Stabilisation of network controlled systems with a predictive approach

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***Abstract*— This paper deals with network controlled systems. A state predictor is used to stabilize such system as in [1], [2]. The full characterization of the prediction horizon is provided, which allows to prove the stability of the closed-loop system with the state predictor (using an estimated horizon).**

***Index Terms*— Network controlled system, time-delay systems, predictive control.**

The importance of time-delay in practice, particularly concerning networked control systems, is now understood as shown for instance in [3], [4].

In most study concerning stabilization of network controlled systems, the time-delay is treated as a constant or time-varying one, but the delay dynamical behavior corresponding to the network characterization is in general not taken into account. This paper deals with the stabilization of such systems, assuming the knowledge of the transmission protocol dynamics, i.e. a "delay model". We aim to provide criteria which ensure stabilization of the network controlled system using a predictive approach. This work follows the previous authors' studies [1], [2], and goes further into details in the characterization of the prediction horizon.

The outline is as follows. In section 2 the considered models for the plant and time-delay behavior are described. In section 3, the state predictor is presented, and the computation of the prediction horizon is detailed in section 4. Section 5 concerns the stabilization proof of the predictive state feedback control law using the estimated horizon. The proof of stability of the complete scheme (which uses the time-delay model) is given in section 6. Some concluding remarks end the paper.

I. PROBLEM STATEMENT

The aim of this paper is to explore how the transmission protocol dynamics can be explicitly used in the design of the control feedback. These dynamics can be described by the general class of systems that write as

$$\dot{z}(t) = f(z(t), u_d(t)), \quad z(0) = z_0 \tag{1}$$
$$\tau(t) = h(z(t), u_d(t)) \tag{2}$$

where $z(t)$ is the internal state of the network (with initial state z_0), $u_d(t)$ is the exogenous input to the system, $f(z(t), u_d(t))$ describes the internal dynamics of the network and $h(z(t), u_d(t))$ gives the resulting delay $\tau(t)$ from the whole model. Note that the description of the network dynamics with a model based on an ordinary differential equation is arbitrary: the proposed results can be applied with a discrete or hybrid model of the network as well. In the specific case of Internet networks, where the emission is regulated by a transfer protocol and a router stores and manages the data packets, we have the following description

- $z(t)$ describes the time evolution of the emitters window size $W_i(t)$ (for $i = 1 \dots N$ sources connected to the network) and the router's queue length $q(t)$. In that case, the state writes as $z(t) = [W_1(t) \ \dots \ W_N(t) \ q(t)]^T$,
- $u_d(t)$ is the number of users N and possibly the router's output capacity C_r, if both are time-varying; we then have $u_d(t) = \{N, C_r\}$,
- $f(z(t), u_d(t))$ is set by the TP on the windows sizes and by the queue management scheme (i.e. TCP and AQM policy),
- $h(z(t), u_d(t))$ determines the delay occurring between the sender and the receiver from network parameters such as the round trip time $R_i(t)$.

Note that (1)-(2) describe an autonomous system with an exogenous input $u_d(t)$. This input is assumed to be known over a certain range of time ahead of the present time (equal to the maximum delay expected τ_{max}). This would be the case if the subsystems of a supply chain act in a predetermined order or if the transfer protocol is set to declare to the network that its source will emit and wait during τ_{max} before starting the emission.

An example of such dynamics is the TCP model described by [5], where a fluid flow model was developed using Poisson counter driven differential equations, with a proportional Active Queue Management (AQM) policy set on the router's site. The AQM is introduced with a packet discard function $p(\cdot)$ and acts as a feedback from the router on the emitter's window size; the proportional scheme is shown to be stable in [6]. The network equations then write as

$$\frac{dW_i(t)}{dt} = \frac{1}{R_i(t)} - \frac{W_i(t)}{2} \frac{W_i(t - R_i(t))}{R_i(t - R_i(t))} p_i(t), \tag{3}$$
$$\frac{dq(t)}{dt} = -C_r + \sum_{i=1}^{N} \frac{W_i(t)}{R_i(t)}, \quad q(t_0) = q_0 \tag{4}$$
$$\tau_i = \frac{R_i(t)}{2}, \quad where \quad R_i(t) \doteq \left[\frac{q(t)}{C_r} + T_{pi}\right]$$

where $p_i(t) = K_p q(t - R_i(t))$ and T_{pi} is the constant propagation delay.

