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Polytopic Control of the Magnetic Flux Profile in a Tokamak Plasma ⋆

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Abstract: In this paper, the problem of steady-state regulation of the magnetic flux profile in a tokamak plasma using the lower hybrid current drive as actuator is considered. Based on a simplified control-oriented model of the magnetic flux dynamics in a tokamak plasma a polytopic control law was developed. Shape constraints on the actuator are considered and conditions to derive a polytopic linear parameter-varying controller that guarantees stability of the closed-loop system are given. Validation and implementation issues are discussed and numerical simulations are presented.

Keywords: Linear parameter-varying model, time-varying systems, distributed-parameter systems, nuclear power

1. INTRODUCTION

Controlled thermonuclear fusion consists in the fusion of two light nuclei to form a heavier one, releasing energy in the process. Given current technical capabilities, these light nuclei are usually hydrogen isotopes. The fusion of a deuterium and a tritium nucleus is of particular interest given its high power output. Since deuterium represents a significant percentage of existing hydrogen, and tritium can be easily bred from lithium, the amount of energy that could be produced is almost unlimited. Considering the potential environmental and safety benefits in comparison with other energy production methods, nuclear fusion is a very attractive prospect and an active research topic.

Although there are several experimental devices capable of achieving nuclear fusion, the tokamak configuration is interesting in light of the ITER program currently under way, which aims to prove the technical feasibility of constructing a power plant based on controlled thermonuclear fusion. This motivates the current international effort to develop and refine the necessary control strategies to sustain the difficult operating conditions required for the fusion process inside a Tokamak to be carried out for long enough and high energy shots. The ultimate goal is to obtain an expected energy amplification factor of 10, see ITER Organization (2010).

A detailed exposition of tokamak physics as well as an overview of current and projected experimental facilities can be found in Wesson (2004). As for existing challenges for tokamak operation, Walker et al. (2008) gives an interesting overview. Also, *advanced* tokamak operation, which allows high confinement and magnetohydrodynamic stability is discussed in Taylor (1997), Gormezano (1999) and Wolf (2003). Profile control in DIII-D and JET facilities is detailed in Moreau et al. (2003), Laborde et al. (2005), Moreau et al. (2008), Ou et al. (2007) and subsequent works and Xu et al. (in press). Advances in Tore-Supra can be found in Martin et al. (2000) and Giruzzi et al. (2009).

To avoid using linear models identified around an operating point, a control-oriented distributed model was developed by Witrant et al. (2007) that takes into account physically relevant dynamics and identified peripheral inputs and state couplings. Based on this model, our aim is now to develop a suitable control law for the regulation of the steady-state magnetic flux profile in the tokamak that allows an easier closed-loop stability analysis than the previous regulator developed by Bribiesca Argomedo et al. (2010). In particular, it is based on a polytopic approach similar to the one described in Briat (2008). For some comprehensive surveys on linear parameter-varying systems (LPV) control and gain scheduling controllers, see Leith and Leithead (2000) and Rugh and Shamma (2000). For some applications of LPV/LMI gain-scheduling controls see Wassink et al. (2005) and Gilbert et al. (2010). The actuation method under consideration is restricted to the use of the non inductive lower hybrid current drive (LHCD), which acts as a current and heat source on the plasma.

The article is organized as follows: in Section 2.1, the control model is introduced and the control problem is stated. In Section 2.2, an overview of the reference model

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for the development of the control law is presented as well as appropriate variable changes. In Section 3, sufficient conditions for the derivation of a stabilizing control law are presented. In Section 4, implementation issues and simulation results are presented and discussed.

2. PROBLEM DESCRIPTION

2.1 Preliminaries

The main focus of this article is the control, by means of three parameters of the LHCD deposit, of the poloidal magnetic flux profile in a tokamak plasma. This variable is defined as the flux per radian of the magnetic field $\mathbf{B}(R, Z, t)$, at time t, passing through a surface S delimited by a horizontal disk centered at the toroidal axis at height Z, with radius R as shown in Fig. 1.

