REMOTE OUTPUT STABILIZATION UNDER TWO CHANNELS TIME-VARYING DELAYS

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Abstract: In this paper we investigate the problem of remote output stabilization via two channels with time-varying delays. This problem arises when the control law is remotely implemented. The exchange of data between the controller and the system is done via two different transmission channels introducing time-varying delays with known dynamics. Assuming a known model for each of the time-delay dynamics, the work in (Witrant *et al.*, 2003) is extended to the case of two different delays appearing: (1) in the output measurement channel and, (2) in the transmission of the control law. This result is extended to output stabilisation by introducing a state observer built upon the delayed output of the plant. Simulation results are also presented.

Keywords: Time delays, time-varying systems, networks, stability, observers.

1. INTRODUCTION

The networked control systems constitute a new class of control systems including specific problems such as delays, loss of information and data process. The problem studied in this paper concerns the remote stabilization by output injection of unstable open-loop systems. The control setup is shown in Figure 1. The sensor, actuator and system are assumed to be remotely commissioned by a controller that interchange measurements and control signals through a communication network. We assume that this communication network has its own dynamics, including the communication channel as well as the coder and decoder, and that we have a known model for it (as the one developed in (Misra *et al.*, 2000) for a TCP network). In addition, it is postulated that the transmission dynamic is not symmetric in the sense that the model dynamics for the channel transmitting the system output to the controller, is different to the one of the channel transmitting the control signal from the controller to the system.

The impact of such network is to introduce a *time-varying* delay in the data transmission between the system and the controller. An inherent difficulty of this type of system is that the time-translation operation is not reversible, i.e. $y(t) \neq y(t + \tau(t) - \tau(t + \tau(t)))$, unless the delay τ is constant.

Most of the the existing control methods (like the Lyapunov-Krasovskii approaches) either assume a *constant time-delay*, or a known upper bound on it (Niculescu *et al.*, 1998). For constant time-delays, methods like the "pole-placement" allows to cope with stable and unstable open-loop plants (Manitius and Olbrot, 1979). The constant delay has also been successfully treated in the context of teleoperation systems in (Anderson and Spong, 1989) and (Niemeyer and Slotine, 1991).

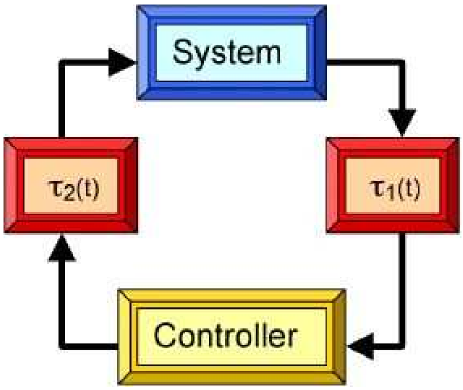

Fig. 1. Two channel delay closed-loop network controlled system.

The case of time-varying or state-dependent delays can be treated along the solutions presented in (Yu, 1999), and (Verriest, 2002) as long as the system is open-loop stable. The variable time delay characterized by a probabilistic distribution or a Markov chain was studied and included in an optimal LQG control in (Nilsson, 1998).

This work presents an extension of (Witrant *et al.*, 2003), where we addressed the problem of remote stabilisation via a network modeled as a time-varying delay from the controller to the plant. The main contribution of this paper is to extend the previous results to a two channels varying delay, meaning that the signal going from the plant to the controller also experiences a delay. The second contribution concerns the design of an observer-based controller, allowing for remote output stabilisation

The aim of this paper is then to explore how the transmission protocol dynamics of the two channels can be explicitly used in the design of the control feedback. Before dealing with a particular transmission protocol dynamics, we aim at exploring how the control design can be elaborated for a system where the transmission delay is given by a particular autonomous stable system for each of the transmission channel . More precisely, we consider systems of the form:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t-\tau_2(t)) && (1)\\
y(t) &= Cx(t) && (2)\\
\dot{z}_i(t) &= W_i(z_i,t) + F_i(z_i,t)\tau_{diz}(t), && (3)\\
\tau_i(t) &= E_i(z_i,t) + R_i(z_i,t)\tau_{di\tau}(t),\ i=1,2 && (4)
\end{aligned}$$