The remotely controlled system has the form:

$$\dot{x}(t) = Ax(t) + Bu(t - \tau(t)) \quad (5)$$
$$y(t) = Cx(t) \quad (6)$$

where $x \in R^n$ is the internal state, $u \in R$ is the control input, $y \in R^m$ is the system output, and A, B, C are matrices of appropriate dimensions. The pairs (A, B) and (A, C) are assumed to be controllable and observable, respectively, but no assumption is made on the stability of A. We assume that all solutions of model (1)-(2) lead to the following properties $\forall t \geq 0$

$$0 \leq \tau(t) \leq \tau_{max} \quad (7)$$
$$\dot{\tau}(t) \leq \nu < 1 \quad (8)$$

where τ_{max} is an upper bound on the time-delay. Note that these conditions are a direct consequence of the lossless property of the network considered.

II. Background on the State Predictor

Due to the inherent time-variation of the delay considered when dealing with networks, it is not possible to design a controller that imposes an invariant closed-loop spectrum. Instead, under certain weak conditions, we are able to set the eigenvalues of a *time-varying shifted* system, or equivalently we transform the time-invariant delayed unstable open-loop system, into a stable time-varying linear system. The control design proposed here is similar to the one used in [7] in an adaptive control scheme.
The system transformation is done by replacing the current time t by the shifted time coordinate $t + \delta(t)$ in (5), which results in

$$x'(t+\delta(t)) = Ax(t+\delta(t)) + Bu(t+\delta(t) - \tau(t+\delta(t))), \quad (9)$$

where $x'(\cdot)$ is the derivative of $x(\cdot)$ with respect to its argument (i.e. $t + \delta(t)$) and $\delta(t)$ is a bounded and positive time-depending function. Defining $\delta(t)$ as

$$\delta(t) = \tau(t + \delta(t)) \quad (10)$$

and considering first the problem of state feedback stabilization, the eigenvalues of the time-varying shifted system (9) are set with the control input

$$\begin{aligned} x(t+\delta) &= e^{A\delta}\left[x(t) + e^{At}\int_t^{t+\delta} e^{-A\theta}Bu(\theta - \tau(\theta))d\theta \right] \\ u(t) &= -Kx(t+\delta(t)). \end{aligned} \quad (11)$$

The resulting closed-loop equation is then

$$x'(t+\delta(t)) = (A - BK)x(t+\delta(t)) = A_{cl}x(t+\delta(t)) \quad (12)$$

where A_{cl} is the closed loop state matrix, that can be made Hurwitz from the controllability hypothesis on (A, B). The stability of this system is established in [8] and reconsidered in [2], where a Lyapunov-based analysis in the time-shifted coordinates $t + \delta(t)$ is proposed. This last result connects the conditions (7)-(8) to the stability of (12).

III. Computation of the Predictor's Horizon

We are now focusing on the solution of the *implicit* equation (10) used to establish the control law. The dynamic computation proposed in [2], allowing for an *explicit* use of the delay dynamics, is detailed here to show that this approach results in the exponential convergence of the predictor's horizon estimation. We exploit the fact that the scalar differential equation $\dot{\delta}(t) = -\delta + g(\delta)$ has only one globally attractive fixed point if the application g has only one fixed point. This is a continuous version of the discrete iteration $\delta_{n+1} = g(\delta_n)$. This approach has the advantage of proposing an explicit solution, where the delay occurs in the state of the controller, and to be more powerful on the level of the computing time necessary to the resolution of the implicit equation (10). We describe how the dynamics of $\delta(t)$ is defined in order to guarantee an asymptotic convergence towards the solution of the implicit equation. Note that this approach motivates the need of a dynamic model of the delay but could also be developed with a discrete or hybrid model of the delay.
First define the functional

$$\hat{s}(t) \doteq \hat{\delta}(t) - \tau(t + \hat{\delta}(t)) \quad (13)$$

where $\hat{\delta}(t)$ is the estimated value of $\delta(t)$. The underlying idea of the proposed approach is to find a variation law for $\hat{\delta}(t)$ such that the surface $s(t) = 0$, where $s(t)$ is the required value of $\hat{s}(t)$ (corresponding to an exact solution of the implicit equation), is rendered attractive and invariant. The result of such a dynamics guarantees the exponential convergence of $\hat{\delta}(t)$ towards $\delta(t)$. Therefore, we build an open loop estimator of $\delta(t)$, solution of the Cauchy's system