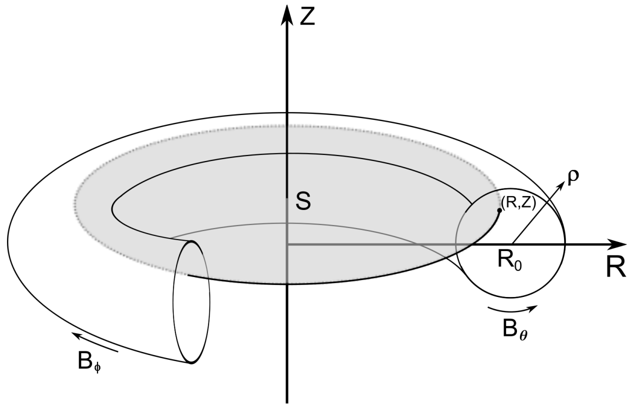


Figure 1. Toroidal plasma centered at R_0, disc coordinates (R,Z) and poloidal magnetic flux surface S.

The poloidal magnetic flux is denoted ψ and defined by:

$$\psi(R, Z, t) \doteq \frac{1}{2\pi} \int_S \mathbf{B}(R, Z, t) \cdot d\mathbf{S}$$

where $\mathbf{B}$ is the magnetic field with poloidal component B_θ and toroidal component B_ϕ.

Using the Grad-Shafranov equilibrium equation, see Blum (1989), Wesson (2004) and Witrant et al. (2007) the magnetic flux $\psi(R, Z, t)$ can be parametrized by time t, and a spatial variable x, which indexes the equilibrium magnetic surfaces.

2.2 From the physical to the LPV Model

The equation representing the time evolution of $\psi(x, t)$ can be spatially discretized as detailed in Witrant et al. (2007). This yields:

$$\dot{\psi} = A(t)\psi + B(t)j_{ni} + W(t) \tag{1}$$

where $A(t)$ is an N×N matrix, $B(t)$ is an N×3 matrix and $W(t)$ is a vector in $\mathbb{R}^N$ representing the evolution of the boundary variables of the poloidal magnetic field. For the rest of this article we may, whenever needed, refer to $\psi(x_i, \cdot)$ simply as $\psi_i(t)$.

Due to the characteristics of the distributed model, the time-varying components of equation (1) can be factorized as a positive definite, diagonal matrix $M(t)$ as follows:

$$\dot{\psi} = M(t)\left(A_{ct}\psi + B_{ct}j_{ni}\right) + W(t) \tag{2}$$

where A_{ct} and B_{ct} are constant matrices of appropriate dimensions. This specific architecture comes from the original non-homogeneous transport PDE model, where a single time and space-varying parameter (the resistivity) multiplies both the diffusive operator and the distributed exogeneous inputs.

New variables are defined around an operating point:

$$\widetilde{\psi} \doteq \psi - \overline{\psi}$$
$$\widetilde{j}_{ni} \doteq j_{ni} - \overline{j}_{ni}$$
$$\widetilde{W} \doteq W - \overline{W}$$

where $(\overline{\psi}, \overline{j}_{ni}, \overline{W})$ is an equilibrium of the original system (that can be obtained through experimental data or numerical simulation).

Using the same hypotheses as in Bribiesca Argomedo et al. (2010), in particular concerning the fact that the plasma current is supposed to be almost constant during steady-state operation of the tokamak, and considering the variations of the bootstrap current around the equilibrium as disturbances, the term $\widetilde{W}$ can be neglected. Furthermore, j_{ni} is reduced to the LHCD current deposition j_{lh} which can be represented, see Witrant et al. (2007), as a function of three parameters $u_p \doteq [\mu_{lh}, \sigma_{lh}, A_{lh}]^T$ resulting in a gaussian distribution:

$$j_{lh}(u_p) = A_{lh}e^{-(x-\mu_{lh})^2/(2\sigma_{lh}^2)} \tag{3}$$

where x is the spatial variable in the distributed system.