where the pairs (A,B) and (A,C) are assumed to be controllable and observable, respectively, but no assumption is made on the stability of A. We then allow for unstable open-loop systems as well. The notation $\dot{x}(t)$ stands for the derivative with respect to time dx/dt. The signal $\tau_{di}(t)$, $z_i(0)$ and the matrices W_i, F_i, E_i, R_i are assumed to be known, and W to be stable. Equations (3) and (4) describe the internal delay dynamics representing the transmission channel model, where $\tau_1(t)$ is the transmission delay from the plant to the control and $\tau_2(t)$ is the delay from the control to the plant. The control can only use delayed information, that is only

$$\underbrace{x(t-\tau_1(t))}_{\text{state feedback}} \quad \text{or} \quad \underbrace{y(t-\tau_1(t))}_{\text{output feedback}}$$

are the available signals for feedback.

We assume that all solutions of model (3)-(4), have the following property

$$\tau_i^{max} \geq \tau_i(t) \geq 0 \quad \forall t \geq 0, \quad \forall, i$$

This property, in the network framework, means that even if a packet is lost it will be re-emited until it reaches its target (the TCP protocol is an example of such network).

2. SINGLE CHANNEL CASE

In this section we revisited the single channel delay case studied in (Witrant *et al.*, 2003) which is used as a base for the subsequent developments. Consider the equations (1)-(4), with $\tau_1 = 0$, and the simple problem of state feedback stabilization, with a full measurable state, i.e. $y(t) = x(t)$.

Defining a new input $v(t)$ as

$$v(t) = u(t - \tau_2(t)) \tag{5}$$

and introducing a bounded time-dependent function $\infty > \delta(t) \geq 0$ (to be defined later), the system (1) shifted by $\delta(t)$ is expressed as

$$\frac{dx}{dt}(t+\delta(t)) = Ax(t+\delta(t)) + Bv(t+\delta(t)) \tag{6}$$

Assuming that the stabilizing control law achieving the pole placement on the time-shifted system given by:

$$v(t+\delta(t)) = -Kx(t+\delta(t)) \tag{7}$$

can be realized, then the resulting closed-loop dynamics is:

$$\frac{dx}{dt}(t+\delta(t)) = (A-BK)x(t+\delta(t)) = A_{cl}x(t+\delta(t)) \tag{8}$$

where $A_{cl} = A - BK$ is the closed-loop state matrix. The eigenvalues of A_{cl} can be placed in the open left-hand plane from the controllability property of the (A,B) pair. Introducing $\zeta(t) = t + \delta(t)$, then, if $\dot{\delta} \neq -1$, (8) gives

$$\frac{dx(\zeta)}{d\zeta} = \gamma(t)A_{cl}x(\zeta), \quad \gamma(t) = \frac{1}{1+\frac{d\delta(t)}{dt}}$$

Note that this equation describes a linear *time-variant* system in the shifted time-coordinate $\zeta(t)$. The stability of this system does not follows directly from the stability of the A_{cl} matrix, but depends on the properties of $\gamma(t)$ as well. They are given in the following Lemma.

Lemma 2.1. (Witrant *et al.*, 2003) Consider the following system

$$\frac{dx}{dt}(t+\delta(t)) = A_{cl}x(t+\delta(t))$$

for $t \geq 0$ and $\delta(0) = \delta_0$. Then, if the following conditions holds:

i) all the real part of the eigenvalues of A_{cl} are in the open left hand side of the complex plane,
ii) $\infty > \delta_M \geq \delta(t) \geq 0$,
iii) $1 > \dot{\delta}(t) > -1$.

then,

$$\lim_{t\to\infty} ||x(t+\delta(t))|| = 0, \quad \forall t \geq \delta_0$$

for all bounded values of $x(\delta_0)$. Furthermore, the converge rate is exponential.

Corollary 2.1. The control law (7) applied to the system (1)-(2), has a bounded solution and exponentially converges to zero.

Therefore, the implementation of the control scheme depends on the possibility to predict $x(t+\delta(t))$, and the possibility of assign $v(t+\delta(t))$ in a causal way.

