Human Friendly Control : an application to Drive by Wire

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Control Objectives

– safety of the driver and performance $\Rightarrow$ passivity and transparency,
– linear approximation of the plant,
– optimal multi-objective synthesis using Linear Matrix Inequalities (LMI).

Outline

1. System Presentation and objectives
2. Control problem expression
3. Control Synthesis
4. Simulations and testing ground

1. System Presentation and objectives

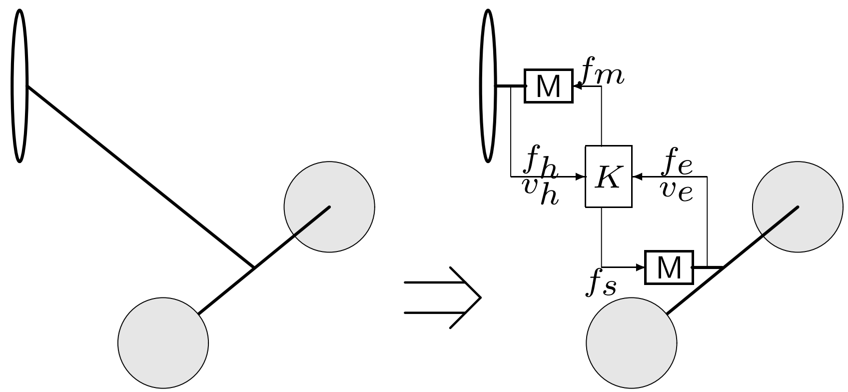

FIG. 1: Schematic principle of the system

The Plant

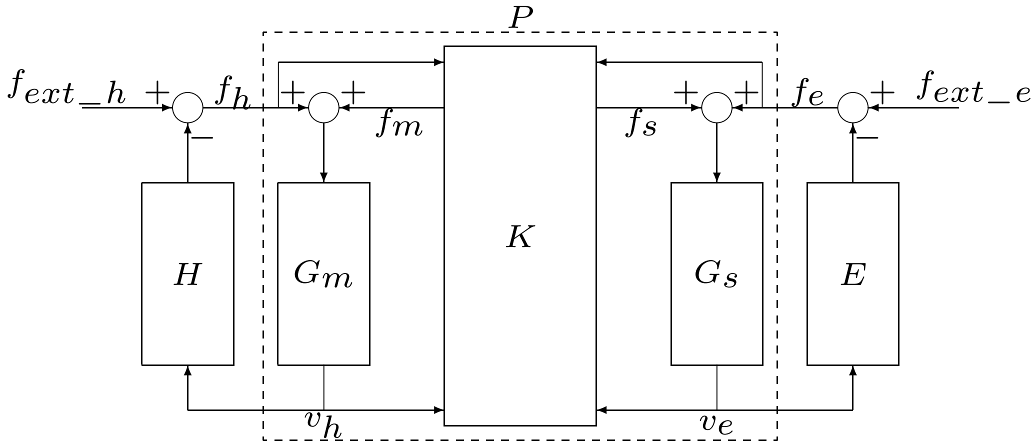

FIG. 2: Typical Structure of a manipulator

Control objectives

The model of Hu-Salcudean-Loewen, 95 :

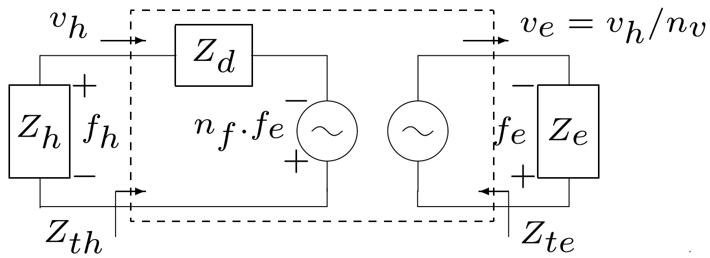

FIG. 3: 2 ports representation of an ideal manipulator idéal

$\Rightarrow$ Use K and n_f, n_v to shape the desired impedance of the system satisfying the control objectives.