Linearizing (3) with respect to a variation of the parameters around $\overline{u}_p \doteq [\overline{\mu}_{lh}, \overline{\sigma}_{lh}, \overline{A}_{lh}]^T$ corresponding to the equilibrium condition $\overline{j}_{lh}$, and defining the variation of the parameters around the equilibrium as $\widetilde{u}_p \doteq u_p - \overline{u}_p$, the resulting equation is:

$$\dot{\widetilde{\psi}} = M(t)\left(A_{ct}\widetilde{\psi} + B_{ct}\nabla_{u_p}j_{lh}\mid_{u_p=\overline{u}_p}\widetilde{u}_p\right) \tag{4}$$

Performing a change of variables $\zeta \doteq M^{-1}(t)\widetilde{\psi}$ and renaming the product $B_{ct}\nabla_{u_p}j_{lh}\mid_{u_p=\overline{u}_p}$ as B_{lin}, the evolution of the new variable ζ is given by:

$$\dot{\zeta} = \left(A_{ct}M(t) - M^{-1}(t)\dot{M}(t)\right)\zeta + B_{lin}\widetilde{u}_p \tag{5}$$

Since $M(t)$ is diagonal and positive definite, $M^{-1}(t)$ always exists and is bounded. Imposing boundedness and differentiability constraints on $M(t)$, we have that the matrix $A_\zeta(t) \doteq A_{ct}M(t) - M^{-1}(t)\dot{M}(t)$ is also bounded. Choosing a nonempty basis $\mathcal{A} = \{A_{\zeta_0}, A_{\zeta_1}, ..., A_{\zeta_{n_p}}\}$, subset of $\mathbb{R}^{N\times N}$, we write:

$$A_\zeta(t) = A_{\zeta_0} + \sum_{i=1}^{n_p}\lambda_i(t)A_{\zeta_i} \tag{6}$$

with $n_p \leq N^2$, $\lambda_i(t) \in [0, 1]$ for all $i \leq n_p$ and all $t \geq 0$. Furthermore, it can be easily shown that, as a consequence of the diagonal structure of $M(t)$, an $n_p \leq 2N$ is enough to exactly represent $A_\zeta(t)$ (a basis of size N to represent the diagonal elements of $M^{-1}(t)\dot{M}(t)$ and another of size N representing each of the columns of A_{ct}).

Using (6) in (5), we get:

$$\dot{\zeta} = \left(A_{\zeta_0} + \sum_{i=1}^{n_p}\lambda_i(t)A_{\zeta_i}\right)\zeta + B_{lin}\widetilde{u}_p \tag{7}$$

In order to reject unmodelled nonlinearities, like the ones represented by the bootstrap current in the system, it is useful to extend this system to include an integrator of a virtual error $\dot{E} = \varepsilon \doteq -C\zeta$, with C in $\mathbb{R}^{N_c \times N}$, as follows:

$$z \doteq \begin{bmatrix} \zeta \\ E \end{bmatrix} \qquad A_0 \doteq \begin{bmatrix} A_{\zeta_0} & 0 \\ -C & 0 \end{bmatrix}$$

$$A_i \doteq \begin{bmatrix} A_{\zeta_i} & 0 \\ 0 & 0 \end{bmatrix} \qquad \forall i \geq 1$$

$$B_e \doteq \begin{bmatrix} B_{lin} \\ 0 \end{bmatrix}$$

The extended system is:

$$\dot{z} = \left(A_0 + \sum_{i=1}^{n_p} \lambda_i(t) A_i \right) z + B_e \widetilde{u}_p \tag{8}$$

Throughout the rest of the article, this will be the reference model for the development of a control law and the corresponding simulations. $N_e = N_c + N$ will denote the size of the vector z.

