

A Parameter Varying Observer for the Enclosed Mass in a Spark Ignited Engine

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Abstract: In this paper, a high gain observer is designed for a parameter varying polytopic model to estimate the enclosed mass in the combustion chamber of a spark ignited engine. The high gain strategy allows the design of an observer that handles the uncertain part of the system. The observer uses the cylinder pressure measurement during the compression stroke to estimate the enclosed mass. An engine compression model is used as a virtual engine to build the observer. The results of the observer are compared to the virtual engine model and a good agreement between the observed variables and the model was obtained.

1. INTRODUCTION

The enclosed mass estimation problem is an interesting and challenging task in the engine control. Indeed, an accurate estimation of the enclosed mass would permit a better control of the associated fuel injection and a better treatment of the pollutant residuals [Butt and Bhatti, 2008].

In automotive control, the variables in the air path are typically used to compute the cylinder characteristics, such as the in-cylinder load and residual mass fraction. Such strategies are usually designed with static approximations (e.g., see [Muller, 2008], [Kang and Chang, 2009], [Senecal et al., 1996] and [Fox et al., 1993]). Alternatively, closed loop observer schemes have also been developed to estimate the engine load during the admission stroke. In [Stotsky and Eriksson, 2002], the uncertainties of the measurements in the intake manifold are introduced and an adaptive learning algorithm is used to track the error between the measured pressure and the estimation. In [Kerkeni et al., 2010], a periodic observer for a class of non-linear models in the discrete Takagi-Sugeno form is designed, using the variables on the engine intake manifold. Both works propose suitable methods to estimate the enclosed mass in the combustion chamber, by computing the in-cylinder mass in the intake manifold, before the valves closure. Other interesting approach can be found in [Chauvin et al., 2008], where the estimation and to control of the masses entering in the cylinders for a diesel Homogeneous Charge Compression Ignition (HCCI) engine. The masses are directly related to the intake manifold pressure, compositions and flow-rates. A nonlinear observer for periodic systems is used as estimation strategy.

The objective of this work is to estimate the total enclosed mass in the combustion chamber, which corresponds to the total air load plus the residual mass after the inlet valve

closure (IVC). In a previous work [Rivas et al., 2012], the authors have proposed a nonlinear high gain observer to estimate the enclosed mass during the compression and combustion strokes using the cylinder pressure measurement. Compared to the previous work, in this paper, the high gain observation strategy is applied to a parameter varying system, to estimate the cylinder temperature during the compression stroke. The enclosed mass is computed using the observed temperature and the cylinder pressure measurement.

The challenge on using a parameter varying polytopic model is the fact that the varying parameters depend on the estimated state and nonlinear terms are added to the system, thus the estimation error contains uncertain elements. Solutions to this class of problems have been explored in [Daafouz et al., 2008], [Maurice et al., 2010] and [Bara et al., 2010]. In those works, interesting strategies to relax the mismatch due to the parameters uncertainties are proposed. The global asymptotical stability of the observer is assured, but for the application considered in this work, this criterion is not enough as the goal is to fully eliminate the mismatch between the observed state and the system states. The contribution of this work is to include a high gain strategy in a linear parameter varying (LPV) observer.

Including the high gain technique improves the observer performance, vanishing the effect of the uncertainties due to the estimated states. It is important to notice that the design of this high gain LPV observer is an open problem. A related work can be found in Gérard et al. [2010], where a high gain observer design is used to consider the level of disturbance attenuation of an LPV functional filter for bilinear systems with a disturbance attenuation specification. The observer gain is written as a function of the estimated state and the high gain observer is computed using Linear Matrix Inequalities (LMI) techniques. In this

work, the high gain approach is used to ensure the stability of the filtering error and to optimize the disturbance attenuation.

This paper is structured as follows. In Section 2, the physical engine model for the compression stroke is designed to build the observer. This model is transformed into an equivalent parameter varying system in Section 3. In Section 4 the high gain observer is implemented: the cylinder enclosed mass is computed in this stage. Simulations of the observed variables are compared to a validated virtual engine to support the results of the observer.

2. COMPRESSION MODEL

The engine compression parameter varying model is based on the engine model proposed by [Rivas et al., 2012]. The combustion chamber is considered as a unique open system and a uniform in-cylinder pressure is assumed. Only the compression stroke, after the inlet valve closure (IVC) is considered. During the engine compression cycle, no energy is transferred between the cylinder and the inlet and outlet ports. The energy equation for the cylinder is inferred from the first thermodynamical principle:

$$dU(t) = -Q_{th}(t) - p(t)dV(t) \quad (1)$$

$U(t)$ is the internal energy of the cylinder gas mixture, $Q_{th}(t)$ expresses the heat transfer of the cylinder contents to the surroundings and $p(t)$ is the cylinder pressure, $dV(t)$ is the variation of the cylinder volume and $p(t)dV(t)$ corresponds to the work delivered by the piston.