From (6) a predictor for $x(t+\delta(t))$ can be constructed. Then using (5) and (7) yields:

$$u(t+\delta(t)-\tau_2(t+\delta(t))) = -Ke^{A\delta(t)}x(t) \quad (9)$$
$$-Ke^{A(t+\delta(t))}\int_t^{t+\delta(t)} e^{-A\theta}Bu(\theta-\tau_2(\theta))d\theta$$

Control law causality holds if $\delta(t)$ is defined as:

$$\delta(t) \doteq max\left\{\delta \geq 0 \;\mid\; \forall\theta \in [t, t+\delta], \quad \theta - \tau_2(\theta) \leq t\right\} \quad (10)$$

which admits a solution $\delta(t)$ such that

$$\delta(t) - \tau_2(t+\delta(t)) = 0 \quad (11)$$

The existence of such a $\delta(t)$ is ensured provided that $0 \leq \tau_2(t) \leq \tau_{2max}$ (see proposition 2.1 of (Witrant *et al.*, 2003)).

Based on the previous definition, the control law (9) can be written in the following computable form

$$u(t) = -Ke^{A\delta(t)}\Big[\, x(t)$$
$$+e^{At}\int_t^{t+\delta(t)} e^{-A\theta}Bu(\theta-\tau_2(\theta))d\theta\,\Big] \quad (12)$$

The previous results leads to the following theorem.

Theorem 2.1. (Witrant *et al.*, 2003) Consider the system

$$\dot{x}(t) = Ax(t) + Bu(t-\tau_2(t))$$

with (A, B) a controllable pair. Assume that the delay dynamics (3)-(4) is such that the following holds for $\tau_2(t)$, and for all $t \geq 0$

$A1)$ $\infty > \tau_2^{max} \geq \tau_2(t) \geq 0$,
$A2)$ There exists a bounded positive function $\delta(t)$ satisfying (10) for all positive t and defined as
$$\delta(t) - \tau_2(t+\delta(t)) = 0$$
$A3)$ $-1 < \dot{\delta}(t) < 1 \quad \forall t \geq t_0$, or equivalently (when $A2$ holds) $-1 < \dot{\tau}_2(t) < 1 \quad \forall t \geq t_0$

Then, the feedback control law (12) ensures that the closed-loop system is bounded, and that the state $x(t)$ converges exponentially to zero.

3. STATE FEEDBACK DESIGN FOR THE TWO CHANNELS DELAYED

In this section we treat the case of state feedback, the output stabilization case will be treated in the next section in connection with the observer design.

The previous result is now extended to the two-channel delay problem by considering the delayed state $x(t-\tau_1(t))$ as the feedback signal.

As before the control goal is to assign the state feedback

$$v(t+\delta(t)) = -Kx(t+\delta(t))$$

such that the closed-loop system is of the form (6). If this is feasible, then the stability properties will be the same as the one stated by Lemma 2.1 and Corollary 2.1.

In the two-channel delay case, the prediction of $x(t+\delta(t))$ is done from $t-\tau_1(t)$ to $t+\delta(t)$ (instead of from t to $t+\delta(t)$ as in the one-channel delay configuration). That is:

$$x(t+\delta(t)) = e^{A(\delta(t)+\tau_1(t))}x(t-\tau_1(t))$$
$$+e^{A(t+\delta(t))}\int_{t-\tau_1(t)}^{t+\delta(t)} e^{-A\theta}Bu(\theta-\tau_2(\theta))d\theta$$

which together with (5), suggest the following expression for the control law:

$$u(t+\delta(t)-\tau_2(t+\delta(t))) = -Kx(t+\delta(t)) \quad (13)$$

Assessing the same definition for $\delta(t)$ as in equation (10) we do fulfill the causality control requirement. The control law (13) can thus be rewritten in the following computable form

$$u(t) = -Ke^{A(\delta(t)+\tau_1(t))}x(t-\tau_1(t)) \qquad (14)$$
$$-Ke^{A(t+\delta(t))}\int_{t-\tau_1(t)}^{t+\delta(t)} e^{-A\theta}Bu(\theta-\tau_2(\theta))d\theta$$

This can be observed by noticing that the integral uses past control information in the moving time-window,

$$[\{t-\tau_1(t)-\tau_2(t-\tau_1(t))\}, \{t+\delta(t)-\tau_2(t+\delta(t))\}]$$

which after using (11) gives

$$[t-\tau_1(t)-\tau_2(t-\tau_1(t)), t]$$

Remark 3.1. During the implementation of control law (14), it is necessary to keep a history of the past control inputs during a time-interval $[t-\tau_1(t)-\tau_2(t-\tau_1(t)), t]$ and to compute $\delta(t)$. For this to be possible, it is necessary to have explicit model for both delays allowing to compute their solutions and predict τ_2 to solve (11).