3. CONTROLLER SYNTHESIS RESULTS

Let us define the set of all partitions of $\mathcal{N}_p \doteq \{1, 2, ...n_p\}$ as $\Omega(\mathcal{N}_p) \doteq \{(\mathcal{C}_j, \mathcal{D}_j) \mid \mathcal{C}_j \cap \mathcal{D}_j = \emptyset, \mathcal{C}_j \cup \mathcal{D}_j = \mathcal{N}_p\}$. It is clear that *card* $\Omega(\mathcal{N}_p) = 2^{n_p}$. Based on this set, let us consider a polytopic control law for a given set of vertex controllers $K_1, ..., K_{2^{n_p}} \in \mathbb{R}^{3 \times N}$ and time-varying parameters $\lambda_1(t), ..., \lambda_{n_p}(t) \in [0, 1]$ as:

$$\widetilde{u}_p = \sum_{j=1}^{2^{n_p}} \beta_j(t) K_j z \tag{9}$$

where:

$$\beta_j(t) = \prod_{k \in \mathcal{C}_j} (1 - \lambda_k(t)) \prod_{l \in \mathcal{D}_j} \lambda_l(t)$$
$$(\mathcal{C}_j, \mathcal{D}_j) \in \Omega(\mathcal{N}_p), \ \forall j \in \mathcal{N}_p$$

Remark 1. For all j in $\mathcal{N}_p$ and all $t \geq 0$: $\beta_j(t) \in [0, 1]$. It can also be shown, by induction on n_p for instance, that $\sum_{j=1}^{2^{n_p}} \beta_j(t) = 1$ for all $t \geq 0$.

Theorem 2. A polytopic control law, as defined in (9), that quadratically stabilizes system (8) can be constructed by setting $K \doteq Q_j W^{-1}$, with $W \in \mathbb{R}^{N_e \times N_e}$ a positive definite symmetric matrix and $Q_j \in \mathbb{R}^{3 \times N_e}$, $j = 1, 2, 3, ..., 2^{n_p}$, full matrices such that the following LMIs are verified [1] :

$$\begin{bmatrix} \varepsilon^{-1} \mathbb{I}_{N_e} & W \\ W & -M_j \end{bmatrix} \succ 0$$
$$\forall j \in \{1, 2, 3, ..., 2^{n_p}\} \tag{10}$$

where, ε is a positive constant and, for all j, M_j is defined as:

$$M_j \doteq \left(A_0 + \sum_{i=1}^{n_p} s_{i,j} A_i \right) W + W \left(A_0 + \sum_{i=1}^{n_p} s_{i,j} A_i \right)^T + B_e Q_j + Q_j^T B_e^T$$

with, for all j, $s_{i,j} = 0$ if $i \in \mathcal{C}_j$, and $s_{i,j} = 1$ otherwise.

[1] Here $\cdot \succ 0$ means that a matrix is positive definite.

Proof. Using the Schur complement, (10) is equivalent to:

$$\left(A_0 + \sum_{i=1}^{n_p} s_{i,j} A_i \right) W + W \left(A_0 + \sum_{i=1}^{n_p} s_{i,j} A_i \right)^T + B_e Q_j + Q_j^T B_e^T + \varepsilon W^2 \prec 0$$
$$\forall j \in \{1, 2, 3, ..., 2^{n_p}\} \tag{11}$$

Set $Q_j = K_j W$ and $P = W^{-1}$. Substituting these in equation (11) and pre- and post-multiplying by P we obtain:

$$P \left(A_0 + \sum_{i=1}^{n_p} s_{i,j} A_i + B_e K_j \right) + \left(A_0 + \sum_{i=1}^{n_p} s_{i,j} A_i + B_e K_j \right)^T P + \varepsilon \mathbb{I}_{N_e} \prec 0$$
$$\forall j \in \{1, 2, 3, ..., 2^{n_p}\} \tag{12}$$