Assuming that the specific heat parameter c_v is constant, the left hand side of (1) can be written as:

$$dU(t) = m(t)c_v dT(t) \quad (2)$$

where $m(t)$ is the total mass of all the species in the cylinder and $T(t)$ corresponds to the temperature of the gas. As the valves are closed, $m(t)$ remains constant and it is equivalent to the enclosed mass.

Solving (1) and (2) for $dT(t)$, an ordinary differential equation governs the system temperature dynamics during the compression:

$$dT(t) = \frac{1}{m(t)c_v} (-p(t)dV(t) - Q_{th}(t)) \quad (3)$$

where $V(t)$ is the gas volume. The ideal gas law is used to find the dynamics of $p(t)$:

$$p(t)V(t) = rmT(t) \quad (4)$$

where r , is the specific gas constant. Taking the derivative of (4) leads to:

$$dp(t) = \frac{rmdT(t)}{V(t)} - \frac{rT(t)mdV(t)}{V(t)^2} \quad (5)$$

Using equations (3) and (5), the in-cylinder pressure dynamics during the compression is modeled as:

$$dp(t) = -\left(\frac{r}{c_v} + 1\right) \frac{dV(t)}{V(t)} p(t) - \frac{r}{V(t)c_v} Q_{th}(t) \quad (6)$$

and replacing $m(t) = \frac{p(t)V(t)}{rT(t)}$ in (3), the temperature dynamics is:

$$dT(t) = -\frac{rdV(t)}{c_v V(t)} T(t) - \frac{r}{c_v V(t)} Q_{th}(t) \frac{T(t)}{p(t)} \quad (7)$$

The wall losses $Q_{th}(t)$ are modeled using the reduced model provided in [Rivas et al., 2012]:

$$Q_{th}(t) = A_w(t)\omega p(t)(k_1 T(t) - k_0 T_w) \quad (8)$$

where $A_w(t)$ is the wall transfer area, ω is the engine speed, k_1 and k_0 are tuning constants and T_w is the cylinder wall temperature.

Defining $p(t) = x_1$ and $T(t) = x_2$ in equations (6) and (7), and replacing the heat wall losses by (8), the state space system is written as:

$$\begin{aligned} \dot{x}_1 &= -\left(\frac{r}{c_v} + 1\right) \frac{dV(t)}{V(t)} x_1 \\ &\quad - \frac{r}{c_v V(t)} A_w(t) \omega x_1 (k_1 x_2 - k_0 T_w) \end{aligned} \quad (9)$$

$$\dot{x}_2 = -\frac{rdV(t)}{c_v V(t)} x_2 - \frac{r}{c_v V(t)} x_2 A_w(t) \omega (k_1 x_2 - k_0 T_w)$$

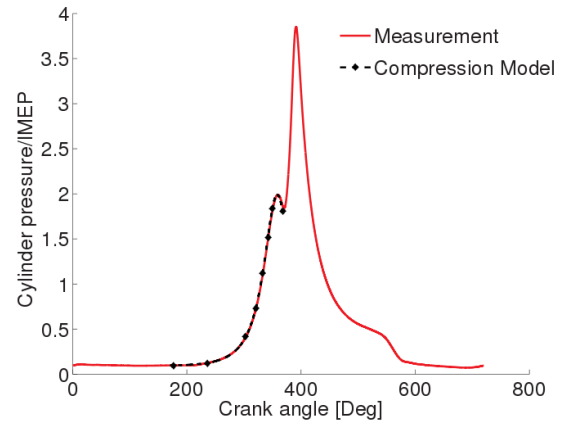


Fig. 1. Cylinder pressure. BMEP=18 bar, N=1500 rpm.

The compression model is tested taking as reference the measurements of a 1.2 liters engine. The data to fit the model is the cylinder pressure. The results presented in this paper correspond to a test performed at $N = 1500 \text{ rpm}$ and $\text{BMEP} = 18 \text{ bar}$.

A good agreement between the measurements and the compression model is obtained, thus the model can be used as virtual engine to build the observer. Results of the validated model are shown in Figure 1. Further results are shown in Figure 2.

