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Validation of plasma current profile model predictive control in tokamaks via simulations

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ABSTRACT

In this work, a new predictive control strategy based on a control-oriented model using a 1D magnetic flux diffusion equation is proposed. The aim is to control the plasma current density to obtain high confinement and good stability of tokamak plasma experiments. The control is designed using both inductive means (variation of magnetic flux at the plasma edge) and non-inductive means (lower hybrid current drive and electron cyclotron current drive). Kinetic variables such as the electronic temperature, usually available in real time, are considered as inputs in particular for the estimation of the plasma resistivity. Successful closed-loop simulations have been performed using Tore Supra parameters and experimental data. Sensitivity tests have also been made by varying several parameters of the reference model, showing the robustness of the proposed strategy. The real-time relevance of the method was also successfully checked.

1. Introduction

The control of the current density profile is now recognised as a key issue to improve the confinement and stability of tokamak plasma experiments. The basic approaches generally consist in tuning PID like control laws to control one single profile shape parameter. The step further consists in using finite dimension multi input multi output linear control models that may be identified from experimental data [1]. However, the control of the entire profile is still an area of lively research that requires a nonlinear model-based approach due to the complex distributed nature of the problem [2].

In this paper, we propose a control design method based on a distributed nonlinear model, namely a simplified version of the 1D resistive diffusion equation of the magnetic flux in the plasma that governs the dynamic evolution of the plasma current density [3].

The control law that was designed is based on a predictive control strategy [4,5]. The principle is the following: at each sampling step, the request to the actuators is derived from the minimization of a criterion based on the error between the current profile target and model prediction at steady state. The actual current profile target is also reprocessed at each sampling step in order to handle the difference between the current profile model prediction and measurements (self-compensator).

This paper is organized as follows: in Section 2, the reference model is introduced with some details given in prospect of an application to Tore Supra. The proposed predictive control law is described in Section 3 and simulation results are given in Section 4. Conclusion and outlook are provided in Section 5.

2. Reference model description

A control-oriented model of the plasma current profile dynamics is required in order to perform predictive control. This model should account for the main physics phenomena at stake but be simple enough to allow an implementation in the real-time control loop. The model presented in [3] and based on [6] fulfils these specifications.

It is based on a partial differential equation (PDE) describing the magnetic flux diffusion in the plasma. Under some assumptions (axisymmetry, MHD equilibrium, and cylindrical approximation), the magnetic flux dynamics can be described by the following equation (ψ denotes the magnetic flux):

$$\frac{\partial \psi}{\partial t}(x,t) - \frac{\eta_{//}(x,t)}{\mu_0 a^2 x}\frac{\partial}{\partial x}\left(x\frac{\partial \psi}{\partial x}\right) = \eta_{//}(x,t) R_0 j_{ni}(x,t) \quad (1)$$

where t and $x \in [0, 1]$ are, respectively, the time and the spatial normalized index of the magnetic surface. The parameters $\eta_{//}$, μ_0, R_0 and a are, respectively, the plasma resistivity, the vacuum perme-

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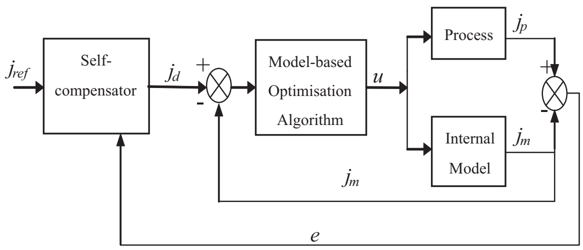

Fig. 1. MPC structure with self-compensator.

ability, and the major and minor plasma radius. The input j_{ni} is the non-inductive source of plasma current. Note that ψ, $\eta_{//}$ and j_{ni} are both space (through x) and time dependent. The plasma current density is obtained from:

$$j = -\frac{1}{\mu_0 R_0 a^2 x}\frac{\partial}{\partial x}\left(x\frac{\partial \psi}{\partial x}\right) \quad (2)$$

The resolution of Eq. (1) requires the knowledge of the plasma resistivity, appearing in the diffusion coefficient as well as the non-inductive current profile source j_{ni}. Regarding the resistivity $\eta_{//}$, we used an analytical expression that is known to be fairly accurate [7,8], with main dependence on the electron temperature T_e ($\eta_{//} = f(T_e{}^{-3/2})$). The spatial–temporal variation of temperature T_e could be modelled with the aid of a second partial differential equation describing the heat transport [6]. But as heat diffusion coefficients are not yet well modelled [9], and as electron temperature is quite well measured on Tore Supra, we chose to consider the temperature as an input of the model.