$$\begin{cases} \dot{s}(t) = 0 \\ s(t=0) = 0 \end{cases}$$

To prevent the numerical instabilities induced by this approach, the dynamics of $\hat{s}(t)$ is defined by

$$\dot{\hat{s}}(t) + \sigma\hat{s}(t) = 0 \quad (14)$$

where σ is a positive constant. Deriving (13) with respect to time and substituting $\dot{\hat{s}}$ in (14), we obtain

$$\dot{\hat{\delta}}(t) - \tau'(\hat{\zeta})(1 + \dot{\hat{\delta}}(t)) + \sigma(\hat{\delta}(t) - \tau(\hat{\zeta})) = 0$$

where $\hat{\zeta}(t) \doteq t + \hat{\delta}(t)$ and $\tau'(\cdot)$ is the derivative of $\tau(\cdot)$ with respect to its argument. The previous equation implies that (14) is satisfied if $\tau'(\cdot) \neq 1$ and if the variation law $\dot{\hat{\delta}}(t)$ is established with

$$\dot{\hat{\delta}}(t) = -\frac{\sigma\hat{\delta}}{1 - \tau'(\hat{\zeta})} + \frac{\tau'(\hat{\zeta}) + \sigma\tau(\hat{\zeta})}{1 - \tau'(\hat{\zeta})} \quad (15)$$

This *explicit* expression of the dynamics of $\hat{\delta}(t)$ ensures that the estimate $\hat{\delta}(t)$ converges towards the desired value $\delta(t)$, and that the function $\hat{s}(t)$ converges exponentially towards zero. We thus directly use dynamics of $\tau(\hat{\zeta})$ given by (1)-(2). It remains to show that the estimation error on $\delta(t)$ induced by the proposed method has the same stability properties as $\hat{s}(t)$, for the type of functions considered. This is established with the following lemma.

Lemma 3.1: Let $x(t) \in \mathcal{X} \subset \mathbb{R}$ be the solution of the implicit equation $x(t) = f(x(t))$, where $f(x(t))$ is a continuous and differentiable function on $\mathcal{X}$ with a Lipschitz coefficient $M < 1$. If $\hat{x}(t)$ is the estimate of this solution, computed from the dynamics

$$\begin{cases} \dot{\hat{s}}(t) &= -\sigma \hat{s}(t) \\ \hat{s}(t) &= \hat{x}(t) - f(\hat{x}(t)) \end{cases}$$

where σ is a positive constant, then the estimation error $\epsilon(t)$ defined by

$$\epsilon(t) \doteq x(t) - \hat{x}(t)$$

satisfies the inequality

$$|\epsilon(t)| \leq \frac{|\hat{s}(t)|}{1-M} \tag{16}$$

and converges exponentially to zero.

Proof: The estimation error [1] ϵ is first expressed as a function of f and $\hat{s}$ with

$$\epsilon = f(x) - \hat{s} - f(\hat{x}) = -\hat{s} + f(x) - f(x - \epsilon)$$

The continuity and differentiability properties of f on $\mathcal{X}$ as well as the mean-value theorem then make it possible to establish that there is one c in the interval $[x - \epsilon, x]$ such that

$$f(x) - f(x - \epsilon) = f'(c)\epsilon$$

This implies

$$\epsilon(1 - f'(c)) = -\hat{s}$$

and consequently

$$\epsilon = -\frac{\hat{s}}{1 - f'(c)}$$

The assumption on the Lipschitz coefficient of f makes it possible to establish that

$$\sup_{x \in \mathbb{R}} f'(x) = M < 1 \Rightarrow f'(c) < 1$$

thus justifying the inequality (16). Finally, the exponential convergence of $\epsilon(t)$ is directly obtained from the dynamic equation defining $s(t)$, which has as the solution $\hat{s}(t) = \hat{s}(0)e^{-\sigma t}$. ∎

Remark 3.1: The preceding result shows equivalently that the tracking error

$$e(t) = s(t) - \hat{s}(t)$$

obeys the law of exponential decay $e(t) = e(0)e^{-\sigma t}$. This is a direct consequence of the fact that the function $s(t)$ is described by a Cauchy's system.

The previous lemma is now be applied to the horizon estimation problem with the following theorem.