Remark 3.2. The main difference between this result and the previous one concerning the single-channel delay is the extension of the prediction horizon range. This results in delaying the time at which the control law can be fully implemented by $\tau_1(t)$. The closed-loop stability properties are similar to those of the one-channel case.

The main contribution of this section is now summarized in the following theorem.

Theorem 3.1. Consider the system

$$\dot{x}(t) = Ax(t) + Bu(t-\tau_2(t))$$

with (A, B) controllable pair. Assume that the delay dynamics(3)-(4) is such that the following holds for $\tau_i(t)$, $i = 1, 2$, $t \geq 0$:

$A1)$ $\infty > \tau_i^{max} \geq \tau_i(t) \geq 0$,
$A2)$ There exists a bounded positive function $\delta(t)$ satisfying for all positive t

$$\delta(t) \doteq max\{\delta \geq 0 \mid \forall\theta \in [t-\tau_1(t), t+\delta],$$
$$\theta - \tau_2(\theta) \leq t\} \qquad (15)$$

$A3)$ $\tau_1(t)$ and $\tau_2(t)$ are such that for all $\delta(t)$ obtained from $(A2)$, the following inequality holds:

$$-1 < \dot{\delta}(t) < 1 \quad \forall t \geq t_0$$

Then, the feedback control law (14) ensures that the closed-loop system is bounded, and that the state $x(t)$ converges exponentially to zero.

Proof. The proof of this theorem follows the same lines than Lemma 2.1. The reader can see (Witrant *et al.*, 2003) for details. ◇◇◇

Remark 3.3. Note that for the particular case when δ satisfies $\delta(t) - \tau_2(t+\delta(t)) = 0$, the assumption $(A3)$ collapses to $-1 < \dot{\tau}_2(t) < 1 \quad \forall t \geq t_0$.

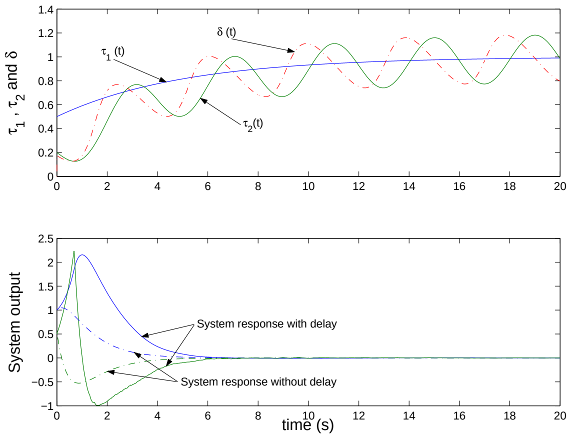

Fig. 2. Time delays and $\delta(t)$ (top) and system response (bottom) to initial conditions x_0.

Example 1: remote stabilisation Consider the mass-spring-damper second order system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-\tau_2(t))$$
$$y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t)$$

for which the controller gain is set to $K = [-2\ -1]$ and the initial conditions to $[1\ 0.5]^T$. The system is simulated in continuous-time while the control is implemented in the discrete form with a sampling period of $T_s = 10ms$ and the integral approximated using the backward rectangular rule. The dynamics of the time delay is given by the following equation:

$$\dot{z}_1 = -\frac{1}{5}z_1 + \frac{2}{5}, \quad \tau(0) = 1$$
$$\tau_1 = z_1$$
$$\dot{z}_2(t) = -\frac{1}{2}z_2(t) + 1, \quad z(0) = 4$$
$$\tau_2(t) = z_2(t) - \frac{1}{2}sin(\frac{1}{2}\pi t)$$

The resulting delays time-evolution is shown at the top of FIG.2 along with the time-evolution of $\delta(t)$. The computation of $\delta(t)$ is done numerically using dichotomy with a desired precision of 10^{-7}.

The resulting system output response $y(t)$ is presented at the bottom of FIG.2 together with the output responses that would be obtained without delay. The simulation shows the effectiveness of the control to stabilise the system for a non-zero initial condition, and the similarity of the dynamics of the time-shifted system compared to those of a system without delay.