Multiplying each inequality by the corresponding β_j and adding them up (remembering that $\sum_{j=1}^{2^{n_p}} \beta_j(t) = 1$) we obtain:

$$P \left(\sum_{j=1}^{2^{n_p}} \beta_j \left[A_0 + \sum_{i=1}^{n_p} s_{i,j} A_i + B_e K_j \right] \right) + \left(\sum_{j=1}^{2^{n_p}} \beta_j \left[A_0 + \sum_{i=1}^{n_p} s_{i,j} A_i + B_e K_j \right] \right)^T P + \varepsilon \mathbb{I}_{N_e} \prec 0 \tag{13}$$

Rearranging the order of the sums, it is easy to see that:

$$\sum_{j=1}^{2^{n_p}} \beta_j \left[\sum_{i=1}^{n_p} s_{i,j} A_i \right] = \sum_{i=1}^{n_p} A_i \left[\sum_{j=1}^{2^{n_p}} \beta_j s_{i,j} \right] \tag{14}$$

And since $\forall j$, $s_{i,j} = 0$ if $i \in \mathcal{C}_j$, and $s_{i,j} = 1$ otherwise, for any given j, we have:

$$\sum_{j=1}^{2^{n_p}} \beta_j s_{i,j} = \lambda_i \prod_{k \in \mathcal{C'}_j} (1 - \lambda_k(t)) \prod_{l \in \mathcal{D'}_j} \lambda_l(t)$$
$$(\mathcal{C'}_j, \mathcal{D'}_j) \in \Omega(\mathcal{N}_p \backslash \{i\}) \ , \ \ \forall j \in \mathcal{N}_p \backslash \{i\} \tag{15}$$

By an argument analogous to the one required to prove that $\sum_{j=1}^{2^{n_p}} \beta_j(t) = 1$, it can be shown that $\sum_{j=1}^{2^{n_p}} \beta_j s_{i,j} = \lambda_i$. And using these two facts, equation (13) is equivalent to:

$$P \left(A_0 + \sum_{i=1}^{n_p} \lambda_i A_i + B_e \sum_{j=1}^{2^{n_p}} \beta_j K_j \right) + \left(A_0 + \sum_{i=1}^{n_p} \lambda_i A_i + B_e \sum_{j=1}^{2^{n_p}} \beta_j K_j \right)^T P + \varepsilon \mathbb{I}_{N_e} \prec 0 \tag{16}$$

which, if pre-multiplied by z^T and post-multiplied by z is actually the time derivative of $V(z) = z^T P z$ for the system (8) under the control law (9). This implies that V is a Lyapunov function for the closed-loop system.

□

It may also be desirable to constrain the gain of the controller (particularly since the application under consideration is based on a linearization around a given input value). Let us denote by $\|\cdot\|_2$ the $\mathcal{L}^2$-norm of a vector or the respective induced norm of a matrix.

Proposition 3. Let $W \in \mathbb{R}^{N_e \times N_e}$ be a positive definite matrix, $K \in \mathbb{R}^{3\times N_e}$ a full matrix and $Q \doteq KW$. A sufficient condition to guarantee that $\|K\|_2 < \sqrt{\gamma}$ is that the following LMIs are satisfied:

$$\begin{bmatrix} -\mathbb{I}_3 & Q \\ Q^T & -\gamma \mathbb{I}_{N_e} \end{bmatrix} \prec 0$$
$$W \succ \mathbb{I}_{N_e} \tag{17}$$

where $\mathbb{I}_l$ represents the $l \times l$ identity matrix.

Proof. Using the Schur complement, the first inequality is clearly equivalent to $Q^TQ - \gamma\mathbb{I}_{N_e} \prec 0$ which in turn implies $z^TQ^TQz < \gamma z^T\mathbb{I}_{N_e}z$, $\forall z \in \mathbb{R}^{N_e}$. That is, $\|Q\|_2 < \sqrt{\gamma}$. In turn, the second LMI implies $\|W\|_2 > 1$. Since $Q = KW$, we have that $\|K\|_2 < \sqrt{\gamma}$.