Theorem 3.1: The solution $\delta(t)$ of the implicit equation (10) can be estimated by the variable $\hat{\delta}(t)$, solution of the dynamic equation (15), with $\hat{\delta}(0) = \hat{\delta}_0 \in [0, \tau_{max}]$ and $\tau(t)$ satisfying the conditions

$P1)$ $0 \leq \tau(t) \leq \tau_{max}$,
$P2)$ $\sup_{t \in \mathbb{R}^+} \dot{\tau}(t) = \nu < 1$.

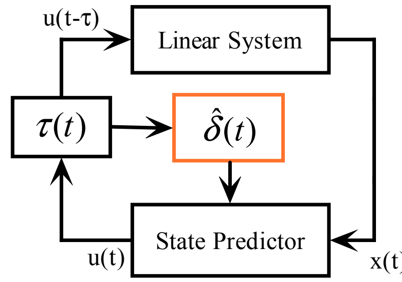

Fig. 1. Control with the estimation of $\delta(t)$.

The error induced by this approximation converges exponentially towards zero and is bounded in the following way:

$$|\epsilon(t)| = |\delta(t) - \hat{\delta}(t)| \leq \frac{|\hat{\delta}_0 - \tau(\hat{\delta}_0)|e^{-\sigma t}}{1 - \nu} \tag{17}$$

where σ is a positive constant.

Proof: This theorem is a direct consequence of the proposed dynamic computation and the properties of the network considered, which make it possible to apply Lemma 3.1. Indeed, the domain considered is on $\mathbb{R}^+$, from the definition of δ (10) and the boundness condition on $\tau(t)$ $(P1)$. The condition on the delay's derivative $(P2)$ ensures that the condition on the Lipschitz coefficient of Lemma 3.1 is satisfied. Finally, the variables $\epsilon(t)$ and $s(t)$ are substituted by their expression in terms of $\delta(\cdot)$, $\hat{\delta}(\cdot)$ and $\tau(\cdot)$. ∎

IV. Predictor with an Estimated Horizon

This section is dedicated to the synthesis of a predictive control law based on an estimated horizon for the stabilization of network controlled systems. We first describe the influence of the horizon estimation on the closed-loop system. Then, the computation method proposed in the previous section is validated by guaranteeing the exponential stability of the closed-loop system. The estimate of the predictor's horizon $\hat{\delta}(t)$ induces a new dynamics which influences the closed-loop system. Indeed, the control law based on the state predictor is now established using the estimate of $\delta(t)$, as presented in Figure 1, and writes as

$$u(t) = -Ke^{A\hat{\delta}(t)}\left[x(t) + e^{At}\int_t^{t+\hat{\delta}(t)} e^{-A\theta}Bu(\theta - \tau(\theta))d\theta\right] \tag{18}$$

or, equivalently,

$$u(t) = -Kx(t + \hat{\delta}(t))$$

with $\hat{\delta}(t)$ defined by its dynamics (15). Using the change in coordinates $t \mapsto t + \delta(t)$ and (18), the system dynamics considered is[2]

$$\begin{aligned} x'(t+\delta) &= Ax(t+\delta) + Bu(t) \\ &= Ax(t+\delta) - BKx(t+\hat{\delta}) \end{aligned} \tag{19}$$

[1] for simplicity sake, the temporal indices of $\epsilon(t)$ are omitted in this proof, the suggested solution remaining true for all t.

[2] the temporal dependence of $\delta(t)$, $\hat{\delta}(t)$ and $\zeta(t)$ is omitted in the sequel for simplicity sake.

Adding and subtracting $BKx(t+\delta)$ on the right side of the previous equality, we obtain the system

$$\Sigma_o : x'(\zeta) = (A-BK)x(\zeta) + BK(x(\zeta) - x(\zeta-\epsilon))$$

where $\zeta(t) = t+\delta(t)$ and $\epsilon(t) = \delta(t) - \hat{\delta}(t)$ has the properties described by Theorem 3.1. Σ_o can be rewritten, by arithmetic equivalence using the formula of Leibniz-Newton,

$$x'(\zeta) = (A-BK)x(\zeta) + BK\int_{-\epsilon}^{0} x'(\zeta+\theta)d\theta$$

The dynamics (19) is then substituted into the integral term to obtain the transformed system

$$\begin{aligned}\Sigma_t : x'(\zeta) &= (A-BK)x(\zeta) + BKA\int_{-\epsilon}^{0} x(\zeta+\theta)d\theta \\ &\quad -(BK)^2\int_{-2\epsilon}^{-\epsilon} x(\zeta+\theta)d\theta\end{aligned}$$