3. PARAMETER VARYING POLYTOPIC SYSTEM REPRESENTATION

The model (9) can be written in the following LPV model:

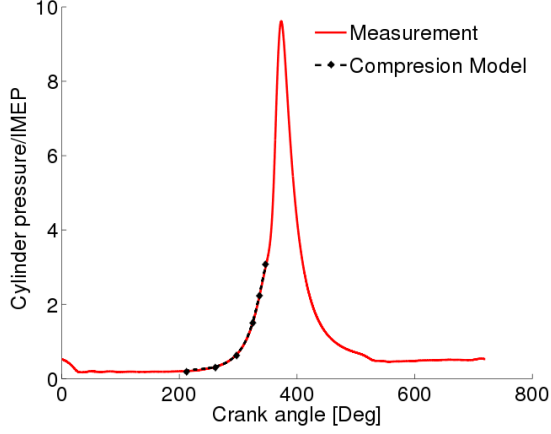


Fig. 2. Cylinder pressure. BMEP=2 bar, N=1200 rpm.

$$\dot{x} = A(\rho(x))x + \phi(x), \quad y = Cx \quad (10)$$

where

$$A(\rho(x)) = \begin{bmatrix} 0 & \frac{r}{c_v}\rho(x) \\ 0 & 0 \end{bmatrix}, \quad C = [1 \ 0] \quad (11)$$

$$\rho(x) = -\frac{1}{V(t)}\omega A_w(t)k_1x_1 \quad (12)$$

and

$$\phi(x) = \begin{bmatrix} \left(-\left(\frac{r}{c_v} + 1\right) \frac{dV(t)}{V(t)} + \frac{r}{c_v V(t)} \omega A_w(t) k_0 T_w \right) x_1 \\ \frac{r}{c_v} \left(-\frac{dV(t)}{V(t)} - \frac{1}{V(t)} \omega A_w(t) (k_1 x_2 - k_0 T_w) \right) x_2 \end{bmatrix} \quad (13)$$

In this study, $\rho(x)$ is assumed to be bounded in the convex set $[\underline{\rho}, \bar{\rho}]$ and $\rho \neq 0$, it allows to represent the system (10) in a polytopic approach. Due to the fact that (11) is a quasi-LPV system since $\rho(x)$ depends on the state x , this assumption induces that the system state is belonging to a bounded set $\Gamma \subset \mathbf{R}^n$. Finally $\phi(x)$ is a Lipschitz continuous function. Under those conditions, the matrix $A(\rho(x))$ can be written in the form

$$A(\rho(x)) = \sum_{i=1}^M \alpha_i(\rho(x)) A_i \quad (14)$$

$M = 2$, is the number of vertexes of the polytope formed by the extremes of the varying parameter $\rho(x)$. $\alpha_i(\rho(x)) \in \mathbf{R}$ is a scheduling function such that $\sum_{i=1}^N \alpha_i(\rho(x)) = 1$ and the matrices $A_i \in \mathbf{R}^{n \times n}$.

The goal is to observe the state x_2 through the measurement x_1 . It is assumed that the uncertainty in x is bounded, thus $|\rho(x) - \rho(\hat{x})| < \Delta$.

Remark 1. It is important to recall that (11) is written in a state space form that allows the design of a high gain observer. Such form is referred to as additive triangular and is detailed next. Two symmetrical choices can be

considered for $A(\rho(x))$, since the bi-linear term x_1x_2 is present. However, x_1 must be included in the parameters in order to be consistent with the triangular additive form required for the high gain observer design.

4. HIGH GAIN OBSERVER

The idea is to extend the notion of a high gain observer to LPV systems under a polytopic representation. For a nonlinear system described in additive triangular form as:

$$\begin{aligned} \dot{x} &= A_0x + \psi(x, u) \\ y &= C_0x \end{aligned} \quad (15)$$

where

$$\begin{aligned} A_0 &= \begin{bmatrix} 0 & a_{21} & 0 \\ 0 & \dots & a_{n-1 \times n} \\ 0 & 0 & 0 \end{bmatrix} \\ C_0 &= [1 \ 0 \ 0 \dots] \end{aligned} \quad (16)$$

a nonlinear high gain observer can be obtained using the following theorem:

Theorem 1. ([Besancon, 2007]). If $\psi(x, u)$ is globally Lipschitz in x and u , thus $\|\psi(x, u) - \psi(\hat{x}, u)\| < \delta\|x - \hat{x}\|$ and such that $\partial\psi_i(x, u)/\partial x_j = 0$, for $j \geq i + 1$, $1 \leq i, j \leq n$, then System (15) admits an observer of the form:

$$\dot{\hat{x}} = A_0\hat{x} + \psi(\hat{x}, u) + \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \dots & \\ 0 & 0 & \lambda_n \end{bmatrix} L_0(C_0\hat{x} - y) \quad (17)$$

with L_0 such that $A_0 - L_0C_0$ is stable and $\lambda, \dots, \lambda_n$ large enough.

The idea of this observer is to use the uniform observability to weight a gain based on the linear part, so as to make the linear dynamics of the observer error to dominate the nonlinear one ([Ljung, 1999]).