4. OBSERVER-BASED CONTROL

In order to release the assumption that the full state is measurable, the aim of this section is to design an observer-based controller. This approach allows us to extend the results to linear systems with an observable (or at least detectable) pair (A, C). The observer state $\hat{x}(t-\tau_1)$ [1] is used to evaluate the state-dependent part of the control law $x(t-\tau_1)$. We introduce the following Luenberger state-observer for system (1)-(2):

$$\begin{aligned}\dot{\hat{x}}(t-\tau_1) &= A\hat{x}(t-\tau_1) + Bu(t-\tau_1-\tau_2(t-\tau_1)) \\ &\quad + H\{y(t-\tau_1) - C\hat{x}(t-\tau_1)\}\end{aligned}$$

The resulting observation error

$$\epsilon(t-\tau_1) \doteq x(t-\tau_1) - \hat{x}(t-\tau_1)$$

has the dynamics

$$\begin{aligned}\dot{\epsilon}(t-\tau_1) &= \dot{x}(t-\tau_1) - \dot{\hat{x}}(t-\tau_1) = (A-HC)\epsilon(t-\tau_1) \\ &= \hat{A}_{cl}\epsilon(t-\tau_1)\end{aligned}$$

The control law can then be expressed as a function of the delayed observation state $\hat{x}(t-\tau_1)$ as:

$$\begin{aligned}u(t) &= -K\left[e^{A(\delta+\tau_1)}\hat{x}(t-\tau_1) + I(t)\right] \\ &= -K\left[e^{A(\delta+\tau_1)}x(t-\tau_1) - e^{A(\delta+\tau_1)}\epsilon(t-\tau_1) + I(t)\right]\end{aligned}$$

where $I(t)$ is the integral part of the control law and $\hat{A}_{cl} = A - HC$ is a matrix with assignable eigenvalues (from the observability property of the system).

Therefore, the complete closed-loop dynamics including the observer is:

$$\begin{aligned}\dot{x}(t+\delta(t)) &= A_{cl}x(t+\delta(t)) + BKe^{A(\delta+\tau_1)}\epsilon(t-\tau_1) \\ \dot{\epsilon}(t-\tau_1) &= \hat{A}_{cl}\epsilon(t-\tau_1)\end{aligned}$$

The stability of this closed-loop system is ensured by the following Lemma.

Lemma 4.1. Consider the following system

$$\dot{x}(\zeta_1) = A_{cl}x(\zeta_1) + BKe^{A(\delta+\tau_1)}\epsilon(\zeta_2) \quad (16)$$

$$\dot{\epsilon}(\zeta_2) = \hat{A}_{cl}\epsilon(\zeta_2) \quad (17)$$

with $\zeta_1 = t + \delta(t)$, $\zeta_2 = t - \tau_1$, for $t \geq 0$ and $\delta(0) = \delta_0$. Then, if the following conditions holds:

i) all the real part of the eigenvalues of A_{cl} and $\hat{A}_{cl}$ are in the open left hand side of the complex plane,
ii) $\infty > \delta_M \geq \delta(t) \geq 0$, and $1 > \dot{\delta}(t) > -1$.
iii) $\infty > \tau_1 \geq 0$, and $1 > \dot{\tau}_1(t) > -1$.

then,

$$\lim_{t\to\infty} ||x(t+\delta(t))|| = 0, \quad \forall t \geq \delta_0$$

and for all bounded values of $\epsilon(\delta_0)$. Furthermore, $\epsilon(t)$ exponentially converges to zero.

[1] For simplicity sake, the time dependency of $\tau_1(t)$, $\tau_2(t)$ and $\delta(t)$ will be omitted. In the sequel, the notation τ_1, τ_2 and δ, will be used instead.

Proof.(outline) The result follows from using Lemma 2.1 in equations (16) and (17). This allows to prove that both states in their corresponding time-arguments ζ_1 and ζ_2 are upperbounded by an exponentially decaying signal. Conditions *(ii)* and *(iii)* ensures such a property. Then, stability of the interconnected system follows from the well-known stability property of cascade connected linear systems.

The existence of the observer-based control is now summarized in the following theorem.

