Lyapunov functions for switched linear hyperbolic systems [*]

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Abstract: Systems of conservation laws and systems of balance laws are considered in this paper. These kinds of infinite dimensional systems are described by a linear hyperbolic partial differential equation with and without a linear source term. The dynamics and the boundary conditions are subject to a switching signal that is a piecewise constant function. By means of Lyapunov techniques some sufficient conditions are given for the exponential stability of the switching system, uniformly for all switching signals. Different cases are considered depending on the presence or not of the linear source term in the hyperbolic equation, and depending on the dwell time assumption on the switching signals. Some numerical simulations are also given to illustrate some main results, and to motivate this study.

Keywords: Hyperbolic system, Lyapunov function, partial differential equation, switched systems.

1. INTRODUCTION

Lyapunov function based techniques are commonly used for the stability analysis of dynamical systems, such as those modeled by partial differential equations (PDEs). The present paper focuses on a class of one-dimensional hyperbolic equations that describe, for example, systems of conservation laws or balance laws (with a sink term). Systems of balance laws are systems of conservation laws with a source term (as in Diagne et al. [2012]). This kind of systems with infinite dimensional dynamics is relevant for a wide range of physical networks having an engineering interest. Among the potential applications, hydraulic networks (see Prieur et al. [2008], Dos Santos and Prieur [2008]), electric line networks, road traffic networks (see e.g., Haut and Bastin [2007]), gas flow in pipeline networks (as in Bastin et al. [2008], Dick et al. [2010]), or flow regulation in deep pits studied in Witrant and Marchand [2008], Witrant and Niculescu [2010] are of prime environmental or industrial importance.

Switching behavior occurs for many control applications when evolution processes involve logical decisions, see El-Farra and Christofides [2004] for the case where a stabilizing feedback is designed by means of Lyapunov techniques applied to a discretization of switched parabolic PDE; see also Hante et al. [2009], where the well-posed issue and the dependence of the solutions on the data of a network of hyperbolic equations with switching as a control are considered. Switching can indeed be an efficient control strategy for many infinite dimensional systems such as the wave equation (Gugat and Tucsnak [2011]), the heat equation (Zuazua [2011]) or other infinite dimensional systems written in abstract form (as in Hante et al. [to appear]).

The stabilizability of such systems is often proved by means of a Lyapunov function, as illustrated by the contributions from Dick et al. [2010], Krstic and Smyshlyaev [2008], Prieur and de Halleux [2004] where different control problems are solved for particular hyperbolic equations. For more general nonlinear hyperbolic equations, the knowledge of Lyapunov functions can be useful for the stability analysis of a system of conservation laws (see Coron et al. [2007]), or even for the design of stabilizing boundary controls (see Coron et al. [2008]).

To demonstrate asymptotic stability of switched linear hyperbolic systems, a Riemann invariant approach is used in Amin et al. [2012]. In the present paper, we derive sufficient conditions on the system parameters to prove Lyapunov stability for classes of hyperbolic equations. The stability property depends on the classes of the switching rules applied to the dynamics (as in Liberzon [2003] for finite dimensional systems). The present paper is also related to Prieur and Mazenc [2012] where unswitched time-varying hyperbolic systems are considered.

In this paper two classes of switched PDE are considered: first the hyperbolic equations, as derived by studying a system of conservation laws, and then the systems written in terms of balance laws (see respectively Sections 2 and 3

below). For both classes of infinite dimensional dynamical systems, some sufficient conditions for the asymptotic stability are stated, with an uniformity property with respect to either any piecewise constant switching signal or piecewise constant switching signals with a sufficiently large dwell time.

The paper is organized as follows. The class of switched linear hyperbolic systems under consideration in this paper is given in Section 2. Switched systems of balance laws are considered in Section 3. In Section 4 an academic example is studied to illustrate one main result (more precisely Theorem 1 below) and the necessity of the dwell time assumption in another main result (namely Theorem 2).

Due to space limitation, the proofs are omitted.