4.1 High Gain Observer for LPV systems

From this part of the document, the temporal dependence notation of the variables is omitted and $\rho(x) = \rho_x$. Given the system (10), an observer for the state x is proposed as:

$$\begin{aligned} \dot{\hat{x}} &= A_0(\hat{\rho}_x)\hat{x} + \Lambda L_0(\hat{\rho}_x)(y - \hat{y}) + \phi(\hat{x}) \\ \hat{y} &= C_0\hat{x} \end{aligned} \quad (18)$$

$$A_0(\hat{\rho}_x) = \sum_{i=1}^N \alpha_i(\hat{\rho}_x) A_i \quad (19)$$

$$L_0(\hat{\rho}_x) = \sum_{i=1}^N \alpha_i(\hat{\rho}_x) L_i \quad (20)$$

and

$$\Lambda = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda^2 & 0 \\ \dots & & \\ 0 & 0 & \lambda^n \end{bmatrix} \quad (21)$$

where $\lambda \in \mathbf{R}$ and $\lambda > 1$.

The dynamics of the estimation error $e = x - \hat{x}$ is governed by:

$$\dot{e} = (A_0(\hat{\rho}_x) - \Lambda L_0(\hat{\rho}_x)C_0)e + \phi(x) - \phi(\hat{x}) + v_x \quad (22)$$

where $v_x = (A_0(\rho_x) - A_0(\hat{\rho}_x))x$, is considered as a bilinear perturbation \mathcal{L}_2 bounded.

Proposition 1. Consider the quasi-LPV system (10) and assume that $\phi(x(t))$ is a Lipschitz continuous function such that $\|\phi(x_a(t)) - \phi(x_b(t))\| < \delta\|x_a(t) - x_b(t)\|$, $\delta > 0$, $x_a, x_b \in \mathbf{D}_x$. If there exists the state feedback $L_0(\hat{\rho}_x)$, the matrices $P = P^T$ such that $P > 0$ and Λ defined in Equation (21), such that the following inequality is satisfied for all $\rho_x \in [\underline{\rho}, \bar{\rho}]$:

$$\begin{bmatrix} \left[A_0(\hat{\rho}_x) - \Lambda L_0(\hat{\rho}_x)C_0 \right]^T P + P \left[A_0(\hat{\rho}_x) \quad P \right] \\ -\Lambda L_0(\hat{\rho}_x)C_0 \\ P \end{bmatrix} + 2\delta P + I \quad \begin{matrix} \\ \\ -\gamma^2 I \end{matrix} < 0 \quad (23)$$

for some constant $\gamma > 0$, then (18) is an observer for System (10) and the estimation error is asymptotically stable and satisfies $\|e(t)\|_2 < \Lambda\gamma\|v_x(t)\|_2$.

Proof: To prove the stability, the auxiliary variable $z(t) = \Lambda^{-1}x(t)$ is introduced and the error dynamics $\epsilon(t) = z(t) - \hat{z}(t)$ is considered:

$$\begin{aligned} \dot{\epsilon}(t) = & \Lambda^{-1} \left[A_0(\hat{\rho}_x) - \Lambda L_0(\hat{\rho}_x)C_0 \right] \Lambda \epsilon + \\ & \Lambda^{-1} \left(\phi(x(t)) - \phi(\hat{x}(t)) \right) + \Lambda^{-1} v_x(t) \end{aligned} \quad (24)$$

Choosing the Lyapunov function $V(t) = \epsilon(t)^T P \epsilon(t)$ where $P = P^T > 0$, the following inequality must be satisfied in order to ensure the observer stability:

$$\dot{V}(t) = \dot{\epsilon}(t)^T P \epsilon + \epsilon(t)^T P \dot{\epsilon}(t) < 0 \quad (25)$$

this yields to:

$$\begin{aligned} \dot{V}(t) = & \epsilon(t)^T \left[\left(\Lambda A_0(\hat{\rho}_x)^T \Lambda^{-1} - \Lambda C_0^T L_0(\hat{\rho}_x)^T \right) P \right. \\ & + P \left(\Lambda^{-1} A_0(\hat{\rho}_x) \Lambda - L_0(\hat{\rho}_x) C_0 \Lambda \right) \left. \right] \epsilon(t) + \\ & \left(\phi(x(t)) - \phi(\hat{x}(t)) \right)^T \Lambda^{-1} P \epsilon(t) + \\ & \epsilon(t)^T P \Lambda^{-1} \left(\phi(x(t)) - \phi(\hat{x}(t)) \right) \\ & + v_x(t)^T \Lambda^{-1} P \epsilon(t) + \epsilon(t)^T P \Lambda^{-1} v_x(t) < 0 \end{aligned} \quad (26)$$