Coupled Transparency (impedance)

Defines a desired mapping between force and speed.

$$v_h^d = y^d \left(f_h + n_f f_e \right) \tag{1}$$

$$v_e^d = y^d \left(\frac{1}{n_v} f_h + \frac{n_f}{n_v} f_e \right) \tag{2}$$

$\Rightarrow$ minimize $\tilde{v}_h \doteq v_h^d - v_h$ and $\tilde{v}_e \doteq v_e^d - v_e$

with adequate filters to set the frequency domain of minimization.

Passivity

Bilateral coupled passivity :

$$< v_h, F_h > \doteq \int_0^\infty v_h^T F_h dt > -\beta \tag{3}$$

$$< v_e, F_e > \doteq \int_0^\infty v_e^T F_e dt > -\beta \tag{4}$$

That is, Z_{th} and Z_{te} have to be passive.

2. Control problem expression

Given a desired admittance $Y_d(s)$, the goal is to find a control K such that :

$i.$ Closed loop transparency, $Y_t(K) = Y_d$.

$ii.$ Closed loop passivity of $Z_{th}(K) = Z_t(K) + \frac{n_f}{n_v} Z_e$ and $Z_{te}(K) = \frac{n_v}{n_f} Z_t(K) + \frac{n_v}{n_f} Z_h$, where $Z_t(K)$ is the actual impedance of the system.

⇒ but it may not have a solution and Z_h, Z_e are unknown,

⇒ relax $i.$ and extend $ii.$:

$iii.$ $\min_K \|Y_t(K) - Y_d\|$, with

$iv.$ $Y_t(K) : F \rightarrow v$ ESPR.

3. Control Synthesis

State-space representation of the plant P and the control K :

$$\Sigma_P : \begin{cases} \dot{x} = Ax + B_w w + Bu \\ z = C_z x + D_{zw} w + D_z u \\ y = Cx + D_w w \end{cases} \qquad \Sigma_K : \begin{cases} \dot{\zeta} = A_K \zeta + B_K y \\ u = C_K \zeta + D_K y \end{cases} \tag{5}$$

Criterion specification :

$$z = T_{zw} w = \begin{pmatrix} T_{vw} \\ T_{\tilde{v}w} \end{pmatrix} w \quad \text{et} \quad \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B}_j \\ \hline \mathcal{C}_j & \mathcal{D}_j \end{array}\right] = \left[\begin{array}{cc|c} A + BD_K C & BC_K & B_j + BD_K F_j \\ B_K C & A_K & B_K F_j \\ \hline C_j + E_j D_K C & E_j C_K & D_j + E_j D_K F_j \end{array}\right]$$

with $B_j = B_w R_j$, $C_j = L_j C_z$, $D_j = L_j D_{zw}$, $E_j = L_j D_z$, $F_j = D_w R_j$.

$\Rightarrow$ Express the system into two transfer functions ; one for each objective.

LMI expression :(S.Boyd et al. 94)

$iii.$: equivalent to $\|T_{\tilde{v}w}\|_\infty < \gamma$, $\Rightarrow$ Bounded Real lemma) :

$$\begin{pmatrix} \mathcal{A}^T\mathcal{P} + \mathcal{P}\mathcal{A} & \mathcal{P}\mathcal{B}_2 & \mathcal{C}_2^T \\ \mathcal{B}_2^T\mathcal{P} & -\gamma I & \mathcal{D}_2^T \\ \mathcal{C}_2 & \mathcal{D}_2 & -\gamma I \end{pmatrix} > 0, \ \mathcal{P} > 0 \tag{6}$$

$iv.$: Positive Real Lemma :

$$\begin{pmatrix} \mathcal{A}^T\mathcal{P} + \mathcal{P}\mathcal{A} & \mathcal{P}\mathcal{B}_1 - \mathcal{C}_1^T \\ \mathcal{B}_1^T\mathcal{P} - \mathcal{C}_1 & -\mathcal{D}_1^T - \mathcal{D}_1 \end{pmatrix} > 0, \ \mathcal{P} > 0 \tag{7}$$

Singularity problem

$$\mathcal{D}_1 \text{ singular} \Rightarrow \begin{pmatrix} \mathcal{A}^T\mathcal{P} + \mathcal{P}\mathcal{A} & \mathcal{P}\mathcal{B}_1 - \mathcal{C}_1^T \\ \mathcal{B}_1^T\mathcal{P} - \mathcal{C}_1 & -\mathcal{D}_1^T - \mathcal{D}_1 \end{pmatrix} > 0, \ \mathcal{P} > 0$$

implies to combine a LMI with a LME.