Notation. The set $\mathbb{R}_+$ is the set of nonnegative real numbers. A continuous function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{K}$ provided that it is increasing and $\alpha(0) = 0$. It belongs to class $\mathcal{K}_\infty$ if, in addition, $\alpha(r) \to \infty$ as $r \to \infty$. Given a matrix G, the transpose matrix of G is denoted as $G^\top$, whereas the matrix $\mathtt{Sym}(G)$ denotes the symmetric part of G: $\mathtt{Sym}(G) = \frac{1}{2}(G + G^\top)$. For each positive integer n, I_n and 0_n are respectively the identity and the null matrix in $\mathbb{R}^{n \times n}$. Given some scalar values $(a_1, \ldots, a_n)$, $\mathtt{diag}(a_1, \ldots, a_n)$ is the matrix in $\mathbb{R}^{n \times n}$ with zero non-diagonal entries, and with $(a_1, \ldots, a_n)$ on the diagonal. The usual Euclidian norm in $\mathbb{R}^n$ is denoted by $|\cdot|$ and the associate matrix norm is denoted $\|\cdot\|$, whereas the set of all functions $\phi : [0, 1] \to \mathbb{R}^n$ such that $\int_0^1 |\phi(x)|^2 < \infty$ is denoted by $L^2(0, 1)$. The set $PC(0, 1)$ is the set of piecewise continuous functions $\phi : [0, 1] \to \mathbb{R}^n$. Following Coron et al. [2008], we introduce the notation, for all matrices $M \in \mathbb{R}^{n \times n}$,

$$\rho_1(M) = \inf\{\|\Delta M \Delta^{-1}\|, \ \Delta \in \mathcal{D}_{n,+}\} , \qquad (1)$$

where $\mathcal{D}_{n,+}$ denotes the set of diagonal positive matrix in $\mathbb{R}^{n \times n}$.

2. SWITCHED LINEAR HYPERBOLIC SYSTEMS

Let us first consider the following switched linear hyperbolic equation:

$$\partial_t y(x, t) + \Lambda_i \partial_x y(x, t) = 0 , \quad x \in [0, 1], t \geq 0 \qquad (2)$$

where, for each $i \in I$, Λ_i is a diagonal (structured) and invertible matrix in $\mathbb{R}^{n \times n}$ such that $\Lambda_i = \mathtt{diag}(\lambda_{i,1}, \ldots, \lambda_{i,n})$, with

$$\lambda_{i,1} < \ldots < \lambda_{i,m} < 0 < \lambda_{i,m+1} < \ldots < \lambda_{i,n} \qquad (3)$$

and m does not depend on the index i in a given finite set I. Let us define the matrices $\Lambda_i^+ = \mathtt{diag}(|\lambda_{i,1}|, \ldots, |\lambda_{i,n}|)$.

The state at time $t \geq 0$ is $y(., t)$, a function that maps the x variable to $\mathbb{R}^n$. The index i is a time-varying function with values in the set I.

From the structure described by (3), let us use the notation: $y = \begin{pmatrix} y_- \\ y_+ \end{pmatrix}$, where y_- is in $\mathbb{R}^m$ and y_+ is in $\mathbb{R}^{n-m}$, $\Lambda_{i,-} = \mathtt{diag}(\lambda_{i,1}, \ldots, \lambda_{i,m})$, and $\Lambda_{i,+} = \mathtt{diag}(\lambda_{i,m+1}, \ldots, \lambda_{i,n})$.

Let us consider the following boundary conditions:

$$\begin{pmatrix} y_-(1, t) \\ y_+(0, t) \end{pmatrix} = G_i \begin{pmatrix} y_-(0, t) \\ y_+(1, t) \end{pmatrix} , \quad t \geq 0 \qquad (4)$$

where, for each $i \in I$, G_i is a matrix in $\mathbb{R}^{n \times n}$.

Given a piecewise continuous function $y^0 : [0, 1] \to \mathbb{R}^n$, the initial condition is

$$y(x, 0) = y^0(x) , \quad x \in [0, 1] . \qquad (5)$$

The first problem considered is the stability analysis of (2) and (4) for different classes of switching signals. Let us define a *switching signal* as a piecewise constant function $i : \mathbb{R}_+ \to I$, and denote by $\mathcal{S}(\mathbb{R}_+, I)$ the set of switching signals such that, for each interval of $\mathbb{R}_+$, the restriction of the function i on this interval has only a finite number of discontinuity points. This allows to avoid the *Zeno behavior*, as described in Liberzon [2003]. Now, given $\tau > 0$, let us denote by $\mathcal{S}_\tau(\mathbb{R}_+, I)$ the set of switching signals with a *dwell time* larger than τ, that is the set of switching signals such that the durations between consecutive switches are no shorter than τ.

The unswitched case is obtained by considering a constant index $\bar{i}$ in Equations (2) and (4). This system is denoted as $(2)_{\bar{i}}$ and $(4)_{\bar{i}}$. It has been considered exhaustively by Coron [2007], Bressan [2000], Li [1994], among others. Switched linear hyperbolic systems are less explored in the literature, although there is a large literature for switched finite-dimensional systems (see Liberzon [2003] and references therein). Given a switching signal i, the existence of a solution for the switched hyperbolic system (2) with the boundary conditions (4) and the initial condition (5) is established by Hante et al. [2009] (see also Amin et al. [2012]) when focusing on piecewise continuous solutions of (2) with the boundary conditions (4) and piecewise continuous initial conditions (5).