with the initial conditions

$$x(\theta) = \phi(\theta),\, \theta\in[t_0-2\epsilon_M,\, t_0],\, (t_0,\,\phi)\in\mathbb{R}^+\times\mathcal{C}^{\upsilon}_{n,\,-2\epsilon_M}$$

where $\epsilon_M \doteq \sup_t \epsilon(t)$, $\mathcal{C}^{\upsilon}_{n,\,\tau} = \{\phi\in\mathcal{C}_{n,\,\tau} : ||\phi||_c < \upsilon\}$, υ is a positive real number, $||\phi||_c = \sup_{-\tau\le t\le 0}||\phi||$, $||\cdot||$ refers to the Euclidian norm and $\mathcal{C}_{n,\tau} = \mathcal{C}([-\tau,\,0],\mathbb{R}^{\ltimes})$ denotes the Banach space of continuous vectorial functions mapping $[-\tau,\,0]$ into $\mathbb{R}^n$ with a uniformly convergent topology (see [9] for more details).

Note that the stability of Σ_t implies that of Σ_o but the reverse is not true (comparison principle), because of the initial conditions prolongation on the temporal space $[\delta(0)-2\epsilon, \delta(0)-\epsilon]$. The stability of the transformed system is guaranteed by the following lemma, which is an application of the results of [10] to the problem considered.

Lemma 4.1: Consider the system Σ_t with appropriate distributed initial conditions. If the following conditions hold

i) A_{cl} is Hurwitz,
ii) $\epsilon(t)$ satisfies (17) and is such that

$$0 < \dot{\epsilon}_M \doteq \sup_t \dot{\epsilon}(t) < \frac{1}{2}$$

then the trajectories of $x(\zeta(t))$ are asymptotically bounded.

Proof: Consider the Lyapunov-Krasovskii functional established for Σ_t

$$\begin{aligned}V(x(\zeta)) &= x(\zeta)^T P x(\zeta) \\ &\quad + \frac{1}{1-\dot{\epsilon}_M}\int_{-\epsilon}^{0}\left[\int_{\zeta+\theta}^{\zeta} x(\mu)^T S x(\mu)d\mu\right]d\theta \\ &\quad + \frac{\alpha}{1-2\dot{\epsilon}_M}\int_{-2\epsilon}^{-\epsilon}\left[\int_{\zeta+\theta}^{\zeta} x(\mu)^T S x(\mu)d\mu\right]d\theta\end{aligned}$$

with P and S some positive definite symmetric matrices, and

$$0 < \alpha < \frac{1-2\dot{\epsilon}_M}{\dot{\epsilon}_M} \tag{20}$$

Deriving $V(\cdot)$ along the trajectories of Σ_t and using the relationships

$$\begin{aligned}\frac{d}{dt}\left[\int_{a(t)}^{b(t)}\int_{t+\theta}^{t} f(\mu)d\mu d\theta\right] &= (b-a)f(t) \\ -(1+\dot{b})\int_a^b f(t+\theta)d\theta &+ (\dot{b}-\dot{a})\int_a^0 f(t+\theta)d\theta\end{aligned}$$

and

$$2u^T v \le u^T S_i^{-1} u + v^T S_i v$$

for $i=1,2$, we obtain the inequality

$$\begin{aligned}\frac{dV(x(\zeta))}{d\zeta} &\le x(\zeta)^T\left[PA_{cl} + A_{cl}^T P + \epsilon a_1 S\right]x(\zeta) \\ &+ \int_{-\epsilon}^{0} a_2 x(\zeta)^T PBKAS^{-1}(PBKA)^T x(\zeta)d\theta \\ &+ \int_{-2\epsilon}^{-\epsilon}\frac{1}{\alpha}x(\zeta)^T P(BK)^2 S^{-1}(P(BK)^2)^T x(\zeta)d\theta \\ &\le x(\zeta)^T\Big[PA_{cl} + A_{cl}^T P + \epsilon a_1 S \\ &+ \epsilon a_2 PBKAS^{-1}(PBKA)^T \\ &+ \epsilon\frac{1}{\alpha}P(BK)^2S^{-1}(P(BK)^2)^T\Big]x(\zeta)\end{aligned} \tag{21}$$