Considering the structure of Λ (Equation (21)) and the fact that $A_0(\hat{\rho}_x)$ is triangular and $C_0 = [1 \dots 0]$, a simple calculation yields to:

$$\begin{aligned} \Lambda A_0(\hat{\rho}_x)^T \Lambda^{-1} - \Lambda C_0^T L_0(\hat{\rho}_x)^T = \\ \lambda \left[A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right]^T \end{aligned} \quad (27)$$

Replacing this result in (26) and taking into account that $\phi(x(t))$ satisfies $\|\phi(x(t)) - \phi(\hat{x}(t))\| < \delta\|x(t) - \hat{x}(t)\|$, the

following inequality must be verified to ensure the error $\epsilon(t)$ converges to 0:

$$\begin{aligned} & \lambda \epsilon(t)^T \left[\left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right)^T P + \right. \\ & P \left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right) \left. \right] \epsilon(t) + 2\delta \epsilon(t)^T P \epsilon(t) \\ & + v_x(t)^T P \Lambda^{-1} \epsilon(t) + \epsilon(t)^T \Lambda^{-1} P v_x(t) < 0 \end{aligned} \quad (28)$$

Considering that $v_x(t)$ is \mathcal{L}_2 bounded, the application of the Bounded Real Lemma leads to the inequality:

$$\begin{bmatrix} G & \Lambda^{-1}P \\ \Lambda^{-1}P & -\gamma^2 I \end{bmatrix} < 0 \quad (29)$$

where

$$\begin{aligned} G = \\ \lambda \left[\left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right)^T P + P \left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right) \right] \\ + 2\delta P + I \end{aligned} \quad (30)$$

therefore it is verified that $\|\epsilon(t)\|_2 < \gamma\|v_x(t)\|_2$ for all $\rho_x \in [\underline{\rho}, \bar{\rho}]$. Since $\epsilon(t) = \Lambda^{-1}e(t)$, (30) becomes $\|e(t)\|_2 < \Lambda\gamma\|v_x(t)\|_2$ and Proposition 1 is proved.

5. DESIGN PROCEDURE

According to Proposition 1, for the observer in (18), Inequality (23) must be accomplished to guarantee the observer stability. However, as there is a nonlinear relation between the parameter Λ and the gain $L_0(\hat{\rho}_x)$, it is not possible to use an LMI solver to obtain the observer feedback. Nevertheless, it has also been proved that if (28) is accomplished, Proposition 1 is satisfied. Thus, using this result, the procedure to obtain Λ and $L_0(\hat{\rho}_x)$ can be solved finding a feedback $L_0(\hat{\rho}_x)$ such that:

$$\begin{aligned} & \left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right)^T P + \\ & P \left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right) < 0 \end{aligned} \quad (31)$$

and a parameter λ large enough, such that (28) is accomplished, similarly as in the classical high gain observation strategy presented before in Theorem 1.

To compute $L_0(\hat{\rho}_x)$ as (20), L_i are deduced from the dual solution of the quadratic stability of an uncertain plant developed in Olalla et al. [2009] and Feron et al. [1992], where a framework for robust linear quadratic regulators (LQRs) control for a convex model of power converters, taking into account uncertainty in the parameters is presented.

In this work, the LQR control problem with uncertain parameters is solved by using an LMI. The dual representation of such controller is used. Thus, the solution to solve the state feedback K for the LQR problem is stated in the following theorem:

Theorem 2. Consider the system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (32)$$

where $A \in \mathbf{R}^{n \times n}$ and $B \in \mathbf{R}^{n \times p}$ contain uncertainties, $u(t) \in \mathbf{R}^p$ is the system input and $C \in \mathbf{R}^{q \times n}$ where q is the number of the system outputs. Given the symmetric matrix $P > 0 \in \mathbf{R}^{n \times n}$, the matrices Y and X and the parameter matrices $W > 0$ and $V = V^T > 0$, the optimal feedback gain K that guarantees that the system (32) is quadratically stable can be found by minimizing the following expression:

$$\min(\text{trace}(X) + \text{trace}(VP)) \quad (33)$$

subject to the following linear matrix inequalities:

$$AP + PA^T + BY + Y^T B^T + I < 0, \quad (34)$$

$$\begin{bmatrix} X & W^{1/2}Y \\ Y^T W^{1/2} & P \end{bmatrix} < 0$$

Once this minimization under constraints is solved, the controller can be recovered by $K = YP^{-1}$.

Remark 2. The polytopic application this theorem consists on replacing the constraints involving matrices A and B by M constraints corresponding to the vertexes of the polytope formed by A_i and B_i , with matrices X_i and Y_i .