The aim of this work is to use Lyapunov functions to state the exponential stability of (2) and (4) for any switching signal in $\mathcal{S}(\mathbb{R}_+, I)$ or in $\mathcal{S}_\tau(\mathbb{R}_+, I)$. Prior to this, let us recall the definition of a Lyapunov function (see e.g. [Luo et al. 1999, Def. 3.62]).

Definition 2.1. *Let $V : PC(0, 1) \to \mathbb{R}$ be a continuously differentiable function, and $\mathcal{S} \subset \mathcal{S}(\mathbb{R}_+, I)$ be a set of switching signals. The functional V is said to be a* Lyapunov functional *for (2) and (4) uniformly for all switching signals in $\mathcal{S}$, if there are two functions κ_S and κ_M of class $\mathcal{K}_\infty$ and one value $\lambda > 0$ such that, for all functions $\phi \in PC(0, 1)$,*

$$\kappa_S\left(|\phi|_{L^2(0,1)}\right) \leq V(\phi) \leq \int_0^1 \kappa_M\left(|\phi(x)|\right) dx \qquad (6)$$

and for all i in $\mathcal{S}$, for all piecewise continuous initial conditions $y^0 : [0, 1] \to \mathbb{R}^n$, for all solutions of (2), (4) and (5), for all $t \geq 0$, the following holds

$$\frac{dV(y(., t))}{dt} \leq -\lambda V(y(., t)) . \qquad (7)$$

In the previous definition the time derivative (7) has to be understood along the characteristics, as in Coron et al. [2008].

Remark 1. When the system (2) and (4) admits a Lyapunov functional V for the set $\mathcal{S}$, then one can check through elementary calculations that, for all solutions of (2), (4) and (5), for all switching signals i in $\mathcal{S}$, and for all instants $t \geq 0$, the inequality

$$|y(.,t)|_{L^2(0,1)} \leq \kappa_S^{-1} \left(e^{-\lambda t} \int_0^1 \kappa_M(|y^0(x)|) dx \right)$$

holds. This inequality is the analogue for the switched system (2), and (4) of the asymptotic stability uniform for all switching signals in $\mathcal{S}$.

When additionally, there exists $c > 0$ such that for all solutions of (2), (4) and (5), for all switching signals i in $\mathcal{S}$, and for all instants $t \geq 0$, the inequality

$$|y(.,t)|_{L^2(0,1)} \leq ce^{-\lambda t}|y^0|_{L^2(0,1)}$$

holds, then the switched system (2) and (4) is said to be *exponentially stable uniformly for all switching signals in $\mathcal{S}$.* ∘

To state the exponential stability some assumptions on the PDE (2) and on the boundary conditions (4) need to be introduced. More precisely the first assumption under interest is the following:

Assumption 1. *For all i in I, the following holds*

$$\rho_1(G_i) < 1 \ . \tag{8}$$

Before stating our first main result, let us shortly comment this assumption. Due to Coron et al. [2008], assuming that the switching signal i is constant, that is $i(t) = \bar{i}$, for all $t \geq 0$, then Assumption 1 implies that the unswitched system $(2)_{\bar{i}}$ and $(4)_{\bar{i}}$ is asymptotically stable. More precisely, under condition (8), due to (1), there exists a diagonal positive definite matrix $\Delta_{\bar{i}}$ such that $\|\Delta_{\bar{i}} G_{\bar{i}} \Delta_{\bar{i}}^{-1}\| < 1$. Then it is shown in Coron et al. [2008] that by letting $Q_{\bar{i}} = \Delta_{\bar{i}}^2(\Lambda_{\bar{i}}^+)^{-1}$, we have

$$\Lambda_{\bar{i}}^+ Q_{\bar{i}} - G_{\bar{i}}^\top Q_{\bar{i}} \Lambda_{\bar{i}}^+ G_{\bar{i}} > 0_n \tag{9}$$

and, by selecting a suitable $\mu > 0$, it is also proven in the same reference that the function $V : y \mapsto \int_0^L e^{-\mu x} y(x)^\top Q_{\bar{i}} y(x) dx$ is a Lyapunov function for the unswitched hyperbolic system $(2)_{\bar{i}}$ and $(4)_{\bar{i}}$.