The non-inductive current density j_{ni} is given by: $j_{ni} = j_{bs} + j_{lh} + j_{ec}$ where j_{lh}, j_{ec} and j_{bs} are, respectively, the LHCD current density, the ECCD current density and the bootstrap current density.

This last one is self-generated by the plasma, it can be modelled by a nonlinear function of the flux (this one can be obtained by an analytical formula [3,10]). The inductive current density or ohmic part is $j_\Omega = j - j_{ni}$. The LHCD and ECCD current densities are modelled by Gaussian functions controlled by engineering parameters (more details are given in [3]). To complete this modelling, both boundary and initial conditions are needed:

- the initial value of the poloidal flux is noted $\psi(x, 0) = \psi_0(x)$,
- at the centre of the plasma, the spatial variation of the flux is null: $\partial\psi/\partial x\,(x = 0, t) = 0$,
- at the plasma edge, the temporal variation is: $\partial\psi/\partial t$ ($x = 1$, $t) = V_{\mathrm{loop}}(t)$, where $V_{\mathrm{loop}}(t)$ is the plasma surface loop voltage.

3. Predictive control strategy

3.1. Basic principle

The model predictive control (MPC) [4,5] is a well suited strategy to control nonlinear processes with variables and/or states constraints. First, a performance criterion, based on the difference between the reference trajectory and the process output on a given time outlook is defined. Then, at each time step, the control method computes and applies the set of control inputs set that minimize this performance criterion while fulfilling the system engineering constraints.

Fig. 1 gives the considered control structure. It is composed of the process with the output j_p, an internal model with the output j_m, a model-based optimization algorithm and a self-compensator. The command μ is applied both to the internal model and to the process. It is calculated by the optimisation algorithm in order to fit j_m to j_d. The self compensator handles the modelling errors between the process and the internal model. It takes into account the difference between j_p and j_m to modify j_d in order to match the steady state reference j_{ref}.

3.2. Definition of constraints and criterion

As mentioned earlier, our objective is to control the current density profile with the control vector μ composed of the loop voltage V_{loop}, the powers (respectively, P_{lh1} and P_{lh2}) and wave refractive indexes (respectively, N_{lh1} and N_{lh2}) of the two Tore Supra LHCD launchers. This objective can be formulated as the following optimization criterion:

$$\min_{V_{\mathrm{loop}}, P_{lh1}, N_{lh1}, P_{lh2}, N_{lh2}} J \doteq \int_0^1 (j_{\mathrm{ref}} - j_p)^2\, dx \quad (3)$$

To take into account the discrepancies between the reference model and the process, a zero-order self-compensator is used: the error e at time k is assumed to be the same at time $k+1$. One obtains according to Fig. 1: $j_d = j_{\mathrm{ref}} - e$, $e = j_p - j_m$, $j_d - j_m = j_{\mathrm{ref}} - j_p$ where $j_m = j_\Omega + j_{lh} + j_{bs}$. From these considerations, the criterion (3) becomes:

$$\min_{V_{\mathrm{loop}}, P_{lh1}, N_{lh1}, P_{lh2}, N_{lh2}} J = \int_0^1 (j_d - (j_\Omega + j_{lh} + j_{bs}))^2\, dx \quad (4)$$

j_Ω and j_{lh} are, respectively, function of the loop voltage V_{loop} and of the LHCD launchers parameters:

$$j_\Omega(x, t \gg t_d) = -\frac{V_{\mathrm{loop}}(t)}{R_0 \eta_{//}(x, t)}$$
$$j_{lh} = f(P_{lh1}, N_{lh1}, P_{lh2}, N_{lh2})$$

where t_d is the diffusion time. Due to engineering limitations, the control variables are bounded:

$$\begin{cases} -5\ \mathrm{V} \le V_{\mathrm{loop}} \le 5\,\mathrm{V} & \\ 0\,\mathrm{MW} \le P_{lh1} \le 1.5\,\mathrm{MW} & 0\,\mathrm{MW} \le P_{lh2} \le 3\,\mathrm{MW} \\ 1.43 \le N_{lh1} \le 2.37 & 1.67 \le N_{lh2} \le 2.33 \end{cases} \quad (5)$$

At each time step, the optimization algorithm looks for the set of control variables that minimize the criterion (4) under the constraints (5), to be applied at the following time step. In practice, a numerical optimization toolbox was used.