□

Combining Theorem 2 and Proposition 3, the following corollary is directly obtained:

Corollary 4. Given $\gamma > 0$, a polytopic control law as defined in (9) that quadratically stabilizes system (8) and has an $\mathcal{L}_2$ gain between the state and control input strictly less than $\sqrt{\gamma}$, can be computed by setting $K_j \doteq Q_jW^{-1}$, with $W \in \mathbb{R}^{N_e\times N_e}$ a positive definite symmetric matrix and $Q_j \in \mathbb{R}^{3\times N_e}$, $j = 1, 2, 3, ..., 2^{n_p}$, full matrices such that the following LMIs are verified:

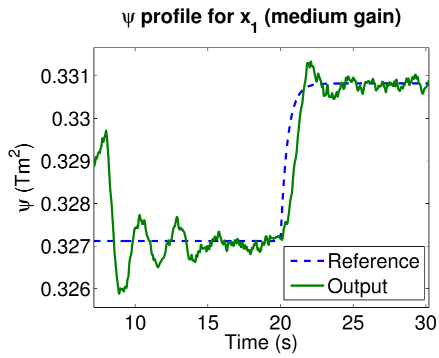
$$\begin{bmatrix} \varepsilon^{-1}\mathbb{I}_{N_e} & W \\ W & -M_j \end{bmatrix} \succ 0$$
$$\begin{bmatrix} -\mathbb{I}_3 & Q_j \\ Q_j^T & -\gamma\mathbb{I}_{N_e} \end{bmatrix} \prec 0$$
$$W \succ \mathbb{I}_{N_e}$$
$$\forall j \in \{1, 2, 3, ..., 2^{n_p}\} \tag{18}$$

where M_j is defined as in Theorem 2.

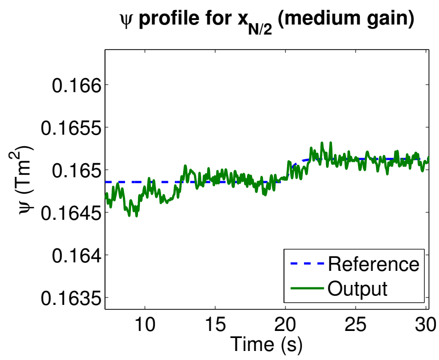
4. VALIDATION

4.1 Implementation

In order to implement the discussed results, a suitable value for n_p had to be determined that could provide a sufficiently good approximation of the original function while reducing the computational cost, particularly since the number of decision variables and the size of the LMIs grows exponentially with the number of parameters used. Choosing an adequate basis, the number of parameters was reduced to 5 while preserving a small approximation error, in average under 1% for the $A_{ct}M(t)$ term and with a peak at around 10% for the $M^{-1}(t)\dot{M}(t)$ term in a few points. The data was fitted using a least squares method with a positivity constraint on the coefficients, to prevent the existence of aberrant vertices having, for instance, a



(a)



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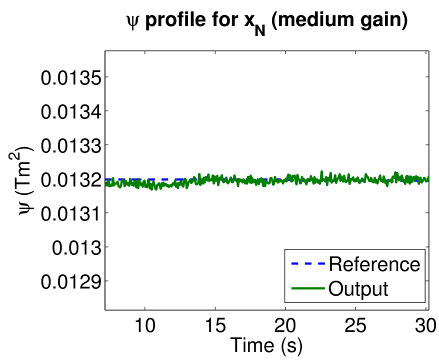

