

# *Data Driven Model Learning*

## Homeworks

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### Lesson 2: Singular Spectrum Analysis

1. Consider the following theorem:

**Theorem 0.1. Singular Value Decomposition.** *Any  $m \times n$  matrix  $A$  can be factored into*

$$A = U\Sigma V^T = (\textit{orthogonal})(\textit{diagonal})(\textit{orthogonal})$$

*The columns of  $U$  ( $m \times n$ ) are eigenvectors of  $AA^T$ , and the columns of  $V$  ( $n \times n$ ) are eigenvectors of  $A^T A$ . The  $r$  singular values on the diagonal of  $\Sigma$  ( $m \times n$ ) are the square roots of the nonzero eigenvalues of both  $AA^T$  and  $A^T A$ .*

Perform the SVD (showing all steps WITHOUT a computer) of the matrix:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \\ 0 & 0 \end{bmatrix}$$

2. Propose an algorithm to “rehankelize” a matrix, e.g. based on the diagonal averaging step as proposed by Nina Golyandina and Anatoly Zhigljavsky in *Singular Spectrum Analysis for Time Series*, Springer 2020. <https://link.springer.com/content/pdf/10.1007/978-3-642-34913-3.pdf>