4. Simulation results

The reference model was implemented using Tore Supra specific parameters (given in [3]). The plasma current profile computed at the end of flat-top of the open-loop pulse TS#35109 was taken as control target. Two kind of tests were performed: to begin with, the reference model was chosen equal to the process in order to get a basic proof of the efficiency of the method. Then, the reference model parameters were modified to address the issue of the robustness of the proposed control against model uncertainties.

4.1. Control strategy validation

Figs. 2–4 show the closed-loop control simulation results. The proposed closed-loop control strategy was actually able to find a set of control variables that is very similar to what was actually applied for several characteristic diffusion time at the end of the flat-top in the open-loop real experiment. Moreover, the steady state is reached after 5 s (with a mean error on the current profile below 2%) while staying within the engineering constraints: the closed-loop control strategy is able to go directly to the control target whereas a scan in wave refractive index of 25 s was actually performed in the open-loop real experiments (point-dashed

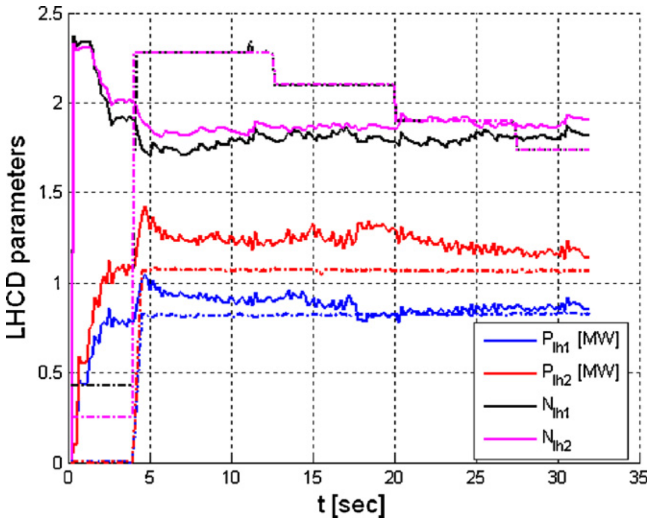

Fig. 2. LHCD parameters in closed loop (solid line) computed in simulation and experimental values (point-dashed) from pulse TS#35109.

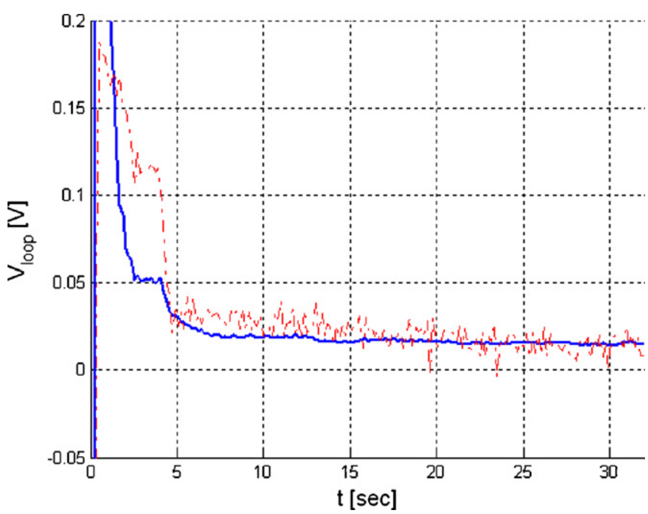

Fig. 3. Loop voltage in closed loop (solid) computed in simulation and experimental value (point-dashed) from pulse TS#35109.

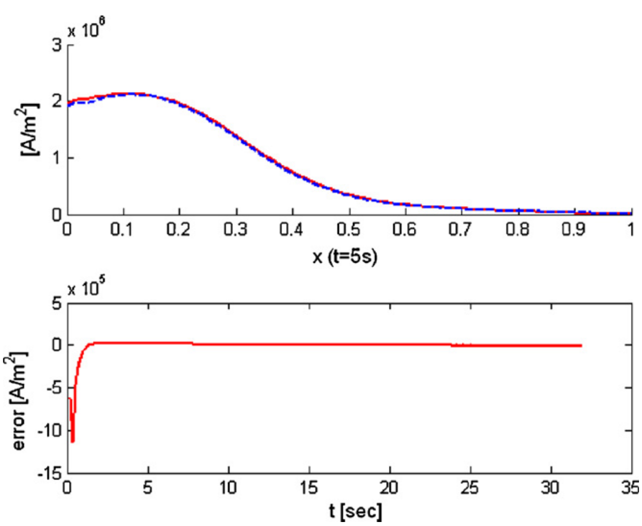

Fig. 4. (Top) Simulated plasma current profile in closed loop (solid) and desired value (dashed) at time $t = 5$ s. (Bottom) Mean control error.