To obtain the observer gain $L_0(\hat{\rho}_x)$, the dual solution of the LQR problem in Theorem 2 for the polytopic case (remark 2) is used to compute L_i , which yields to the following proposition:

Proposition 2. Consider the observer (18). Given the symmetric matrix $P > 0 \in \mathbf{R}^{n \times n}$, the matrices Y_i and X_i and the parameter matrices W and $V = V^T > 0$, such that the following expression is minimized:

$$\min(\text{trace}(X_i) + \text{trace}(VP)) \quad (35)$$

subject to the linear matrices inequalities:

$$A_i^T P + PA_i + C_0^T Y_i + Y_i^T C_0 + I < 0 \quad (36)$$

$$\begin{bmatrix} X_i & W^{1/2}Y_i \\ Y_i^T W^{1/2} & P \end{bmatrix} < 0 \quad (37)$$

for $i = 1, 2$, the matrices $L_i = Y_i P^{-1}$ guarantee that $(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0)^T P + P(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0) < 0$ for all $\rho_x \in [\underline{\rho}, \bar{\rho}]$.

This theorem satisfies Inequality (31). The parameters W and V are chosen as:

$$W = 0.05, \quad V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (38)$$

To complete the observer design, the parameters in the matrix Λ have to be chosen large enough to guarantee the estimation error to converge to 0. In this thesis, $\lambda = 150$, thus:

$$\Lambda = \begin{bmatrix} 150 & 0 \\ 0 & 150^2 \end{bmatrix} \quad (39)$$

thus, $L_0(\hat{\rho}_x)$ and Λ have been designed such that Proposition 1 is satisfied.

Remark 3. Notice that observing (10)-(13) implies the estimates:

$$\hat{\rho}_x = -\frac{1}{V(t)}\omega A_w(t)k_1 \hat{x}_1(t) \quad (40)$$

and

$$\phi(\hat{x}(t)) = \begin{bmatrix} \left(-\left(\frac{r}{c_v} + 1\right) \frac{dV(t)}{V(t)} + \frac{r}{c_v V(t)} \omega A_w(t) k_0 T_w \right) \hat{x}_1(t) \\ \frac{r}{c_v} \left(-\frac{dV(t)}{V(t)} - \frac{1}{V(t)} \omega A_w(t) (k_1 \hat{x}_2(t) - k_0 T_w) \right) \hat{x}_2(t) \end{bmatrix} \quad (41)$$

However, since $y(t) = x_1(t)$ is a measured signal, $\hat{\rho}_x$ may indeed be replaced by:

$$\hat{\rho}_x = -\frac{1}{V(t)}\omega A_w(t)k_1 \hat{y}(t) \quad (42)$$

Remark 4. If the perturbation $v_x = 0$, Inequality (28) yields:

$$\lambda \epsilon(t)^T \left[\left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right)^T P + P \left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right) \right] \epsilon(t) + 2\delta \epsilon(t)^T P \epsilon(t) < 0 \quad (43)$$

As it has been shown in Proposition 2, it is possible to find a feedback such Inequality (31) is accomplish, moreover, the dual solution of Theorem 2 guarantees the quadratic stability of an observer using such a feedback, thus the first term on Inequality (43) satisfies:

$$\lambda \epsilon(t)^T \left[\left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right)^T P + P \left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right) \right] \epsilon(t) < -\lambda \alpha \|\epsilon(t)\|^2 \quad (44)$$

for some constant values $\alpha > 0$. Using this result and bounding the second term, Inequality (44) yields to:

$$\lambda \epsilon(t)^T \left[\left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right)^T P + P \left(A_0(\hat{\rho}_x) - L_0(\hat{\rho}_x)C_0 \right) \right] \epsilon(t) + 2\delta \epsilon(t)^T P \epsilon(t) < -\lambda \alpha \|\epsilon(t)\|^2 + \beta \|\epsilon(t)\|^2 \quad (45)$$

for some constant $\beta > 0$. Thus a bound value for λ can be obtained as $\lambda > \frac{\beta}{\alpha}$ that ensures the exponential stability of $\epsilon(t)$.

Remark 5. A restriction of this synthesis is the fact that the LQR dual adaptation to obtain the gains L_i has brought two more calibration parameters besides the values of Λ . Such parameters are the matrices V and W , present in the synthesis of the dual solution of Theorem 2, presented as the Proposition 2.

5.1 Simulation results

Using the observed states \hat{x}_1 and \hat{x}_2 , the enclosed mass in the cylinder is computed using the ideal gas law (6):

$$\hat{m} = \frac{\hat{x}_1 V}{r \hat{x}_2} \quad (46)$$

The results are compared to the virtual engine model proposed by [Rivas et al., 2012]. Figures 3 and 4 show the cylinder pressure and temperature estimations during the compression stroke when the valves are closed and Figure 5 shows the result of the mass estimation, the operating conditions are $N = 4000$ rpm and $IMEP = 8$ bar. The second simulation case is shown in Figures 6, 7 and 8, where the operating conditions are changed to $N = 1200$ rpm and $IMEP = 2$ bar.

