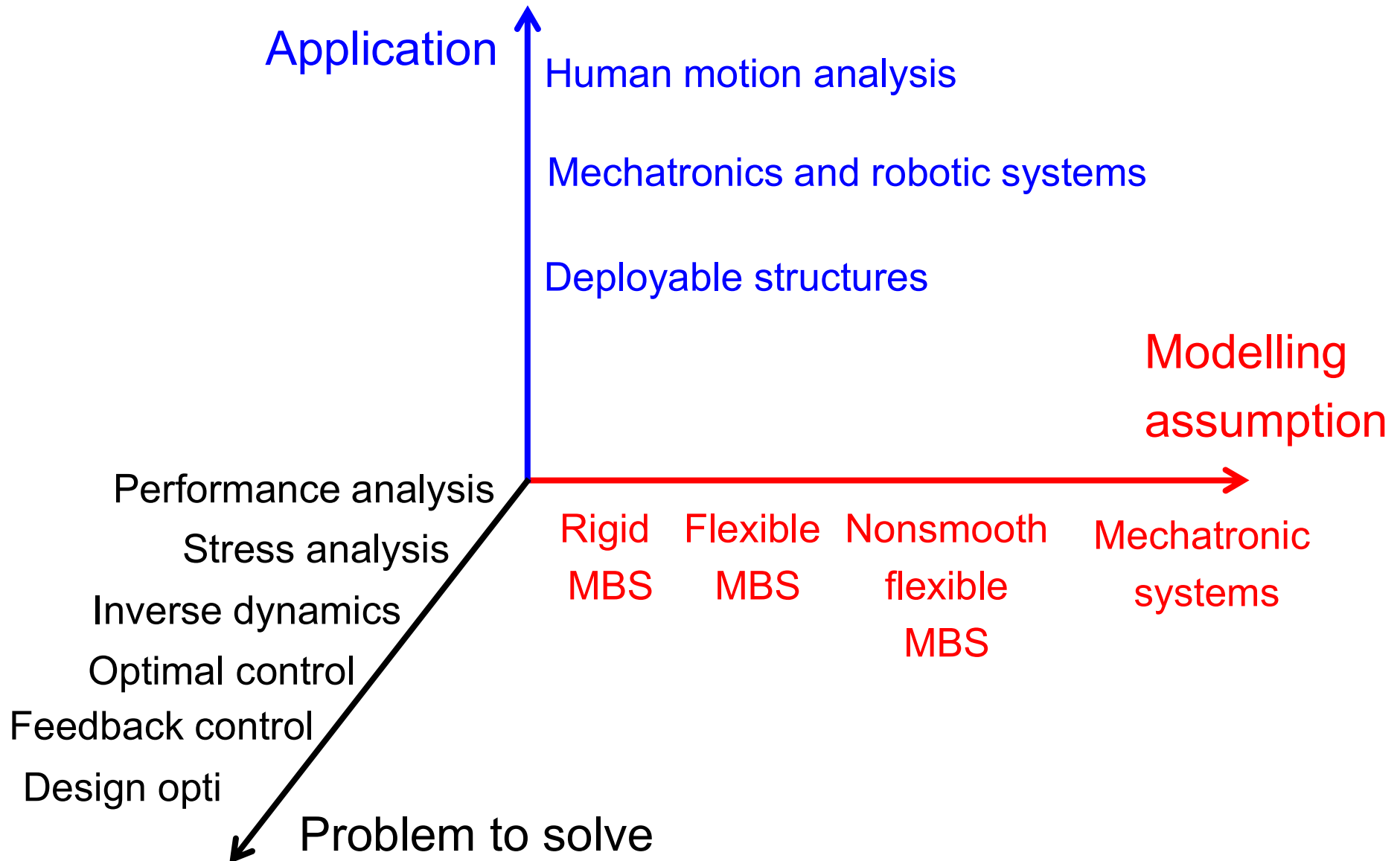
Actual output trajectories before and after optimization

Intermediate summary 4

Inverse dynamics of flexible systems

- A stable inversion is needed to obtain a bounded solution
- The FE approach leads to an implicit DAE formulation (no need to derive the I/O normal form)
- Formulation as a
 - DAE BVP on a Lie group
 - DAE optimization problem on a Lie group
- Numerical solution by
 - multiple shooting
 - direct collocation
- The method was successfully applied to the model of a 3D parallel kinematic manipulator with flexible links

Multibody & Mechatronic Systems Lab



Merci de votre attention !

Flexible multibody dynamics:
From FE formulations to control and optimization

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Department of Aerospace and Mechanical Engineering (LTAS)

University of Liège, Belgium



INRIA Rhône-Alpes, Grenoble, July 3, 2017