Our first main result shows that under a dwell time property on the switching signal, and under Assumption 1, the switched system (2) and (4) is uniformly exponentially stable:

Theorem 1. *Under Assumption 1, there exists $\bar{\tau} > 0$ such that for all $0 < \bar{\tau} < \tau$, the switched system (2) and (4) is exponentially stable uniformly for all switching signals in $\mathcal{S}_\tau(\mathbb{R}_+, I)$.*

In comparison with Assumption 1, a stronger assumption is thus to assume that the matrix $Q_{\bar{i}}$ in (9) does not depend on the index $\bar{i}$. This motivates the following assumption:

Assumption 2. *There exists a diagonal positive definite matrix Q in $\mathbb{R}^{n \times n}$ such that, for all i in I,*

$$\Lambda_i^+ Q - G_i^\top Q \Lambda_i^+ G_i > 0_n \ , \tag{10}$$

and

$$\Lambda_i^+ Q > 0_n \ , \tag{11}$$

hold.

This assumption allows us to prove the exponential stability for a larger set of switching signals:

Theorem 2. *Under Assumption 2, the switched system (2) and (4) is exponentially stable uniformly for all switching signals in $\mathcal{S}(\mathbb{R}_+, I)$.*

3. STABILITY OF SWITCHED SYSTEMS OF BALANCE LAWS

The aim of this section is to consider the case of switched systems of balance laws. First let us recall that a unswitched system of balance laws is given by

$$\partial_t y(x,t) + \Lambda \partial_x y(x,t) = F y(x,t) \ , \quad x \in [0,1], \ t \geq 0 \tag{12}$$

where F is a constant matrix of appropriate dimension, which introduces a source or a sink effect, in contrast with

$$\partial_t y(x,t) + \Lambda \partial_x y(x,t) = 0 \ , \quad x \in [0,1], \ t \geq 0 \tag{13}$$

for an unswitched hyperbolic system of conservation laws. Both kinds of system are considered with the following boundary conditions:

$$\begin{pmatrix} y_-(1,t) \\ y_+(0,t) \end{pmatrix} = G \begin{pmatrix} y_-(0,t) \\ y_+(1,t) \end{pmatrix} \ , \quad t \geq 0 \ . \tag{14}$$

The stability study for a system of balance laws is motivated as follows:

Remark 2. Using Coron et al. [2008] (see also Prieur and Mazenc [2012], Prieur et al. [2008]), we may prove that if the condition $\rho_1(G) < 1$ for the stability of the system (13) with the boundary conditions (14) is satisfied then, for any sufficiently small matrix F, the system (12) with the boundary conditions (14) is also asymptotically stable.

However, when the matrix F is not sufficiently small, the stability of the system (13) with the boundary conditions (14) does not imply the stability the system (12) with the boundary conditions (14), even when F is a Hurwitz matrix. See Appendix A below for an example of such a system of balance laws. This motivates the study of the stability of systems of balance laws by considering the complete dynamics and the coupling between the hyperbolic system (that is the left-hand side of (12)) and the source term (that is the right-hand side of (12)). ∘

Let us introduce the following switched system of balance laws:

$$\partial_t y(x,t) + \Lambda_i \partial_x y(x,t) = F_i y(x,t) \ , \quad x \in [0,1], t \geq 0 \tag{15}$$

where, for each $i \in I$, Λ_i is a diagonal and invertible matrix in $\mathbb{R}^{n \times n}$ such that $\Lambda_i = \mathtt{diag}(\lambda_{i,1}, \ldots, \lambda_{i,n})$, and such that (3) holds, and F_i is a matrix in $\mathbb{R}^{n \times n}$.

Let us consider again the boundary conditions (4) where, for each $i \in I$, G_i is a matrix in $\mathbb{R}^{n \times n}$. Given a piecewise continuous function $y^0 : [0,1] \to \mathbb{R}^n$, the initial condition is (5).

To study the stability condition of the switched system of balance laws (15) with the boundary conditions (4), with and without any dwell time assumption on the switching signal, we need the following:

Assumption 3. *There exist a positive value $\underline{\kappa}$, a diagonal positive definite matrix Q in $\mathbb{R}^{n \times n}$ such that, for all i in I and for all $\kappa \geq \underline{\kappa}$,*

$$\Lambda_i^+ Q - G_i^\top Q \Lambda_i^+ G_i > 0_n \ , \tag{16}$$

and

$$\mathtt{Sym}(\Lambda_i^+ Q - \kappa Q F_i) > 0_n \tag{17}$$

hold.