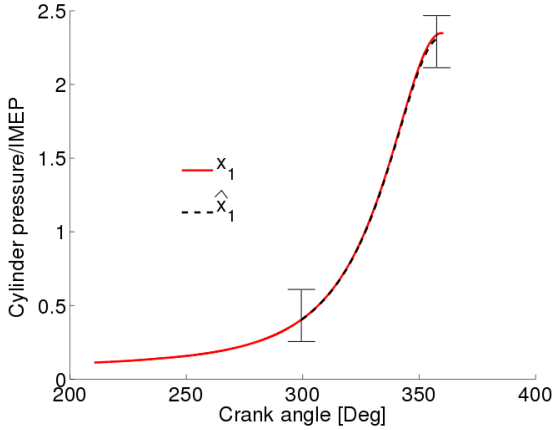


Fig. 3. Cylinder pressure. Case 1: $N=4000$ rpm $IMEP=8$ bar, $IT=2.25$ CAD

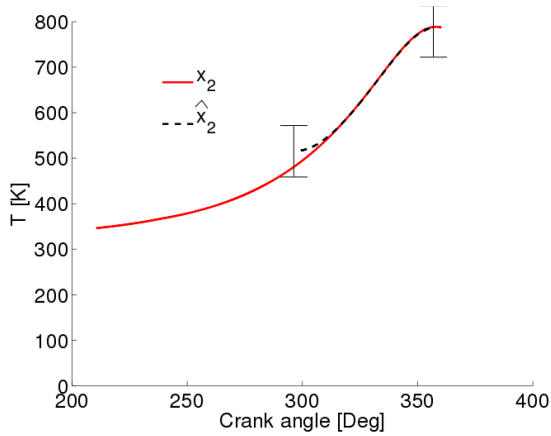


Fig. 4. Cylinder Temperature. Case 1: $N=4000$ rpm $IMEP=8$ bar, $IT=2.25$ CAD

The vertical bars in the figures represent the interval where the observer is valid: the observer is enabled only during the compression (valves closed), in the remaining parts of the cycle the observer is disabled and reset. One engine cycle is enough to accurately observe the variables of interest \hat{x}_1 and \hat{x}_2 .

The compression stroke might be short in comparison with the whole engine cycle, limiting the available time for the estimation. In this thesis, the interest is to compute the enclosed mass before the ignition. The observer settling time to compute the enclosed mass is 1.7 ms. At 1200 rpm the compression stroke lasts 6.5 ms for an $IT = 15$ CAD, at 5500 rpm the compression stroke lasts 1.4 ms for an

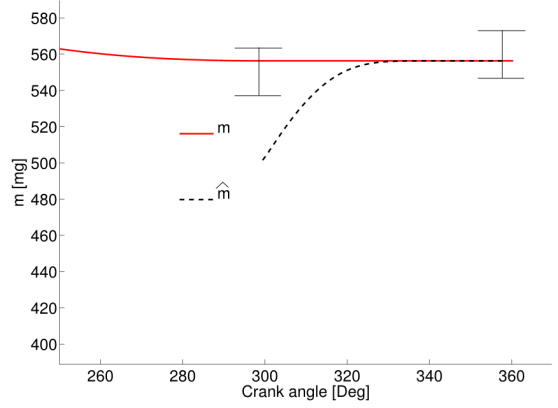


Fig. 5. Cylinder mass estimation. Case 1: $N=4000$ rpm $IMEP=8$ bar, $IT=2.25$ CAD