where

$$a_1 \doteq \frac{1+\alpha(1-\dot{\epsilon}_M)}{(1-\dot{\epsilon}_M)(1-2\dot{\epsilon}_M)},\quad a_2 \doteq \frac{1-2\dot{\epsilon}_M}{1-(2+\alpha)\dot{\epsilon}_M} \tag{22}$$

are some positive constants. Defining the matrices

$$\begin{aligned}R &\doteq a_1 S + a_2 PBKAS^{-1}(PBKA)^T \\ &\quad + \frac{1}{\alpha}P(BK)^2S^{-1}(P(BK)^2)^T \\ Q &\doteq -(PA_{cl} + A_{cl}^T P)\end{aligned}$$

which are positive definite by construction and by the assumption *i*), respectively, the previous inequality writes as

$$\begin{aligned}\frac{dV(x(\zeta))}{d\zeta} &\le -x(\zeta)^T Q x(\zeta) + \epsilon x(\zeta)^T R x(\zeta) \\ &\le (-\lambda_m(Q) + |\epsilon|\lambda_M(R))||x(\zeta)||^2\end{aligned}$$

The convergence of the function $\epsilon(t)$ ensures that there is a time t_c such as

$$|\epsilon(t)| < \frac{\lambda_m(Q)}{\lambda_M(R)}$$

for all $t > t_c$, and thus that the Lyapunov-Krasovskii functional converges for all $x(\zeta)\in\{x(\zeta(t)) : t > t_c\}$. From the fact that the system considered is linear and cannot diverge in finite time, we conclude that the trajectories of the functional differential equation Σ_t are asymptotically stable. ■

Remark 4.1: Although the method used to establish the previous lemma can seem conservative, in particular concerning the bounds imposed on the error variation, it remains suitable to support the matter of this section. Indeed, these limits are determined by an appropriate choice of the constant σ, which must be selected such that

$$\sigma < \frac{1-\nu}{2|\hat{\delta}_0 - \tau(\hat{\delta}_0)|}$$

This limits the possible convergence speed of the estimation error.

Remark 4.2: The maximum acceptable variation of the error $\dot{\epsilon}_M$ is given by the precision of the network model or can be set with the transfer algorithm if a buffer is introduced at the receiver's input (use of the transfer protocol for the control requirements).

V. EXPLICIT USE OF THE NETWORK MODEL

The last step aims at describing the control law with an *explicit* use of the network model and at showing that the stability is ensured for a delay satisfying (7)-(8). This is established by the following theorem.

Theorem 5.1: Consider the system described by

$$\dot{x}(t) = Ax(t) + Bu(t - \tau(t))$$

where (A, B) is a controllable pair. Suppose that the delay dynamics described by (1)-(2) and the positive constant σ are such that the following conditions hold for all t

$A1)$ $0 \leq \tau(t) \leq \tau_{max}$,
$A2)$ $\dot{\tau}(t) \leq \nu < 1$,
$A3)$ $0 < \dot{\epsilon}_M \doteq \sup_t \dot{\epsilon}(t) < \frac{1}{2}$

then the state feedback control law

$$\begin{aligned}
u(t) &= -Ke^{A\hat{\delta}(t)} \left[x(t) + e^{At} \int_t^{t+\hat{\delta}(t)} e^{-A\theta} Bu(\theta - \tau(\theta)) d\theta \right] \\
\dot{\hat{\delta}}(t) &= -\frac{\sigma}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}} \hat{\delta} + \frac{d\tau(\hat{\zeta})/d\hat{\zeta} + \sigma\tau(\hat{\zeta})}{1 - d\tau(\hat{\zeta})/d\hat{\zeta}} \\
\frac{d\tau}{d\hat{\zeta}}(\hat{\zeta}) &= \frac{dh}{d\hat{\zeta}}(z(\hat{\zeta}), u_d(\hat{\zeta})) \\
\frac{dz}{d\hat{\zeta}}(\hat{\zeta}) &= f(z(\hat{\zeta}), u_d(\hat{\zeta})), \quad z(0) = z_0
\end{aligned}$$

with $\hat{\zeta} = \hat{\zeta}(t) = 1 + \hat{\delta}(t)$ and $\hat{\delta}(0) = \hat{\delta}_0 \in [0, \tau_{max}]$, ensures that the closed-loop system trajectories are asymptotically stable.

Proof: This theorem directly follows from the results obtained in the previous sections. The assumptions $(A1)$ to $(A3)$ allow

- to guarantee the exponential convergence of the estimation error since the conditions of Theorem 3.1 are satisfied,
- to satisfy the condition (ii) of Lemma 4.1.