We can now state the last main results of this paper:

Theorem 3. *Under Assumption 1, and for sufficiently small matrices F_i, there exists $\bar{\tau} > 0$ such that for all $0 < \bar{\tau} < \tau$, the switched system (15) and (4) is exponentially stable uniformly for all switching signals in $\mathcal{S}_\tau(\mathbb{R}_+, I)$.*

Theorem 4. *Under Assumption 3, the switched system (15) and (4) is exponentially stable uniformly for all switching signals in $\mathcal{S}(\mathbb{R}_+, I)$.*

4. ILLUSTRATING EXAMPLE

In this example, to illustrate Theorems 1 and 2, we consider the following switched hyperbolic system:

$$\partial_t y(x,t) + \partial_x y(x,t) = 0 , \quad x \in [0,1], t \geq 0 \qquad (18)$$

where $y(x,t) \in \mathbb{R}^2$. There is no switch in the dynamical equation (18), but the boundary conditions are defined from a switching signal $i : \mathbb{R}_+ \to \{1,2\}$ as

$$y(0,t) = G_i y(1,t) , t \geq 0 \qquad (19)$$

where $G_1 = \begin{pmatrix} 0.1 & 0.6 \\ -1.2 & 0.1 \end{pmatrix}$ and $G_2 = \begin{pmatrix} 0.1 & 1.2 \\ -0.6 & 0.1 \end{pmatrix}$.

Considering the diagonal positive definite matrices $D_1 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$, and $D_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 \end{pmatrix}$, it can be verified that, for each $\bar{i} \in \{1,2\}$, $\|D_{\bar{i}} G_{\bar{i}} D_{\bar{i}}^{-1}\| < 1$ and thus $\rho_1(G_{\bar{i}}) < 1$. Therefore, with Coron et al. [2008], for each $\bar{i} \in \{1,2\}$, the unswitched hyperbolic system $(18)_{\bar{i}}$ and $(19)_{\bar{i}}$ is globally exponentially stable, and Assumption 1 holds.

Moreover, using Theorem 1, there exists $\bar{\tau} > 0$ such that for all $0 < \bar{\tau} < \tau$, the switched system (18) and (19) is exponentially stable uniformly for all switching signals in $\mathcal{S}_\tau(\mathbb{R}_+, I)$.

To illustrate that, let us consider the following switching signal:

$$i(t) = \begin{cases} 1, & \text{if } t \in \cup_{n \in \mathbb{N}}[4n, 4n+2) , \\ 2, & \text{if } t \in \cup_{n \in \mathbb{N}}[4n+2, 4n+4) , \end{cases} \qquad (20)$$

and let us numerically compute the solution of (18) and (19) with this switching signal and with the following initial condition:

$$y(x,0) = \begin{pmatrix} \sin(x) \\ 0 \end{pmatrix} , \quad \forall x \in [0,1] . \qquad (21)$$

It can be checked on Figure 1 that the solution converge to 0 as t increases.

Now to illustrate that the attractivity does not hold for all switching signals without any positive dwell time, let us consider the following function:

$$i(t) = \begin{cases} 1, & \text{if } t \in \cup_{n \in \mathbb{N}}[2n, 2n+1) , \\ 2, & \text{if } t \in \cup_{n \in \mathbb{N}}[2n+1, 2n+2) . \end{cases} \qquad (22)$$

By numerically computing the solution of the system (18) and (19) and the initial condition (21) with this switching signal, it can be observed on Figure 2 that the solution seems to diverge.

5. CONCLUSION

In this paper, some sufficient conditions have been derived for the exponential stability of hyperbolic PDE with switching signals defining the dynamics and the boundary conditions. The set of hyperbolic PDE is written either as
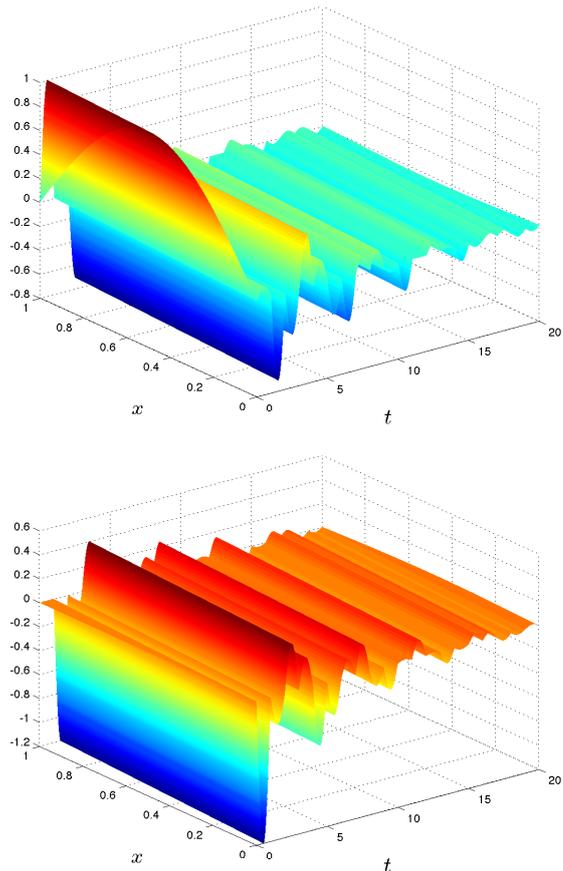
Fig. 1. Time evolution of the first component y_1 (top) and of the second component y_2 (bottom) of the solution of the system (18) and (19) with the initial condition (21) and the switching signal (20)