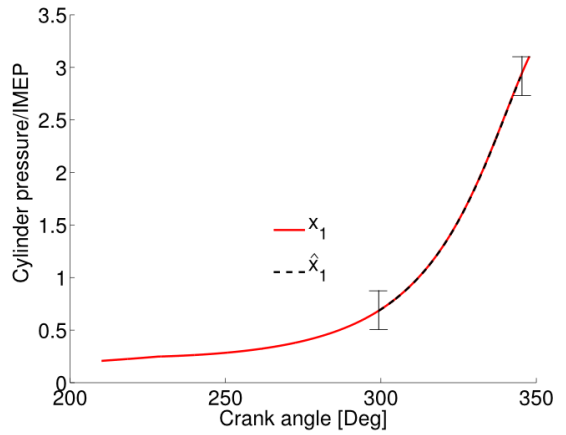


Fig. 6. Cylinder pressure. Case 2: $N=1200$ rpm $IMEP=2$ bar, $IT=15$ CAD

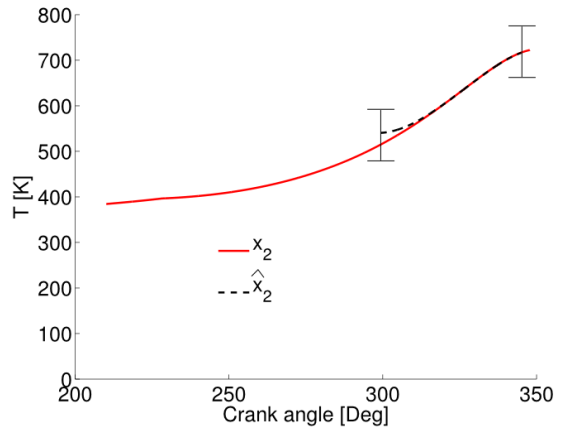


Fig. 7. Cylinder Temperature. Case 2: $N=1200$ rpm $IMEP=2$ bar, $IT=15$ CAD

$IT = 13$ CAD and at 4500 rpm, the compression stroke lasts around 2.1 ms. It shows that even if the initial error is important when the observer is initialized, the estimated mass converges soon enough before the ignition timing for an engine speed $N < 4500$ rpm, always that the compression stroke lasts more than 1.7 ms. For engine speeds above 4500 rpm, the convergence of the observer before the ignition timing is not ensured. An example of such limitation is shown in Figure 9, where a test at an

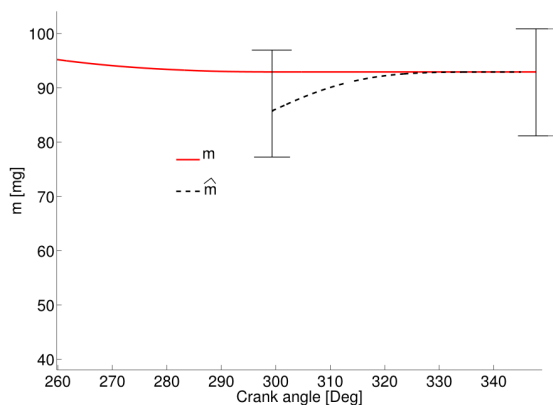


Fig. 8. Cylinder mass estimation. Case 2: $N=1200$ rpm
IMEP= 2 bar, IT=15 CAD

engine speed of 5500 rpm is presented. Only the enclosed mass estimation is plotted. As the engine speed is high, the duration of the compression is short: 1.4 ms. Thus, the observer has not converged when the combustion starts, and the mass estimation cannot be achieved.

Remark 6. Notice that if the IT is too advanced and/or the IVC is too delayed, it might shorten the compression stroke by some crank angle degrees.

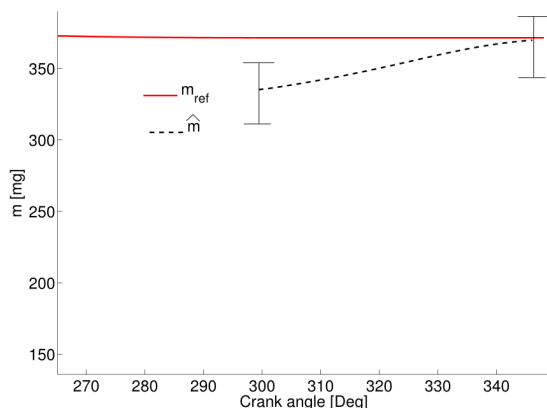


Fig. 9. Cylinder mass estimation. Case 3: $N=5500$ rpm
IMEP= 12 bar, IT=13.875

6. CONCLUSION

This work presents a new method to estimate the cylinder enclosed mass during the compression stroke when the engine valves are closed. A high gain nonlinear observer based on a parameter varying model of the cylinder temperature during the compression stroke is built and the enclosed mass is computed using the observed temperature and the cylinder pressure measurement through the ideal gases law. The preliminary results have shown to be effective to handle the strong non linearity of the compression model. One engine cycle computation is enough to obtain the mass estimation.

7. NOMENCLATURE

All variables are in S.I Metric Units.

ω : Engine speed (rad/s)
 IT : Ignition timing
 A_w : Heat transfer wall area
 CAD : Crank angle degrees
 c_v : Specific heat at constant volume
 h_c : Heat transfer coefficient for wall losses
 H_p : Piston height
 k_0 : Calibration constant
 k_1 : Calibration constant
 N : Engine speed (rpm)
 m : Total mass in the combustion chamber
 p : Pressure
 r : Specific gases constant
 T : Temperature
 T_w : Wall temperature
 U : Energy
 V : Cylinder volume

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