Theorem 4.1. Consider the system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t-\tau_2(t)) \\ y(t) &= Cx(t)\end{aligned}$$

with (A, B) controllable pair and (A, C) an observable pair. Assume that the delay dynamics(3)-(4) is such that the following holds for $\tau_i(t), i = 1, 2.$, and for all $t \geq 0$

A1) $\infty > \tau_i(t) \geq 0$,
A2) There exists a bounded positive function $\delta(t)$ satisfying (15) for all positive t,
A3) $\tau_1(t)$ and $\tau_2(t)$ are such that for all $\delta(t)$ obtained from $(A2)$, the following inequality holds:
$$-1 < \dot{\delta}(t) < 1 \quad \forall t \geq t_0$$
A4) $1 > \dot{\tau}_1(t) > -1$

Then, the observer-based feedback control law

$$\begin{aligned}u(t) &= -Ke^{A(\delta(t)+\tau_1(t))}\hat{\bar{x}}(t) \\ &\quad -Ke^{A(t+\delta(t))}\int_{t-\tau_1(t)}^{t+\delta(t)} e^{-A\theta}Bu(\theta-\tau_2(\theta))d\theta \\ \dot{\hat{\bar{x}}}(t) &= A\hat{\bar{x}}(t) + Bu(t-\tau_1-\tau_2(t-\tau_1)) \\ &\quad + H\{y(t-\tau_1) - C\hat{\bar{x}}(t)\}\end{aligned}$$

with $\hat{\bar{x}}(t) \doteq \hat{x}(t-\tau_1(t))$ ensures that the closed-loop system is bounded, and that the state $x(t)$ converges exponentially to zero.

Example 2: remote output stabilisation Consider the same second order system, initial conditions, control gain and time delays as presented in Example 1, the only difference being the system output

$$y(t) = [1 \;\; 0]x(t)$$

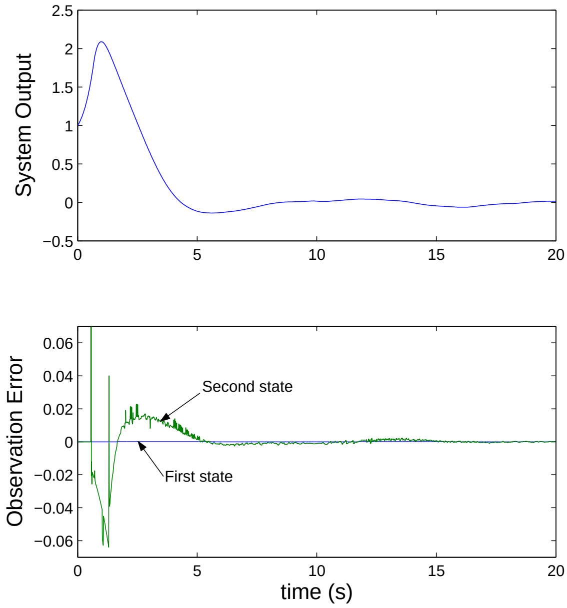

Fig. 3. System response (top) to initial conditions x_0 using the observer-based control, and observer error (bottom).

The observer gain H is chosen to place the poles of $\hat{A}_{cl}$ both at -1000. The resulting system output response $y(t)$ is presented at the top of FIG.3 together with the observer error ϵ on both states at the bottom, for a sampling period of 10 ms.

Comparing these results with the ones obtained in Example 1, it appears that the observer error affects the transient response of the system, as could be forecast by the closed-loop shifted system dynamics. Furthermore, the influence of the sampling period on the observer error illustrates the influence of the integral's approximation on the error and the transient dynamics. This problem was already reported in (Mirkin, 2003), where a solution for the case of constant delays was proposed, but will not be further investigated here. Finally, the simulation shows the effectiveness of the observer-based control to stabilize the system for a non-zero initial condition.

5. CONCLUSIONS

In this paper we have investigated the problem of remote output stabilization via transmission channels with time-varying delay, which is formulated as the problem of stabilizing an open-loop unstable system with two different time-varying delay and known dynamics.

The proposed observer-based controller results in an exponentially converging closed-loop system, under relatively weak assumptions. The controller is based on a $\delta(t)$-step ahead predictor, where $\delta(t)$ is the solution of the implicit equation $\delta - \tau_2(t + \delta) = 0$, which is shown to be solved if the time delay is bound.

We have presented a certain number of simulations showing the capability of this controller to stabilize the system and the impact of the numerical approximation when the delayed state is provided by an observer.

6. ACKNOWLEDGEMENTS

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