a system of conservation laws or as a system of balance laws. This stability analysis has been done with Lyapunov functions and exploiting the dwell time assumption, if it holds, of the switching signals.

This work lets many questions open and may have natural applications on physical applications. In particular, exploiting the sufficient conditions for the derivation of switching stabilizing boundary controls (as in Dos Santos and Prieur [2008]) seems to be a natural extension. The computation of an optimal switching control, as in Hante et al. [2009] seems also a challenging issue.

Appendix A. AN EXAMPLE ILLUSTRATING REMARK 2

The aim of this section is to exhibit an example of an unstable unswitched system of balance laws as given by (12) such that the associate hyperbolic system (13) is exponentially stable and such that the matrix F is Hurwitz. This simple example motivates the study of the stability of systems of balance laws by itself (and not by deducing it from the stability of hyperbolic systems, obtained by letting $F = 0$ in the system of balance laws).

To do that, let us consider the following specific 2D system:

$$\partial_t y(x,t) + \Lambda \partial_x y(x,t) = 0 , \quad x \in [0,1], t \geq 0 \qquad (A.1)$$

with the boundary conditions
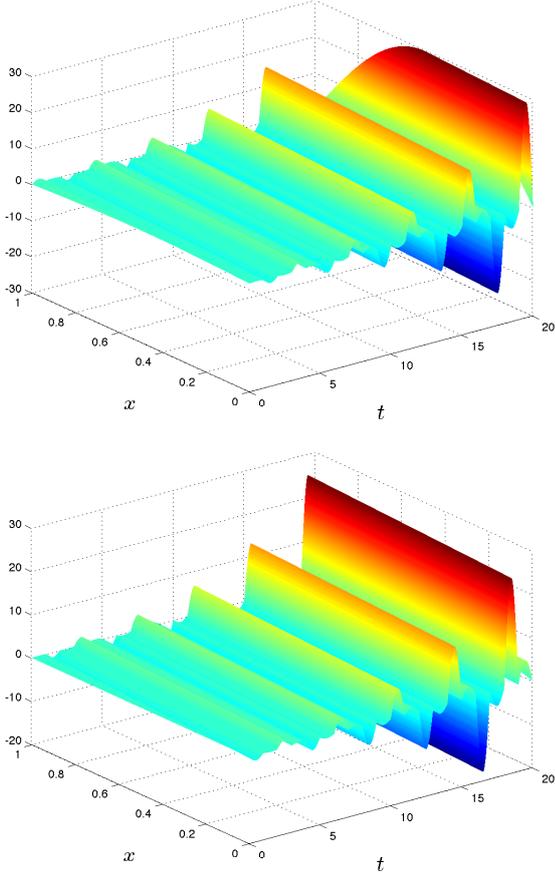
Fig. 2. Time evolution of the first component y_1 (top) and of the second component y_2 (bottom) of the solution of the system (18) and (19) with the initial condition (21) and the switching signal (22)

$$y(0) = Gy(1) \qquad (A.2)$$

where $G = \begin{pmatrix} 0.6 & -0.6 \\ 0.3 & 0 \end{pmatrix}$ and $\Lambda = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. Denoting by $\rho(|G|)$ the spectral radius of the matrix which entries are the absolute values of G, the condition $\rho(|G|) < 1$ holds, thus, $\rho_1(|G|) < 1$ holds also (see [Coron et al. 2008, Prop. 3.2]). Therefore with Coron et al. [2008] (or with Li [1994]), the system (A.1) with the boundary conditions (A.2) is exponentially stable. Using a Lax-Friedrichs method and the solver described in Shampine [2005], we may check the attractivity of this system with the initial condition:

$$y^0(x) = \begin{pmatrix} 1.6x - 0.6 \\ 1.7x + 0.3 \end{pmatrix} , \quad x \in [0, 1] . \qquad (A.3)$$

See Figure A.1 for the time evolution of the two components.