To solve this problem (Khalil, 96) :

- Introduce a fictitious sector bound non-linearity simulating some uncertainties,
- use the circle criterion to design K such that the closed-loop fictitious system is SPR,
- conclude that the original system is SPR.

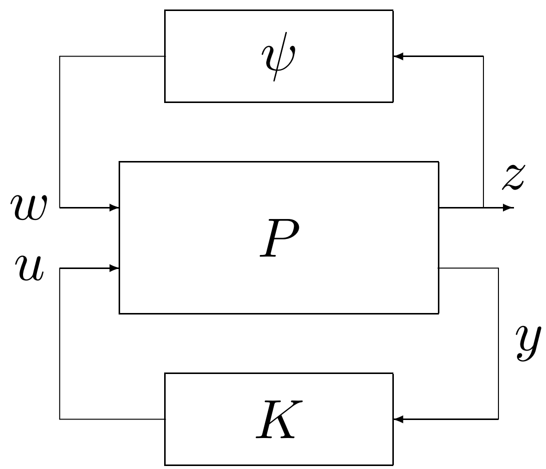

FIG. 4: Fictitious system

Where ψ is a sector bound non-linearity defined by :

$$[\psi(t)z - Q_{min}z]^T[\psi(t)z - Q_{max}z] \leq 0, \quad \forall\ t \geq 0, \quad \forall\ z \in \Gamma \subset I\!R^p \quad (8)$$

The resulting fictitious system is :

$$\left(\begin{array}{c|cc} \bar{A} & \bar{B}_w & \bar{B} \\ \hline \bar{C}_z & \bar{D}_{zw} & \bar{D}_z \\ \bar{C} & \bar{D}_w & \bar{D} \end{array}\right) = \left(\begin{array}{c|cc} A - B_w Q_{min} C_z & B_w & B \\ \hline (Q_{max} - Q_{min}) C_z & I & 0 \\ C - D_w Q_{min} C_z & D_w & 0 \end{array}\right) \tag{9}$$

The identity matrix in "D" allows for the use of the positive real lemma.

Optimal multi-objective synthesis using LMI (Scherer-Gahinet-Chilali, 97)

- Two LMIs to solve simultaneously, with $\mathcal{P} > 0$,
- a linearizing change of variables to include the closed-loop expression,
- A, B et C real and fictitious have to be the same $\Rightarrow Q_{min} = 0$,
- a unique control K can be found, its order is the same as the system,
- two parameters still need to be fixed : γ et Q_{max}.

Analysis tools

- passivity : Nyquist plot (SPR),
- transparency : $\mathcal{H}_\infty$ norm and Bode plot,

4. Simulations and testing ground

$G_m(s)$ was given by :

$$G_m(s) = \frac{1}{a_m s + b_m} = \frac{1}{0,0222s + 0,0042} \qquad (10)$$

The desired admittance was chosen as :

$$y^d = \frac{1}{0,1s + 1} \qquad (11)$$

Some 'good' values for the remaining parameters :

$\gamma = 0.7$, $qh_{max} = 0.892$ and $qe_{max} = 0.874$.

The Control obtained has the form :

$$K = \frac{1}{D}\left[\begin{array}{cccc} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{array}\right] \qquad (12)$$

with D and K_{ii} some 4^{th} order polynomials.