Table 1
Uncertainty intervals on each parameter.

Parameters	δ_{min}	δ_{max}	Parameters	δ_{min}	δ_{max}
w_{lh1}	−0.99	>1	η_{lh2}	−0.85	>1
w_{lh2}	−0.99	>1	$\bar{Z}$	−0.75	0.99
μ_{lh1}	−0.99	0.65	T_e	−0.63	>1
μ_{lh2}	−0.99	0.65	$\bar{n}$	−0.99	0.85
η_{lh1}	−0.85	>1	$\eta_{//}$	−0.7	>1

lines in Fig. 2). Also note that the control request is quiet smooth (no strong oscillations or overshoots). On the real experiment, LH was applied at 4.8 s leading to an increase of the electronic temperature. In the simulation, the experimental open-loop electronic temperature was used to compute plasma resistivity, which means that the control has to manage an unexpected evolution of this parameter at time 4.8 s before reaching steady state.

4.2. Robustness assessment

Further simulations were performed to test the robustness of the proposed control strategy, in particular to check its behaviour in case of discrepancies between the reference model and the process. Two kinds of discrepancies were considered: firstly, errors in the plasma state measurements taken as input in the reference model, such as the mean electron density $\bar{n}$, the effective value of the plasma charge $\bar{Z}$ or the temperature profile T_e. Secondly, uncertainties in the scaling laws used for the estimation of j_{lh} and $\eta_{//}$. For each launcher $i \in [1, 2]$, the current j_{lh} is given by:

$$j_{lhi}(x) \propto \frac{\eta_{lhi} P_{lhi}}{\bar{n} R_0} \exp\left(\frac{\mu_{lhi} - x}{2\sigma_{lhi}}\right)$$
$$\sigma_{lhi} = \frac{(\mu_{lhi} - w_{lhi})}{2\ln(2)}$$

where the parameters η_{lhi}, μ_{lhi}, w_{lhi} are given by scaling laws and are dependant of $\bar{Z}$, N_{lhi}, the total plasma current I_p and the poloidal magnetic field B_θ, see [3]. The parameters vector X_{par} is defined as following:

$$X_{par} = (\, w_{lh1} \quad w_{lh2} \quad \mu_{lh1} \quad \mu_{lh2} \quad \ldots \quad \eta_{lh1} \quad \eta_{lh2} \quad \bar{Z} \quad T_e \quad \bar{n} \quad \eta_{//}\,)^T$$

For the reference model, a different parameters vector X'_{par} was used:

$$X'_{par} = (I_n + \Delta_{par}) X_{par}, \qquad \Delta_{par} = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_{10})$$

where I_n and δ_i, $i \in [1, 10]$ are, respectively, the identity matrix and the relative errors made on each parameters. For each parameter, ones after the other, we performed simulation to define the minimum and maximum deviation that the control can cope with, i.e. keeping time response satisfactory. If δ_i is set out of the interval presented in Table 1, either the system becomes unstable or the current profile is too far from the reference or the control request is too noisy. The actual uncertainties are generally much smaller than these ones. Thus, good performance in practice could be expected.

With this sensibility study, we are able to find which parameters are important for the control loop. For the modelling of the coupling between the plasma and the LHCD launchers, the main parameters seem to be μ_{lh1} and μ_{lh2}. Despite that errors on $\bar{Z}$, T_e, $\bar{n}$, $\eta_{//}$ result in a difference between the dynamics of the reference model and the process, the control law is highly robust against these parameters variations. Series of simulation with randomly distributed model uncertainties up to ±20% have been performed showing the same robustness. However, no theoretical proof of stability with several model uncertainties is given.

5. Conclusion

In this paper a model-based predictive control strategy has been presented for the control of the plasma current density profile. The controller is based on a control-oriented model using a 1D partial differential equation and scaling laws to model magnetic flux diffusion, plasma resistivity and LHCD current drive source. The main idea is to use as much as possible the physics understanding to design the control.

This control strategy provides good results in simulation both in the nominal case and with model uncertainties: the steady state is quickly reached and the system is robust against modelling errors. The next step would be to test it on a more sophisticated plasma response simulator, like the CRONOS suite of codes (see [11]), before final validation on real experiments, where the additional issue of current profile real-time identification from existing measurements will also have to be dealt with.

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