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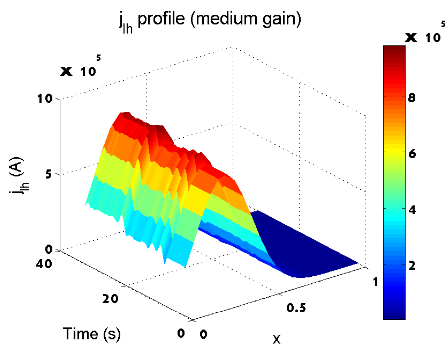
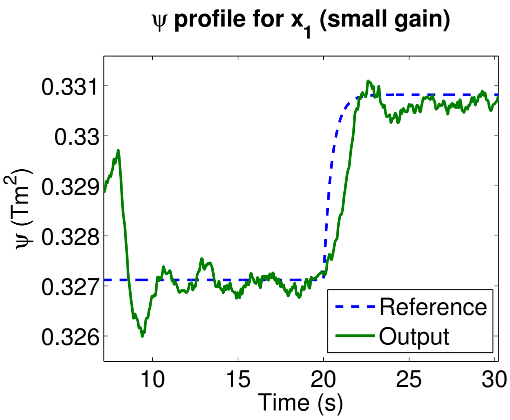
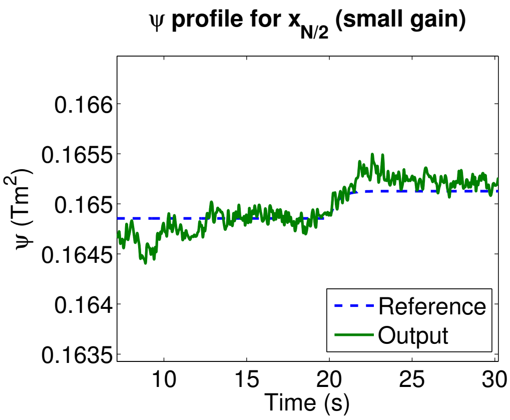


(d)

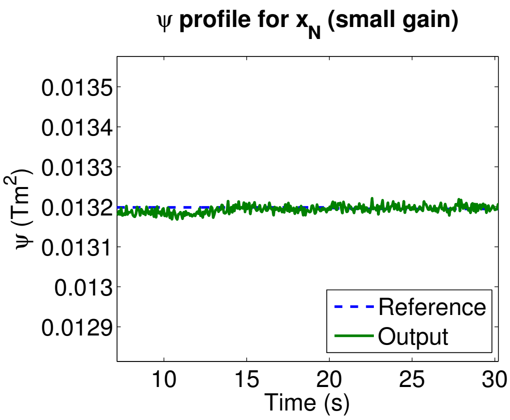
Figure 2. Regulation around $\overline{\psi}$ with LMI controller with intermediate gain (plain line: numerical simulation, dashed line: the reference). (a) Evolution and reference of the state ψ_1; (b) evolution and reference of the state $\psi_{N/2}$; (c) evolution and reference of the state ψ_N; (d) applied j_{lh} profile.



(a)



(b)



(c)

Figure 3. Regulation around $\overline{\psi}$ with LMI controller with a more restricted gain (plain line: numerical simulation, dashed line: the reference). (a) Evolution and reference of the state ψ_1; (b) evolution and reference of the state $\psi_{N/2}$; (c) evolution and reference of the state ψ_N.

negative diffusion coefficient. As a further development a recursive least-squares algorithm could be implemented.

The parameter ε ensures that the real part of all closed-loop poles is less than $-\varepsilon$, indirectly allowing the controller to be more robust with respect of modeling errors due, for instance, to the use of only 5 parameters in the approximation of $A_{ct}M(t)$ and $M^{-1}(t)\dot{M}(t)$. The necessary systems of LMIs were solved using YALMIP, see Löfberg (2004), and SeDuMi (2011).