Moreover let us consider the following finite-dimensional system:

$$\partial_t y(x, t) = Fy(x, t) , \quad x \in [0, 1], t \geq 0 \qquad (A.4)$$

where $F = \begin{pmatrix} -4 & 5 \\ -3 & 3 \end{pmatrix}$ (with eigenvalues having a negative real part). No boundary conditions are specified since it is an ordinary differential equation (x can be seen as parameter). Since the matrix F is Hurwitz the system (A.4) is exponentially stable.
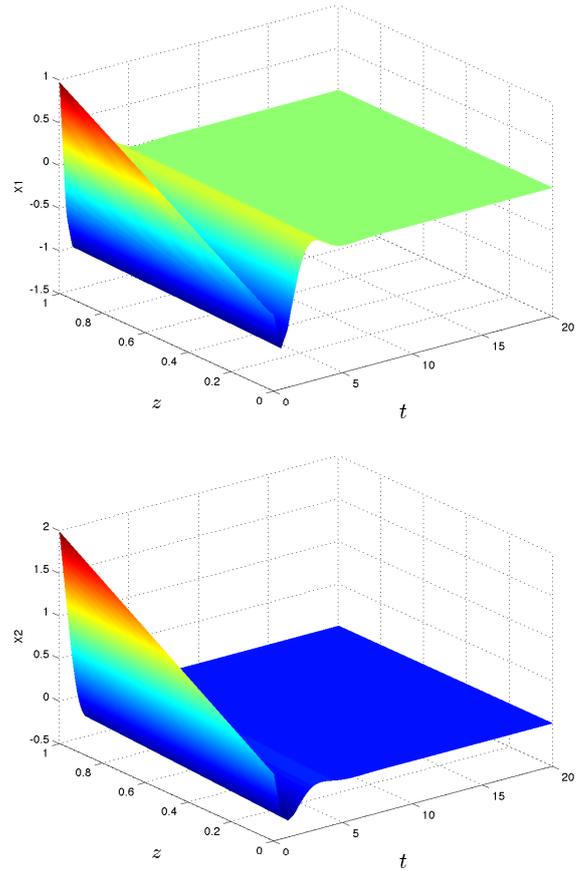
Fig. A.1. Time evolution of the first component y_1 (top) and of the second component y_2 (bottom) of the solution of the system (A.1) and (A.2) with the initial condition (A.3)

Now combining the two previous systems leads to

$$\partial_t y(x, t) + \Lambda \partial_x y(x, t) = Fy(x, t) , \quad x \in [0, 1], t \geq 0 \quad (A.5)$$

with the boundary conditions (A.2). This system seems to be unstable by considering the initial condition (A.3) (see Figure A.2).

The system (A.5) and (A.2) gives an example illustrating Remark 2.

REFERENCES

Amin, S., Hante, F., and Bayen, A. (2012). Exponential stability of switched linear hyperbolic initial-boundary value problems. *IEEE Transactions on Automatic Control*, 57(2), 291–301.

Bastin, G., Coron, J.M., and d'Andréa Novel, B. (2008). Using hyperbolic systems of balance laws for modeling, control and stability analysis of physical networks. In *Lecture notes for the Pre-Congress Workshop on Complex Embedded and Networked Control Systems*. 17th IFAC World Congress, Seoul, Korea.

Bressan, A. (2000). *Hyperbolic systems of conservation laws: the one-dimensional Cauchy problem*, volume 20. Oxford University Press, USA.

Coron, J.M. (2007). *Control and Nonlinearity*, volume 136 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI.
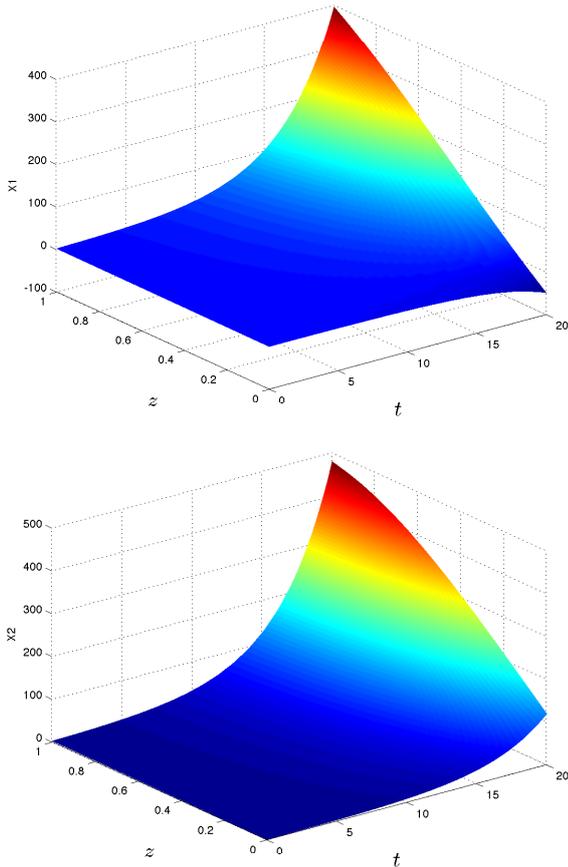
Fig. A.2. Time evolution of the first component y_1 (top) and of the second component y_2 (bottom) of the solution of the system (A.5) and (A.2) with the initial condition (A.3)