The fact that the pair (A, B) is controllable ensures that there exists a gain K such that $A - BK$ is Hurwitz. We can thus apply Lemma 4.1 and conclude on the asymptotic convergence of the system trajectories. ■

Remark 5.1: This control law requires to keep in memory the control signals emitted during the time interval $[t - \tau_{max}, t]$. Moreover, the calculation of the predictor horizon implies a knowledge of the delay on the interval $[t, t + \tau_{max}]$. This last assumption is most restrictive; it is satisfied

- for periodic systems, thanks to the knowledge of the system behavior during the following period,
- for entirely deterministic systems, by using a state predictor on the delay (possibly nonlinear),
- in a more general way, by combining an observer and a predictor on the delay.

Note that the transfer protocol algorithm can be used on this level to make an aperiodic network totally deterministic. Indeed, let us suppose that a source emits a preliminary signal, of negligible size, informing of its intention to use the network and waits during a time τ_{max} before emitting. The number of sources planing to use the network is thus known in advance and a model of the emission protocols and of the queue management can be used in order to precisely predict the delay behavior.

VI. CONCLUSIONS

In this paper, a state predictive approach has been used to deal with network controlled system in the case where the transmission protocol dynamic is explicitly used in the model formulation. We have proved that such a system can be stabilized with a state predictor, and have mainly focused on the characterization of the prediction horizon.

Most part of this work are developed in [11], where an application to the stabilization of an inverted pendulum is presented. In this application, an observer is also needed to estimation the state variable, in order to implement the state predictive control law. The design of this observer, in the framework of varying time-delay, and in connection to the considered control scheme, is currently studied.

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REFERENCES

[1] E. Witrant, C. Canudas-de-Wit, and D. Georges, "Remote output stabilization under two channels time-varying delays," in *Proc. of the 4^{th} IFAC Workshop on Time Delay Systems*, Rocquencourt, France, sep 2003.

[2] E. Witrant, C. Canudas-de-Wit, D. Georges, and M. Alamir, "Remote stabilization via time-varying communication network delays: Application to TCP networks," in *Proc. of the IEEE Conference on Control Applications*, Taipei, Taiwan, sep 2004.

[3] B. Zhang, M. Branicky, and M. Phillips, "Stability of networked control systems," *IEEE Control Systems Magazine*, pp. 84–99, Feb. 2001.

[4] D.-S. Kim, Y. Lee, W. Kwon, and H. Park, "Maximum allowable delay bounds of networked control systems," *Control Engineering Practice*, vol. 11, no. 11, pp. 1301–1313, 2003.

[5] V. Misra, W.-B. Gong, and D. Towsley, "Fluid-based analysis of a network of aqm routers supporting tcp flows with an application to red," in *Proc. of ACM SIGCOMM'00*, Stockholm, Sweden, Sept. 2000.

[6] C. V. Hollot and Y. Chait, "Nonlinear stability analysis for a class of tcp/aqm networks," in *Proc. of the 40^{th} IEEE Int. Conf. on Decision and Control*, Orlando, Florida, USA, Dec. 2001.

[7] M. T. Nihtilä, "Adaptive control of a continuous-time system with time-varying input delay," *Systems and Control Letters*, no. 12, pp. 357–364, 1989.

[8] Z. Artstein, "Linear systems with delayed control: a reduction," *IEEE Transactions on Automatic Control*, vol. ac-27, no.4, pp. 869–879, Aug. 1982.

[9] S.-I. Niculescu, E. Verriest, L. Dugard, and J.-M. Dion, "Stability and robust stability of time-delay systems: A guided tour," in *Stability and Control of Time-delay Systems*, ser. Lecture Notes on Control and Information Sciences, L. Dugard and E. Verriest, Eds. New York: Berling Springer, 1998, vol. LNCIS 228, pp. 1–71.

[10] S.-I. Niculescu, C. de Souza, L. Dugard, and J.-M. Dion, “Robust exponential stability of uncertain systems with time-varying delays,” *IEEE Transactions on Automatic Control*, vol. 43, no. 5, pp. 743–748, May 1998.

[11] E. Witrant, “Stabilisation des systèmes commandés par réseaux.” Ph.D. dissertation, INPG/Laboratoire d’Automatique de Grenoble, Grenoble, France, september 2005.