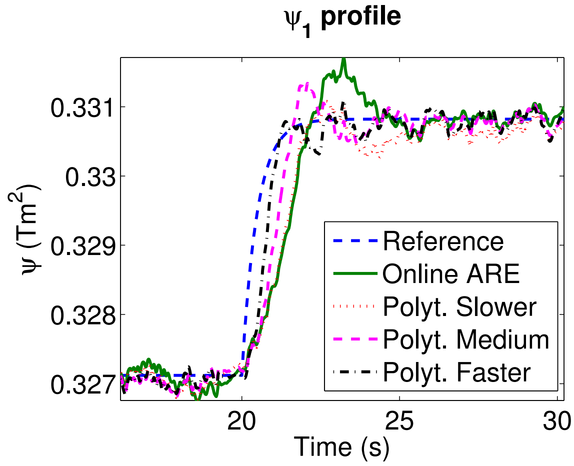


Figure 4. Regulation around $\overline{\psi}$ for ψ_1. The thin dashed line is the reference, the solid line is the ARE-based simulation and the others are polytopic regulators with different γ values.

4.2 Simulation Results

In order to test the proposed method by numerical simulation, a reference was chosen at three points of the ψ profile, taken from estimations based on Tore-Supra shot TS-35109 and equal to the one used in Bribiesca Argomedo et al. (2010). The global parameters of the simulation were also taken from shot TS-35109 ($I_p = 0.6$ MA, power input around 1.8 MW). The system was discretized in 8 points for the controller calculation and in 22 for the simulation. In all the simulations, the system is taken close to the desired operating point by applying the open-loop control sequence of the actual shot TS-35109, and then, at $t = 8$ s activating the controller. A change of reference is then applied at $t = 20$ s.

In Figure 2, a controller with medium gain $\gamma = 2.75 \times 10^{-6}$ was chosen with a settling time comparable to that presented in Bribiesca Argomedo et al. (2010). Figures 2(a), (b) and (c) show the evolution of the points ψ_1, $\psi_{N_e/2}$ and ψ_{N_e}, respectively; 2(d) shows the applied j_{lh} profile. It can be seen that j_{lh} is a gaussian curve, the parameters of which are calculated as $u + \overline{u}_p$.

To show the versatility of the proposed gain limit, another controller was calculated with stricter gain limitations by reducing the value of the γ parameter in the LMI system to 2.5×10^{-6}. The results are shown in Figure 3: (a), (b) and (c) show the evolution of the points ψ_1, $\psi_{N_e/2}$ and ψ_{N_e}, respectively. The settling time is greater than in the previous case, which is to be expected when restricting the gain.

To further illustrate the interest of the proposed scheme, Figure 4 shows a comparison of the behaviour of ψ_1 for three different polytopic controllers with different values of γ and the online ARE approach used in Bribiesca Argomedo et al. (2010). It should be noted that the tuning of the LMI-based polytopic controllers is much easier than finding appropriate weighting matrices for the LQR computation used in Bribiesca Argomedo et al. (2010). It is also important to mention that the online computational cost of the polytopic controller is much less than the ARE-based one, since it only requires to estimate the current values of the parameters λ_i whereas the latter requires to solve an ARE at each sampling time. Nevertheless, the polytopic approach does require the prior solution of the system of LMIs which can be done offline and only needs to be done once.

5. CONCLUSIONS AND PERSPECTIVES

In this paper, a polytopic controller was developed for the regulation of the magnetic flux profile in a tokamak plasma based on a physically relevant, distributed but simplified model of the system. After discretization of the model, a change of variables was performed that allowed for the simple construction of a polytopic control law. The theoretical results apply to the case of bounded parameters with bounded time derivatives and are then tested under simulation with a more complete model to test the robustness of the approach with respect to unmodeled dynamics, disturbances and approximation errors.

Further work will focus on avoiding the discretization when computing the control and obtaining useful results in the infinite-dimensional setting. Some work will address as well the dependance of the control in the derivative of the resistivity coefficients, thus removing the need for noise-filtering. Experimental validation on Tore-Supra is also expected.

6. ACKNOWLEDGMENTS

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