Coron, J.M., Bastin, G., and d'Andréa Novel, B. (2008). Dissipative boundary conditions for one-dimensional nonlinear hyperbolic systems. *SIAM Journal on Control and Optimization*, 47(3), 1460–1498.

Coron, J.M., d'Andréa Novel, B., and Bastin, G. (2007). A strict Lyapunov function for boundary control of hyperbolic systems of conservation laws. *IEEE Transactions on Automatic Control*, 52(1), 2–11.

Diagne, A., Bastin, G., and Coron, J.M. (2012). Lyapunov exponential stability of linear hyperbolic systems of balance laws. *Automatica*, 48(1), 109–114.

Dick, M., Gugat, M., and Leugering, G. (2010). Classical solutions and feedback stabilization for the gas flow in a sequence of pipes. *Networks and Heterogeneous Media*, 5(4), 691–709.

Dos Santos, V. and Prieur, C. (2008). Boundary control of open channels with numerical and experimental validations. *IEEE Transactions on Control Systems Technology*, 16(6), 1252–1264.

El-Farra, N. and Christofides, P. (2004). Coordinating feedback and switching for control of spatially distributed processes. *Computers & chemical engineering*, 28(1-2), 111–128.

Gugat, M. and Tucsnak, M. (2011). An example for the switching delay feedback stabilization of an infinite dimensional system: The boundary stabilization of a string. *Systems & Control Letters*, 60(4), 226–233.

Hante, F., Sigalotti, M., and Tucsnak, M. (to appear). On conditions for asymptotic stability of dissipative infinite-dimensional systems with intermittent damping. *Journal of Differential Equations*.

Hante, F., Leugering, G., and Seidman, T. (2009). Modeling and analysis of modal switching in networked transport systems. *Applied Mathematics & Optimization*, 59(2), 275–292.

Haut, B. and Bastin, G. (2007). A second order model of road junctions in fluid models of traffic networks. *Networks and Heterogeneous Media*, 2(2), 227–253.

Krstic, M. and Smyshlyaev, A. (2008). Backstepping boundary control for first-order hyperbolic PDEs and application to systems with actuator and sensor delays. *Systems & Control Letters*, 57, 750–758.

Li, T.T. (1994). *Global classical solutions for quasilinear hyperbolic systems*, volume 32 of *RAM: Research in Applied Mathematics*. Masson, Paris.

Liberzon, D. (2003). *Switching in systems and control*. Springer.

Luo, Z.H., Guo, B.Z., and Morgul, O. (1999). *Stability and stabilization of infinite dimensional systems and applications*. Communications and Control Engineering. Springer-Verlag, New York.

Prieur, C. and de Halleux, J. (2004). Stabilization of a 1-D tank containing a fluid modeled by the shallow water equations. *Systems & Control Letters*, 52(3-4), 167–178.

Prieur, C. and Mazenc, F. (2012). ISS-Lyapunov functions for time-varying hyperbolic systems of balance laws. *Mathematics of Control, Signals, and Systems*, 24(1), 111–134.

Prieur, C., Winkin, J., and Bastin, G. (2008). Robust boundary control of systems of conservation laws. *Mathematics of Control, Signals, and Systems*, 20(2), 173–197.

Shampine, L. (2005). Solving hyperbolic PDEs in Matlab. *Appl. Numer. Anal. & Comput. Math.*, 2, 346–358.

Witrant, E. and Marchand, N. (2008). Modeling and feedback control for air flow regulation in deep pits. In S. Sivasundaram (ed.), *Mathematical Problems in Engineering, Aerospace and Sciences*, Scientific Monographs and Text Books. Cambridge Scientific Publishers.

Witrant, E. and Niculescu, S.I. (2010). Modeling and control of large convective flows with time-delays. *Mathematics in Engineering, Science and Aerospace*, 1(2), 191–205.

Zuazua, E. (2011). Switching control. *European Mathematical Society*, 13, 85–117.