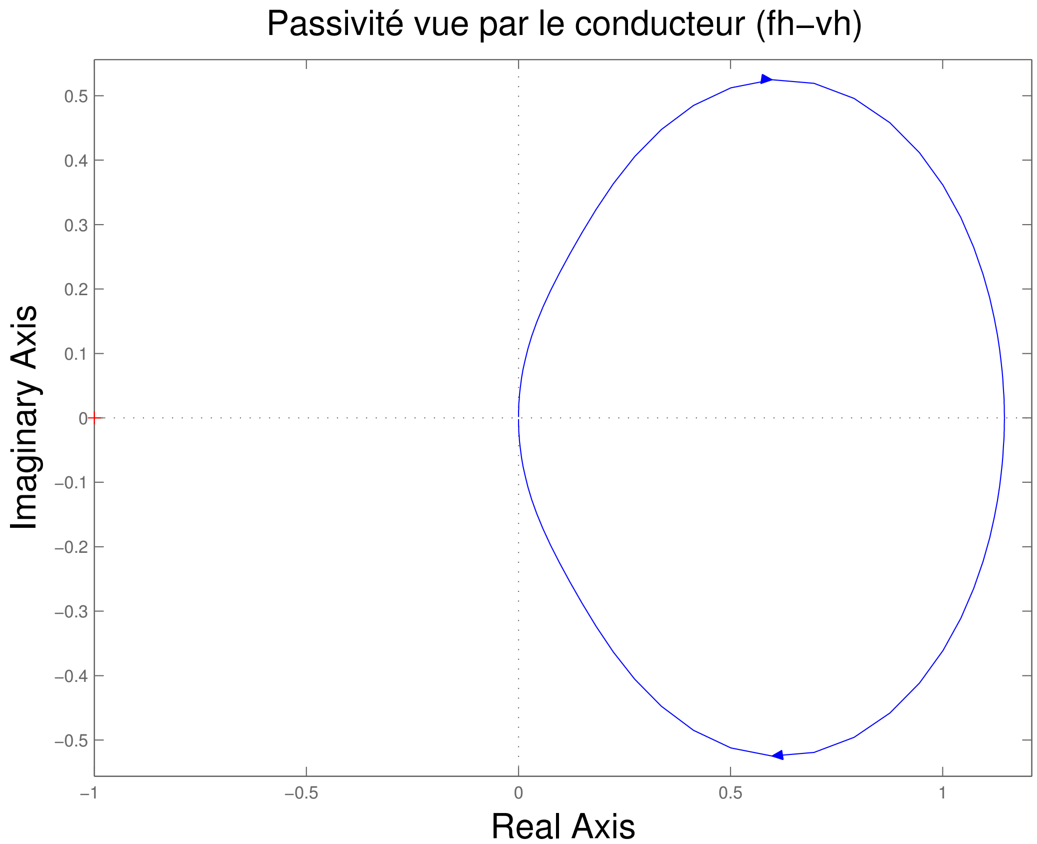

FIG. 5: Passivity of the closed-loop system

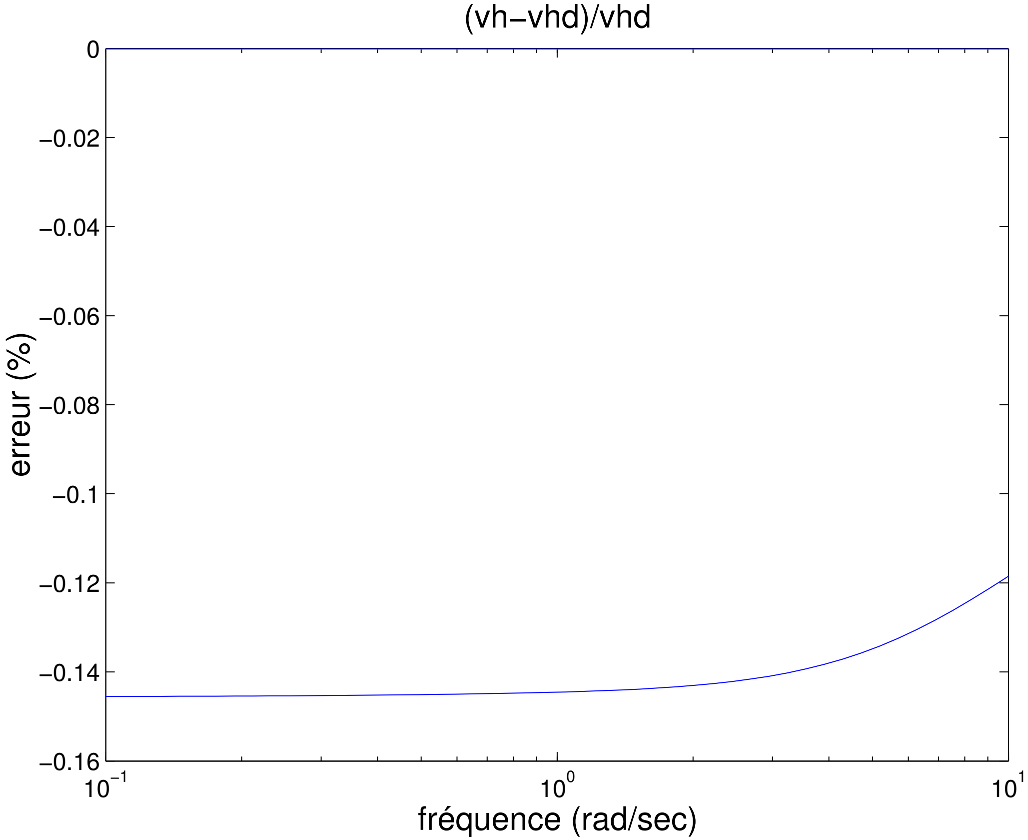

FIG. 6: Minimized criterion $\tilde{v}$

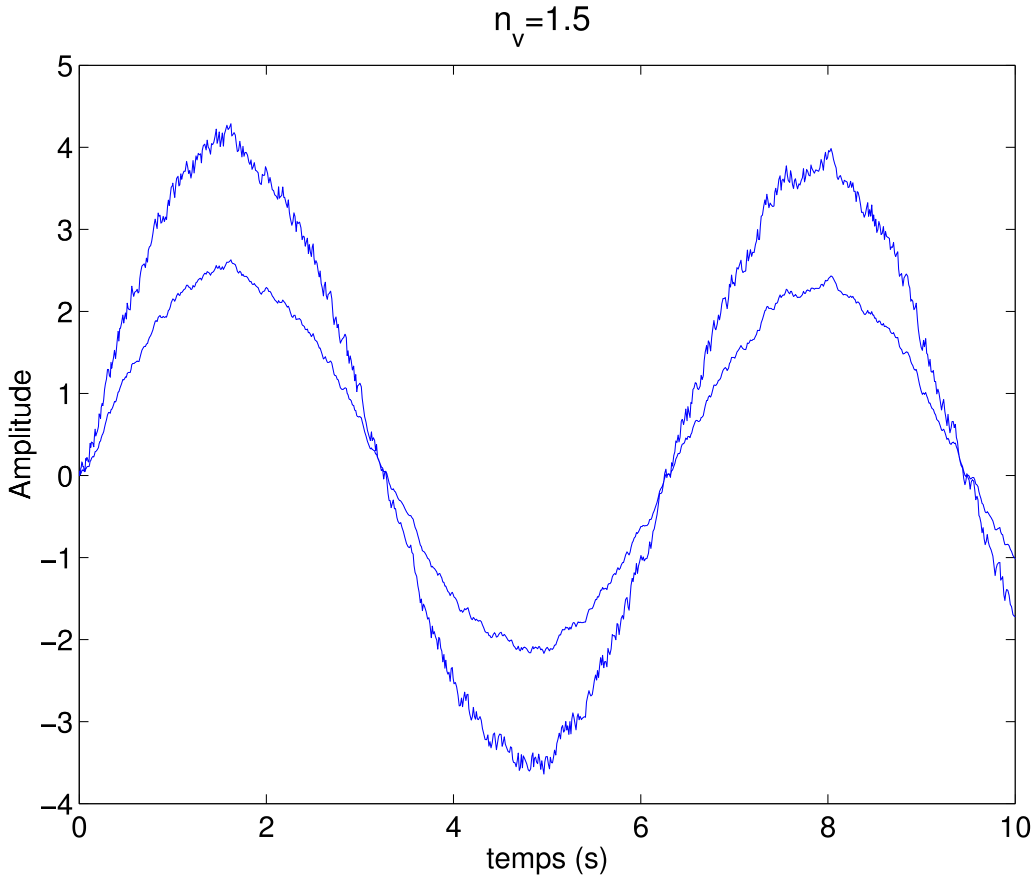

FIG. 7: v_h and v_e with a speed factor $n_v = 1,5$ (input force = sinusoïde + noise)

FIG. 8: Testing ground

Conclusions

– Different approaches were explored for the transparency,
– the control has been obtained with some tuning parameters, allowing for the freedom of the user,
– the simulations results are satisfying and the procedure is validated,
– experimental results were also obtained.